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International Conference
Current Developments and new directions in Kähler Geometry

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Talk titles and abstracts

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Daniele Angella (Florence)
Special Hermitian non-Kähler metrics
In the tentative move from the Kähler to the non-Kähler setting, we consider Hermitian non-Kähler metrics on complex manifolds with several special curvature properties and/or characterized by cohomological conditions. In particular, we focus on locally conformally structures. (Based on joint works with Simone Calamai, Antonio Otal, Alexandra Otiman, Cristiano Spotti, Nicoletta Tardini, Luis Ugarte, Raquel Villacampa.)

Tamas Darvas (Maryland)
Complex Monge-Ampère equations with prescribed singularity type
Given a Kähler manifold (X, ω), finding smooth solutions to the equation $(\omega + i\partial\bar\partial u)^n = f\omega^n$ goes back to Yau's solution of the Calabi conjecture in the seventies. In joint work with E. Di Nezza and C.H. Lu, we proposed to solve this same equation with the added constraint that $u \in \mathrm{PSH}(X, \omega)$ has prescribed singularity type. As it turns out, this problem is well posed only for a certain class of (model) singularity types that we characterize, and we also solve the corresponding equation. Our results extend to the case of big cohomology classes as well.

Thibaut Delcroix (ENS Paris)
K-stability of Fano spherical varieties
The resolution of the Yau-Tian-Donaldson conjecture for Fano manifolds, that is, the equivalence of the existence of Kähler-Einstein metrics with K-stability, raises the question of determining when a given Fano manifold is K-stable. I will present a combinatorial criterion of K-stability for Fano spherical manifolds. These form a very large class of almost-homogeneous manifolds, containing toric manifolds, homogeneous toric bundles, and classes of manifolds for which the Kähler-Einstein existence question was not solved yet, for example equivariant compactifications of (complex) symmetric Spaces.

Ruadha Dervan (Cambridge)
Relative K-stability for Kähler manifolds
We study the existence of extremal Kähler metrics on Kähler manifolds. After introducing a notion of relative K-stability for Kähler manifolds, we prove that Kähler manifolds admitting extremal Kähler metrics are relatively K-stable. We also prove a general L^p lower bound on the Calabi functional, generalising Donaldson's work. Both of these results improve the known results for projective manifolds (due to Donaldson, Chen, Szkelyhidi, Stoppa and others).

Eleonora Di Nezza (Imperial College)
The space of Kähler metrics on singular varieties
The geometry and topology of the space of Kähler metrics on a compact Kähler manifold is a classical subject, first systematically studied by Calabi in relation with the existence of extremal Kähler metrics. Then, Mabuchi proposed a Riemannian structure on the space of Kähler metrics under which it (formally) becomes a non-positive curved infinite dimensional

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space. Chen later proved that this is a metric space of non-positive curvature in the sense of Alexandrov and its metric completion was characterized only recently by Darvas.

In this talk we will talk about the extension of such a theory to the setting where the compact Kähler manifold is replaced by a compact singular normal Kähler space.

As one application we give an analytical criterion for the existence of Kähler-Einstein metrics on certain mildly singular Fano varieties, analogous to a criterion in the smooth case due to Darvas and Rubinstein. This is based on a joint work with Vincent Guedj.

Zakarias Sjostrom Dyrefelt (Toulouse)
K-stability and Kähler manifolds with transcendental cohomology class

Gregory Edwards (Northwestern)
Metric contraction of the cone divisor by the conical Kähler-Ricci flow
The conical Kähler-Ricci flow is a parabolic flow of Kähler metrics with cone singularities along a codimension one complex submanifold which deforms the smooth part of the metric by the Ricci curvature and keeps the conic boundary conditions fixed. On Hirzebruch surfaces, we analyze solutions of the flow with symmetry and show that the flow always reaches a finite time singularity which either contracts the cone divisor to a single point and the flow Gromov-Hausdorff converges to a projective orbifold, or the flow converges to either the Riemann sphere or a single point. This phenomenon fits into a conjectural framework that characterizes finite time non-collapsing singularities of the flow on complex surfaces.

Henri Guenancia (Stony Brook)
The Miyaoka-Yau inequality for minimal models
I will discuss a recent collaboration with Behrouz Taji where we establish Miyaoka-Yau inequality for log-minimal models. Miyaoka-Yau inequality is an inequality dating back to the seventies involving the first and second Chern classes of a compact Kähler manifold with positive canonical bundle. I will explain some generalizations that were obtained during the last thirty years before moving on to our joint result and, if time allows, to the ideas of its proof.

Bin Guo (Columbia)
Geometric estimates for complex Monge-Ampere equation
We will discuss some geometric estimates for a family of degenerate complex Monge-Ampère equations, in particular, we show that the diameters of the Kähler metrics defined by the MA equations are uniformly bounded. We will also derive some geometric applications of these estimates.

Sławomir Kołodziej (Jagiellonian)
Monge-Ampère and Hessian equations on compact Hermitian manifolds
Let (X, ω) be a compact Hermitian manifold of complex dimension n. I shall discuss some recent results concerning weak solutions to the complex Monge-Ampère equation and, the more general, complex Hessian equation:

$$(\omega + dd^c\phi)^k \wedge \omega^{n-k} = c_k f \omega^n$$

(where $0 \leq f$ belongs to some L^p space) including existence, stability and Hölder continuity. They were obtained in collaboration with Sławomir Dinew and Cuong Ngoc Nguyen. I would like to highlight interesting open problems.

Eveline Legendre (Toulouse)

Some applications of the Duistermaat-Heckman localization formula in Sasaki geometry

I will explain the Duistermaat-Heckman localization formula, an extension of it and apply on the study of the Einstein-Hilbert functional on the Sasaki cone (i.e cone of Reeb vector fields of a given Sasaki manifold/Kähler cone).

Gang Liu (Northwestern)

On Yau's uniformization conjecture

Let M^n be a complete noncompact Kähler manifold with nonnegative bisectional curvature and maximal volume growth. We prove M is biholomorphic to $\mathbf{C}^n$. This confirms the uniformization conjecture of Yau when the manifold has maximal volume growth.

Heather Macbeth (MIT)

Kähler-Ricci solitons on crepant resolutions

By a gluing construction, we produce steady Kähler-Ricci solitons on crepant resolutions of $\mathbf{C}^n/G$, where G is a finite subgroup of $\mathrm{SU}(n)$, generalizing Cao's construction of such a soliton on a resolution of $\mathbf{C}^n/\mathbf{Z}_n$. This is joint work with Olivier Biquard.

Tommy Murphy (Cal State Fullerton)

Complex Riemannian foliations of Kähler manifolds

For many natural problems arising in Riemannian geometry, the Kähler setting is restrictive enough to allow concrete classification results. In this vein I will outline joint work with Paul-Andi Nagy classifying complex Riemannian foliations of any open subset of a Hermitian symmetric space of compact type. General results restricting such foliations on any Kähler manifold are also derived.

Sebastien Picard (Columbia)

The Hull-Strominger system on fibrations over a Riemann surface

In the 1980s, C. Hull and A. Strominger introduced a system of partial differential equations characterizing the compactification of heterotic superstrings with torsion. From the point of view of complex geometry, this system of equations on non-Kähler 3-folds with trivial canonical bundle is an analog of Ricci-flat metrics on Kähler Calabi-Yau manifolds. The Kähler condition is replaced by the conformally balanced condition, and the curvature condition is an equation on $(2,2)$ forms which is quadratic in the Riemannian curvature tensor. We will discuss a class of solutions on hyperkahler fibrations over a Riemann surface. These solutions have infinitely many topological types. This is joint work with Teng Fei and Zhijie Huang.

Valentino Tosatti (Northwestern)

$C^{1,1}$ estimates for complex Monge-Ampère equations

I will discuss a method that we recently introduced in collaboration with J. Chu and B. Weinkove which gives interior $C^{1,1}$ estimates for the non-degenerate complex Monge-Ampère equation on compact Kähler manifolds (possibly with boundary). The method is sufficiently robust to also give $C^{1,1}$ regularity of geodesic segments in the space of Kähler metrics (thus resolving a long-standing problem originating from the work of X. Chen), of quasi-psh envelopes in Kähler and nef and big classes (solving a conjecture of R. Berman), and of geodesic rays that arise from test configurations (improving results of D.H. Phong and J. Sturm), and it even applies in the almost-complex case.

Cristiano Spotti (Aarhus)

Explicit degenerations of Kähler-Einstein Fano manifolds

I will describe how to find some refined constraints on the singularities of Gromov-Hausdorff limits of KE Fanos, and I will explain how such information can be useful both to study existence of KE metrics in explicit families of Fano varieties and to find concrete examples of KE Fano moduli compactifications (Fano K-moduli). The talk is based on joint work with Song Sun.

Song Sun (Stony Brook)

Singularities in Kähler geometry

David Witt Nyström (Gothenburg)

Restricted volumes of big classes and deformations of Kähler manifolds

Let L be a big line bundle on a projective manifold X of dimension n, and let Y be a subvariety of X. Then the restricted volume of L along Y measures the asymptotic growth of the spaces of sections of kL restricted to Y that extend to the whole of X. Boucksom-Favre-Jonsson and independently Lazarsfeld-Mustata proved that if Y is a divisor intersecting the ample locus of L, then the restricted volume is equal to $(1/n)$ times the derivative of $\mathrm{vol}(L + tY)$ at $t = 0$ (here vol denotes the ordinary volume). Since the ordinary volume of L is known to be a numerical invariant we get as an important corollary that the restricted volume only depends on the numerical class of the divisor Y.

The study of big line bundles on projective manifolds can be generalized to the study of big $(1,1)$ cohomology classes on compact Kähler manifolds. Fundamental work of e.g. Boucksom and Demailly has showed that the notions of volume and restricted volume have natural analogues for big classes. I will discuss work in progress to prove the generalized version of the above mentioned result on restricted volumes, and I will also mention some applications to the study of deformations of Kähler manifolds.

Michela Zedda (Salento)

Projectively induced Kähler-Einstein metrics

A classical problem in Kähler geometry is to classify Kähler-Einstein manifolds admitting a holomorphic and isometric immersion into the complex projective space. The aim of this talk is to summarize known results in this direction and to discuss open conjectures, with particular attention to the case when the manifolds involved are not compact.

Xiangwen Zhang (UC Irvine)

The Anomaly flow and Hull-Strominger system

The anomaly flow is a geometric flow which implements the Green-Schwarz anomaly cancellation mechanism originating from superstring theory, while preserving the conformally balanced condition of Hermitian metrics. We will discuss the criteria for long time existence and convergence of the flow on toric fibrations with the Fu-Yau ansatz. This is joint work with D.H. Phong and S. Picard.