On the affect of the Laplacian in equlibration dynamics of the Spherical Model

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Consider a system of N linearly interacting degrees of freedom. We collect them into a vector $\bar{\mathbf{x}}(t) = (\, x_1(t), \cdots, x_N(t)\,)$ and represent their interactions with random coupling matrix $\mathbf{J}(t)$. We subject them to a global constraint: lying on an N-dimensional sphere of radius N. These degrees of freedom may also experience constant bias $\mathbf{h}(t)$ external noise $\boldsymbol{\xi}(t)$. We write down the Langevin Equation of motion, dropping the explicit time dependence for brevity.

$$\partial_t \bar{\mathbf{x}} = (\mathbf{J} - \mu)\bar{\mathbf{x}} + \mathbf{h} + \boldsymbol{\xi} \tag{1}$$

$$\text{where} \quad \mu = \frac{1}{N}\bar{\mathbf{x}}^\top (\mathbf{J}\bar{\mathbf{x}} + \mathbf{h} + \boldsymbol{\xi}) \quad \text{enforces constraint} \quad |\bar{\mathbf{x}}(t)|^2 = N \tag{2}$$