# Inference of network actions for phenotypic reprogramming

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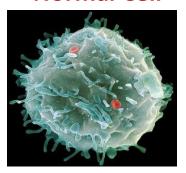
IBISC - UEVE

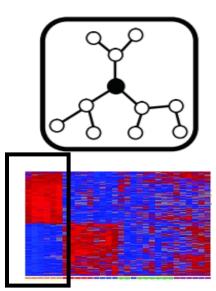


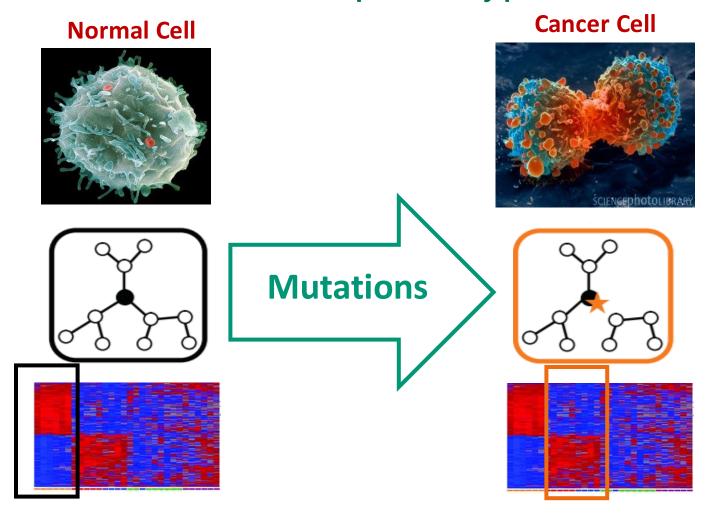


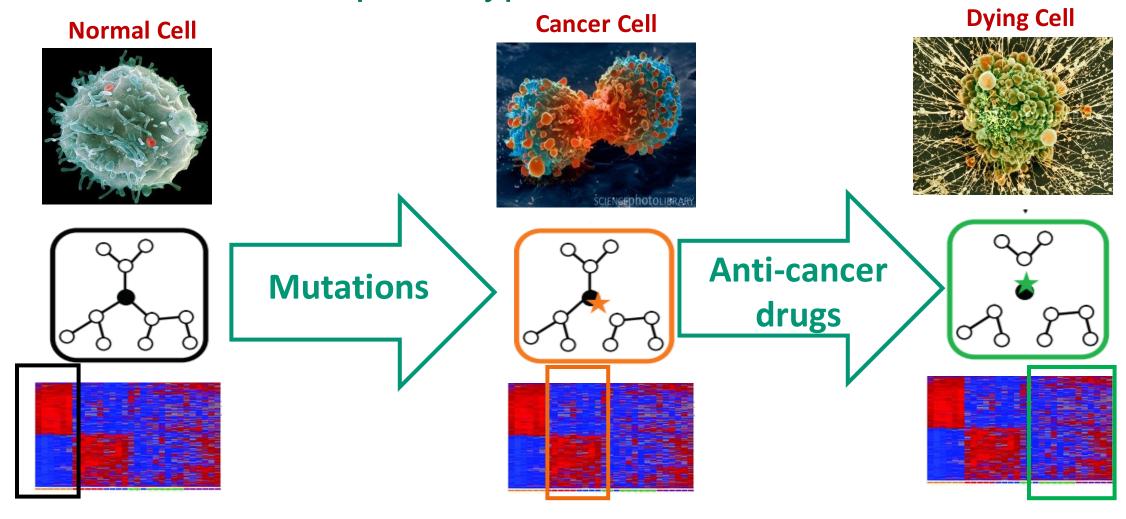


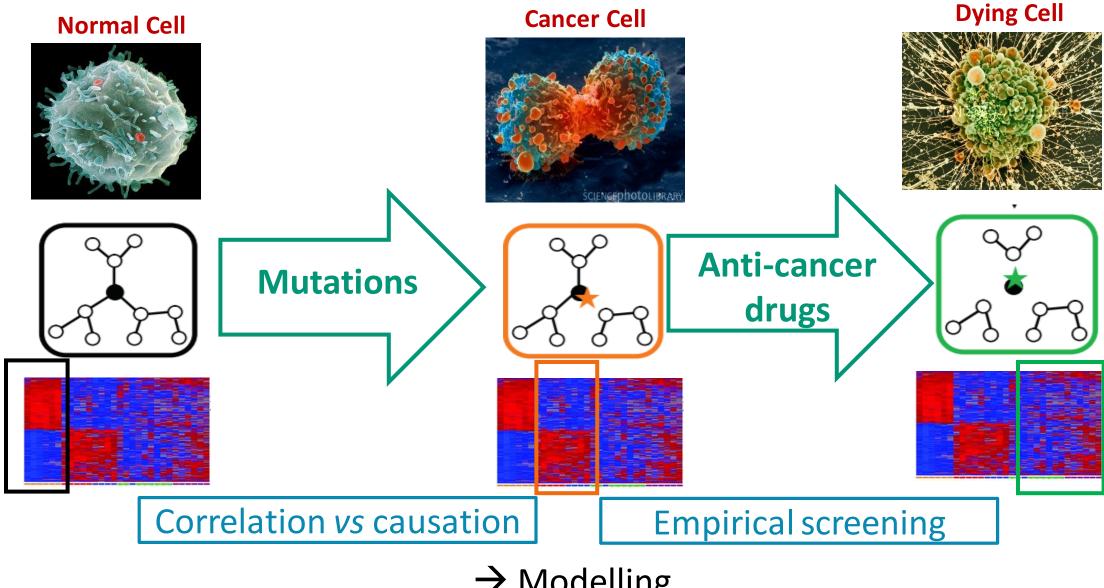
**Normal Cell** 











→ Modelling...

# Dynamical systems reprogramming

$$F = \begin{cases} x_1 = f_1(x_1, ..., x_n) \\ ... \\ x_i = f_i(x_1, ..., x_n) \\ ... \\ x_n = f_n(x_1, ..., x_n) \end{cases}$$
Structural actions
$$G = \begin{cases} x_1 = g_1(x_1, ..., x_n) \\ ... \\ x_i = g_i(x_1, ..., x_n) \\ ... \\ x_n = g_n(x_1, ..., x_n) \end{cases}$$

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# Boolean network Reprogramming

Theoretical Framework

Reprogramming Specification

Inference of actions

### Boolean Networks - Definition

### **Boolean Variables:**

$$X = \{x_1, \dots, x_n\}$$

### **Network:**

$$F = \begin{cases} x_1 = x_2 \lor x_3 \\ x_2 = \neg x_3 \\ x_3 = \neg x_2 \land x_1 \end{cases}$$

### State:

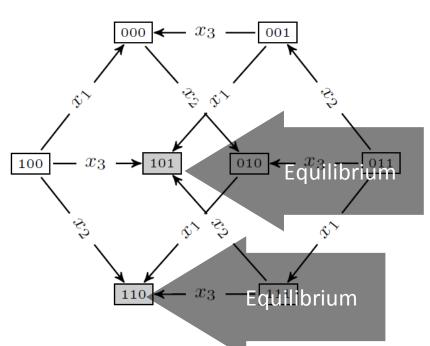
$$s: X \to \{0,1\}$$

### Model of dynamics:

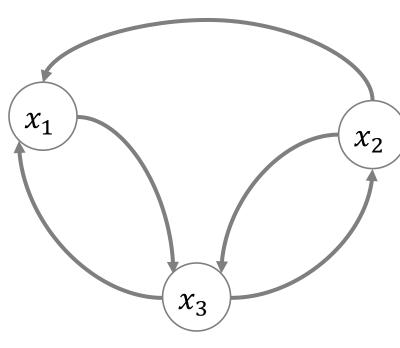
$$\langle \rightarrow, X, S \rangle$$

### **Transition Relation:**

$$\rightarrow \subseteq S \times X \times S$$



### Interaction Graph:



# Boolean Control networks for reprogramming

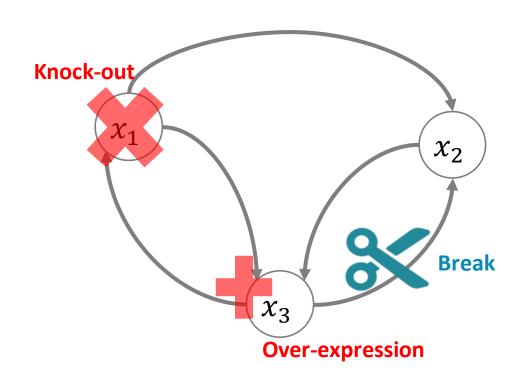
 $U = \{u_1, \dots u_m\}$ : Control parameters

$$F_{U} = \begin{cases} x_{1} = f_{1}(x_{1}, ..., x_{n}, u_{1}, ..., u_{m}) \\ x_{i} = f_{i}(x_{1}, ..., x_{n}, u_{1}, ..., u_{m}) \\ ... \\ x_{n} = f_{n}(x_{1}, ..., x_{n}, u_{1}, ..., u_{m}) \end{cases}$$

Control input  $\mu: U \to \{0,1\}$ 

# Structural control category

### Convention: an active control parameter is equal to 0



### Node Action: D-freezing

$$x_i = f_i(x_1, \dots, x_n) \wedge d_i^0$$

$$x_i = f_i(x_1, \dots, x_n) \lor \neg d_i^1$$

### **Edge Action: U-freezing**

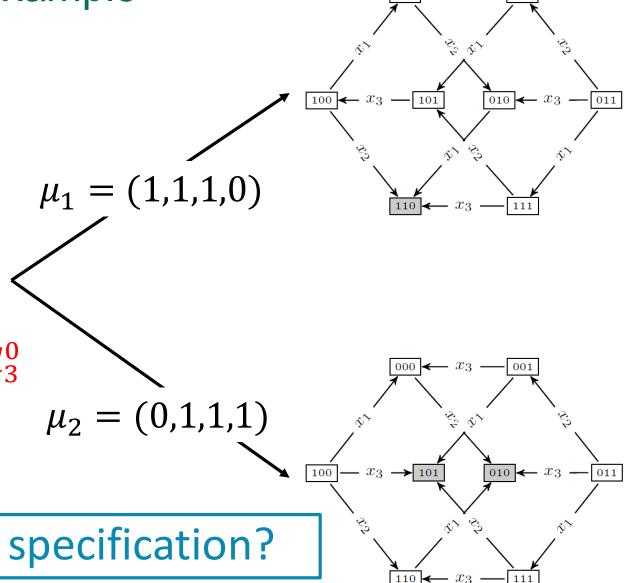
$$x_j = f_j((x_1, \dots, x_i \wedge u_{i,j}^0, \dots, x_n)$$

$$x_j = f_j((x_1, \dots, x_i \vee \neg u_{i,j}^1, \dots, x_n)$$

# Boolean control network - Example

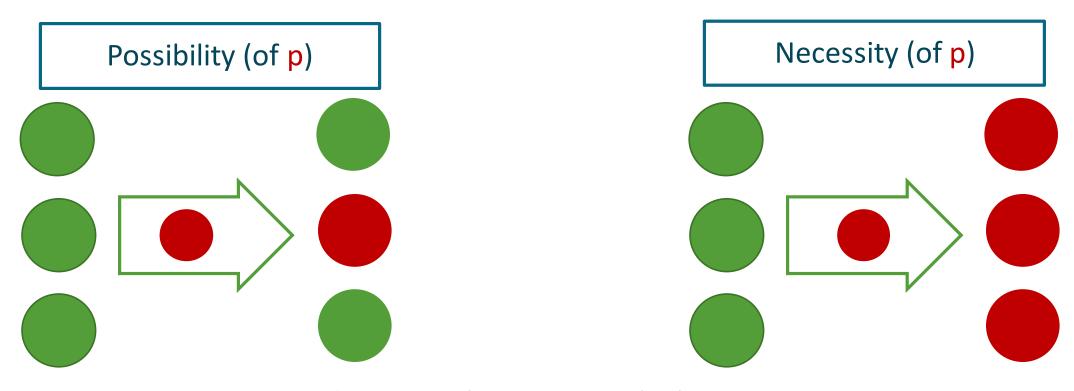
Set of control parameters *U* 

$$F_{U} = \begin{cases} x_{1} = (x_{2} \wedge u_{2,1}^{0}) \vee x_{3} \\ x_{2} = \neg(x_{3} \vee \neg u_{3,2}^{1}) \\ x_{3} = ((\neg x_{2} \wedge x_{1}) \vee \neg d_{3}^{1}) \wedge d_{3}^{0} \end{cases}$$



Phenotypic reprogramming specification?

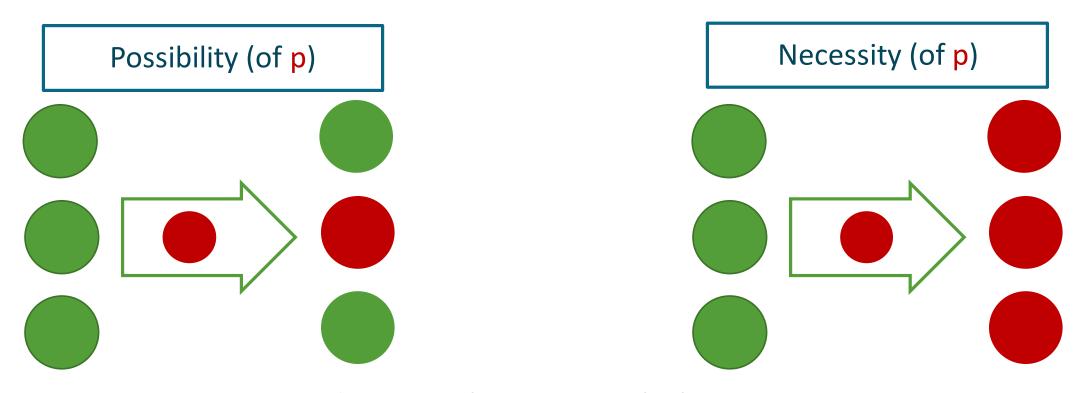
# Possibility/Necessity of a property



Find a control input  $\mu$  such that:

$$\exists s \in S : STBL_{F_{\mu}}(s) \land p(s) \qquad \forall s \in S : STBL_{F_{\mu}}(s) \Rightarrow p(s)$$

# Possibility/Necessity of a property



Find a control input  $\mu$  such that:

$$\exists s \in S : STBL_{F_{\mu}}(s) \land p(s) \qquad \forall s \in S : STBL_{F_{\mu}}(s) \Rightarrow p(s)$$

 $\rightarrow$  Inference of  $\mu$ ?

# Possibility and Necessity as abduction problems

### **THEOREM**

Finding  $\mu$  such that:

$$\exists s \in S : STBL_{F_{\mu}}(s) \land p(s)$$

$$\forall s \in S : STBL_{F_{\mu}}(s) \Rightarrow p(s)$$

are (respectively) equivalent to finding a cube  $C_{\mu}$  such that:

$$(C_S \wedge C_\mu) \wedge \phi \vDash STBL_{F_U} \wedge p$$

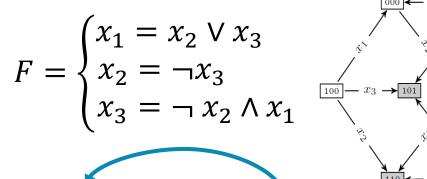
$$C_{\mu} \wedge \phi \models STBL_{F_U} \Rightarrow p$$

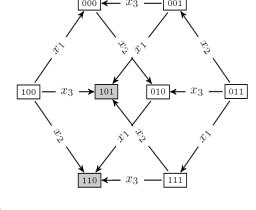
where  $C_s$ ,  $C_\mu$  are the minterms of s,  $\mu$  and  $STBL_{F_{II}} \stackrel{\text{def}}{=} \bigwedge_{i=1} (x_i \Leftrightarrow f_i(x_1, ..., x_n, u_1, ..., u_m))$ 

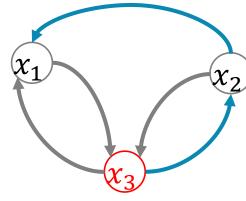
# Protaxion library (Mathematica) Example

Boolean Network

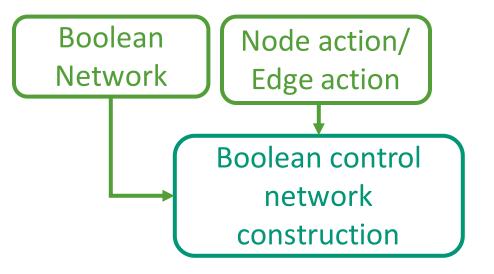
Node action/ Edge action

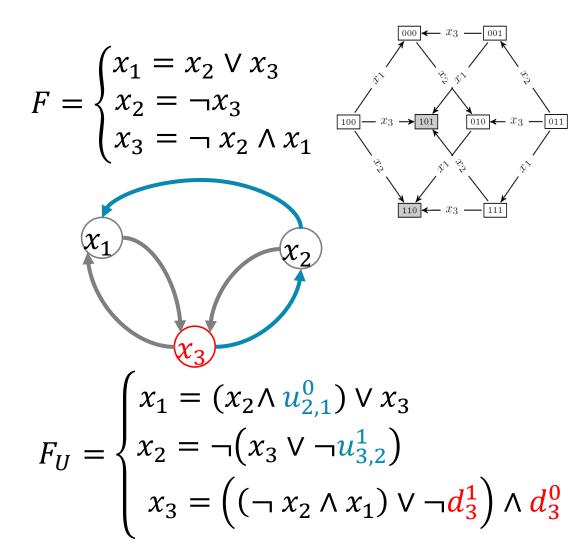




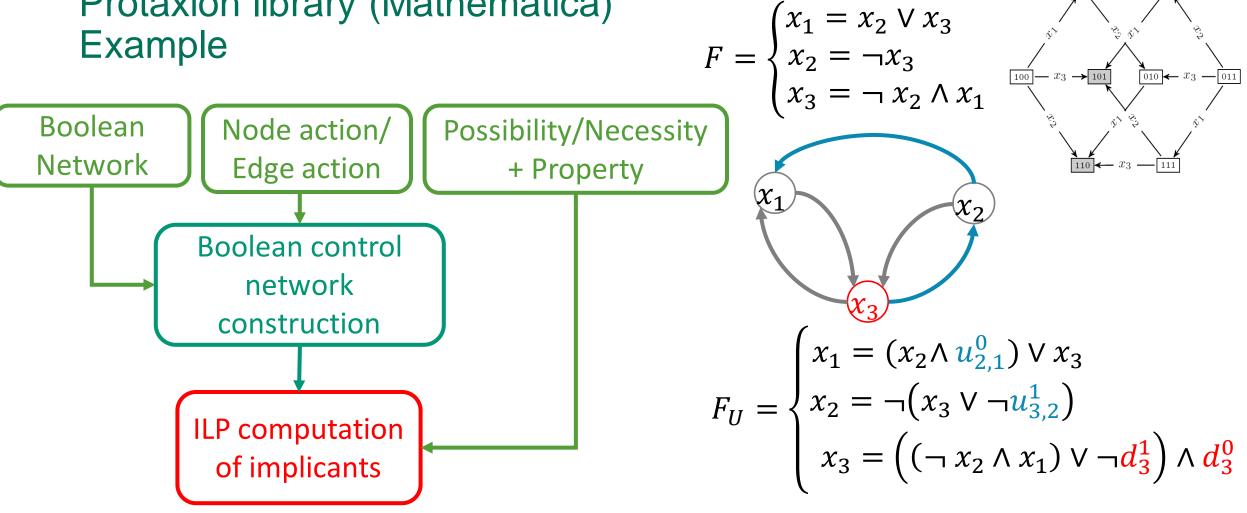


# Protaxion library (Mathematica) Example





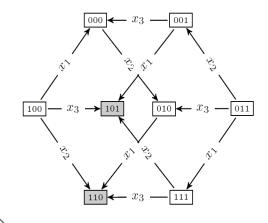
# Protaxion library (Mathematica) Example



Necessary Loss of 101 stable state

# Protaxion library (Mathematica) Example

 $F = \begin{cases} x_1 = x_2 \lor x_3 \\ x_2 = \neg x_3 \\ x_3 = \neg x_2 \land x_1 \end{cases}$ 





Possibility/Necessity

+ Property

Boolean control network construction

**ILP** computation of implicants

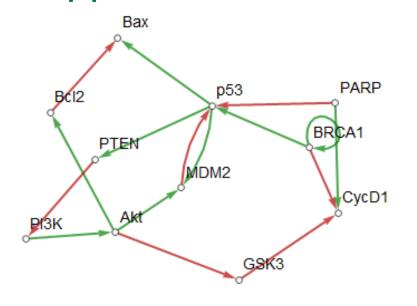
 $F_{U} = \begin{cases} x_{1} = (x_{2} \wedge u_{2,1}^{0}) \vee x_{3} \\ x_{2} = \neg(x_{3} \vee \neg u_{3,2}^{1}) \\ x_{3} = ((\neg x_{2} \wedge x_{1}) \vee \neg d_{3}^{1}) \wedge d_{3}^{0} \end{cases}$ 

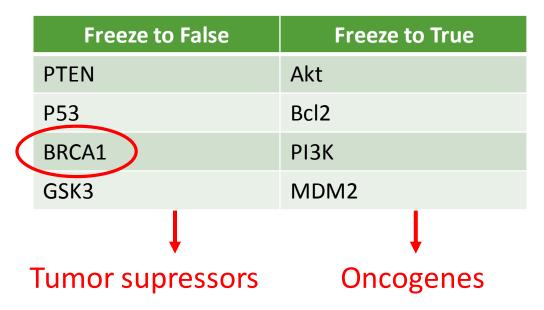
Minimal set of causal actions

Necessary Loss of 101 stable state

3 minimal solutions: Inhibit x3, Break x2-> x1, (1,1,1,0),(0,1,1,1),(1,0,1,1) Break x3-> x2

# Application to mutations prediction in Breast cancer





Akt → PI3K

Bax → ¬ Bcl2 ∧ p53

Bcl2 → Akt

BRCA1 → BRCA1

CycD1 → ¬ GSK3 ∨ (¬ BRCA1 ∧ PARP)

GSK3 → ¬ Akt

MDM2 → Akt ∧ p53

p53 → ¬ MDM2 ∧ (BRCA1 ∨ ¬ PARP)

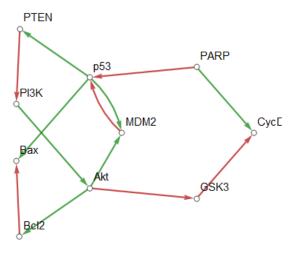
PARP → True

PI3K → ¬ PTEN

PTEN → p53

- 1) Actions on all nodes except markers
- 2) Necessary loss of apoptosis
- → Marking: CycD1=0, Bax=1

# Application to drug targets prediction in Breast cancer



BRCA1

Akt → PI3K

Bax → ¬ Bcl2 ∧ p53

Bcl2 → Akt

BRCA1 → False

CycD1 → ¬ GSK3 ∨ (¬ BRCA1 ∧ PARP)

GSK3 → ¬ Akt

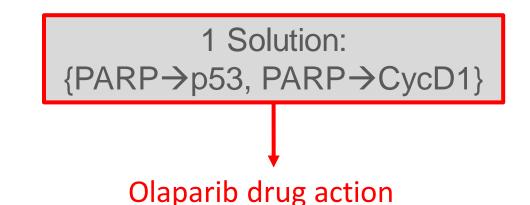
MDM2 → Akt ∧ p53

p53 → ¬ MDM2 ∧ (BRCA1 ∨ ¬ PARP)

PARP → True

PI3K → ¬ PTEN

PTEN → p53



- 1) Actions on Edges
- 2) Possible Gain of apoptosis
- → Marking: CycD1=0, Bax=1

# Conclusion & Perspectives

- \*Cancer and drug targets can be found in molecular networks
- ❖Dynamical systems reprogramming
- ☆Control Boolean Network
- \*Algorithmic resolution of inference of control
- \* Library developped in Mathematica
- Application to the prediction of mutated genes in Breast Cancer and of their synthetic lethal partner
- ☆Add weights to actions
- Application to the prediction of driver genes in Triple negative Breast Cancer

# THANK YOU