

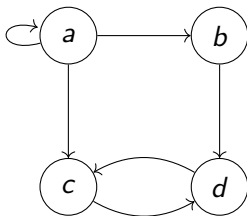
# On The Cost Of Simulating A Parallel Boolean Automata Networks By A Sequential One

Florian Bridoux

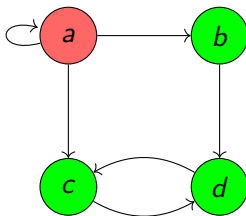
Thesis first year student in Marseille University

Collaborators: Sylvain Sené (LIF) et Guillaume Theyssier (I2M),  
Adrien Richard (I3S), Pierre Guillon (I2M), Kévin Perrot (LIF)

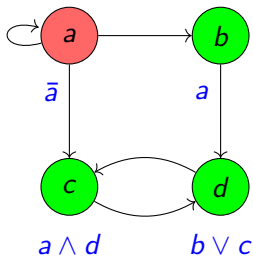
# Boolean automata networks (BAN)



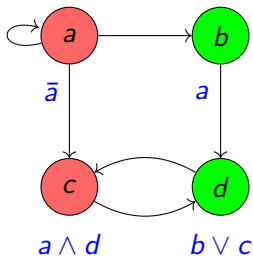
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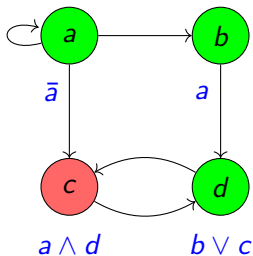
# Boolean automata networks (BAN)



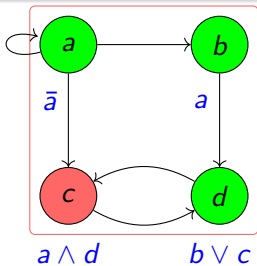
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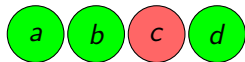
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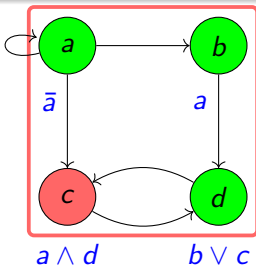
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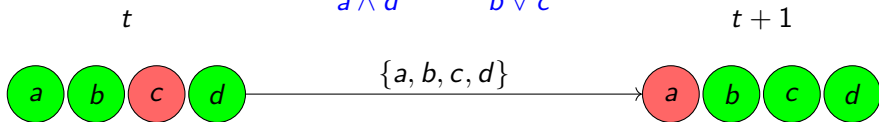
$t$



# Boolean automata networks (BAN)

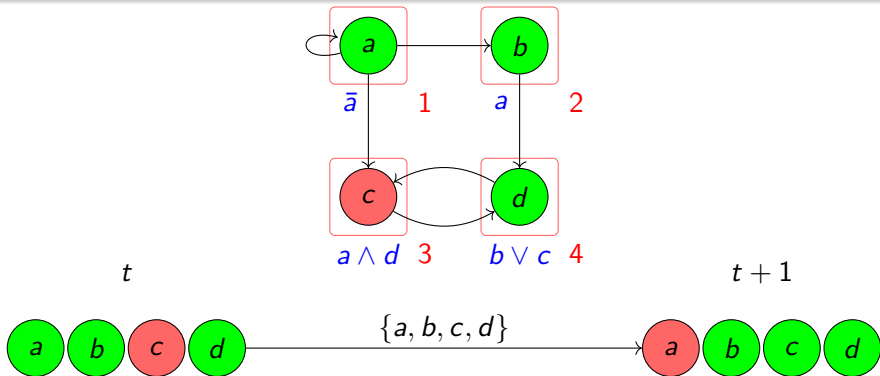


Parallel

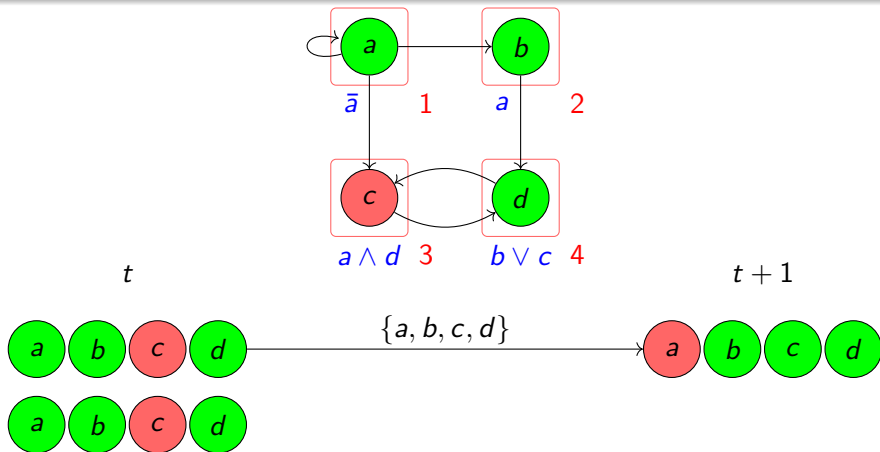




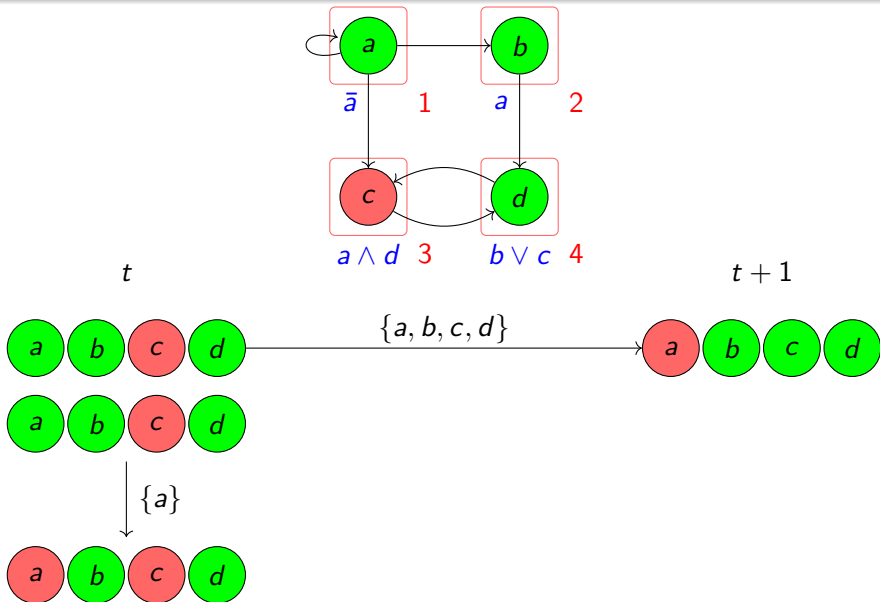
# Boolean automata networks (BAN)



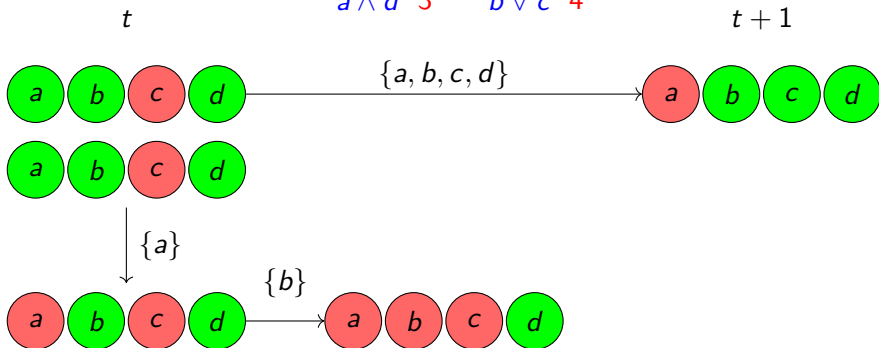
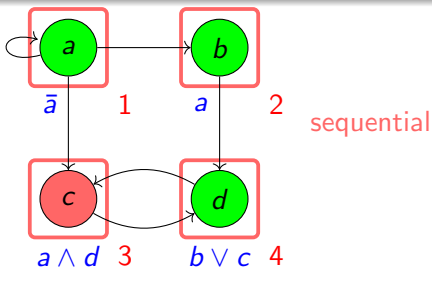
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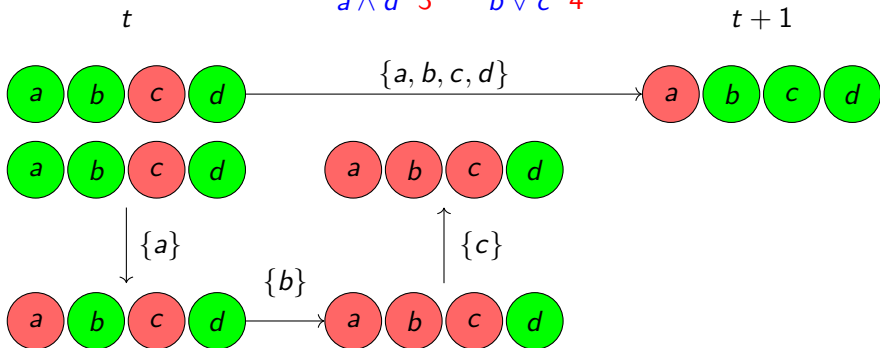
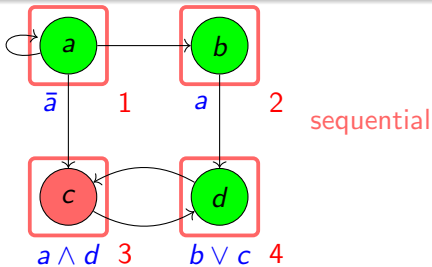
# Boolean automata networks (BAN)



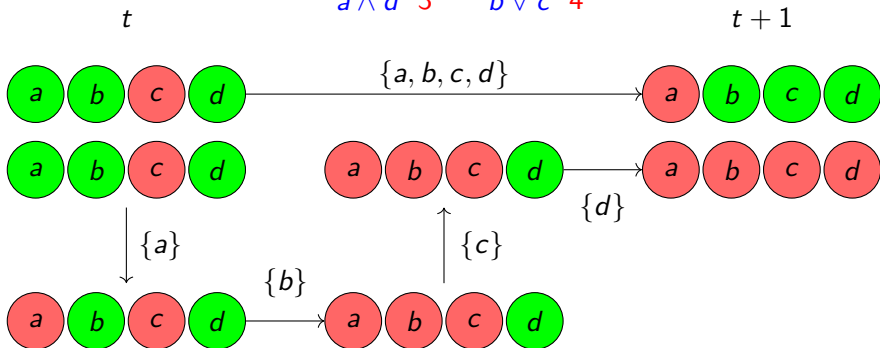
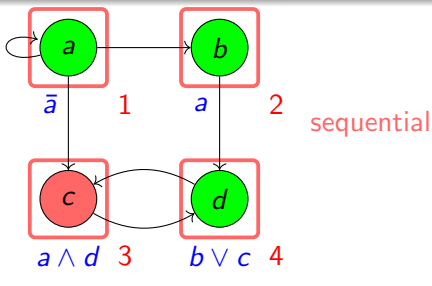
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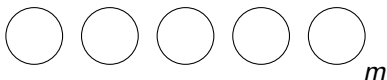
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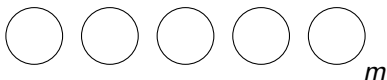


$(N', S')$

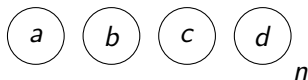


# Intrinsic simulation

$(N', S')$

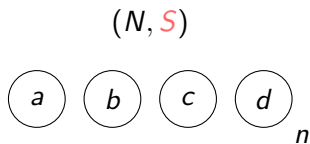
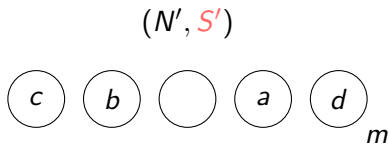


$(N, S)$

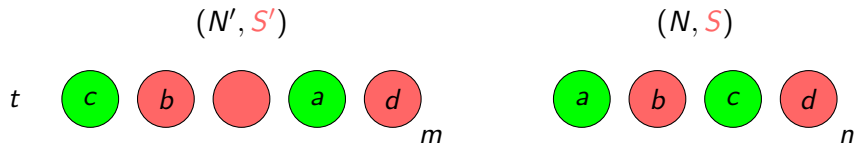




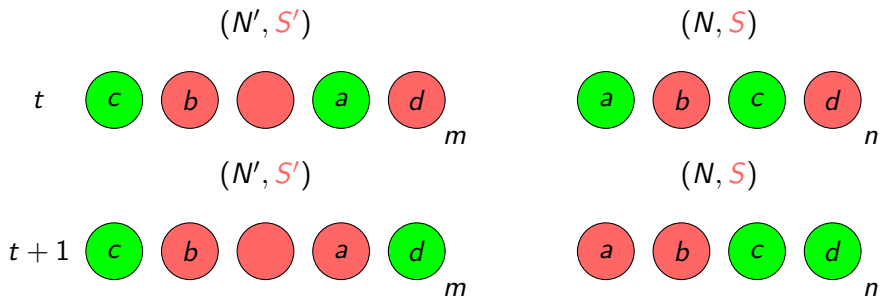
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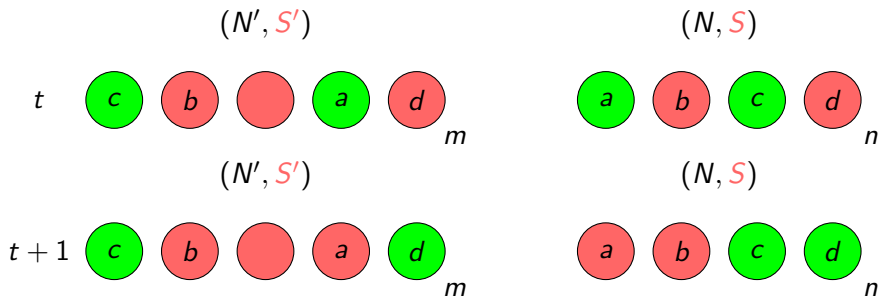
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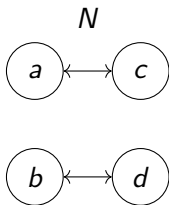


# Intrinsic simulation

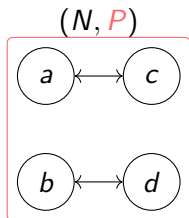


$$(N', S') \triangleright (N, S)$$

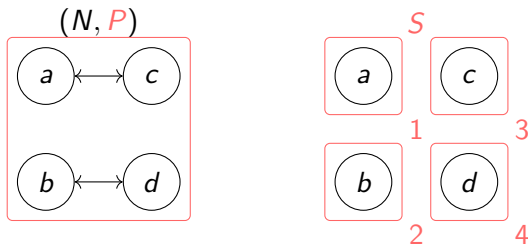
# The Cost of simulation $K^+$ and $K$



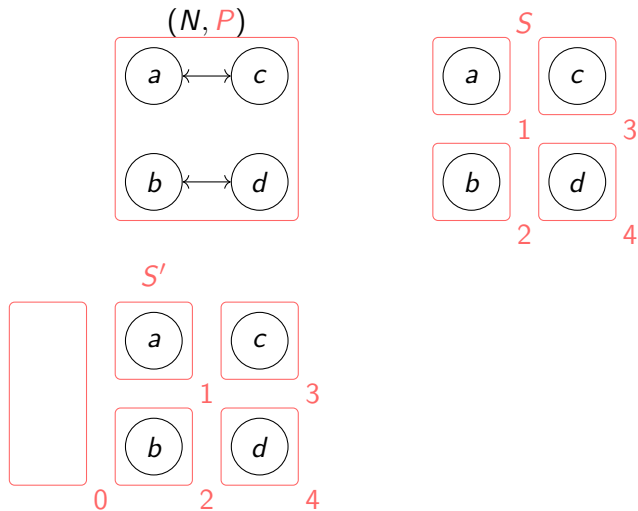
# The Cost of simulation $K^+$ and $K$



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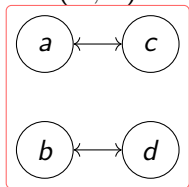
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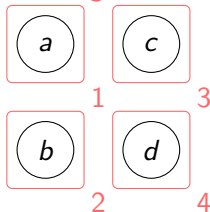


# The Cost of simulation $K^+$ and $K$

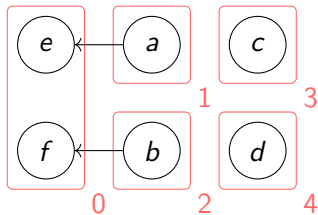
$(N, P)$



$S$

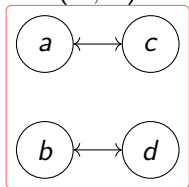


$(N', S')$

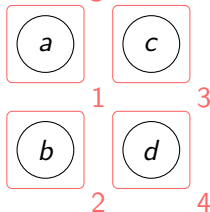


# The Cost of simulation $K^+$ and $K$

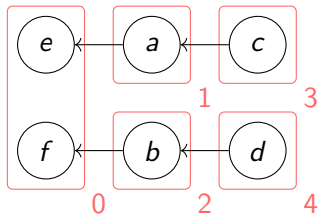
$(N, P)$



$S$

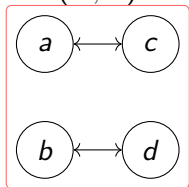


$(N', S')$

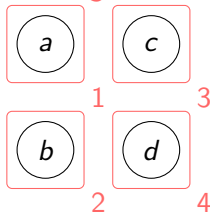


# The Cost of simulation $K^+$ and $K$

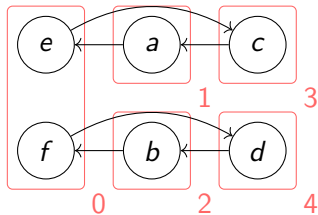
$(N, P)$



$S$

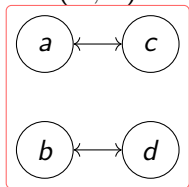


$(N', S')$

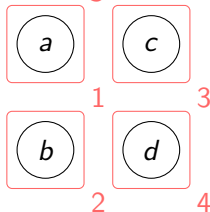


# The Cost of simulation $K^+$ and $K$

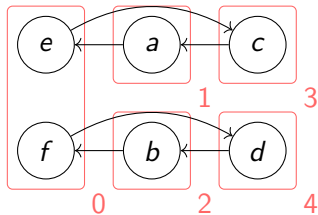
$(N, P)$



$S$

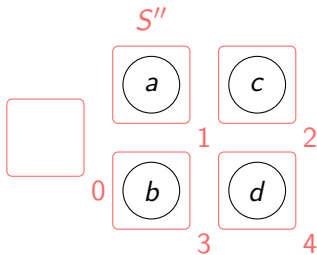
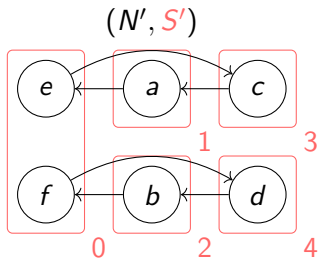
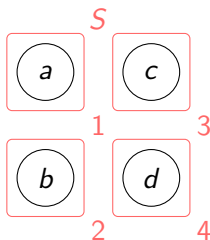
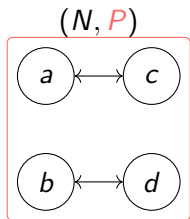


$(N', S')$



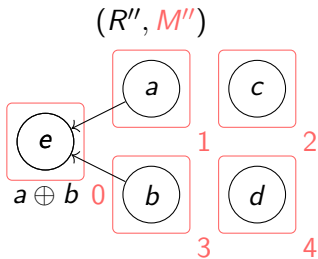
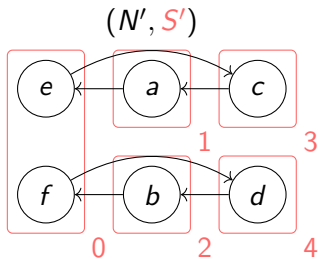
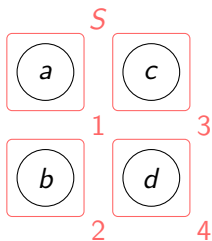
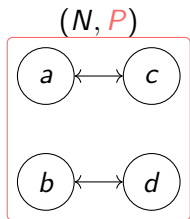
$$K^+(N, S) \leq 2$$

# The Cost of simulation $K^+$ and $K$



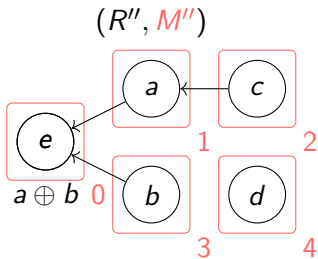
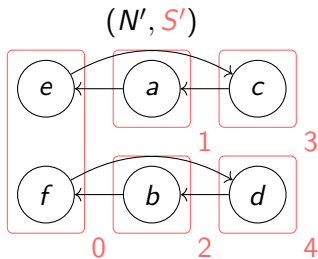
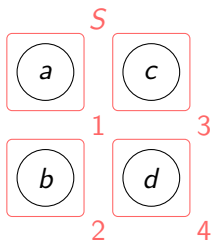
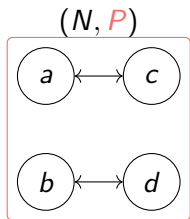
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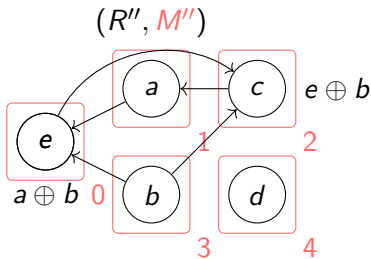
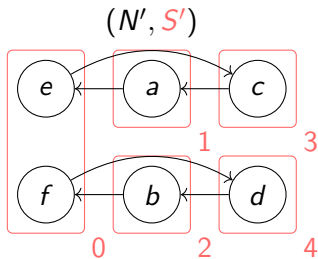
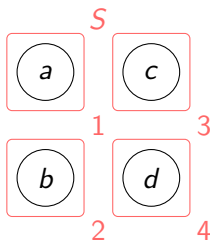
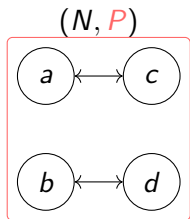
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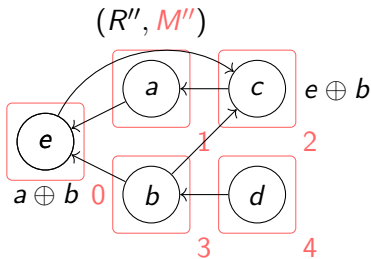
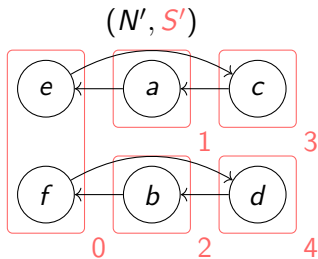
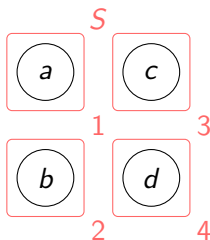
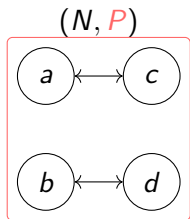
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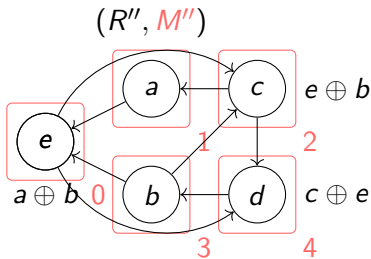
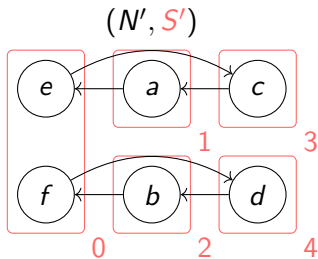
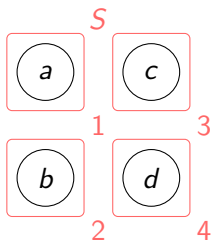
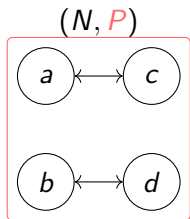


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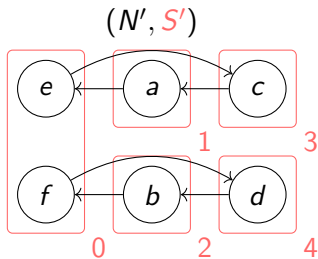
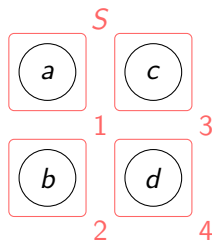
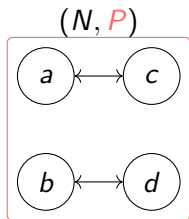
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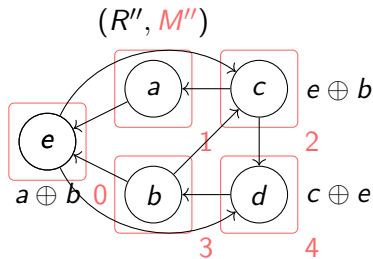


$$K^+(N, S) \leq 2$$

# The Cost of simulation $K^+$ and $K$

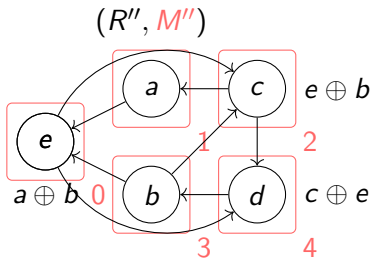
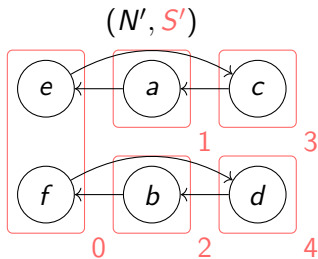
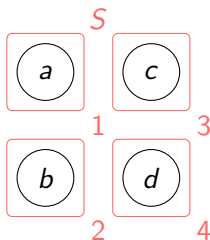
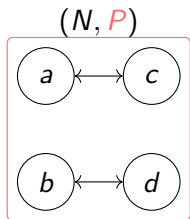


$$K^+(N, S) \leq 2$$



$$K(N, S) \leq 1$$

# The Cost of simulation $K^+$ and $K$

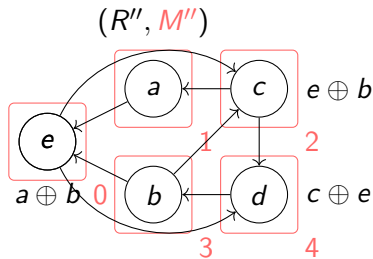
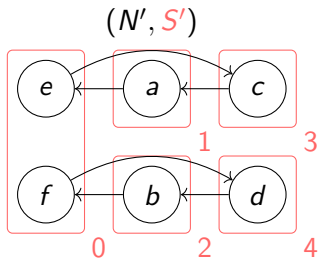
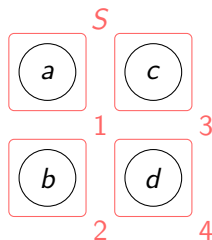
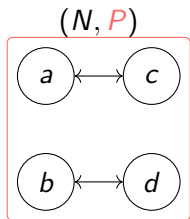


$$K^+(N, S) \leq 2$$

$$K^+(N, S) \geq K(N, S)$$

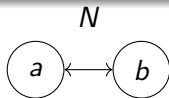
$$K(N, S) \leq 1$$

# The Cost of simulation $K^+$ and $K$

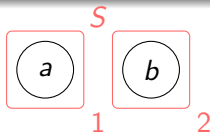
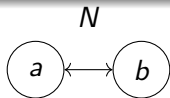


$$K^+(N, S) = 2 \quad K^+(N, S) \geq K(N, S) \quad K(N, S) = 1$$

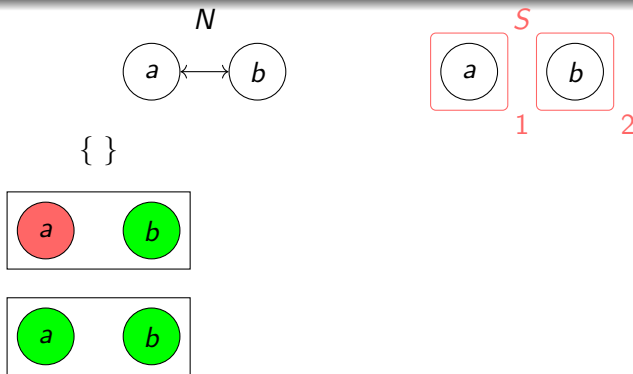
# Confusion graph $G_{N,s}$



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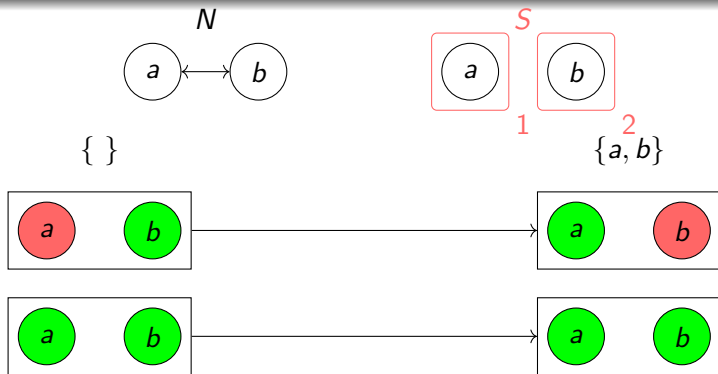


# Confusion graph $G_{N,S}$

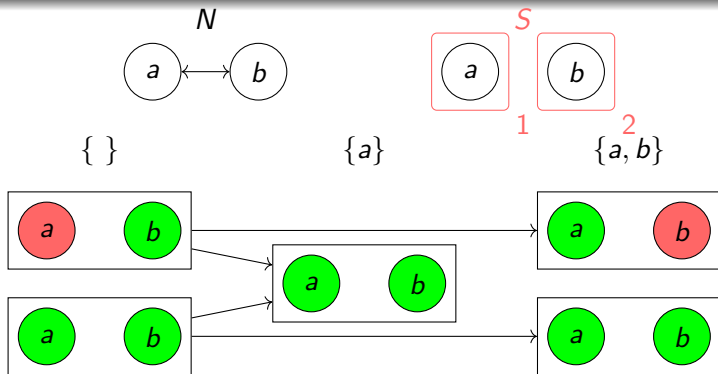




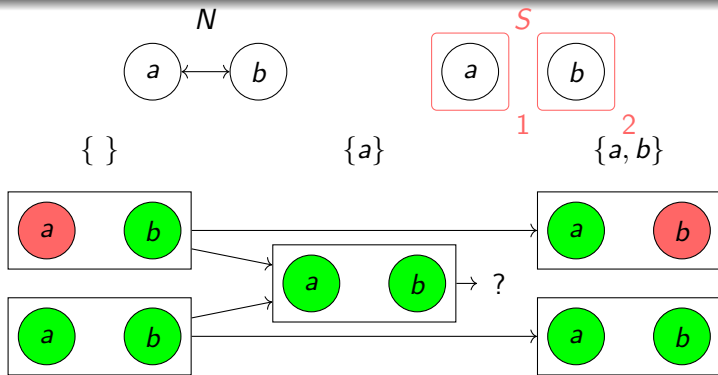
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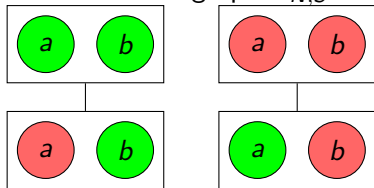
# Confusion graph $G_{N,S}$



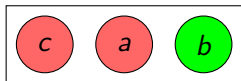
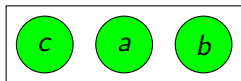
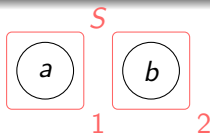
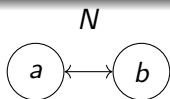
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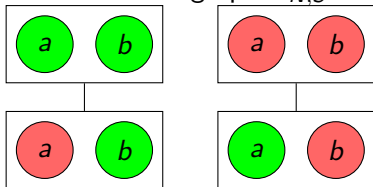
Confusion graph  $G_{N,S}$  :



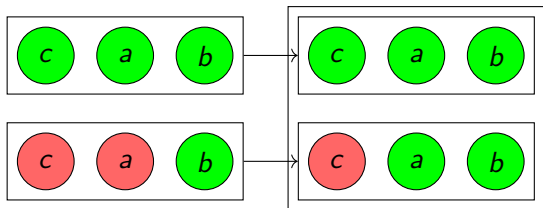
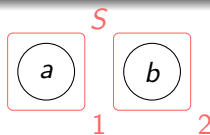
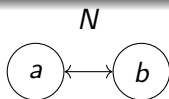
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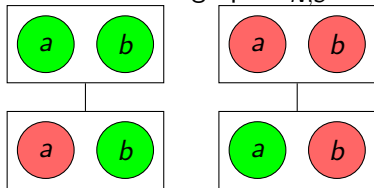
Confusion graph  $G_{N,S}$  :



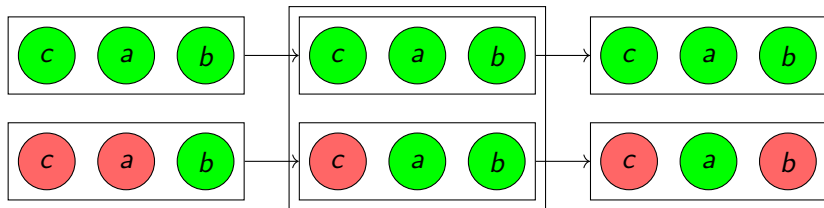
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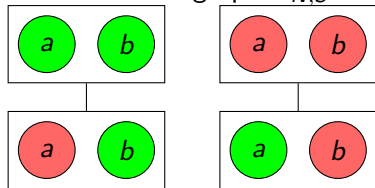
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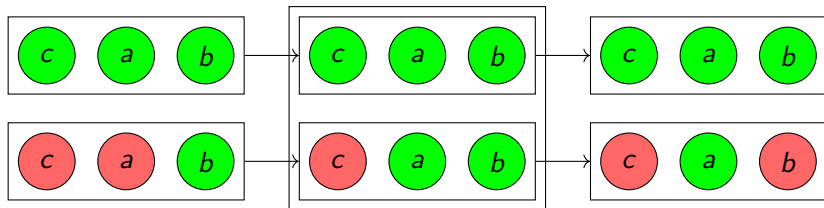
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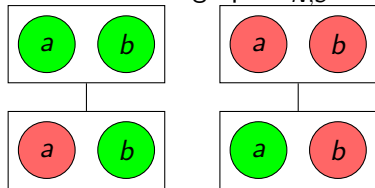
Confusion graph  $G_{N,S}$  :



# Confusion graph $G_{N,S}$

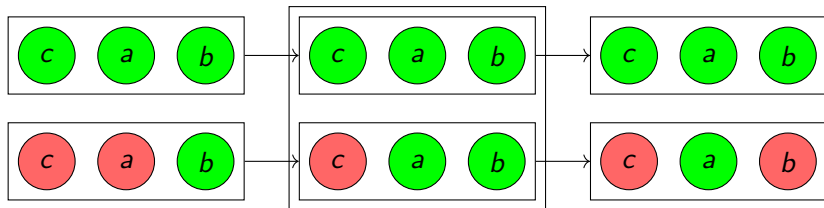


Confusion graph  $G_{N,S}$  :

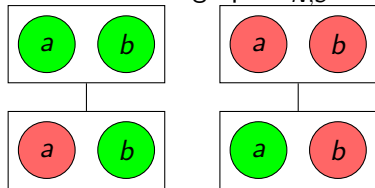


**Lemma.**  $K_{N,S}^+ \geq \lceil \log_2(\chi(G_{N,S})) \rceil$

# Confusion graph $G_{N,S}$



Confusion graph  $G_{N,S}$  :



**Theorem.**  $K_{N,S}^+ = \lceil \log_2(\chi(G_{N,S})) \rceil$



## Principal results :

- Regarding  $K_n^+$  :
  - $K^+(N, S) = \log(\chi(G_{N,S}))$ .
  - $\lfloor n/2 \rfloor \leq K_n^+ \leq 2n/3 + 2$ .
  - $\omega(G_{N,S}) \leq \lfloor n/2 \rfloor$ .
  - If  $N$  is bijective then  $K_n^+ \leq \lfloor n/2 \rfloor$ .
- Regarding  $K_n$  :
  - $\lfloor n/3 \rfloor \leq K_n \leq K_n^+$ .
  - $K_n \leq \tau$ .