Solving Partial Differential Equations on Noisy Quantum Computers

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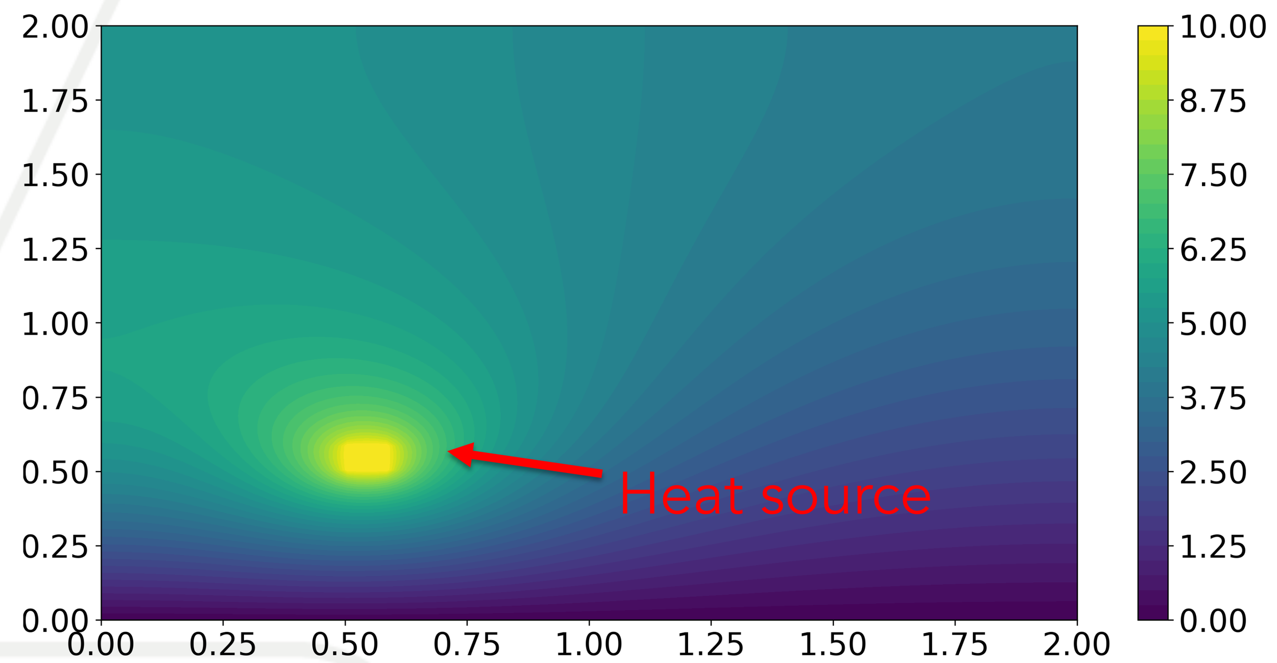
Introduction

Many scientific problems demand solutions to PDEs. Algorithms have been proposed to solve such PDEs on quantum hardware. Most algorithms verified on quantum simulators, and so quantum noise is ignored. However, noise is intrinsic do their operation.

How do such quantum algorithms perform on actual hardware or noisy simulators?



Consider 2D Poisson Problem:

$$\nabla^2\phi(x,y) = \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} = f(x,y)$$

with boundary conditions:

$$\phi_{\text{bot}} = 0, \frac{\partial\phi_{\text{top}}}{\partial x} = \frac{\partial\phi_{\text{left}}}{\partial y} = \frac{\partial\phi_{\text{right}}}{\partial y} = 0$$

Finite difference scheme in space,

$$\nabla_5^2\phi_{i,j} \approx \frac{1}{h^2}(\Phi_{i-1,j} + \Phi_{i+1,j} + \Phi_{i,j-1} + \Phi_{i,j+1} - 4\Phi_{i,j})$$

the differential equation is approximated by linear system

$$A\Phi = F$$

Methods

1. Encoding normalized linear system into a Hamiltonian

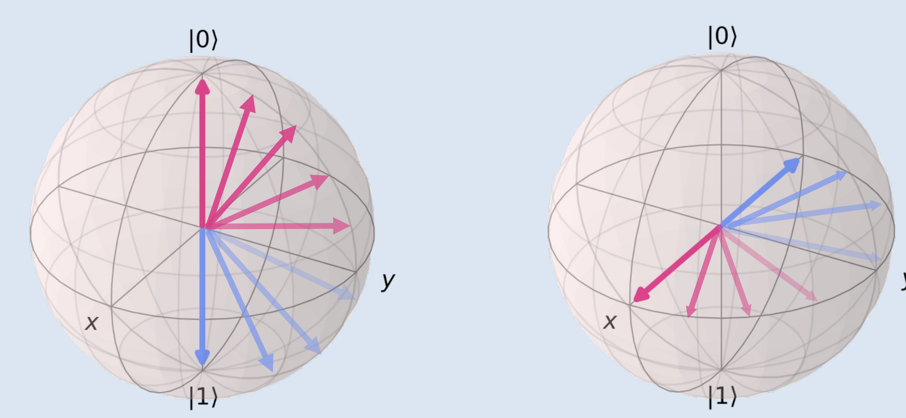
$$H_{\text{eff}} = A^\dagger(|F\rangle\langle F| - I)A$$

Solution is encoded into ground eigenstate of effective Hamiltonian with eigenvalue $\lambda = 0$.

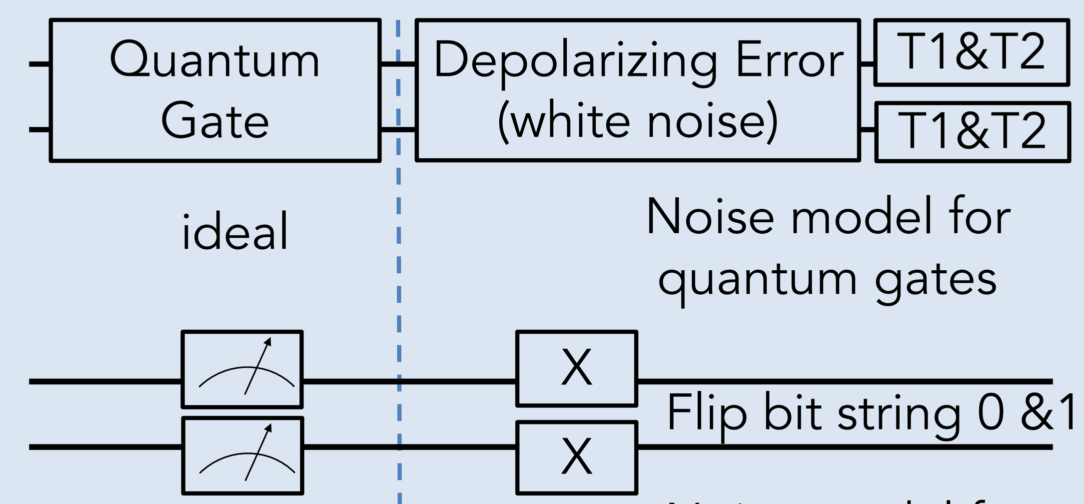
2. Use Variational Quantum Eigensolver (VQE) [1] to find ground state (encoded solution). VQE adopts parametrized quantum circuit $U(\theta)$ to prepare quantum state and classical optimizer to minimize the loss

$$\min_{\vec{\theta}} \mathcal{L}(\vec{\theta}) = \min_{\vec{\theta}} \langle 0^{\otimes N}|U^\dagger(\vec{\theta})H_{\text{eff}}U(\vec{\theta})|0^{\otimes N}\rangle$$

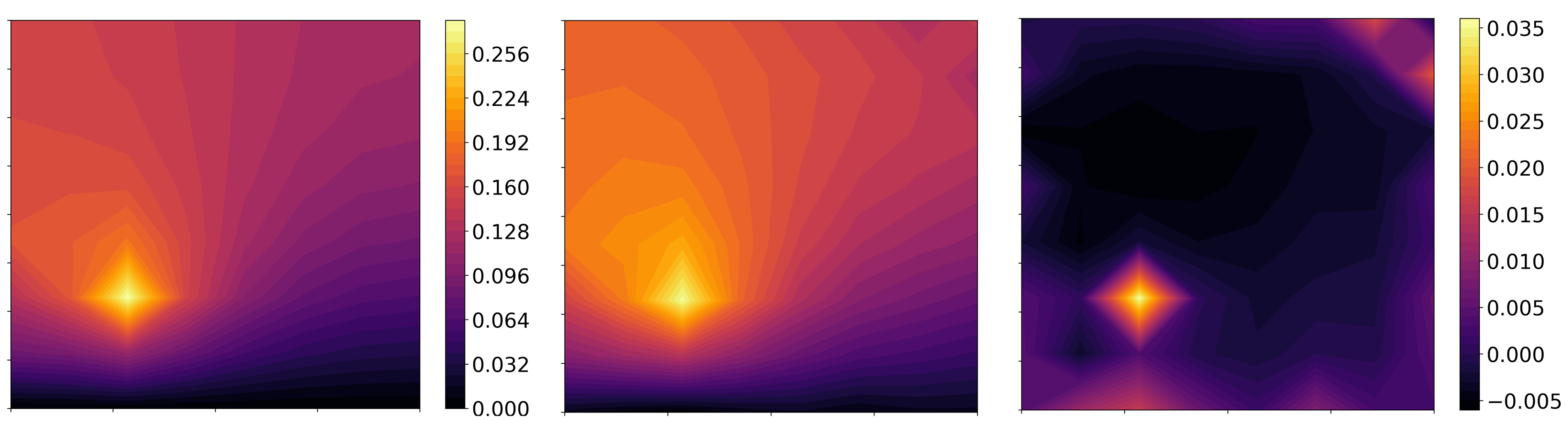
3. Quantum operations and noise model



Energy Relaxation (T1) Dephasing (T2)

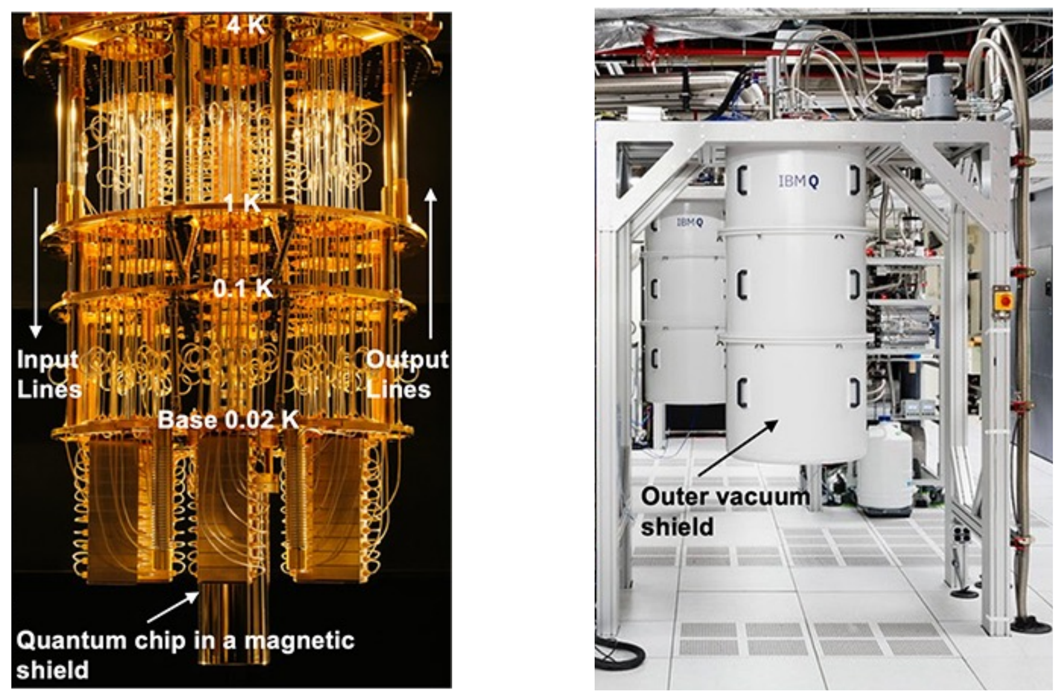


Results



Classical solution downscaled to 8x8 grid | Noisy simulator results using 6 qubits on ibmq_jakarta | Error (residual) ~10%

Quantum Hardware



(left) IBM's superconducting quantum computer operates at microwave and mK regimes. (right) Dilution fridge that houses and cools down superconducting quantum processors.

Conclusion

- Quantum approach requires much fewer resources for same mesh size
- Accuracy sensitive to hardware noise
- Next step: Adopt error suppression method to enhance on-hardware performance

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[1] Peruzzo, Alberto, et al. "A variational eigenvalue solver on a photonic quantum processor." *Nature Communications* 5.1 (2014): 4213.

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