Georgia Tech Research Institute

Optimization with Near-term Quantum Computers

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Quantum computing: cutting through the hype

Known quantum algorithms

- Shor's factoring ("quantum Fourier transform") – exponential: N^3 vs 2^N
- Grover's search ("reverse phonebook") – polynomial: $(\sqrt{N})$ vs. $N/2$
- For more see, e.g., quantumAlgorithmZoo.org

Simulating quantum systems

- E.g. Chemical dynamics, condensed matter ("quantum materials"), quantum field theory, open quantum systems – believed to be superpolynomial

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Bottom line

Complexity theory has proven significant speedup for a handful of specific applications (assuming a perfect, large quantum computer)

NISQ vs fault tolerance

Preskill, J. Quantum Computing in the NISQ era and beyond. *Quantum* **2**, 79 (2018).

"Fault tolerance" is needed to realize complexity advantage

- Existing quantum computers make LOTS of mistakes (1 in 100 typical, 1 in 10,000 world record)
- GTRI 2-qubit gate: 6×10^{-4}
  Clark, C. R. *et al.* High-Fidelity Bell-State Preparation with 40-Ca+ Optical Qubits. *Phys. Rev. Lett.* **127**, 130505 (2021).
- Error correction:
  - Form "logical qubits" from tens of noisy qubits
  - First demonstrated entanglement of logical qubits (Quantinuum):
    Ryan-Anderson, C. *et al.* Implementing Fault-tolerant Entangling Gates on the Five-qubit Code and the Color Code. arXiv:2208.01863 (2022).

NISQ = Noisy Intermediate-Scale Quantum

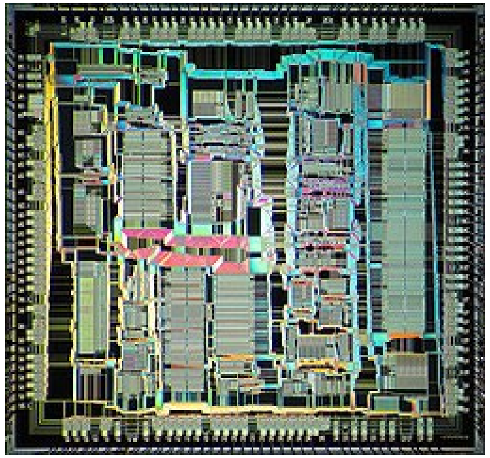
- On the way to large-scale fault tolerant quantum computers, we'll have thousands of noisy qubits.
- Could there be a computational advantage without full fault tolerance?
- In 2018, Preskill suggested there could be for:
  - Optimization
  - Matrix inversion, recommendation systems
  - Quantum simulation

Combinatorial optimization problems

- *"For example, we might **formulate m constraints on n bits**, and seek an **n-bit string which solves as many of the m constraints as possible**."*

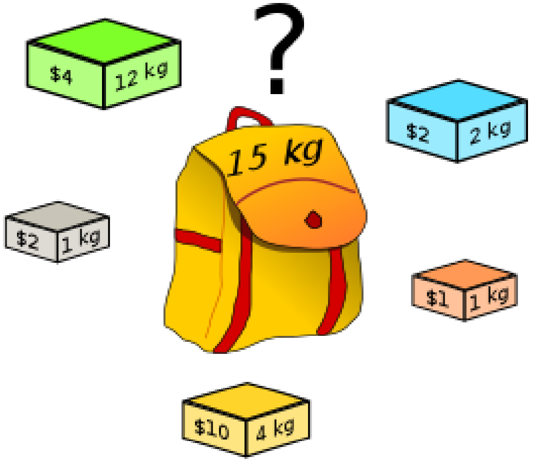
Preskill, "Quantum Computing in the NISQ Era and Beyond", Quantum **2**, 79 (2018), arXiv:1801.00862.

- E.g.



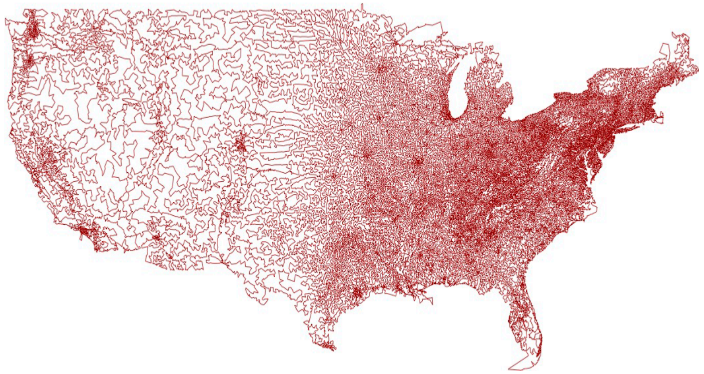
https://commons.wikimedia.org/wiki/File:VLSI_Tandem_CLX_800_CPU_die.jpg

Satisfiability (SAT) of Boolean clauses



https://commons.wikimedia.org/wiki/File:Knapsack.svg

Knapsack (packing)



http://www.math.uwaterloo.ca/tsp/usa50/index.html

Traveling Salesperson

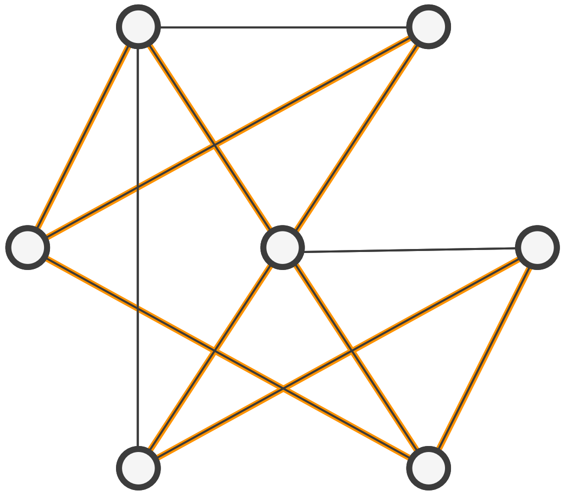
- Others: nurse scheduling, graph coloring, max cut, vertex cover, max independent set
- State-of-the-art: many classical heuristics that find good approximate solutions

OPTIQ project approach

Optimization with Trapped Ion Qubits

- OPTIQ hardware uses global entangling operations to efficiently implement MaxCut QAOA for densely connected graphs

- Non-trivial native graphs; arbitrary graphs programmed through addressed one-qubit bit flips

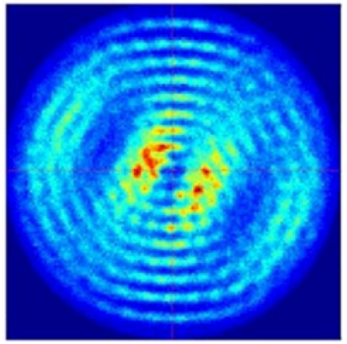


Divide vertices into two subsets so that as many edges as possible span the subsets (are “cut”).

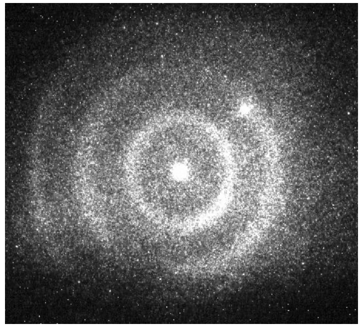
Maximum Cut (MaxCut)

ONISQ program

10 equally-spaced $^{171}Yb^{+}$ ions (GTRI)



100s of $^{9}Be^{+}$ ions
Bollinger (NIST)



~100 $^{40}Ca^{+}$ ions (GTRI)

Dual hardware approach

QAOA

- Farhi, Goldstone, Gutman proposed a Quantum Approximate Optimization Algorithm in 2014 (arXiv:1411:4028)
- Hybrid algorithm: quantum operations parameterized by vectors $\vec{\gamma}, \vec{\beta}$, classically optimized

Quantum operations

- Initial state: $|+ + \cdots +\rangle = \sum_b |b\rangle$, where b is any bit string of 0's and 1's

  superposition of all possible answers
- Alternately apply (p times):
  - Cost (γ_i): Apply cost operator, modifies phase between bit strings
  - Mixer (β_i): Flip 0's and 1's, probability depends on β

  Together these can amplify (diminish) certain bit strings likelihood of being observed
- Measure a particular bit string (repeat thousands of times)

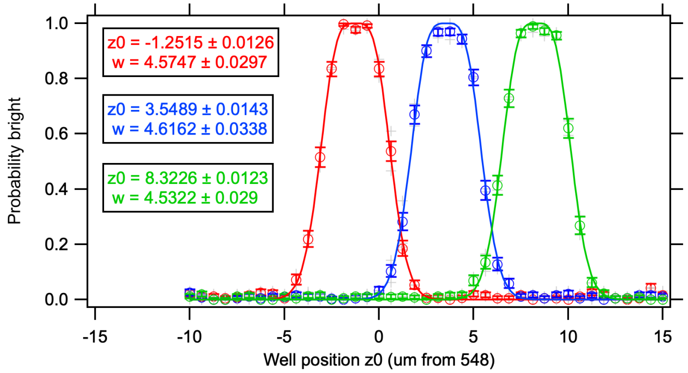
Classical optimization loop

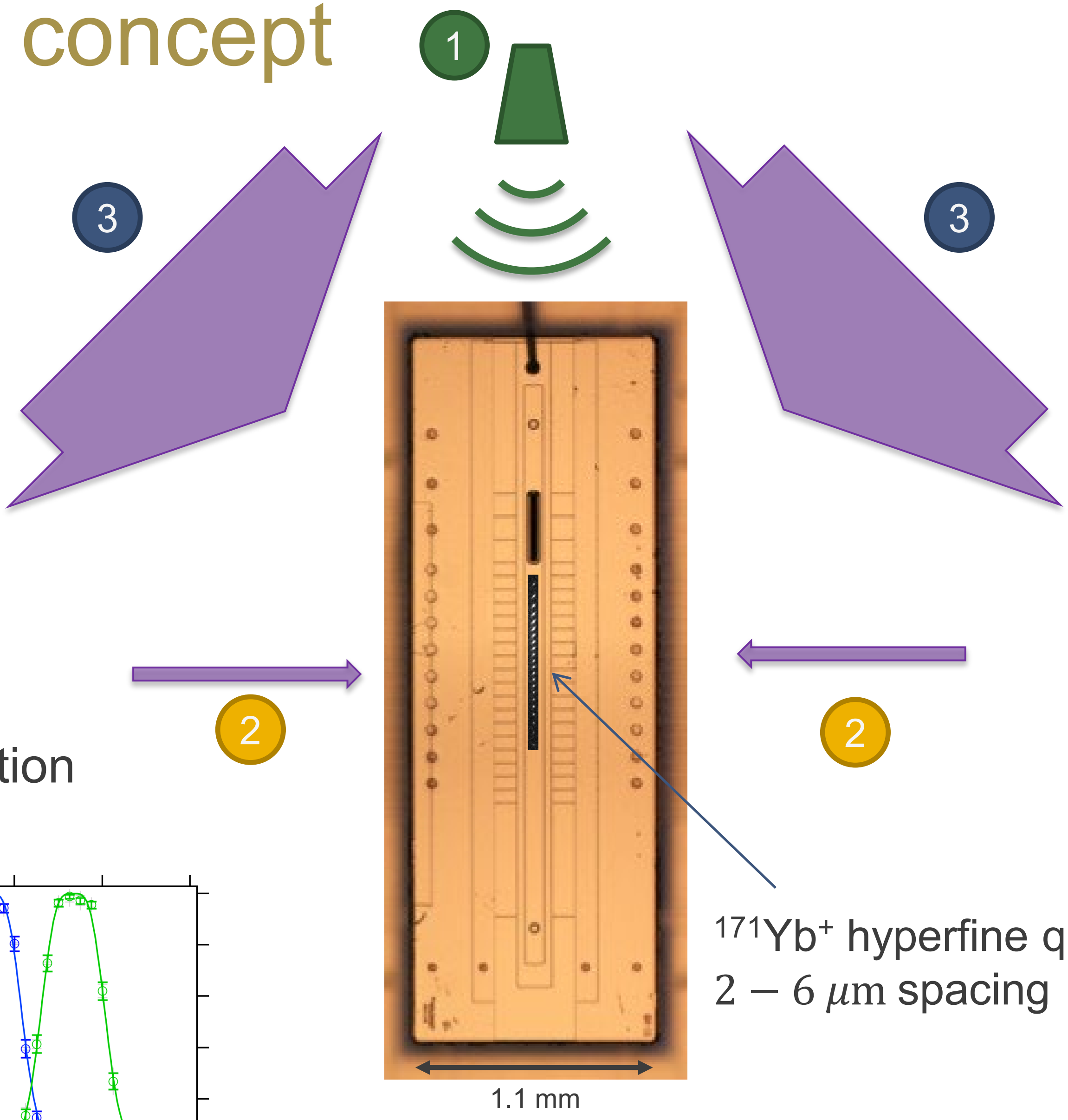
- Compute expected cost $\langle C \rangle$
- Vary $\vec{\gamma}, \vec{\beta}$ and optimize to increase likelihood of measuring a good answer
- Stop when a suitably good answer is observed

Linear trap: proof of OPTIQ concept

1. **Microwaves**
   - Global 1Q rotations (direct)
   - Very high fidelity
   - Initial state preparation & mixer
2. **Focused beams**
   - Individually addressed 1Q phase shifts
   - Qubit bit flips for cost function generation
   - Ion transport to hit any ion
3. **Global beams**
   - Global 1Q rotations (Raman)
   - Mølmer-Sørensen (MS) entangling interaction $X_i X_j$ for cost function generation

e.g. 3-ion addressing,
<1% crosstalk

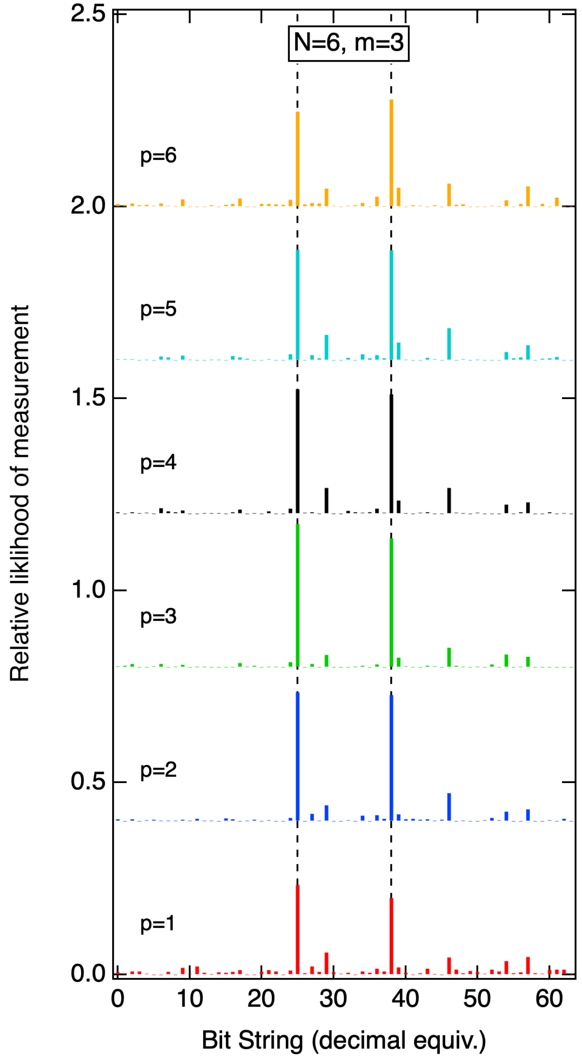


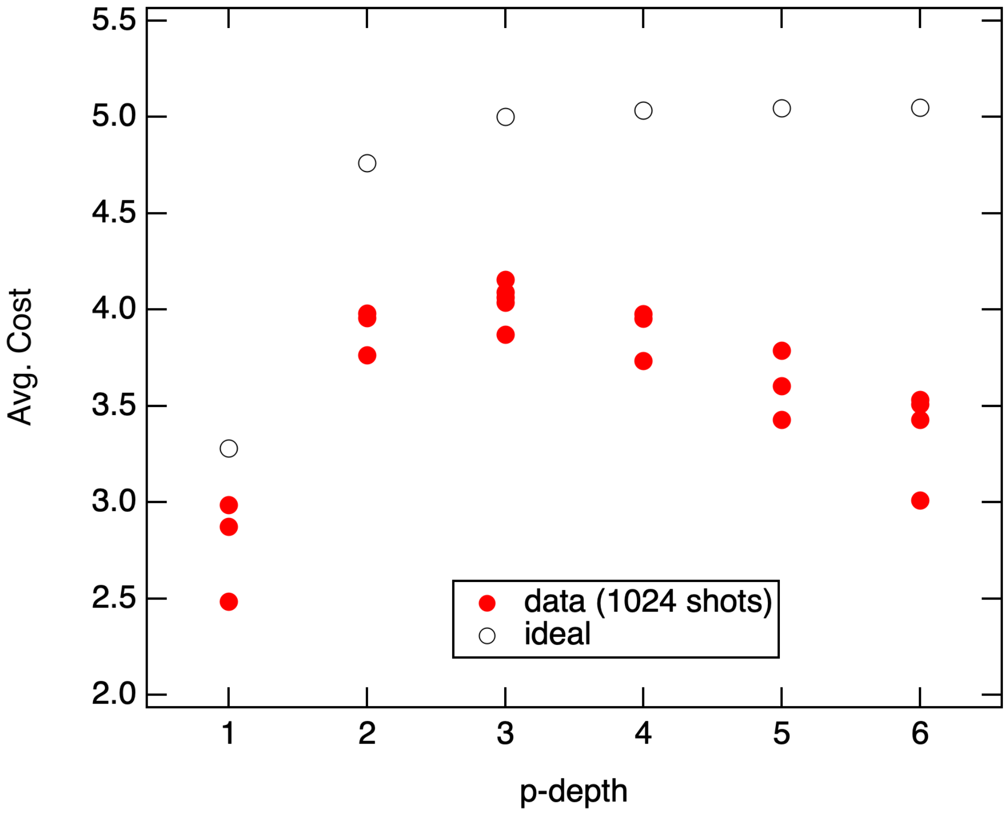


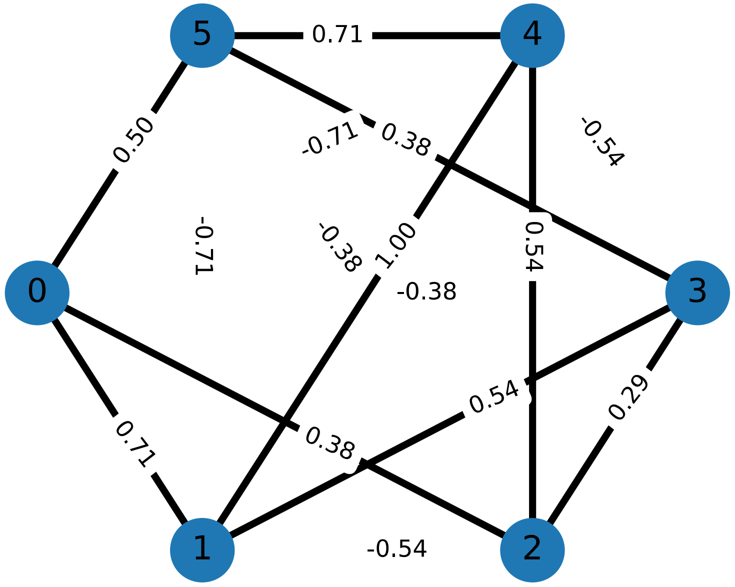
$^{171}Yb^+$ hyperfine qubits,
$2 - 6\ \mu m$ spacing

GTRI/Honeywell BGA trap

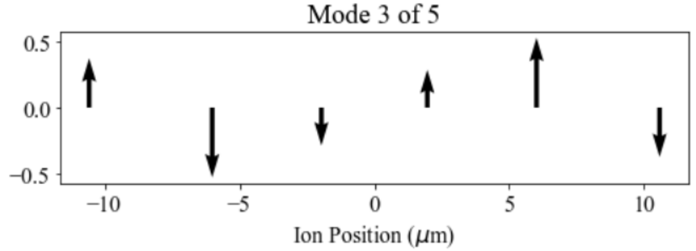
6-ion native graph at $\vec{\gamma}^*, \vec{\beta}^*$





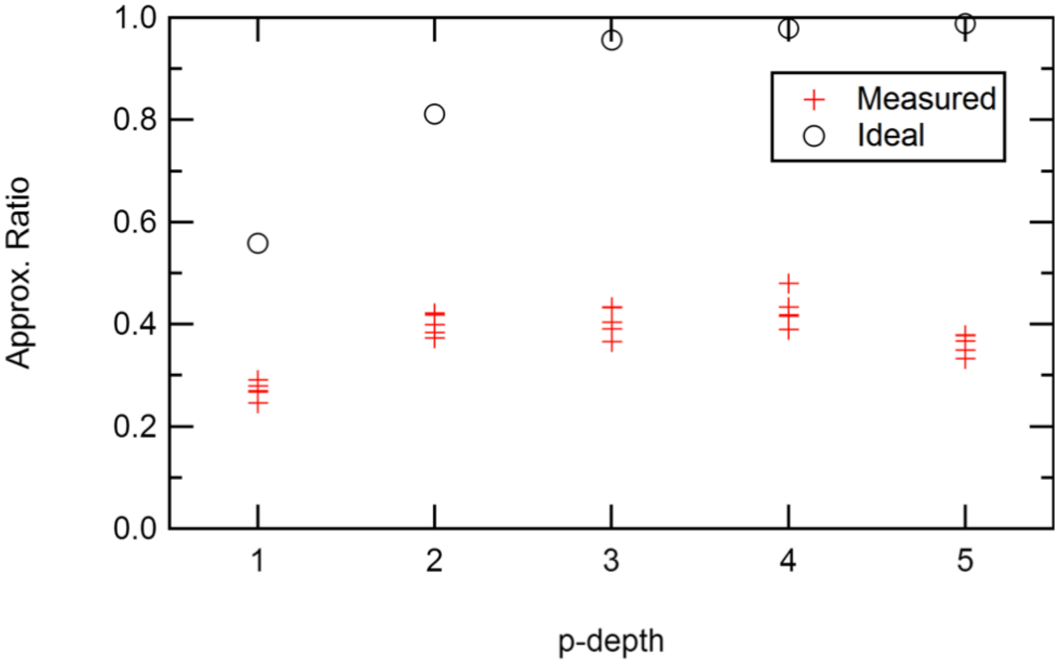


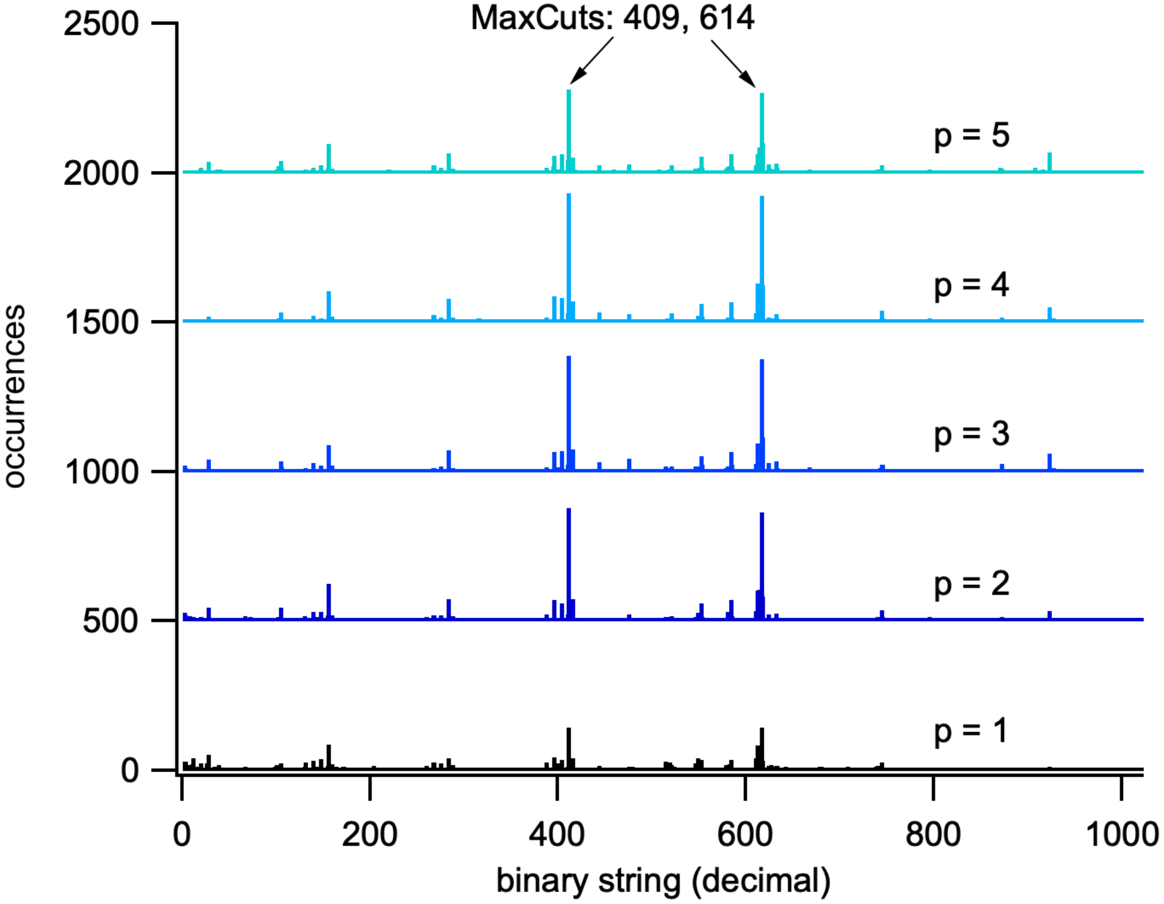
MaxCut of 5.05 for $|011001\rangle, |100110\rangle$



10-ion native graph at $\vec{\gamma}^*, \vec{\beta}^*$

N=10, mode=5 native graph





- Observed $\langle C \rangle$ improves to p=4

Compare to:
Harrigan, M. P. *et al.* Quantum Approximate Optimization of Non-Planar Graph Problems on a Planar Superconducting Processor. *Nat. Phys.* **17**, 332–336 (2021).

Ebadi, S. *et al.* Quantum optimization of maximum independent set using Rydberg atom arrays. *Science* **doi:10.1126/science.abo6587** (2022). arXiv:2202.09372

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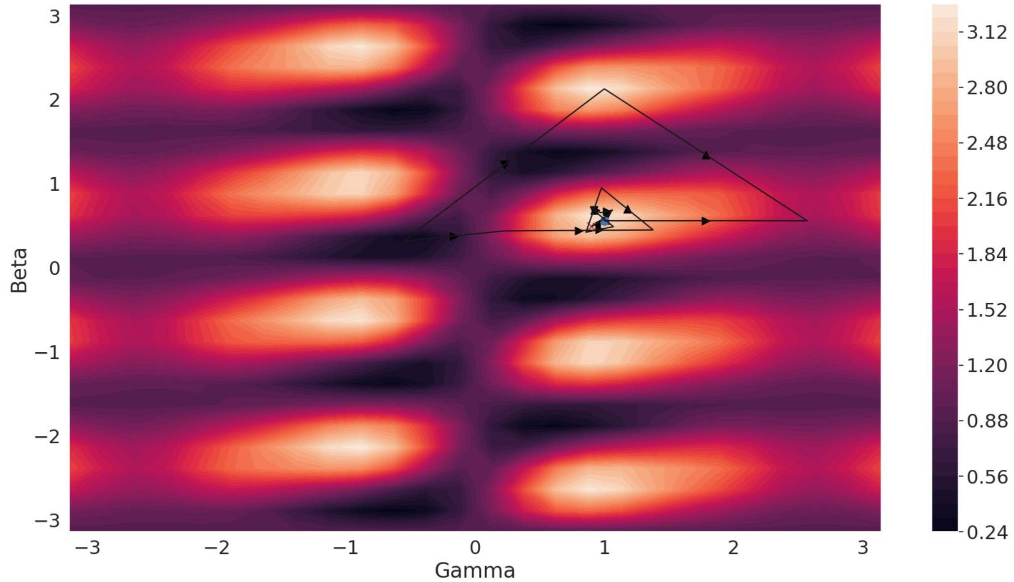
Classical loop: optimizing QAOA parameters

- We use existing XACC software (open-source, led by ORNL) as a quantum programming and compilation framework

XACC
QUANTUM FRAMEWORK

- XACC supports hybrid optimization methods, supervises classical loop of QAOA
- XACC supports multiple execution targets: IBM, Rigetti, IonQ, simulators, GTRI (private branch for OPTIQ hardware)

- **E.g. N=6, p=1 optimization over γ_1, β_1**
- We use COBYLA as it's more robust to noisy costs



Lavrijsen, W., Tudor, A., Müller, J., Iancu, C. & de Jong, W. Classical Optimizers for Noisy Intermediate-Scale Quantum Devices. *arXiv:2004.03004 [quant-ph]* (2020).

Scaling QAOA to large N

- QAOA easily simulated for data shown (N<=10)
- Plus, straightforward classical solution of MaxCut
- Expect 100s of qubits needed to be competitive with classical solvers for some problems, 1000s generally
- 1D array limited to 50-100 ions

Zhang, J. *et al.* Observation of a many-body dynamical phase transition with a 53-qubit quantum simulator. *Nature* **551**, 601–604 (2017).

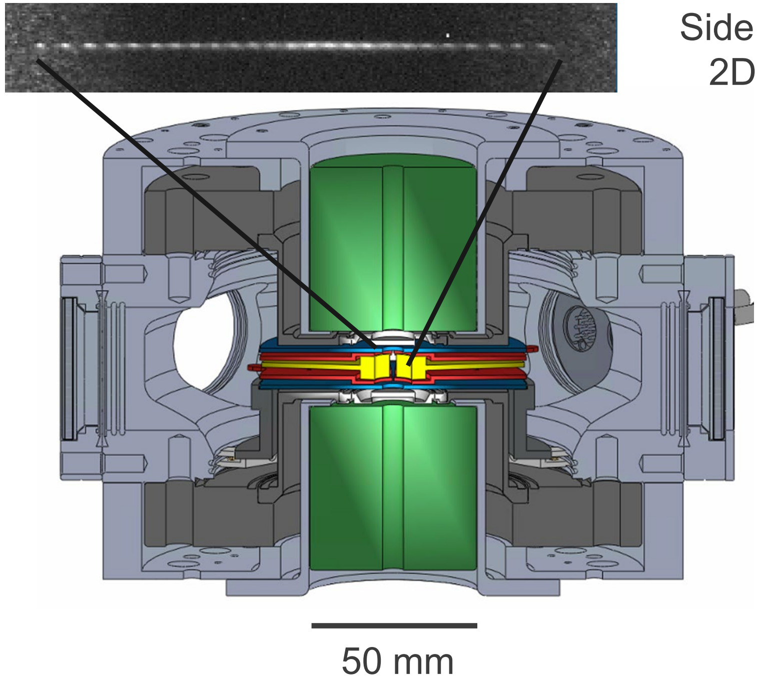
- **Transition to 2d array**

McMahon, B. J., Volin, C., Rellergert, W. G. & Sawyer, B. C. Doppler-cooled ions in a compact reconfigurable Penning trap. *Phys. Rev. A* **101**, 013408 (2020).

McMahon, B. J. & Sawyer, B. C. Second-Scale $^9Be^+$ Spin Coherence in a Compact Penning Trap. (2021).

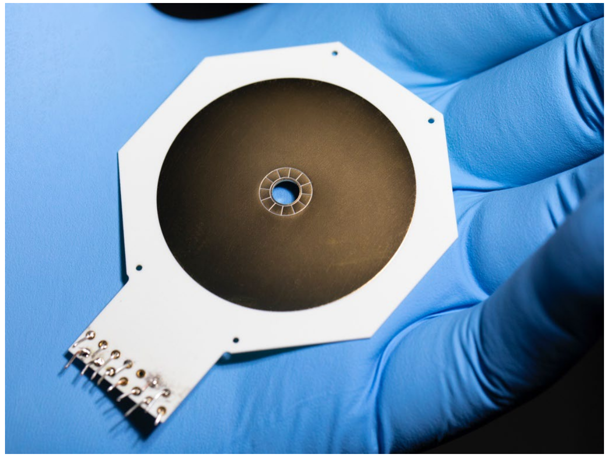
GTRI Compact Penning Trap

- Developed permanent magnet based Penning trap (ONR)

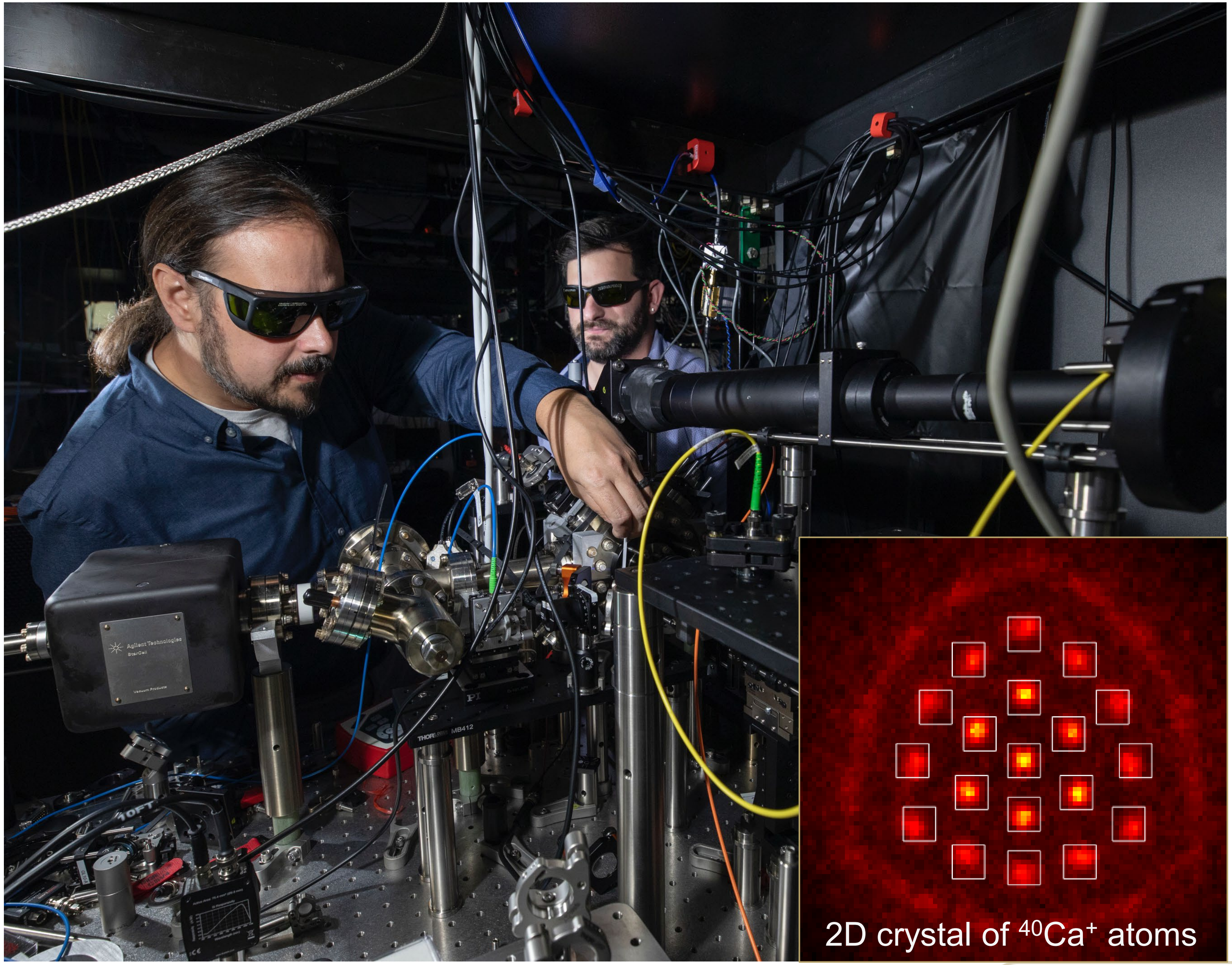


GTRI Compact Penning Trap (PCB)

Dr. Brian Sawyer (left) adjusts a laser beam alignment as Dr. Brian McMahon (right) looks on (with trepidation!)



PCP Penning trap electrodes



2D crystal of $^{40}Ca^{+}$ atoms

Related work from the OPTIQ team

Warm-starts for QAOA (GTRI + GT)

- Tate, R., Farhadi, M., Herold, C., Mohler, G. & Gupta, S. "Bridging Classical and Quantum with SDP initialized warm-starts for QAOA." ACM Transactions on Quantum Computing, https://doi.org/10.1145/3549554 (2022).
- Tate, R., Gard, B., Mohler, G. & Gupta, S. "Classically-inspired Mixers for QAOA Beat Goemans-Williamson's Max-Cut at Low Circuit Depths." *arXiv:2112.11354* (2021).

Addressing method with deformable mirror (NIST)

- Polloreno, A. M., Rey, A. M. & Bollinger, J. J. "Individual qubit addressing of rotating ion crystals in a Penning trap." *arXiv:2203.05196* (2022).

Noise Modeling (ORNL + GTRI)

- Lotshaw, P. C. *et al.* Modelling noise in global Molmer-Sorensen interactions applied to quantum approximate optimization. arXiv:2211.00133 (2022).

Generating arbitrary Max-Cut graphs (GTRI + GT)

- Rajakumar, J. *et al.* "Generating Target Graph Couplings for QAOA from Native Quantum Hardware Couplings." *Phys. Rev. A* **106**, 022606 (2022).

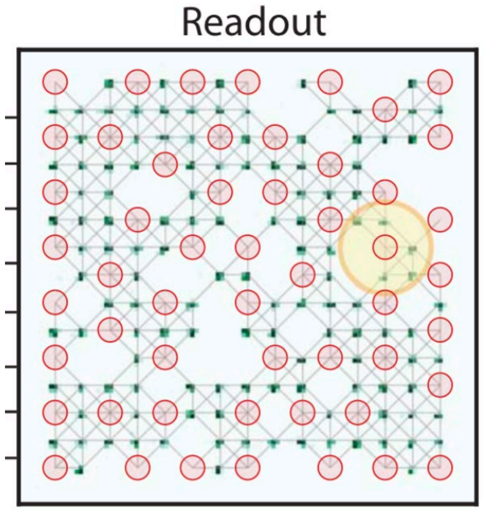
Related QAOA theory (UT Knoxville + ORNL)

- Lotshaw, P. C. *et al.* "Scaling Quantum Approximate Optimization on Near-term Hardware." *arXiv:2201.02247* (2022).
- Shaydulin, R., Lotshaw, P. C., Larson, J., Ostrowski, J. & Humble, T. S. "Parameter Transfer for Quantum Approximate Optimization of Weighted MaxCut." *arXiv:2201.11785* (2022).

Other ONISQ results

- Lukin group (Harvard) have built a computer based on individually trapped Rydberg atoms

- Maximum Independent Set (Rydberg blockade)



- Recent paper (right) shows better performance as analog simulator compared to QAOA

Ebadi, S. *et al.* Quantum optimization of maximum independent set using Rydberg atom arrays. *Science* **376**, 1209–1215 (2022).

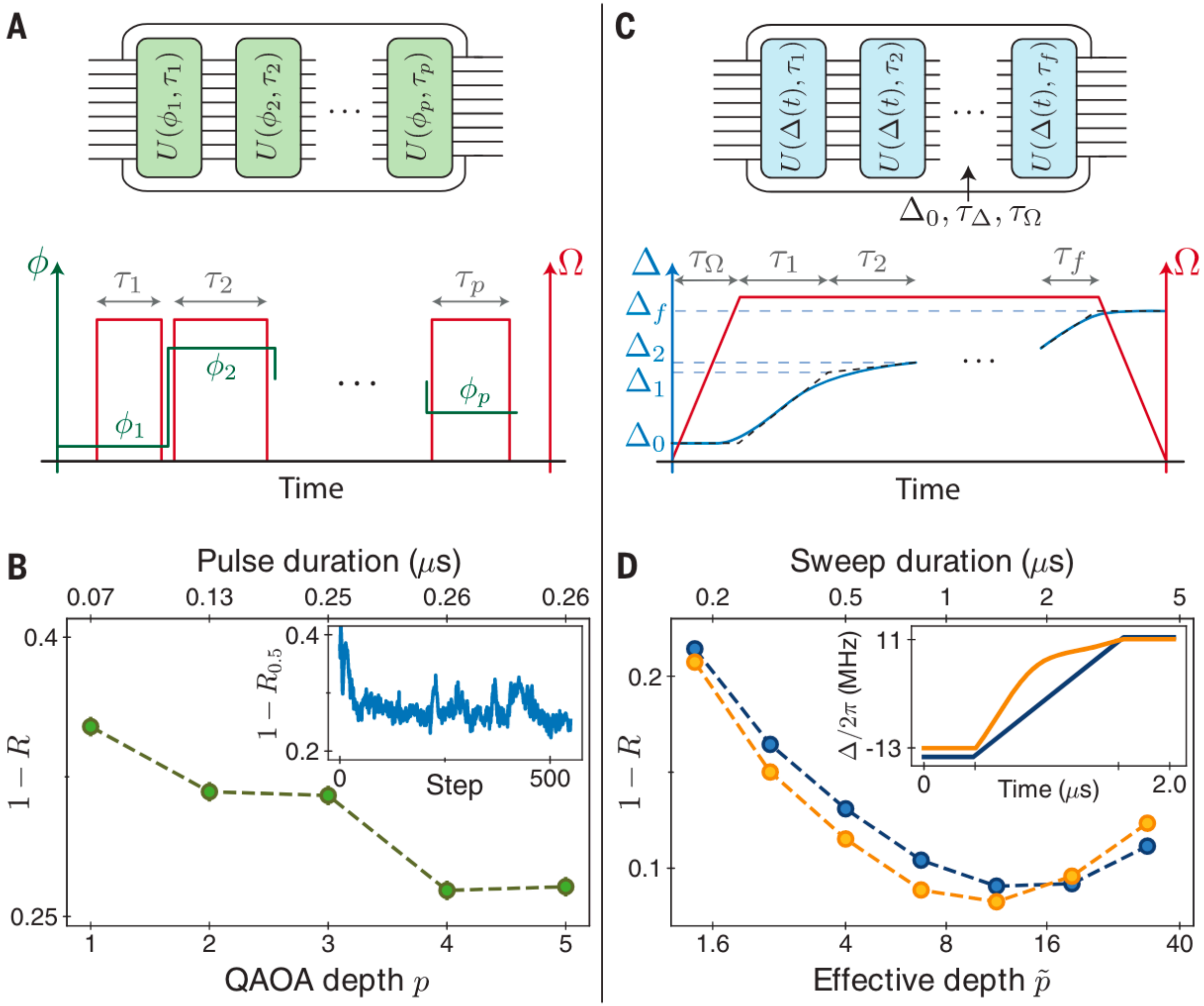


Fig. 2. Testing variational quantum algorithms. (**A**) Implementation of the quantum approximate optimization algorithm (QAOA), consisting of sequential layers of resonant pulses with variable duration τ_i and laser phase ϕ_i. (**B**) Variational optimization of QAOA parameters results in a decrease in approximation error $1 - R$, up to depth $p = 4$ (inset: example performance of quantum-classical closed-loop optimization

NISQ computing prospects

- Despite a flurry of effort over the past four years, no general proof of complexity speedup for NISQ applications

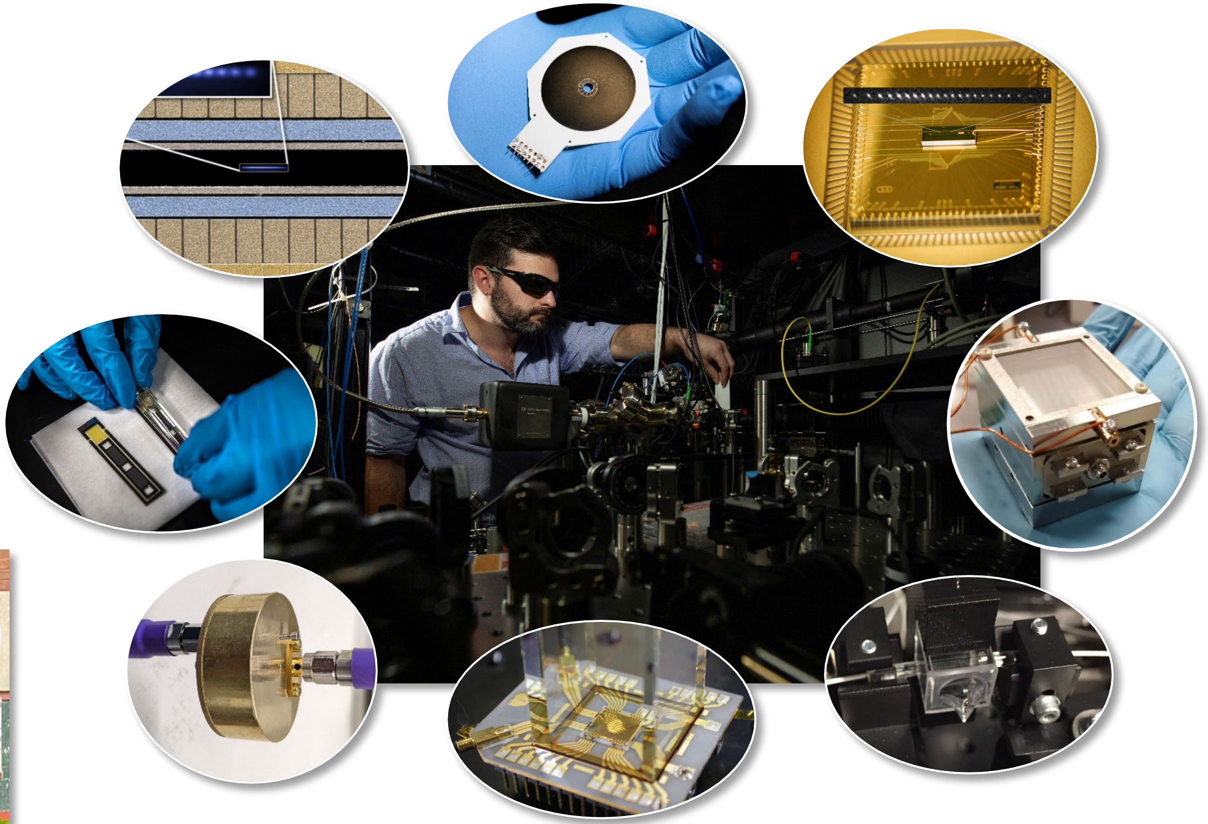
- Expect special-purpose hardware to solve particular problems better than conventional computers, likely (analog) quantum simulation

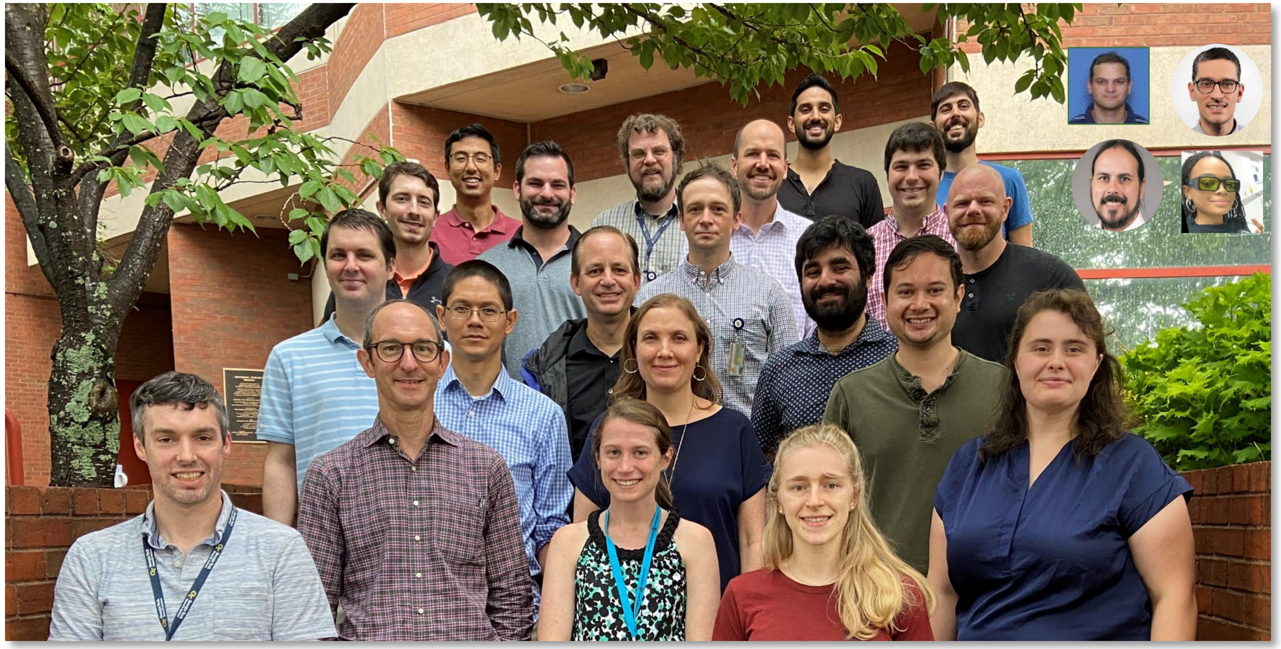
- Unlikely that digital quantum computers (that implement 1- and 2-qubit gates) will be competitive for optimization

  Lotshaw, P. C. *et al.* “Scaling Quantum Approximate Optimization on Near-term Hardware.” *arXiv:2201.02247* (2022).

- Still potential for surprise if new NISQ application found

GTRI Quantum Systems Division





26 Research Scientists and Engineers

Work with us!

GTRI's Quantum Systems Division is hiring at the bachelor, master, and PhD level. Active postings for:

- Applied Physicist (BS/MS)
- Applied Physicist (PhD)
- Also interested in Optical, Mechanical, Electrical, and Software Engineers

Our research portfolio includes funded programs in trapped-ion quantum computing, simulation, and sensing; quantum magnetometry and electrometry; molecular data storage technology; mass spectrometry; and chemical sensing.

gtri.gatech.edu/careers (category: Quantum Computing)