HW 9

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1.1

$$h := f\ (head\ X\ l)$$

$$t := m\ (tail\ X\ l)$$

$$body := fix\ \lambda\ m^{[X]\to[Y]}\ l^{[X]}.\ \text{if}\ isnil\ X\ l\ \text{then}\ nil\ Y\ \text{else}\ cons\ Y\ h\ t$$

$$\dfrac{\dfrac{}{X^\star, Y^\star, f^{X\to Y} \vdash f : X \to Y}\ \text{Var} \qquad \dfrac{\dfrac{\dfrac{}{X^\star \vdash X : \star}\ \text{Var} \quad \dfrac{}{head : \forall X.\,[X] \to X}\ \text{Def}}{X^\star \vdash head\ X : [X] \to X}\ \text{TApp} \quad \dfrac{}{X^\star, l^{[X]} \vdash l : X}\ \text{App}}{X^\star, l^{[X]} \vdash head\ X\ l : X}\ \text{App}}{X^\star, Y^\star, f^{X\to Y}, l^{[X]} \vdash h : Y}$$

$$\dfrac{\dfrac{}{X^\star, Y^\star, m^{[X]\to[Y]} \vdash m : [X] \to [Y]}\ \text{Var} \qquad \dfrac{\dfrac{\dfrac{}{X^\star \vdash X : \star}\ \text{Var} \quad \dfrac{}{\vdash tail : \forall X.\,[X] \to [X]}\ \text{Def}}{X^\star \vdash tailX : [X] \to [X]}\ \text{TApp} \quad \dfrac{}{X^\star, l^{[X]} \vdash l : [X]}\ \text{Var}}{X^\star, l^{[X]} \vdash tail\ X\ l : [X]}\ \text{App}}{X^\star, Y^\star, m^{[X]\to[Y]}, l^{[X]} \vdash t : [Y]}\ \text{App}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{Y^\star \vdash Y : \star}\ \text{Var} \quad \dfrac{}{\vdash cons : \forall X.X \to [X] \to [X]}\ \text{Def}}{Y^\star \vdash cons\ Y : Y \to [Y] \to [Y]}\ \text{TApp} \quad \dfrac{}{X^\star, Y^\star, f^{X\to Y}, l^{[X]} \vdash h : Y}}{X^\star, Y^\star, f^{X\to Y}, l^{[X]} \vdash cons\ Y\ h : [Y] \to [Y]}\ \text{App} \quad \dfrac{}{X^\star, Y^\star, m^{[X]\to[Y]}, l^{[X]} \vdash t : [Y]}}{X^\star, Y^\star, f^{X\to Y}, m^{[X]\to[Y]}, l^{[X]} \vdash cons\ Y\ h\ t : [Y]}\ \text{App}$$

$$\dfrac{\dfrac{\dfrac{}{X^\star \vdash X : \star}\ \text{Var} \quad \dfrac{}{\vdash isnil : \forall X.\,[X] \to Bool}\ \text{Def}}{X^\star \vdash isnil\ X : [X] \to Bool}\ \text{TApp}}{X^\star, l^{[X]} \vdash isnil\ X\ l : Bool}\ \text{App}$$

$$\dfrac{\dfrac{}{X^\star, l^{[X]} \vdash isnil\ X\ l : Bool}\ \text{App}\dfrac{\dfrac{}{Y^\star \vdash Y : \star}\ \text{Var}\quad\dfrac{}{\vdash nil : \forall Y.[Y]}\ \text{Def}}{Y^\star \vdash nil\ Y : [Y]}\ \text{TApp}\quad\dfrac{}{X^\star, Y^\star, f^{X\to Y}, m^{[X]\to[Y]}, l^{[X]} \vdash cons\ Y\ h\ t : [Y]}}{\begin{array}{l} X^\star, Y^\star, f^{X\to Y}, m^{[X]\to[Y]}, l^{[X]} \vdash \text{if } isnil\ X\ l \text{ then } nil\ Y \\ \qquad\qquad \text{else } cons\ Y\ \ (f\,(head\ X\ l)\ \ (m\,(tail\ X\ l))) \\ \qquad\qquad : [Y] \end{array}}\ \text{If}$$

$$\dfrac{\begin{array}{l} X^\star, Y^\star, f^{X\to Y} \vdash \lambda\ m^{[X]\to[Y]}\ l^{[X]}.\ \text{if } isnil\ X\ l \text{ then } nil\ Y \\ \qquad\qquad \text{else } cons\ Y\ \ (f\,(head\ X\ l)\ \ (m\,(tail\ X\ l))) \\ \qquad\qquad : ([X]\to[Y])\to[X]\to[Y] \end{array}}{X^\star, Y^\star, f^{X\to Y} \vdash body : [X]\to[Y]}\ \text{Fix}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\begin{array}{l} X^\star, Y^\star, f^{X\to Y}m^{[X]\to[Y]} \vdash \lambda\ l^{[X]}.\ \text{if } isnil\ X\ l \text{ then } nil\ Y \\ \qquad\qquad \text{else } cons\ Y\ \ (f\,(head\ X\ l)\ \ (m\,(tail\ X\ l))) \\ \qquad\qquad : [X]\to[Y] \end{array}}{X^\star, Y^\star, f^{X\to Y} \vdash \lambda\ m^{[X]\to[Y]}\ l^{[X]}.\ \text{if } isnil\ X\ l \text{ then } nil\ Y \\ \qquad\qquad \text{else } cons\ Y\ \ (f\,(head\ X\ l)\ \ (m\,(tail\ X\ l)))\\ \qquad\qquad : ([X]\to[Y])\to[X]\to[Y]}\ \text{Abs}}{X^\star, Y^\star, f^{X\to Y} \vdash body : [X]\to[Y]}\ \text{Fix}}{X^\star, Y^\star \vdash \lambda f^{X\to Y}.\ body : (X\to Y)\to[X]\to[Y]}\ \text{Abs}}{X^\star \vdash \Lambda Y.\lambda f^{X\to Y}.\ body : \forall Y.(X\to Y)\to[X]\to[Y]}\ \text{TAbs}}{\vdash \Lambda X\ Y.\lambda f^{X\to Y}.\ body : \forall X.\forall Y.(X\to Y)\to[X]\to[Y]}\ \text{TAbs}$$

1.2

$$reverse = \Lambda X.fix\Big(\lambda r^{[X]\to[X]\to[X]} a^{[X]} l^{[X]}.\text{if } isnil\ X\ l \text{ then } nil\ X \text{ else } r\ \ (cons\ X\ \ (head\ X\ l)\ \ a)\,(tail\ X\ l)\Big)\,(nil\ X)$$

1.3

$$insert = \Lambda X.\lambda c^{X\to X\to Bool} e^X.fix\Big(\lambda r^{[X]\to[X]} l^{[X]}.\ \text{if } isnil\ X\ l \text{ then } cons\ X\ e\ \ (nil\ X)\ \text{ else}$$
$$\text{if } c\ e\ \ (head\ X\ l)\ \text{ then } cons\ X\ e\ l \text{ else } cons\ X\ \ (head\ X\ l)\,(r\ \ (tail\ X\ l)))$$

$$sort = \Lambda X.\lambda c^{X\to X\to Bool}.fix\Big(\lambda r^{[X]\to[X]} l^{[X]}.\ \text{if } isnil\ X\ l \text{ then } nil\ X \text{ else } insert\ X\ c\ \ (head\ X\ l)\,(r\,(tail\ X\ l))\Big)$$