

# RANDOM NUMBERS

- *pseudo* random
- uniformity and correlation
- repeatable (*seed*)

Try:

```
import random
```

```
import pylab as plt
```

```
import numpy as np
```

```
random.seed(-1001)
```

```
random.uniform(0,1)
```

```
random.uniform(0,1)
```

```
random.seed(-1003)
```

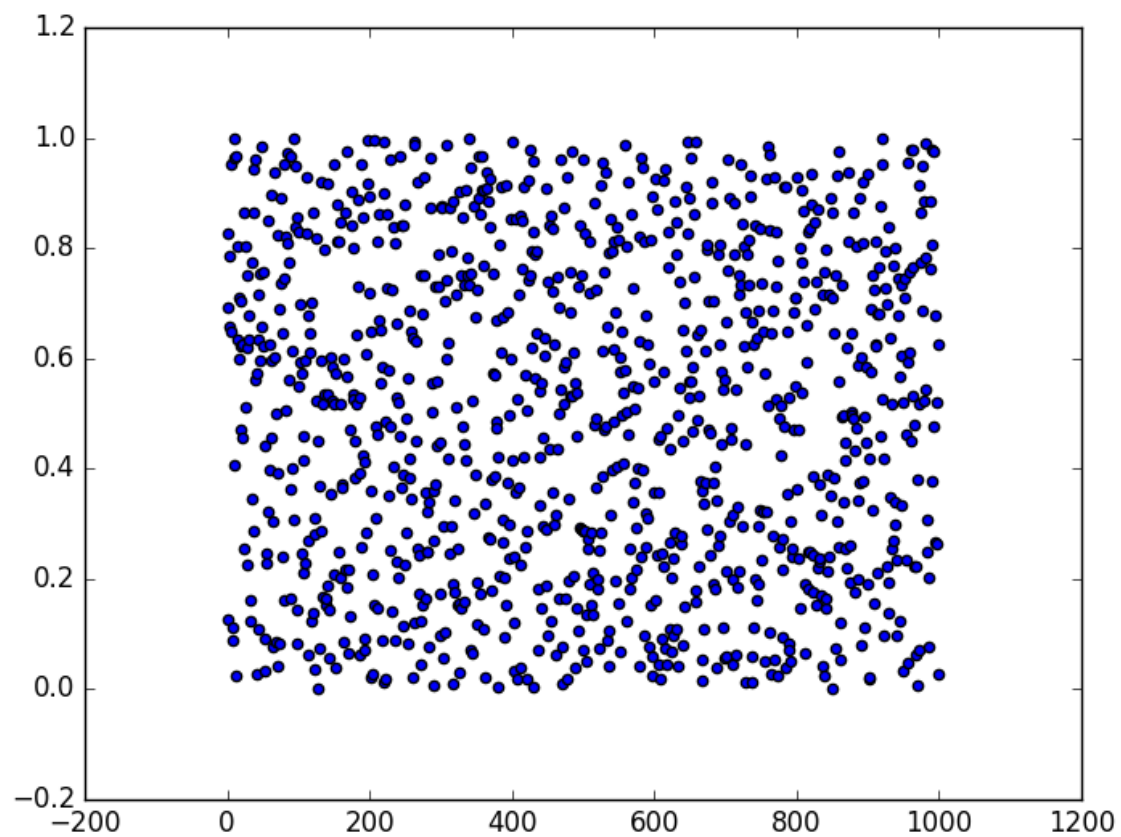
```
random.uniform(0,1)
```

```
random.uniform(0,1)
```

Exercise: plot a distribution of 1000 uniform random numbers in  $[0,1)$

```
rn=np.random.uniform(0,1,1000)
```

```
plt.scatter(range(1000), rn)
```



Exercise: plot a distribution of 1000 random number with Gaussian distribution (hint: use `random.normal`)

Exercise: make a histogram of the two distributions (using `plt.hist`)

..look at the available distributions using `random`?

## MAKING A SPHERE

Let's plot a 2D uniform distribution of 10,000 points with center (Xc,Yc) and size L; for example, (10,10) and L=50

**N=10000**

**R=50**

**XC=10**

**YC=10**

**rnx=np.random.uniform(0,R,N)-R/2+XC**

**rny=np.random.uniform(0,R,N)-R/2+YC**

**plt.scatter(rnx,rny)**

**plt.show()**

How would you extract a circle from this? Hint: use np.where()

For instance:

**x=np.random.uniform(0,1,1000)**

**i=np.where(x<0.5)**

**plt.scatter(range(len(x[i])),x[i])**

**plt.show()**

np.where() is a *very powerful* function! Not that, for giving more conditions, the syntax is the following:

**j=np.where( (x>0.2) & (x<0.5) )**

with the *binary* operators (&, | ...)

**Problem:** in this way, a number of “attempts” are wasted.

Solution: use spherical coordinates and sample  $r$ ,  $\phi$ :

```
r=np.random.uniform(0,1,1000)
```

```
phi=np.random.uniform(0,6.28,1000)
```

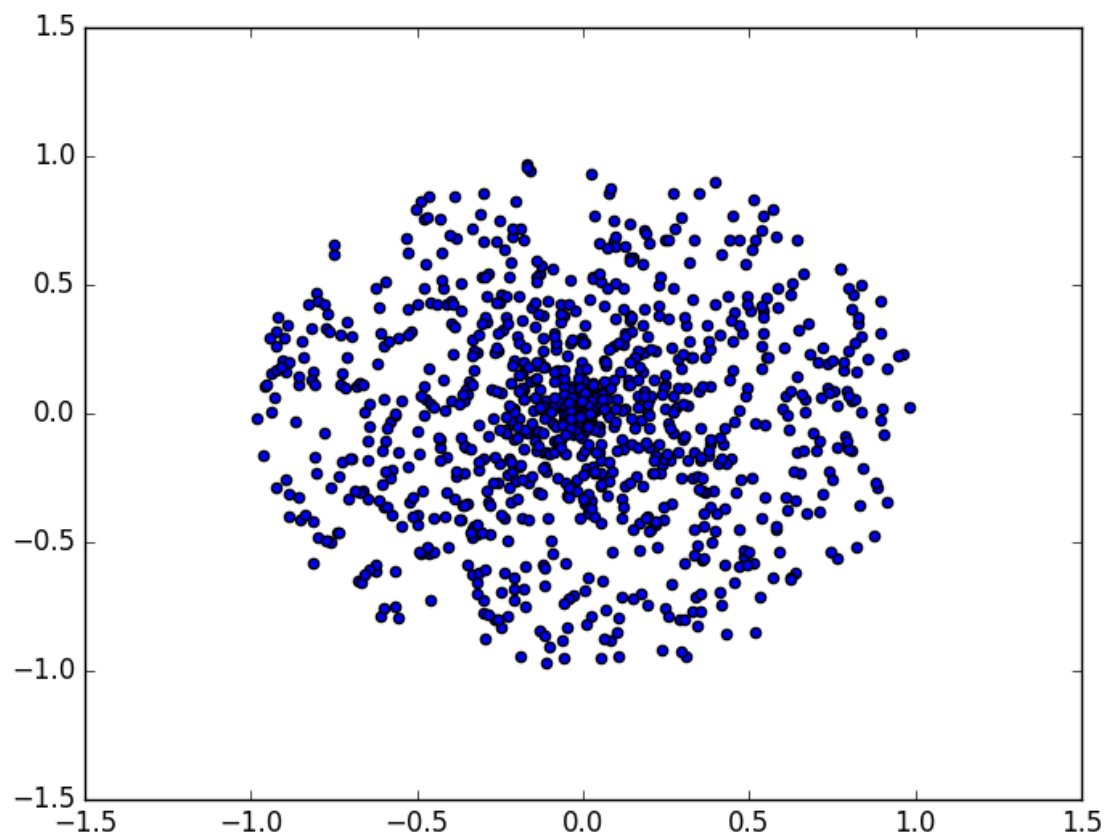
```
rnx=r*np.cos(phi)
```

```
rny=r*np.sin(phi)
```

*but....*

```
plt.scatter(rnx,rny)
```

```
plt.show()
```



and this is *NOT* uniform!

The reason is that in Cartesian coordinates the volume of the circle is

$$\int_0^{R/2} \int_0^{\sqrt{R^2 - 4x^2}} dx dy$$

in polar coordinates is

$$\int_0^{R/2} \int_0^{2\pi} r dr d\varphi$$

thus, in Cartesian coordinates x,y must be uniform; in polar, r dr, d  $\varphi$  !

To sample a *non*-uniform distribution from a uniform one, one must equate their PDF;

if U(u) is the uniform distribution and F(r) the needed one, we need F(r)=U(u)

where r is our radius and u our uniform number. Thus:

$$\int r dr = \int du \quad 1/2 r^2 = u \quad r = 2\sqrt{u}$$

**R=4**

**r=np.sqrt(2.\*np.random.uniform(0,R\*R/2.,1000))**

**phi=np.random.uniform(0,6.28,1000)**

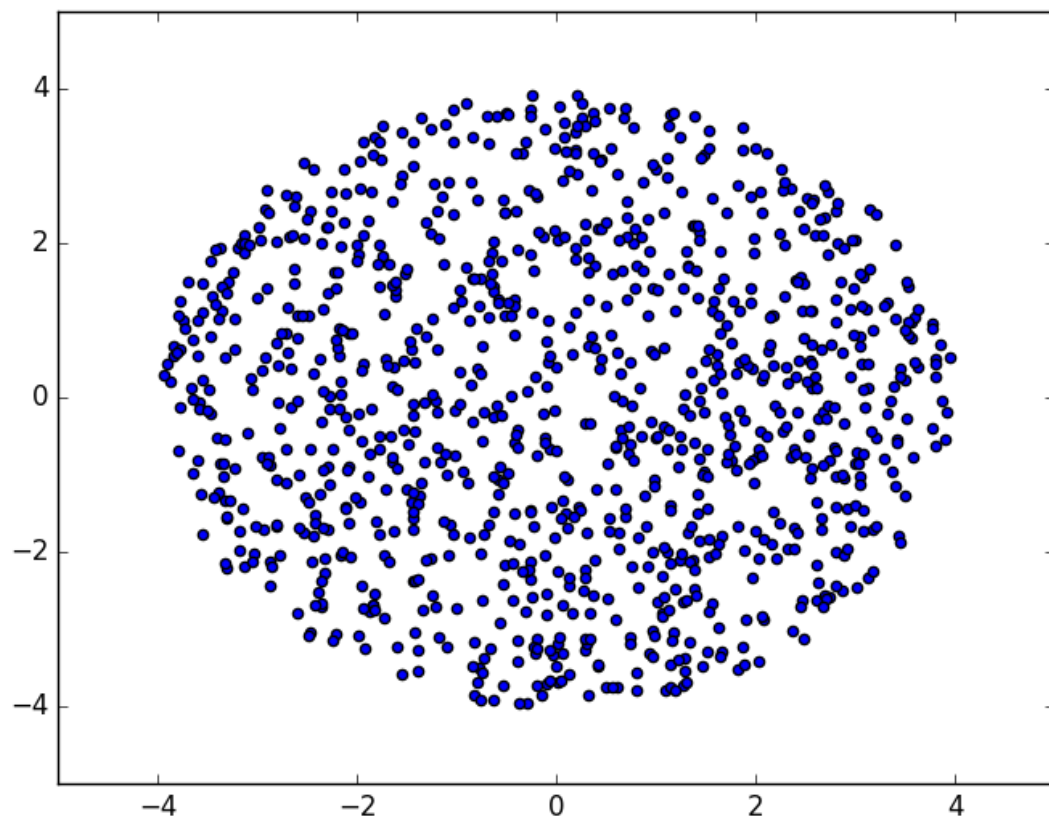
**rnx=r\*np.cos(phi)**

**rny=r\*np.sin(phi)**

**plt.scatter(rnx,rny)**

**plt.show()**

*..note the extremes...*



Exercise: knowing that the volume of a sphere (3D) is

$$\int_0^R dr \int_0^{2\pi} d\varphi \int_{-1}^1 d\cos\vartheta$$

produce a distribution of points uniformly distributed in the sphere and plot the three projection, with 1000, 10000 and 100000 points.

**NEXT: ROUNDOFF ERRORS!**