Hashing

❖ Associative Indexing

Regular array indexing is positional (`[0]`, `[1]`, `[2]`, ...):

- can access items by their position in an array
- but generally don't know position for an item with key K
- we need to search for the item in the collection using K
- search can be linear ($O(n)$) or binary ($O(\log_2 n)$)

An alternative approach to indexing:

- use the key value as an index ... no searching needed
- access data for item with key K as **`array[K]`**

❖ ... Associative Indexing

Difference between positional and associative indexing:

Positional (normal) indexing

	[0]	[1]	[2]	[3]	[4]	
`courses[]`	data about COMP1511	data about COMP1521	data about COMP1531	data about COMP2511	data about COMP2521	.....

`courses[4]` *gives access to COMP2521 data*

Associative indexing

	["COMP1511"]	["COMP1521"]	["COMP1531"]	["COMP2511"]	["COMP2521"]	
`courses[]`	data about COMP1511	data about COMP1521	data about COMP1531	data about COMP2511	data about COMP2521	.....

`courses["COMP2521"]` *gives access to COMP2521 data*

❖ Hashing

Hashing allows us to get close to associative indexing

Ideally, we would like ...

```
courses["COMP3311"] = "Database Systems";
printf("%s\n", courses["COMP3311"]);
```

but C doesn't have non-integer index values.

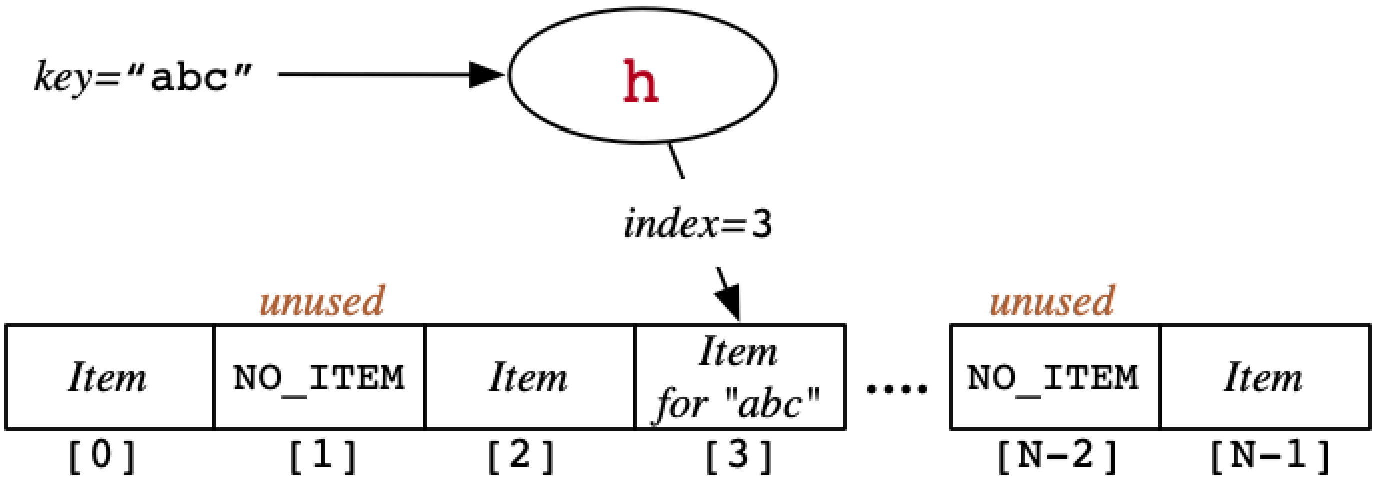
Something almost as good:

```
courses[h("COMP3311")] = "Database Systems";
printf("%s\n", courses[h("COMP3311")]);
```

Hash function **h** converts key to integer and uses that as the index.

❖ ... Hashing

What the **h** (hash) function does



Converts a key value (of any type) into an integer index.

Sounds good ... in practice, not so simple ...

❖ ... Hashing

Reminder: what we'd like ...

```
courses[h("COMP3311")] = "Database Systems";
printf("%s\n", courses[h("COMP3311")]);
```

In practice, we do something like ...

```
key = "COMP3311";
item = {"COMP3311","Database Systems",...};
courses = HashInsert(courses, key, item);
printf("%s\n", HashGet(courses, "COMP3311"));
```

❖ ... Hashing

To use arbitrary values as keys, we need ...

- set of `Key` values *dom(`Key`)*, each key identifies one `Item`
- an array (of size N) to store `Item`s
- a hash function *h()* of type *dom(`Key`)* $\to$ $[0..N\text{-}1]$
  - requirement: if $(x = y)$ then $h(x) = h(y)$
  - requirement: $h(x)$ always returns same value for given x

A problem: array is size N, but *dom(`Key`)* $\gg N$

So, we also need a collision resolution method

- collision = $(x \neq y \ \ but \ \ h(x) = h(y))$

❖ Hash Table ADT

Generalised ADT for a collection of **`Item`**s

Interface:

```
typedef struct CollectionRep *Collection;

Collection newCollection();     // make new empty collection
Item *search(Collection, Key); // find item with key
void insert(Collection, Item); // add item into collection
void delete(Collection, Key);  // drop item with key
```

Implementation:

```
typedef struct CollectionRep {
    ... some data structure to hold multiple Items ...
} CollectionRep;
```

... Hash Table ADT

For hash tables, make one change to "standard" Collection interface:

```
typedef struct HashTabRep *HashTable;
// make new empty table of size N
HashTable newHashTable(int);
// add item into collection
void HashInsert(HashTable, Item);
// find item with key
Item *HashGet(HashTable, Key);
// drop item with key
void HashDelete(HashTable, Key);
// free memory of a HashTable
void dropHashTable(HashTable);
```

i.e. we specify max # elements that can be stored in the collection

❖ ... Hash Table ADT

Example hash table implementation:

```
typedef struct HashTabRep {
   Item **items; // array of (Item *)
   int  N;       // size of array
   int  nitems;  // # Items in array
} HashTabRep;

HashTable newHashTable(int N)
{
   HashTable new = malloc(sizeof(HashTabRep));
   new->items = malloc(N*sizeof(Item *));
   new->N = N;
   new->nitems = 0;
   for (int i = 0; i < N; i++) new->items[i] = NULL;
   return new;
}
```

❖ ... Hash Table ADT

Hash table implementation (cont)

```
void HashInsert(HashTable ht, Item it) {
   int h = hash(key(it), ht->N);
   // assume table slot empty!?
   ht->items[h] = copy(it);
   ht->nitems++;
}
Item *HashGet(HashTable ht, Key k) {
   int h = hash(k, ht->N);
   Item *itp = ht->items[h];
   if (itp != NULL && equal(key(*itp),k))
      return itp;
   else
      return NULL;
}
```

`key()` and **`copy()`** come from **`Item`** type; **`equal()`** from **`Key`** type

❖ ... Hash Table ADT

Hash table implementation (cont)

```
void HashDelete(HashTable ht, Key k) {
   int h = hash(k, ht->N);
   Item *itp = ht->items[h];
   if (itp != NULL && equal(key(*itp),k)) {
      free(itp);
      ht->items[h] = NULL;
      ht->nitems--;
   }
}
void dropHashTable(HashTable ht) {
   for (int i = 0; i < ht->N; i++) {
      if (ht->items[i] != NULL) free(ht->items[i]);
   }
   free(ht);
}
```

❖ Hash Functions

Characteristics of hash functions:

- converts **`Key`** value to index value [0..N-1]
- deterministic (key value *k* always maps to same value)
- use **`mod`** function to map hash value to index value
- spread key values **uniformly** over address range
  (assumes that keys themselves are uniformly distributed)
- as much as possible, $h(k) \neq h(j)$ if $j \neq k$
- cost of computing hash function must be cheap

❖ ... Hash Functions

Basic mechanism of hash functions:

```
int hash(Key key, int N)
{
   int val = convert key to 32-bit int;
   return val % N;
}
```

If keys are **int**s, conversion is easy (identity function)

How to convert keys which are strings? (e.g. **"COMP1927"** or **"John"**)

Definitely prefer that *hash*(**"cat"**,N) ≠ *hash*(**"dog"**,N)

Prefer that *hash*(**"cat"**,N) ≠ *hash*(**"act"**,N) ≠ *hash*(**"tac"**,N)

❖ ... Hash Functions

Universal hashing uses entire key, with position info:

```
int hash(char *key, int N)
{
   int h = 0, a = 31415, b = 21783;
   char *c;
   for (c = key; *c != '\0'; c++) {
      a = a*b % (N-1);
      h = (a * h + *c) % N;
   }
   return h;
}
```

Has some similarities with RNG.   Aim: "spread" hash values over [0..N-1]

❖ ... Hash Functions

A real hash function (from PostgreSQL DBMS):

```
hash_any(unsigned char *k, register int keylen, int N)
{
   register uint32 a, b, c, len;
   // set up internal state
   len = keylen;
   a = b = 0x9e3779b9;
   c = 3923095;
   // handle most of the key, in 12-char chunks
   while (len >= 12) {
      a += (k[0] + (k[1] << 8) + (k[2] << 16) + (k[3] << 24));
      b += (k[4] + (k[5] << 8) + (k[6] << 16) + (k[7] << 24));
      c += (k[8] + (k[9] << 8) + (k[10] << 16) + (k[11] << 24));
      mix(a, b, c);
      k += 12; len -= 12;
   }
   // collect any data from remaining bytes into a,b,c
   mix(a, b, c);
   return c % N;
}
```

❖ ... Hash Functions

Where **`mix`** is defined as:

```
#define mix(a,b,c) \
{ \
  a -= b; a -= c; a ^= (c>>13); \
  b -= c; b -= a; b ^= (a<<8);  \
  c -= a; c -= b; c ^= (b>>13); \
  a -= b; a -= c; a ^= (c>>12); \
  b -= c; b -= a; b ^= (a<<16); \
  c -= a; c -= b; c ^= (b>>5);  \
  a -= b; a -= c; a ^= (c>>3);  \
  b -= c; b -= a; b ^= (a<<10); \
  c -= a; c -= b; c ^= (b>>15); \
}
```

i.e. scrambles all of the bits from the bytes of the key value

❖ Problems with Hashing

In ideal scenarios, search cost in hash table is *O(1)*.

Problems with hashing:

- hash function relies on size of array ($\Rightarrow$ can't expand)
  - changing size of array effectively changes the hash function
  - if change array size, then need to re-insert all **`Items`**
- items are stored in (effectively) random order
- if size(KeySpace) $\gg$ size(IndexSpace), collisions inevitable
  - collision: k $\neq$ j && hash(k,N) = hash(j,N)
- if **`nitems`** > **`nslots`**, collisions inevitable

Produced: 9 Jul 2020