Abstract Data Types

❖ Abstract Data Types

A data type is ...

- a set of values (atomic or structured values)
- a collection of operations on those values

An abstract data type is ...

- an approach to implementing data types
- separates interface from implementation
- users of the ADT see only the interface
- builders of the ADT provide an implementation

E.g. do you know what a (`FILE *`) looks like? do you want/need to know?

❖ DTs, ADTs, GADTs

We want to distinguish ...

DT = (non-abstract) data type (e.g. C strings)

- internals of data structures are visible (e.g. `char s[10];`)

ADT = abstract data type (e.g. C files)

- can have multiple instances (e.g. `Set a, b, c;`)

GADT = generic (polymorphic) abstract data type

- can have multiple instances (e.g. `Set<int> a, b, c;`)
- can have multiple types (e.g. `Set<int> a; Set<char> b;`)
- not available natively in the C language

❖ Interface/Implementation

ADT interface provides

- a user-view of the data structure (e.g. **`FILE*`**)
- function signatures (prototypes) for all operations
- semantics of operations (via documentation)
- a contract between ADT and its clients

ADT implementation gives

- concrete definition of the data structures
- definition of functions for all operations

❖ Collections

Many of the ADTs we deal with ...

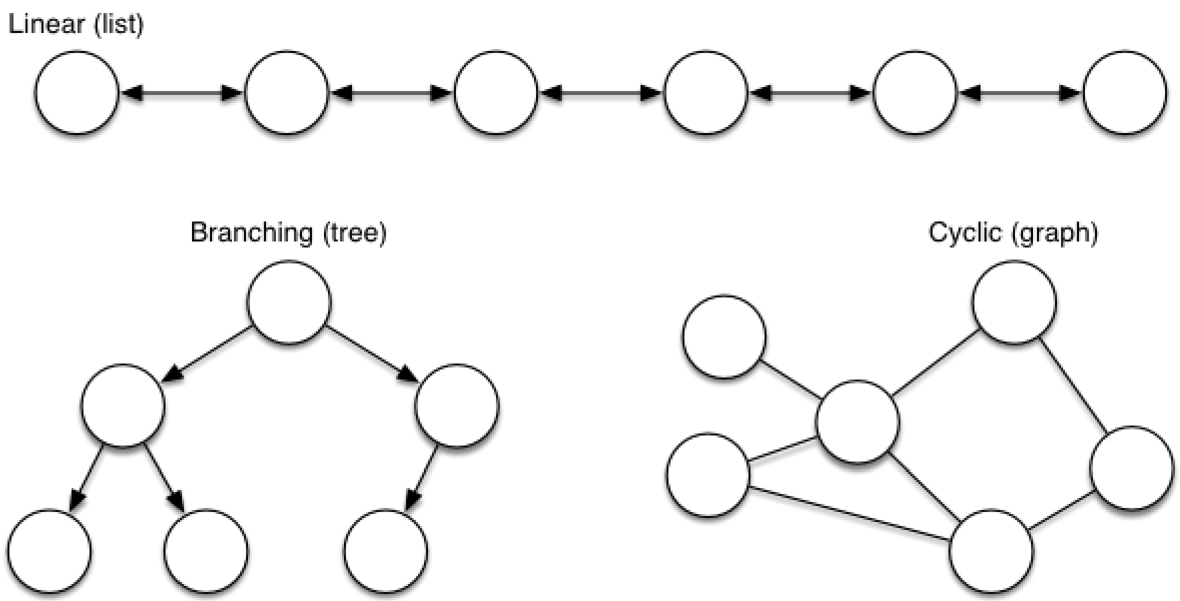
- consist of a collection of items
- where each item may be a simple type or an ADT
- and items often have a key (to identify them)

Collections may be categorised by ...

- structure: linear (list), branching (tree), cyclic (graph)
- usage: set, matrix, stack, queue, search-tree, dictionary, ...

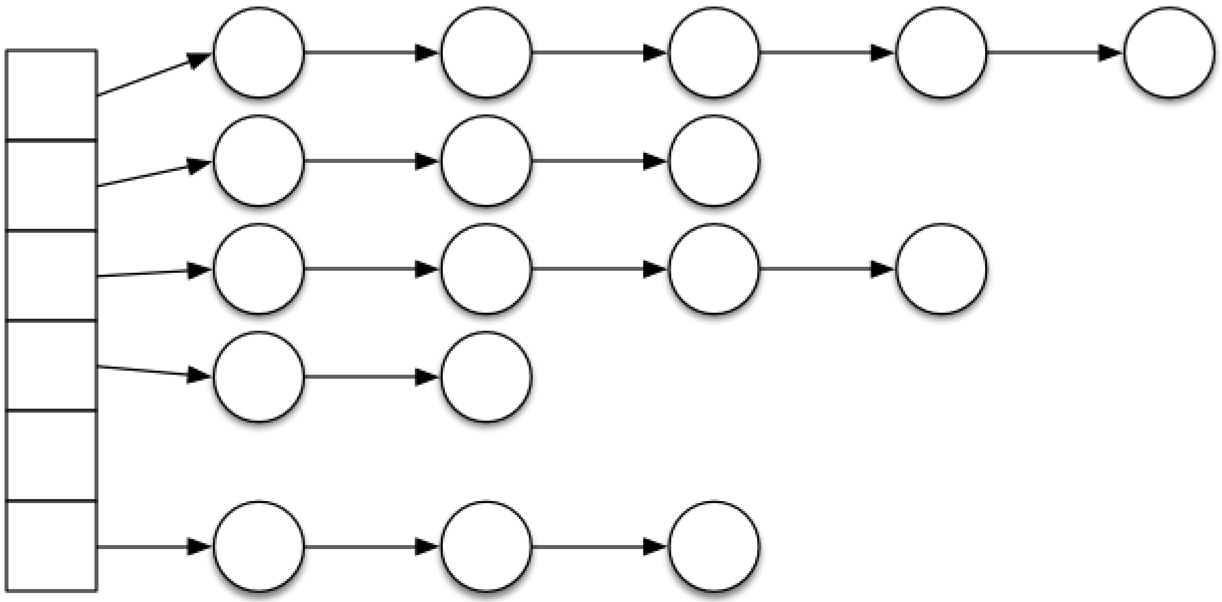
❖ ... Collections

Collection structures:



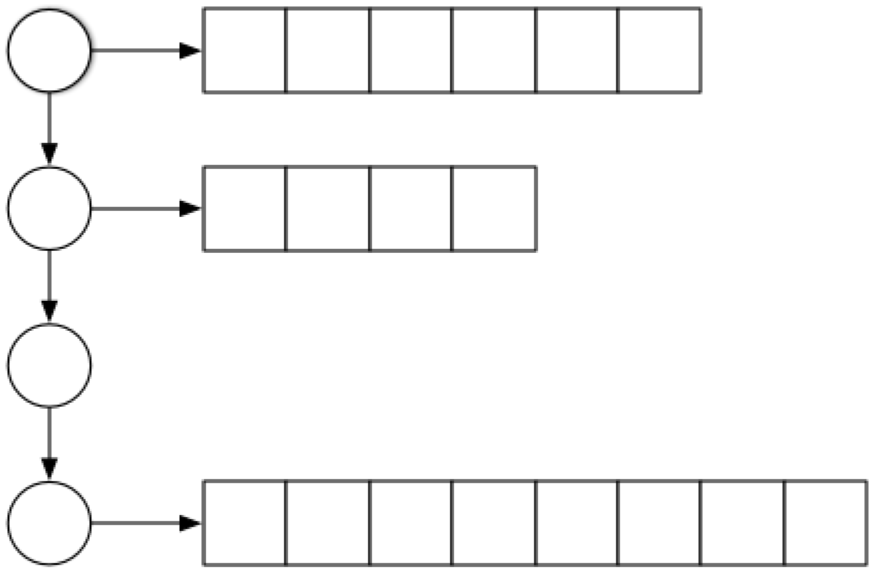
❖ ... Collections

Or even a hybrid structure like ...



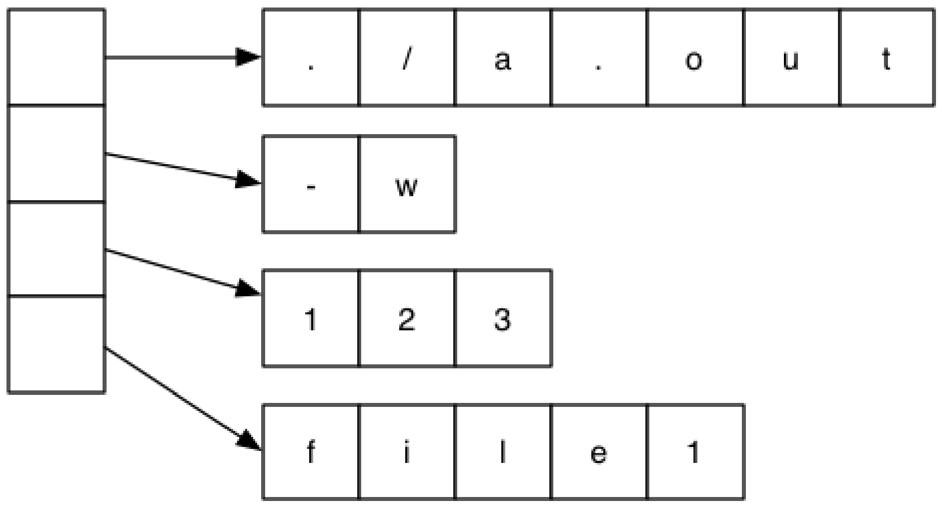
❖ ... Collections

Or this ...



❖ ... Collections

Or this ...



❖ ... Collections

Typical operations on collections

- create an empty collection
- insert one item into the collection
- remove one item from the collection
- find an item in the collection
- check properties of the collection (size,empty?)
- drop the entire collection
- display the collection

❖ Example: Set ADT

Set data type: collection of unique integer values.

"Book-keeping" operations:

- `Set newSet()` ... create new empty set
- `void dropSet(Set)` ... free memory used by set
- `void showSet(Set)` ... display as `{1,2,3...}`

Assignment operations:

- `void readSet(FILE*,Set)` ... read+insert set values
- `Set SetCopy(Set)` ... make a copy of a set

❖ ... Example: Set ADT

Data-type operations:

- `void SetInsert(Set,int)` ... add number into set
- `void SetDelete(Set,int)` ... remove number from set
- `int SetMember(Set,int)` ... set membership test
- `Set SetUnion(Set,Set)` ... union
- `Set SetIntersect(Set,Set)` ... intersection
- `int SetCard(Set)` ... cardinality (#elements)

Note: union and intersection return a newly-created `Set`

❖ Set ADT Interface

```
// Set.h ... interface to Set ADT

#ifndef SET_H
#define SET_H

#include <stdio.h>
#include <stdbool.h>

typedef struct SetRep *Set;

Set newSet();                   // create new empty set
void dropSet(Set);              // free memory used by set
Set SetCopy(Set);               // make a copy of a set
void SetInsert(Set,int);        // add value into set
void SetDelete(Set,int);        // remove value from set
bool SetMember(Set,int);        // set membership
Set SetUnion(Set,Set);          // union
Set SetIntersect(Set,Set);      // intersection
int SetCard(Set);               // cardinality
void showSet(Set);              // display set on stdout
void readSet(FILE *, Set);      // read+insert set values
```

```
#endif
```

❖ ... Set ADT Interface

Example set client: set of small odd numbers

```
#include "Set.h"
...
Set s = newSet();
for (int i = 1; i < 26; i += 2)
    SetInsert(s,i);
showSet(s); putchar('\n');
```

Outputs:

```
{1,3,5,7,9,11,13,15,17,19,21,23,25}
```

❖ Set Applications

Example: eliminating duplicates

```
#include "Set.h"
...
// scan a list of items in a file
int item;
Set seenItems = newSet();
FILE *in = fopen(FileName,"r");
while (fscanf(in, "%d", &item) == 1) {
   if (!SetMember(seenItems, item)) {
      SetInsert(seenItems, item);
      process item;
   }
}
fclose(in);
```

❖ Set ADT Pre/Post-conditions

Each **`Set`** operation has well-defined semantics.

Express these semantics in detail via statements of:

- what conditions need to hold at start of function
- what will hold at end of function (assuming successful)

Could implement condition-checking via **`assert()`**s

But only during the development/testing phase

- **`assert()`** does not provide useful error-handling

At the very least, implement as comments at start of functions.

❖ ... Set ADT Pre/Post-conditions

If **x** is a variable of type **T**, where **T** is an ADT

- $\mathrm{ptr}(x)$ is the pointer stored in **x**
- $\mathrm{val}(x)$ is the abstract value represented by ***x**
- $\mathrm{valid}(T, x)$ indicates that
    - the collection of values in ***x**
      satisfies all constraints on "correct" values of type **T**
- x' is an updated version of x (note: $\mathrm{ptr}(x') == \mathrm{ptr}(x)$)
- res is the value returned by a function

Can also use math/logic notation as used in pseudocode.

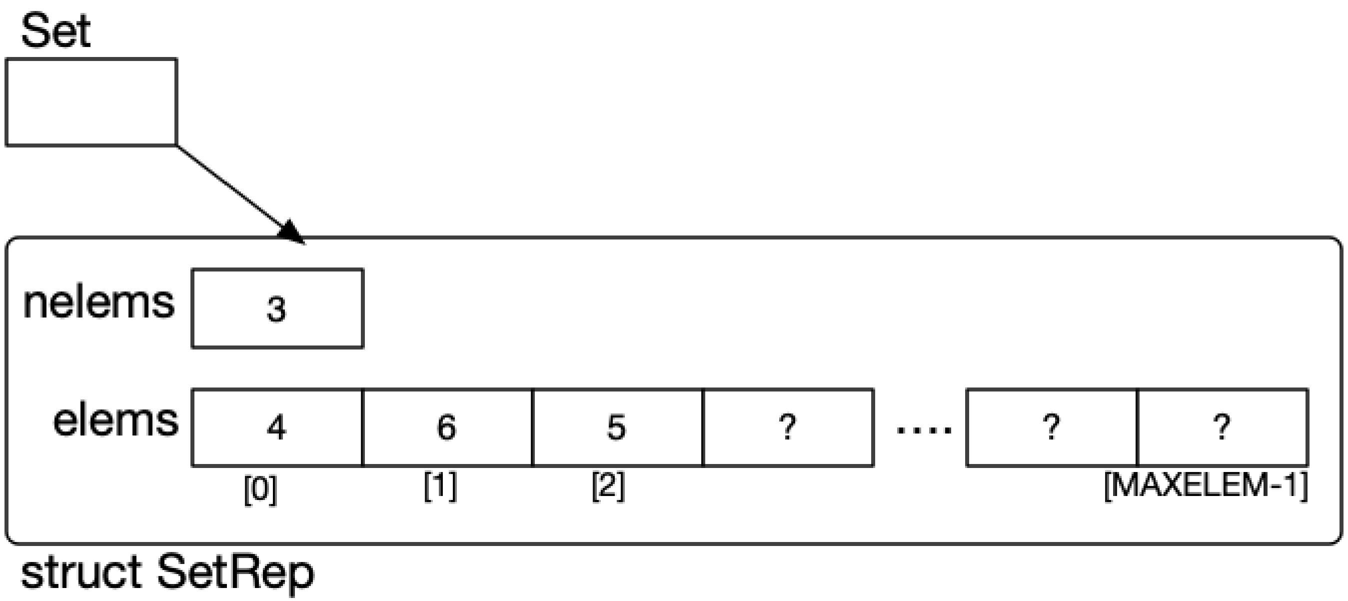
❖ ... Set ADT Pre/Post-conditions

Examples of defining pre-/post-conditions:

```
// pre:  true
// post: valid(Set,res) and res = {}
Set newSet() { ... }

// pre:  valid(Set,s) and valid(int n)
// post: n ∈ s'
void SetInsert(Set s, int n) { ... }

// pre:  valid(Set,s1) and valid(Set,s2)
// post: ∀ n ∈ res, n ∈ s1 or n ∈ s2
Set SetUnion(Set s1, Set s2) { ... }

// pre:  valid(Set,s)
// post: res = |s|
int SetCard(Set s)  { ... }
```

❖ Sets as Unsorted Arrays

Concrete data structure:



❖ ... Sets as Unsorted Arrays

Concrete data structure (in C):

```
#define MAXELEMS 1000

// concrete data structure
struct SetRep {
    int elems[MAXELEMS];
    int nelems;
};
```

Need to set upper bound on number of elements

Could do statically (as above) or dynamically

```
Set newSet(int maxElems) { ... }
```

❖ ... Sets as Unsorted Arrays

Set creation:

```
// create new empty set
Set newSet()
{
   Set s = malloc(sizeof(struct SetRep));
   if (s == NULL) {
      fprintf(stderr, "Insufficient memory\n");
      exit(EXIT_FAILURE);
   }
   s->nelems = 0;
   // assert(isValid(s));
   return s;
}
```

❖ ... Sets as Unsorted Arrays

Checking membership:

```
// set membership test
int SetMember(Set s, int n)
{
   // assert(isValid(s));
   int i;
   for (i = 0; i < s->nelems; i++)
      if (s->elems[i] == n) return TRUE;
   return FALSE;
}
```

❖ ... Sets as Unsorted Arrays

Costs for set operations on unsorted array:

- card: read from struct; constant cost *O(1)*
- member: scan list from start; linear cost *O(n)*
- insert: duplicate check, add at end; linear cost *O(n)*
- delete: find, copy last into gap; linear cost *O(n)*
- union: copy s1, insert each item from s2; quadratic cost *O(nm)*
- intersect: scan for each item in s1; quadratic cost *O(nm)*

Assuming: s1 has *n* items, s2 has *m* items

❖ Sets as Sorted Arrays

Same data structure as for unsorted array.

Differences in

- membership test ... can use binary search
- insertion ... binary search and then shift up and insert
- deletion ... binary search and then shift down

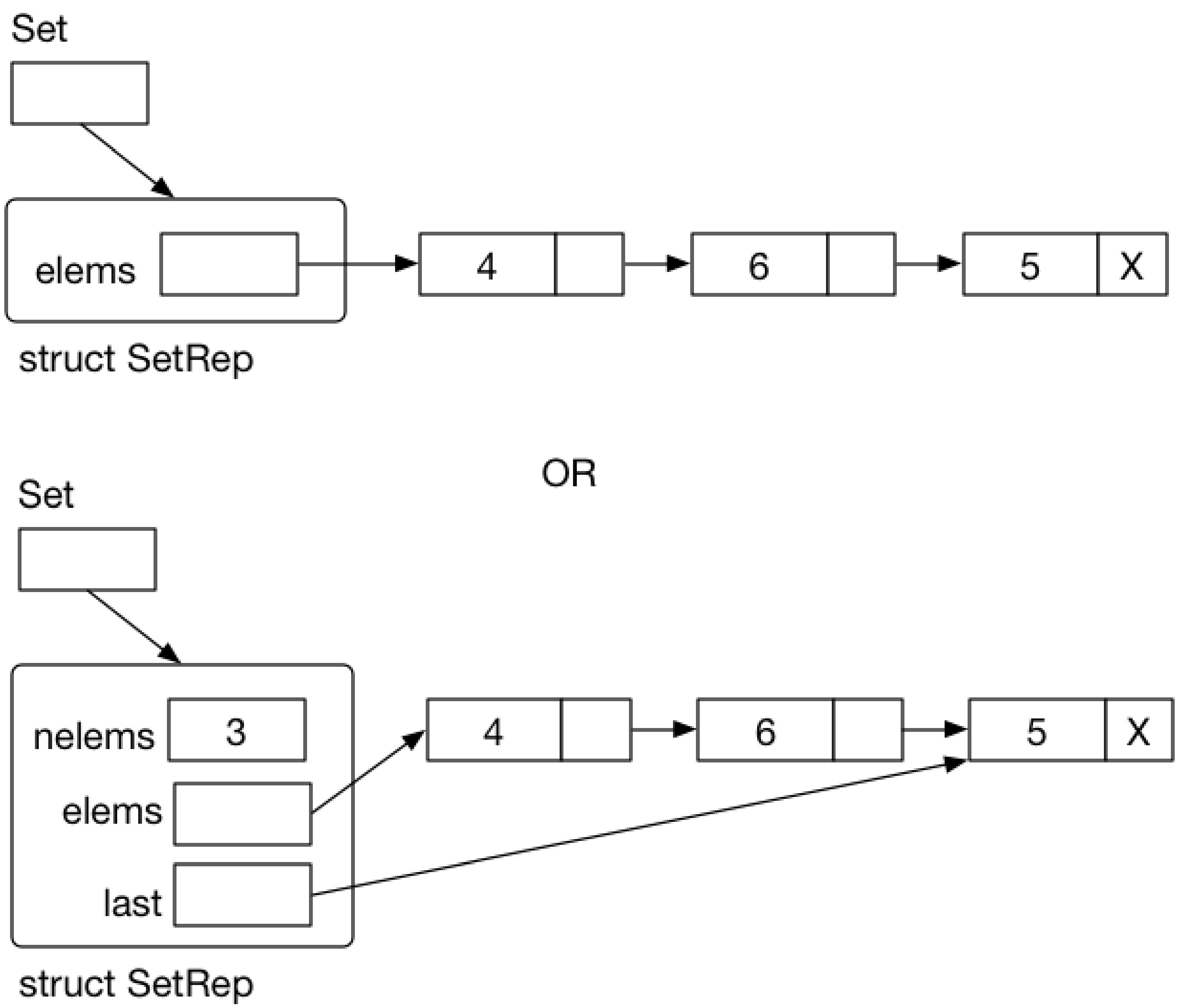
❖ ... Sets as Sorted Arrays

Costs for set operations on sorted array:

- card: read from struct; *O(1)*
- member: binary search; *O(log n)*
- insert: find, shift up, insert; *O(n)*
- delete: find, shift down; *O(n)*
- union: merge = scan s1, scan s2; *O(n)* (technically *O(n+m)*)
- intersect: merge = scan s1, scan s2; *O(n)* (technically *O(n+m)*)

❖ Sets as Linked Lists

Concrete data structure:



... Sets as Linked Lists

Concrete data structure (in C):

```
typedef struct Node {
   int  value;
   struct Node *next;
} Node;

struct SetRep {
   Node *elems;  // pointer to first node
   Node *last;   // pointer to last node
   int nelems;   // number of nodes
};
```

❖ ... Sets as Linked Lists

Set creation:

```
// create new empty set
Set newSet()
{
   Set s = malloc(sizeof(struct SetRep));
   if (s == NULL) {...}
   s->nelems = 0;
   s->elems = s->last = NULL;
   return s;
}
```

❖ ... Sets as Linked Lists

Checking membership:

```
// set membership test
int SetMember(Set s, int n)
{
   // assert(isValid(s));
   Node *cur = s->elems;
   while (cur != NULL) {
      if (cur->value == n) return true;
      cur = cur->next;
   }
   return false;
}
```

❖ ... Sets as Linked Lists

Costs for set operations on linked list:

- insert: duplicate check, insert at head; *O(n)*
- delete: find, unlink; *O(n)*
- member: linear search; *O(n)*
- card: lookup; *O(1)*
- union: copy s1, insert each item from s2; *O(nm)*
- intersect: scan for each item in s1; *O(nm)*

Assume n = size of $s1$, m = size of $s2$

If we don't have **`nelems`**, card becomes *O(n)*

❖ Sets as Bit-strings

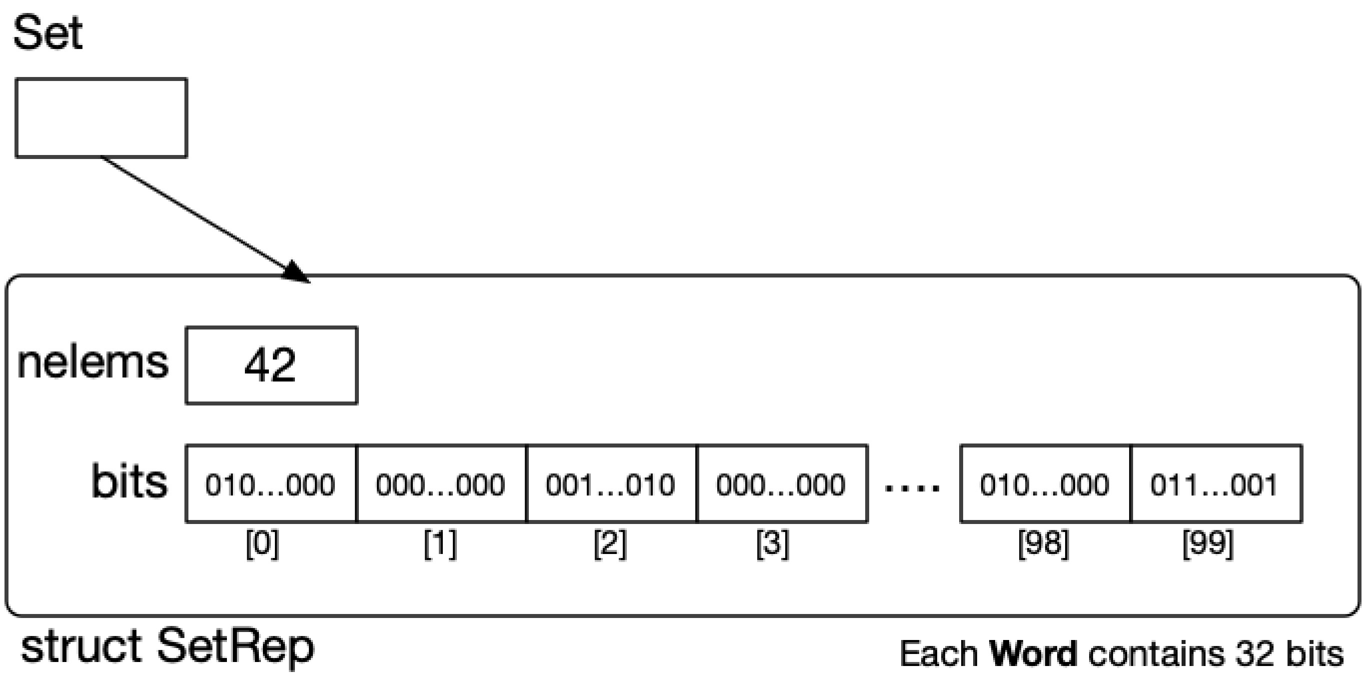
Set is a very long bit-string, typically an array of **`words`**.

Restrict possible values that can be stored in the Set

- typically restricted to 0..N-1, (where N%32 == 0)
- represent each value by position in large array of bits
- insertion means set a bit to 1 (**`bit|1`**)
- deletion means set a bit to 0 (**`bit&0`**)
- bit position for value **`i`** is easy to compute

❖ ... Sets as Bit-strings

Concrete data structure:



❖ ... Sets as Bit-strings

Concrete data structure (in C):

```
#define NBITS 1024
#define NWORDS (NBITS/32)
typedef unsigned int Word;
typedef Word Bits[NWORDS];

struct SetRep {
   int nelems;
   Bits bits;  // Word bits[NWORDS]
};
```

`Set`s defined like this can hold values in range 0..1023

❖ ... Sets as Bit-strings

Implementation as bit-strings requires extra functions:

- **`getBit(Bits b, int i)`** ... get value of i'th bit, 0 or 1
- **`setBit(Bits b, int i)`** ... ensure i'th bit is set to 1
- **`unsetBit(Bits b, int i)`** ... ensure i'th bit is set to 0

Can be implemented efficiently, e.g.

```
getBit(Bits b, int i) {
   int whichWord = i / 32;
   int whichBit  = i % 32;
   Word mask = (1 << whichBit)
   return (b[whichWord] & mask) >> whichBit;
}
```

❖ Setting and unsetting bits

Setting and unsetting bits by **&** and **|**

```
unsigned char x, y, z;

x 00000111   y 10000001   z = x & y;              z 00000001

x 00000111   y 10000001   z = x | y;              z 10000111

x 00000011                z = x & 0xFF;           z 00000011

x 00000001                z = x | 0xFF;           z 11111111

x 00000000                z = x | (1 << 2);       z 00000100

x 11111111                z = x & ~(1 << 2);      z 11111011
```

The last two switch on/off bit 2

❖ ... Setting and unsetting bits

Powers of two by bit-shifting - don't use **`pow(...)`** from **`math.h`**!

x	operation	x'
00000001	`x = x << 1`	00000010
00000001	`x = x << 2`	00000100
00000001	`x = x << 99`	00000000
00000010	`x = x >> 1`	00000001
00011000	`x = x >> 2`	00000110
11111111	`x = x >> 99`	00000000

`x = x << n`

is

$x = x * 2^n$

assume:

```
unsigned char x;
```

`x = x >> n`

is

$x = x / 2^n$

❖ Performance of Set Implementations

Performance comparison:

Data Structure	insert	delete	member	∪, ∩	storage
unsorted array	*O(n)*	*O(n)*	*O(n)*	*O(n.m)*	*O(N)*
sorted array	*O(n)*	*O(n)*	$O(\log_2 n)$	*O(n+m)*	*O(N)*
unsorted linked list	*O(n)*	*O(n)*	*O(n)*	*O(n.m)*	*O(n)*
sorted linked list	*O(n)*	*O(n)*	*O(n)*	*O(n+m)*	*O(n)*
bit-maps	*O(1)*	*O(1)*	*O(1)*	*O(N)*	*O(N)*

n,m = #elems, *N* = max #elems,

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