Project Report – Synth Filter Design

Name	Md Golam Tanvir Zim
Student No	180715305
Course	Real Time DSP (ECS732P)

Project Description

The objective of the project was to design a Resonant Lowpass Filter, which passes low frequencies and attenuates or blocks high frequeny components. The term resonant refers to high Q-factor which means there is an audible peak in the spectrum around the cutoff frequency point.

Design Process

The Resonant Lowpass filter is a 4^{th} order IIR filter, in which two 2^{nd} order filters are cascaded in series. The filter is a user-adjustable filter, in which the cutoff frequency and the Q factor can be varied using potentiometer for tuning.

The continious-time transfer function for the 2^{nd} order filter is as follows:

$$H(s) = \frac{\omega^2}{s^2 + 2 * \frac{\omega}{Q} * s + \omega^2}$$

Here, ω = Angular CutOff Frequency = 2*pi*f_c

Q = Quality Factor, which determines the resonance.

Using bilinear transformation, the continious time transfer function is converted into discrete-time transfer function which is as follows:

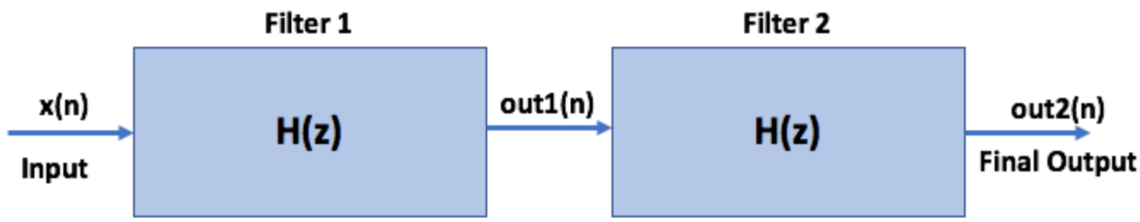
$$H(z) = \frac{b_0 + b_1 * z^{-1} + b_2 * z^{-2}}{a_0 + a_1 * z^{-1} + a_2 * z^{-2}}$$

The values of coefficients are dependant on three variable, sampleRate, cutoff frequeny and Q-factor. The derivation from H(s) to H(z) and calculation of the six co-efficients will be described in another section.

After determining the discrete-time transfer function H(z) = Y(z)/X(z), the normalized difference equation in discrete-time domain is realized as follows:

y (n) = - a1 * y(n-1) - a2 * y(n-2) + b0 * x(n) + b1 * x(n-1) + b2 * x(n-2)

This is the difference equation for 2nd order filter. The 4th order filter is realized by cascading two 2nd order filters in series in the following way:



So, the following two differecne equations were implemeted to get the 4th order filter output:

out1(n) = - a1 *out1(n-1) – a2 * out1(n-2) + b0 * x(n) + b1 * x(n-1) + b2 * x(n-2)

out2(n) = - a1 *out2(n-1) – a2 * out2(n-2) + b0 * out1(n) + b1 *out1(n-1) + b2 * out1(n-2)

The input was taken initially via generating a sawtooth wave. Then the system was tasted with a sample wav file. Finally, the input was taken via audio input port of Bela Kit using `audioRead()` function. After calculating the co-efficients in render functon and filtering the input, the output was obtained by writting to the audio output of Bela Kit via `audioWrite()` function.

Deriving the Transfer Function

The continious-time transfer fucntion (1) is as follows:

$$H(s) = \frac{\omega^2}{s^2 + 2 * \frac{\omega}{Q} * s + \omega^2}$$

Using bilinear transformation, $s = \frac{1}{k} * \frac{z-1}{z+1}$ [k = Sampling Period/2], equation (1) yields,

$$\Rightarrow H(z) = \frac{k^2 * \omega^2 * (z+1)^2}{(z-1)^2 + \left(\omega * \frac{k}{Q}\right) * (z-1) * (z+1) + k^2 * \omega^2 * (z+1)^2}$$

$$\Rightarrow H(z) = \frac{z^2 * k^2 * \omega^2 + z * 2 * k^2 * \omega^2 + (k^2 * \omega^2)}{z^2 * \left(1 + w * \frac{k}{Q} + k^2 * \omega^2\right) + z * (-2 + 2 * k^2 * \omega^2) + \left(1 - w * \frac{k}{Q} + k^2 * \omega^2\right)}$$

$$\Rightarrow H(z) = \frac{b_0 + b_1 * z^{-1} + b_2 * z^{-2}}{a_0 + a_1 * z^{-1} + a_2 * z^{-2}}$$

After normalization (dividing by a_0),

$b_0 = k^2 * w^2$ / denominator;

$b_1 = 2 * b_0$;

$b_2 = b_0$;

$a_0 = 1$;

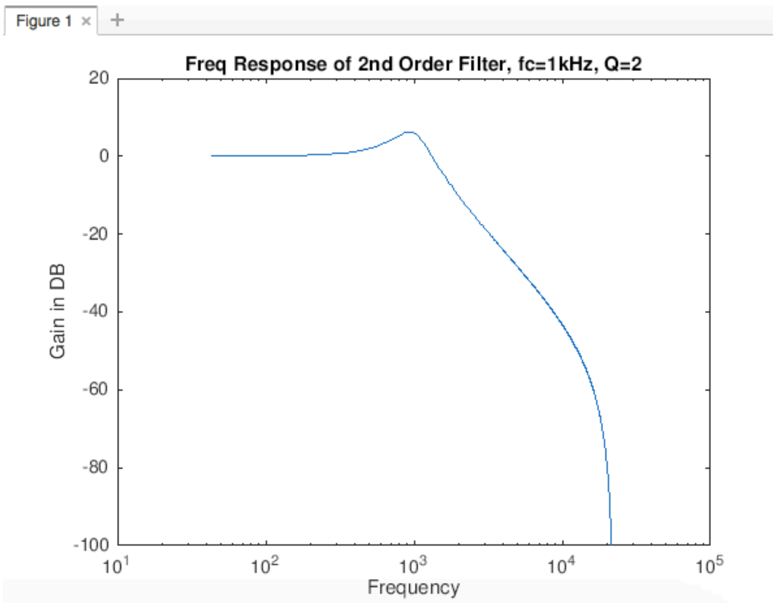
$a_1 = 2 * (k^2 * w^2 – 1)$ / denominator;

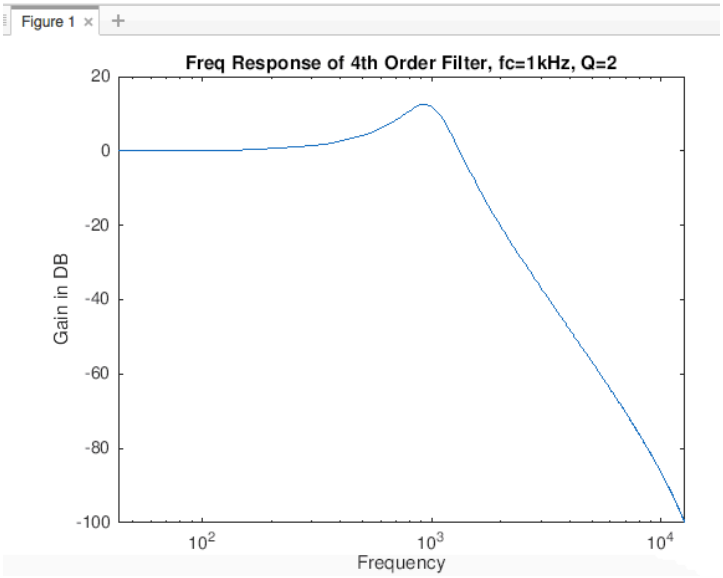
$a_2 = (1 – w*k/Q + k^2 * w^2)$ / denominator;

Here, denominator = $1 + w*k/Q + k^2 * w^2$;

Using this co-efficients, the Frequency Response of the filter were plotted in Matlab and it yielded the following results:

For Cutoff Frequency = 1000Hz, Q = 2 and Sampling Rate = 44100Hz, following frequency responses were obtained:





Implementation of the Design

Controlling the Cutoff Frequency and Q-factor

In the design, initially the Cutoff Frequency and Q-factor was taken as input using Bela GUI. Later two potentiometer circuit were connected to take two analog input for controlling these two variables. The analog reading range (0 ~ 1) was mapped into the desired ranges using the `map()` function.

The range of Cutoff Frequency = 100 ~ 4000 Hz, implemented in logarithmic scale.

The range of Q = 0.5 ~ 10, implemented in linear scale.

Input

The input to the system was 3.5 mm audio input jack. The bela kit was connected to pc, signal genarator and it succesfully took the input.

Storing the variables of previous inputs and outputs

The present output of the filter is dependant on previous outputs, present and previous inputs. So, the following sequence was used to store the data after each rendering.

x(n-2) = x(n-1)

x(n) = x(n)

out1(n-2) = out1(n-1)

out1(n-1) = out1(n-1)

out2(n-2) = out2(n-1)

out2(n-2) = out2(n-1)

Testing and Verification

Due to difficulty in using the oscilloscope, accurate measurement of the gain and cutoff frequency could not be done. To test the filter, tone of different frequencies were used and using headphone the output was analyzed. Findings are as follows:

- Increasing the input frequency resulted the output to be attenuated after certain extent, which indicates the filter is a low pass filter.
- Increasing the cutoff frequency resulted those attenuated frequencies to be audible, which indicates the controlling the cutoff frequency did work properly.
- Increasing the Q factor did produce resonance clearly, so the Q-factor had the desired response in the filter.