

Problem Statement 1:

A test is conducted which is consisting of 20 MCQs (multiple choices questions) with every MCQ having its four options out of which only one is correct. Determine the probability that a person undertaking that test has answered exactly 5 questions wrong.

Solution 1: Let X be the random variable representing number of questions answered wrong in a given MCQs test

$X \sim \text{Bin}(20, 0.75)$, therefore the probability is given by

$$\begin{aligned} P(X = 5) &= \binom{n}{x} * p^x * (1 - p)^{n-x} \\ &= \binom{20}{5} * 0.75^5 * (1 - 0.75)^{20-5} \\ &= 15504 * 0.2373 * (0.25)^{15} = 0.00000342 \end{aligned}$$

Problem Statement 2:

A die marked A to E is rolled 50 times. Find the probability of getting a "D" exactly 5 times.

Solution 2: Let X be the random variable representing number of questions answered wrong in a given MCQs test

$X \sim \text{Bin}(50, 0.2)$, therefore since n is large, the distribution is approximated to the normal distribution $X \sim N(10, 8)$, (with continuity correction)

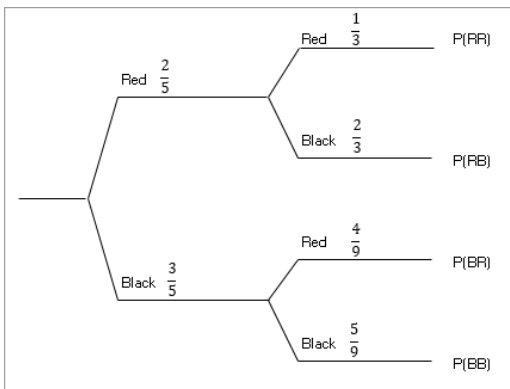
$$\begin{aligned} P(X = 5) &= P\left(Z = \frac{x - \mu}{\sigma}\right) \\ &= P\left(\frac{4.5 - 10}{\sqrt{8}} \leq Z \leq \frac{5.5 - 10}{\sqrt{8}}\right) = 0.0299 \end{aligned}$$

Problem Statement 3:

Two balls are drawn at random in succession without replacement from an urn containing 4 red balls and 6 black balls. Find the probabilities of all the possible outcomes.

Solution 3:

These are independent events, below is a tree diagram to represent the scenario



Outcome 1: $P(\text{two red balls}) = \frac{2}{5} \times \frac{1}{3} = \frac{2}{15}$

Outcome 2: $P(\text{red ball first and black ball second})$

$$= \frac{2}{5} \times \frac{2}{3} = \frac{4}{15}$$

Outcome 3: $P(\text{black ball first and red ball second})$

$$= \frac{3}{5} \times \frac{4}{9} = \frac{4}{15}$$

Outcome 4: $P(\text{two black balls}) = \frac{3}{5} \times \frac{5}{9} = \frac{1}{3}$