

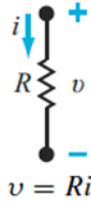
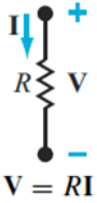
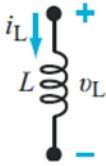
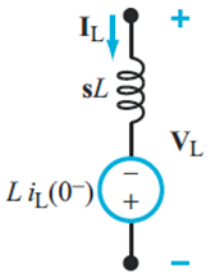
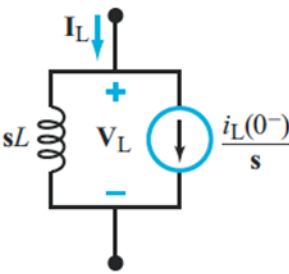
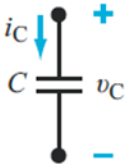
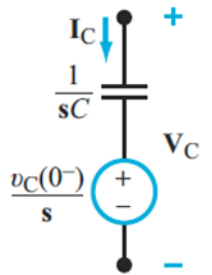
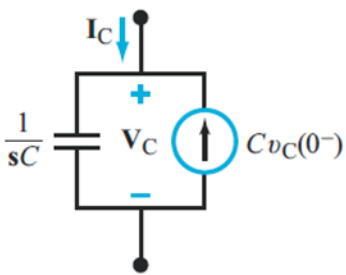
Table 3-2: Examples of Laplace transform pairs. Note that $x(t) = 0$ for $t < 0^-$ and $T \geq 0$.

Laplace Transform Pairs		
	$x(t)$	$X(s) = \mathcal{L}[x(t)]$
1	$\delta(t)$	$\longleftrightarrow 1$
1a	$\delta(t - T)$	$\longleftrightarrow e^{-Ts}$
2	$u(t)$	$\longleftrightarrow \frac{1}{s}$
2a	$u(t - T)$	$\longleftrightarrow \frac{e^{-Ts}}{s}$
3	$e^{-at} u(t)$	$\longleftrightarrow \frac{1}{s + a}$
3a	$e^{-a(t-T)} u(t - T)$	$\longleftrightarrow \frac{e^{-Ts}}{s + a}$
4	$t u(t)$	$\longleftrightarrow \frac{1}{s^2}$
4a	$(t - T) u(t - T)$	$\longleftrightarrow \frac{e^{-Ts}}{s^2}$
5	$t^2 u(t)$	$\longleftrightarrow \frac{2}{s^3}$
6	$t e^{-at} u(t)$	$\longleftrightarrow \frac{1}{(s + a)^2}$
7	$t^2 e^{-at} u(t)$	$\longleftrightarrow \frac{2}{(s + a)^3}$
8	$t^{n-1} e^{-at} u(t)$	$\longleftrightarrow \frac{(n-1)!}{(s + a)^n}$
9	$\sin(\omega_0 t) u(t)$	$\longleftrightarrow \frac{\omega_0}{s^2 + \omega_0^2}$
10	$\sin(\omega_0 t + \theta) u(t)$	$\longleftrightarrow \frac{s \sin \theta + \omega_0 \cos \theta}{s^2 + \omega_0^2}$
11	$\cos(\omega_0 t) u(t)$	$\longleftrightarrow \frac{s}{s^2 + \omega_0^2}$
12	$\cos(\omega_0 t + \theta) u(t)$	$\longleftrightarrow \frac{s \cos \theta - \omega_0 \sin \theta}{s^2 + \omega_0^2}$
13	$e^{-at} \sin(\omega_0 t) u(t)$	$\longleftrightarrow \frac{\omega_0}{(s + a)^2 + \omega_0^2}$
14	$e^{-at} \cos(\omega_0 t) u(t)$	$\longleftrightarrow \frac{s + a}{(s + a)^2 + \omega_0^2}$
15	$2e^{-at} \cos(bt - \theta) u(t)$	$\longleftrightarrow \frac{e^{j\theta}}{s + a + jb} + \frac{e^{-j\theta}}{s + a - jb}$
15a	$e^{-at} \cos(bt - \theta) u(t)$	$\longleftrightarrow \frac{(s + a) \cos \theta + b \sin \theta}{(s + a)^2 + b^2}$
16	$\frac{2t^{n-1}}{(n-1)!} e^{-at} \cos(bt - \theta) u(t)$	$\longleftrightarrow \frac{e^{j\theta}}{(s + a + jb)^n} + \frac{e^{-j\theta}}{(s + a - jb)^n}$

Table 3-1: Properties of the Laplace transform for causal functions; i.e., $x(t) = 0$ for $t < 0^-$.

Property	$x(t)$	$X(s) = \mathcal{L}[x(t)]$
1. Multiplication by constant	$K x(t)$	$\longleftrightarrow K X(s)$
2. Linearity	$K_1 x_1(t) + K_2 x_2(t)$	$\longleftrightarrow K_1 X_1(s) + K_2 X_2(s)$
3. Time scaling	$x(at), \quad a > 0$	$\longleftrightarrow \frac{1}{a} X\left(\frac{s}{a}\right)$
4. Time shift	$x(t - T) u(t - T), T \geq 0$	$\longleftrightarrow e^{-Ts} X(s)$
5. Frequency shift	$e^{-at} x(t)$	$\longleftrightarrow X(s + a)$
6. Time 1st derivative	$x' = \frac{dx}{dt}$	$\longleftrightarrow s X(s) - x(0^-)$
7. Time 2nd derivative	$x'' = \frac{d^2x}{dt^2}$	$\longleftrightarrow s^2 X(s) - s x(0^-) - x'(0^-)$
8. Time integral	$\int_{0^-}^t x(t') dt'$	$\longleftrightarrow \frac{1}{s} X(s)$
9. Frequency derivative	$t x(t)$	$\longleftrightarrow -\frac{d}{ds} X(s) = -X'(s)$
10. Frequency integral	$\frac{x(t)}{t}$	$\longleftrightarrow \int_s^\infty X(s') ds'$
11. Initial value	$x(0^+)$	$= \lim_{s \rightarrow \infty} s X(s)$
12. Final value	$\lim_{t \rightarrow \infty} x(t) = x(\infty)$	$= \lim_{s \rightarrow 0} s X(s)$
13. Convolution	$x_1(t) * x_2(t)$	$\longleftrightarrow X_1(s) X_2(s)$

Table 4-1: Circuit models for R , L , and C in the s-domain.

Time-Domain	s-Domain
<p>Resistor</p>  <p>$v = Ri$</p>	 <p>$V = RI$</p>
<p>Inductor</p>  <p>$v_L = L \frac{di_L}{dt}$</p> <p>$i_L = \frac{1}{L} \int_{0^-}^t v_L dt' + i_L(0^-)$</p>	<div style="display: flex; align-items: center; justify-content: space-around;">  <p>OR</p>  </div> <p>$V_L = sLI_L - L i_L(0^-)$</p> <p>$I_L = \frac{V_L}{sL} + \frac{i_L(0^-)}{s}$</p>
<p>Capacitor</p>  <p>$i_C = C \frac{dv_C}{dt}$</p> <p>$v_C = \frac{1}{C} \int_{0^-}^t i_C dt' + v_C(0^-)$</p>	<div style="display: flex; align-items: center; justify-content: space-around;">  <p>OR</p>  </div> <p>$V_C = \frac{I_C}{sC} + \frac{v_C(0^-)}{s}$</p> <p>$I_C = sCV_C - C v_C(0^-)$</p>