Homework

3.1

1.) a.) Invalid

Row 2; both hypothesis are true but conclusion is false.

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

~~For every row $p \vee q$ is true~~

b.) Valid

When $p \leftrightarrow q$ is true and $p \vee q$ is true, p is true

p	q	$p \leftrightarrow q$	$p \vee q$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	T	F

c.) Valid

Row 1

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

d.) Invalid

Row 2

p	q	$p \vee q$	$\neg q$	$p \leftrightarrow q$
T	T	T	F	T
T	F	T	T	F
F	T	T	F	F
F	F	F	T	T

Homework

3.1

e.) Invalid

Row 4 and Row 6

p	q	r	$(p \vee q) \to r$	$(p \wedge q) \to r$
T	T	T	T	T
T	T	F	F	F
T	F	T	T	T
T	F	F	F	T
F	T	T	T	T
F	T	F	F	T
F	F	T	T	T
F	F	F	T	T

g.) Invalid

Row 4

p	q	$q \to p$	$\neg q$
T	T	T	F
T	F	T	T
F	T	F	F
F	F	T	T

2.) a.)

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

Row 3; $p \to q$ is true and q is true, p is false

b.)

p	q	$p \to q$	$\neg p$	$\neg q$
T	T	T	F	F
T	F	F	F	T
F	T	T	T	F
F	F	T	T	T

Row 3; $p \to q$ is true, $\neg p$ is true, $\neg q$ is false.

Homework

3.2

1.)

a.) ~~Invalid~~ Valid

p	q	$p \vee q$	$\neg p$
T	T	T	F
T	F	T	F
F	T	T	T
F	F	F	T

Disjunctive Syllogism

~~b.)~~ c.) Valid

Addition

d.) Valid Modus tollens.

2.) a.) $p \to q$ hypethesis

1 $q \to r$ hypethesis

$\neg r$ hypethesis.

2 $\Rightarrow p \to r$ hypathetical Syllogism 1,2

$\neg r$ Modus tollens 2,3

3 $\Rightarrow \neg p$

b.) $p \to (q \wedge r)$ Hypethesis

1 $\neg q$ Hypothesis,

Simplification 1

2 $\Rightarrow p \to q$ Modus tollens 2

$\neg q$

3 $\neg p$

Homework

3.2

1.)

$$1\begin{cases}(p \wedge q) \rightarrow r \\ \neg r \\ q\end{cases}$$

Hypothesis
Hypothesis
Hypothesis.

$2 \Rightarrow p \rightarrow r$ Simplification 1, 1 again

$\neg r$ Modus tollens 2

* Simplify q into $(p \wedge q)$

$3 \Rightarrow \neg p$

5.) a.) $j \rightarrow (c \wedge h)$

$\neg h$

$\therefore \neg j$

j: will get a job
c: will buy a new car
h: will buy a house.

The argument is valid When $j \rightarrow (c \wedge h)$ is true, and $\neg h$ is true, then $\neg j$ is also true.

j	c	h	$j \rightarrow (c \wedge h)$	$\neg h$	$\neg j$
T	T	T	T	F	F
T	T	F	F	T	F
T	F	T	F	F	F
T	F	F	F	T	F
F	T	T	T	F	T
F	T	F	T	T	T
F	F	T	T	F	T
F	F	F	T	T	T

c.) $(c \wedge h) \rightarrow j$

$\neg j$

$\therefore \neg c$

Invalid; when $(c \wedge h) \rightarrow j$ is true, and $\neg j$ is true, $\neg c$ is false.

j	c	h	$(c \wedge h) \rightarrow j$	$\neg j$	$\neg c$
T	T	T	T	F	F
T	T	F	T	F	F
T	F	T	T	F	T
T	F	F	T	F	T
F	T	T	F	T	F
F	T	F	T	T	F
F	F	T	T	T	T
F	F	F	T	T	T

Homework

3.3

1.) a.) $\forall x(h(x) \lor b(x))$
$\exists x(\neg h(x))$
$\therefore \exists x(b(x))$

$\left.\begin{array}{l}\forall x(h(x) \lor b(x)) \\ \exists x(\neg h(x))\end{array}\right\}$ Hypothesis.

c is arbitrary
h: practices hard
b: plays badly

$\forall x(h(x) \lor b(x))$
$h(c) \lor b(c)$

Universal instantiation

$\exists x(\neg h(x))$
$\neg h(c)$

Existential instantiation

$\Rightarrow \left.\begin{array}{l}h(c) \lor b(c) \\ \neg h(c)\end{array}\right\}$ Disjunctive Syllogism

$\Rightarrow b(c)$ Existential generalization

$\Rightarrow \exists x(b(x))$

Homework

3.3

c: is arbitrary
L: lives in the city
F: owns a Ferrari
S: speeding ticket

b.) $L(x)$
$F(x)$
$\forall x(F(x) \to S(x))$
$\therefore \exists x(S(x))$

Hypothesis.

$\Rightarrow L(x)$
$L(x) \wedge F(x)$
$F(x)$

Commutative
Simplification

$\Rightarrow F(x)$
$\forall x(F(x) \to S(x))$
$\therefore S(x)$

Universal Instantiation

$\Rightarrow F(c)$
$F(c) \to S(c)$

Modus Ponens

$\Rightarrow S(c)$

Existential generalization

$\Rightarrow \exists x(S(x))$

Homework

3.3

d.) $\forall x (P(x) \to F(x))$
$\forall x (P(x))$
$\therefore \forall x (F(x))$

Hypothesis

c is arbitrary
$P(x)$: has permission slip
$F(x)$: can go on field trip

$\forall x (P(x) \to F(x))$
$P(c) \to F(c)$

$\forall x (P(x))$
$P(c)$

Universal Instantiation

$\Rightarrow P(c) \to F(c)$
$P(c)$

Modus Ponens

$\Rightarrow F(c)$

Universal generalization

$\Rightarrow \forall x (F(x))$

2.) a.) Invalid

Since it is said a student is getting an A, one student must. However, John does not have to be the student since it could be another student.

b.) Valid

If Suzy sold 50 boxes of cookies, then she could go. Since she could get a prize, she must have sold at least 50.

Homework

3.3 + 3.4

4.) a.) Valid $\exists x(P(x) \wedge Q(x))$ Hypothesis

c is arbitrary

$\Rightarrow P(c) \wedge Q(c)$ Existential instantiation

$\Rightarrow P(c)$ Simplification

$\Rightarrow \exists x(P(x))$ Existential instantiation

$\Rightarrow Q(c) \wedge P(c)$ Commutative

$\Rightarrow Q(c)$ Simplified

$\Rightarrow \exists x(Q(x))$ Existential instantiation

$\Rightarrow \exists x(P(x)) \wedge \exists x(Q(x))$ Conjunction.

d.) Not valid

~~c is arbitrary~~

	P	Q
a	F	F
b	T	F

3.4

4.) a.) 1, 7

d.) 1, 3, 5, 15, 25, 75

e.) 1, 2, 3, 5, 6, 10, 15, 30

Homework

3.4 + 3.5

5.) a.) Neither

b.) Neither

c.) Prime

f.) Composite 2 is a divisor of 56328

3.5

2.) a.) $n=0$ $(0+1)^2 > 0$ true

$n=1$ $(1+1)^2 > 1$ true

$n=2$ $(2+1)^2 > 8$ true

$n=3$ $(3+1)^2 > 27$ False

c.) $n=1$ $(1+1)^3 \geq 3^1$ true

$n=2$ $(2+1)^3 \geq 3^2$ true

$n=3$ $(3+1)^3 \geq 3^3$ true

$n=4$ $(4+1)^3 \geq 3^4$ true

3.) a.) February has 28 or 29.

b.) If $n=2$ then $n^2=4$ and is divisible by 4
However, 2 is not divisible by 4.

5.) a.) If $x=1$ and $y=1$ then $\frac{1}{1}+\frac{1}{1}=2$

b.) If $x=3$ then $1+2=3$ is true

c.) If $m=n=0$ then $\sqrt{0+0}=0$ and $\sqrt{0}+\sqrt{0}=0$

d.) If $c=3$ and $d=-4$ then $7(3)+5(-4)=1$

Homework

3.5 + 3.6 + 3.7

g.) If $z=-1$, then any pair of the same number will $= 0$

3.6

1.) a.) $a^3/b \Rightarrow a^3/ka^3$ $\quad$ $\left.\begin{matrix} 1/k \\ 1/j \end{matrix}\right\} 1/jk$

$b^2/c \Rightarrow b^2/jb^2$

$a^6/c \Rightarrow a^6/(jk^2)a^6$

Since a^3/b, and b^2/c can be simplified to a simplified a^6/c, then they equal when $k=j=1$

~~b.)~~

~~b.) Set m to the integer 0. this will set~~

b.) If k and m are direct inverses ~~then~~ as $k = 1/6$ and $m = 6/1$ then $n=1$. $m/n = 6$ and $5n^3 - 2n^2 + 3n = 6$.

2.) a.) It states $z = ky$ without proving that as true.

3.7

1.) a.) We assume the sum of an odd and even integer = odd $(2k+1 + 2k = 4k+1)$

Let $x=1$ (odd integer) and $y=2$ (even integer)

then $1+2=3$ (odd integer)

this proves odd + even = odd.

Homework

3.7

b.) Assume two odd integers = even integer.
~~[illegible]~~ ($2K+1 + 2K+1 = 4K+2$)

Let $x=1$, and $y=3$ (both odd integers)

then $1+3=4$ (even integer)

this proves odd integer + odd integer = even integer

c.) Prove the square of an odd integer is an odd integer ~~[illegible]~~ ($\sqrt{(2K+1)^2} = 2K+1$)

Let $x=25$ (odd integer)

then $\sqrt{25} = 5$ (an odd integer)

this proves the square of an odd integer equals an odd integer.

d.) Prove the product of two odd integers is an odd integer ($2K+1 \cdot 2K+1$) $=$ ~~[illegible]~~ $4K^2+4K+1$

Let $x=3$, and $y=5$. (both odd integers)

then $xy \Rightarrow 3 \cdot 5 = 15$ (an odd integer)

this proves the product of two odd integers is an odd integer.

Homework

3.7

e.) ~~Prove if x~~ Assume x is an even integer, and y is an odd integer; then Prove $x^2 + y^2 =$ an odd integer.

Let $x = 2$ and $y = 3$

then $2^2 + 3^2 \Rightarrow 4 + 9 = 13$ (odd integer)

this proves $x^2 + y^2 =$ odd integer when x is even integer and y is an odd integer.

2.) a.) Prove the product of two rational numbers is a rational number.

Let $x = \frac{1}{2}$ and $y = \frac{2}{3}$ (both rational)

then $xy \Rightarrow 1/2 \cdot 2/3 = \frac{2}{6}$ or $\frac{1}{3}$. (rational)

this proves product of two rational numbers is rational

b.) Prove the quotient of a rational number and a non zero rational number = a rational number.

Let $x = y = 1/2$

then $(1/2) \div (1/2) = 1$ (rational)

this proves rational number $\div$ rational number is rational if the bottom $\neq 0$.

Homework

3.7

f.) Prove the average of two rational numbers, is rational.

Let $x=2$, and $y=3$ (both rational)

then $\frac{2+3}{2} = \frac{5}{2}$ (rational)

This proves the average of two rational numbers is also rational.

4.) a.) true

Let $x=2$ and $y=4$

then $x+y \Rightarrow 2+4=6$

b.) False

Let $x=1$ $y=3$

then $x+y \Rightarrow 1+3=4$ (an even integer), but x and y are both odd integers.

c.) False

Let $x=2$ and $y=-2$

then $x^2 \Rightarrow 2^2=4$ and $y^2 \Rightarrow -2^2=4$ so $x^2=y^2$
However, $x \neq y$ since $2 \neq -2$.

Homework

3.7 + 3.8

f.) False

Let $x=1$, $y=3$

then average of 1 and $3=2$ (even integer)

h.) true

Let $x=3$, $y=5$

then average 3 and 5 to get 4. If the $x \neq y$ but are odd, the answer will be an even integer. If $x=y$, the average $=x$.

3.8

1.) a.) Prove if n^2 is odd then n is odd.
$\Rightarrow$ Contrapositive: Prove if n is even, then n^2 is even.

Let $x=2$

then $x^2 \Rightarrow 2^2=4$ (even)

This proves if n^2 is odd, then n is odd by Contrapositive.

Homework

3.8

b.) Prove if n^3 is even, then n is even.
=> Contrapositive: If n is odd, then n^3 is odd.

Let $x=3$

then $x^3 \Rightarrow 3^3 = 27$

this proves if n^3 is even then n is even by contrapositive.

c.) Prove if $5n+3$ is even, then n is odd.
=> Contrapositive: If n is even, then $5n+3$ is odd

Let $x=2$ (even)

then $5x+3 \Rightarrow 5(2)+3 = 13$ (odd)

this proves if $5n+3$ is even, then n is odd by contrapositive.

2.) a.) Prove that if $3 \nmid xy$ then $3 \nmid x$
=> Contrapositive: If $3 \mid x$, then $3 \mid xy$.

Let $x=3$ and $y=2$

then $3 \mid xy \Rightarrow 3 \mid 6$ and $3 \mid x \Rightarrow 3 \mid 3$

this proves if $3 \nmid xy$ then $3 \nmid x$ is true due to Contrapositive.

Homework

3.8

5.) a.) Prove the product of any integer with an even integer is even.

Let $x = 1, y = 2$ (odd and even)

then $xy \Rightarrow 1 \cdot 2 = 2$

Let $x = 4, y = 2$ (even and even)

then $xy \Rightarrow 4 \cdot 2 = 8$

This is a direct proof that proves the product of any integer with an even integer is even

b.) Prove if $p > 2$ and p is a prime number, then p is odd.

$\Rightarrow$ Contrapositive: If p is a prime number and $p \leq 2$, then p is even

Let $x = 2$

then the only prime number ≤ 2 is 2, and 2 is even. This proves by contrapositive, if $p > 2$ ~~and~~ and is a prime number, then p is odd.

Homework

3.8 + 3.9

9.) Prove the difference of two rational numbers is a rational number.

let $x = \frac{3}{2}$ and $y = \frac{1}{2}$

then $x - y \Rightarrow \frac{3}{2} - \frac{1}{2} = \frac{2}{2}$ or 1

this is a direct proof showing if two rational numbers are subtracting, the answer is also rational.

3.9

1.) Prove that $\sqrt{2}/2$ is irrational.
Assume it to be rational. Then it can be in form $\frac{x}{y}$.

squaring both sides gives $2/4 = \frac{x^2}{y^2}$
$\frac{1}{4}y^2 = x^2$

$\frac{1}{4}y^2 = 2K^2 \Rightarrow y^2 = 8K^2$

thus y^2 must be even. Since both x and y are even, they are divisible by 2. this contradicts $\frac{x}{y}$ is in its lowest form.

So we must conclude the assumption $\sqrt{2}/2$ is rational, is false.

Homework

3.9

b.) Prove $2-\sqrt{2}$ is irrational.

~~First Prove $\sqrt{2}$ is Irr~~

Assume $2-\sqrt{2}$ is rational

Let $2-\sqrt{2} = 2-\frac{x}{y}$ this means $\sqrt{2}=\frac{x}{y}$

squaring gives $2=\frac{x^2}{y^2} \Rightarrow 2y^2=x^2$

x^2 must be even since it is a multiple of 2.

Now $2y^2=4k^2$ so $y^2=2k^2$
this shows y^2 must also be even.

This shows both x and y are divisible by 2 and are not in its simplest form. This contradicts $2-\sqrt{2}=2-\frac{x}{y}$.
We conclude $2-\sqrt{2}$ is rational, is false.

5.) a.) Statement: If x and y are a pair of consecutive integers, then x and y have opposite parity.
Contrapositive: If x and y ~~has~~ don't have opposite parity, then they are not a pair of consecutive integers.
Contradiction: If x and y are a pair of consecutive integers, then x and y ~~have~~ don't have opposite parity. (Should be false).

Homewerk.

3.9 + 3.10

6.) a.) ~~If 9x = 100t then 3x(9 x = 12t~~

Assume there is ^{not} a kid who won at least 12 trophies. x.

$x = P/q \quad \Rightarrow \frac{x}{2} = P/2q$

d.) Assume there is a smallest integer x.

$x = P/q$ where P and q are integers.

$x/2 = P/2q$ since x is an integer $x/2$ is also.

$x/2 < x$ which contradicts x as the smallest integer.

3.10

1.) a.) Every real number x, $x^2 \geq 0$

Case 1: $x \geq 0$

Let $x = 2$

then $x^2 \geq 0$, $2^2 = 4 \geq 0$

Case 2: $x \leq 0$

Let $x = -2$

then $x^2 \geq 0$, $-2^2 = 4 \geq 0$.

Homework

3.10

b.) For every integer n, $n^2 \geq n$
Case 1: smallest integer set $x=1$
let $x=1$, then $1^2 \geq 1$ since $1=1$

Case 2: negative integer
Let $x=-5$, then $-5^2 = 25 \geq -5$

2.) a.) If x is an integer, then x^2+5x-1 is odd.
Case 1: x^2+5x-1 is odd
Let $x=2$ (even number)
then $2^2+5(2)-1=13$ (odd number).

Case 2: x^2+5x-1 is odd
Let $x=3$ (odd number)
then $3^2+5(3)-1=23$ (odd number)

c.) If integers are consecutive $x<y$, then they have opposite parity
Case 1: $x=2$, even number.
then y must equal 3
2 is even, 3 is odd

Case 2: $x=3$ odd number
then y must equal 4
3 is odd, 4 is even.

Homework

3.10

9.) If x and y are numbers that xy and $x+y$ are both even, then x and y are both even.

Case 1: x and y are even
Let $x=2$, $y=4$
then $xy=8$ (even) and $x+y=6$ (even)

Case 2: x and y are odd
Let $x=3$, $y=5$
then $xy=15$ (odd) and $x+y=8$ (even)
This is False, which shows xy and $x+y$ must both be even.

3.) a.) For any real number x, $|x| \geq 0$
Case 1: x is positive
Let $x=2$
then $|2| = 2 \geq 0$

Case 2: x is negative
Let $x=-2$
then $|-2| = 2 \geq 0$.

b.) For any real number x, $|x| \geq x$ and $|x| \geq -x$
Case 1: Positive x
Let $x=2$
then $|2|=2 \geq 2$ and $|2|=2 \geq -2$

Case 2: Negative x
Let $x=-2$
then $|-2|=2 \geq -2$ and $|-2|=2 \geq 2$