Homework

# 7.1

1.) a.) $1, \sqrt{2}, \sqrt{3}, \sqrt{4}=2, \sqrt{5}, \sqrt{6}, \sqrt{7}, \sqrt{8}, 3, \sqrt{10}$

Non-increasing, Non-decreasing

b.) 1, 1, 2, 3, 5, 8, 13, 21, 34, 55

Non-Decreasing

c.) 2, 1

Non-increasing, decreasing

d.) $1/1, 1/2, 1/3, 1/4, 1/5, 1/6, 1/7, 1/8, 1/9, 1/10$

Non-increasing, decreasing

e.) 3, 3, 3, 3, 3, 3, 3, 3, 3, 3

Non-increasing, Non-decreasing.

f.) 1, 4, 9, 16, 25, 36, 49, 64, 81, 100

Non-decreasing, increasing

g.) $0, 1, \log_2(3), 2, \log_2(5), \log_2(6), \log_2(7), 3, \log_2(9), \log_2(10)$

Increasing, Non-decreasing.

Homework

# 7.1

~~h.) 0, 2, 3.16, 4, 4.64, 5.16, 5.62, 6, 6.34, 6.64~~

h.) $1, 2, 2^{1.58}, 4, 2^{2.32}, 2^{2.58}, 2^{2.81}, 8, 2^{3.17}, 2^{3.32}$

Increasing, Non-decreasing

i.) $-1/2, -2/2$ or $-1$, $-3/2, -4/2$ or $-2$, $-5/2, -6/2$ or $-3$, $-7/2$,
$-8/2$ or $-4$, $-9/2, -10/2$ or $-5$.

Increasing, Non-decreasing.

2.) a.) Increasing, Non-decreasing

b.) Non-Increasing, Non-decreasing.

c.) Non-Increasin, Non-decreasing.

d.) Increasing, Non-decreasing.

e.) Decreasing, Non-Increasing.

3.) a.) 2, 6, 18, 54, 162, 486

b.) 2, 5, 8, 11, 14, 17

c.) 27, 9, 3, 1, 1/3, 1/6

d.) 3, 2.5, 2, 1.5, 1, 0.5

Homework

# 7.2 + 7.3

1.) a.) 1, 2, 2, 4, 8, 32

b.) 1, 5, 13, 41, 121, 365

c.) ~~[illegible]~~ 2, 1, 5, 21, 110, 681

d.) 4, 5, 20, 100, 2,000, 200,000

e.) 1, 3, -4, -25, -53, -178

f.) 1, 1, 2, 5, 27, 734

g.) 0, 2, 10, 46, 210, 958

7.3

1.) a.) 31

b.) 31

c.) -27

d.) 40

e.) 60,702

f.) 1,277.8356

g.) 25,553

h.) 454,730

Homework

# 7.3

2.) a.) $\sum_{k=-2}^{7} k^5$

b.) $\sum_{k=-2}^{5} k$

c.) $\sum_{k=2}^{8} 2^k$

d.) $\sum_{k=0}^{17} k^3$

e.) $\sum_{k=1}^{15} k^3$

f.) $\sum_{k=0}^{50} 2k+1$

4.) a.) $\sum_{j=0}^{n} (j+2) \Rightarrow \sum_{j=2}^{n+2} (i) \Rightarrow \sum_{j=2}^{n+2} (j)$

b.) $\sum_{k-1}^{n-2} 2^{k-1}$

c.) $\sum_{j=-1}^{n} (j-1)\cdot\cancel{(j-1)} \Rightarrow \sum_{j=-1}^{n} (j-1)$

d.) $\sum_{j=0}^{13} (j)$

e.) $\sum_{k=15}^{25} \left(5k-4/5\right)$

Homework

7.4

1.) a) $= \frac{\overset{9+3)(6+1)}{3(3+1)(2(3)+1)}}{6} \Rightarrow 42$

b.) $\sum_{j=1}^{n} j^2 = \frac{K(K+1)(2K+1)}{6}$

c.) $\sum_{j=1}^{n} j^2 = \frac{(K+1)[(K+1)+1][2(K+1)+1]}{6}$

d.) Must be proven true when $n=1$ (or $j=1$)

e.) Must be proven true that

$= j^2 = \frac{[K+1][(K+1)+1][2(K+1)+1]}{6}$

f.) $\frac{(K+n)[(K+n)+1][2(K+n)+1]}{6}$

g.) * Check base case

$1^2 = 1$ $\qquad \frac{1(1+1)(2(1)+1)}{6} = \frac{6}{6} = 1$

so, $1=1$

Assume $n=K$ $\qquad \frac{K(K+1)(2K+1)}{6} + K+1$

$= \frac{K+1(K+1+1)(2(K+1)+1)}{6}$

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# Homework

## 7.4

g.) Continued. Set $k+1 \cdot \frac{6}{6}$

$$K(K+1)(2K+1) + 6(K+1) = K+1(K+2)(2(K+1)+1)$$

$$\Rightarrow 2K^3+3K^2+K + 6K+6 = 2K^2+9K^2+13K+6$$

2.) a.) $$\left(\frac{K(K+1)}{2}\right)^2 + \frac{2(K+1)}{2} = \left(\frac{(K+1)[(K+1)+1]}{2}\right)^2$$

$\Rightarrow K^2 + \cancel{K^2} + 2K +$ [illegible] $+$ [illegible] $K + 2 = K^2+3K+2$

$K^2+3K+2$

b.) $$\cancel{(K+1)}2^{K+1} + 2 + \cancel{(K+1)} \cdot 2^{K+1} = \cancel{(K+1-1)}2^{K+1+1} + 2$$

True

$K^2+2K+1-1$

c.) $$\frac{K(K^2-1)}{3} + \frac{3(K+1)(K+\cancel{1+1})}{3} = \frac{K+1((K+1)^2-1)}{3}$$

$K(3K+3)$ $\qquad$ $(K+1)(K^2+2K)$

$$\Rightarrow K^3-K + 3K^2+3K = K^3+2K^2+K^2+2K$$

$$\Rightarrow K^3+3K^2+2K = K^3+3K^2+2K$$ True

d.) $$\cancel{K}\frac{1}{K+1} + \frac{1}{(K+1)(K+1+1)} = \cancel{K}\frac{1}{K+1+1}$$

$$\Rightarrow K+1 + K^2+2K+K+2 = K+2$$

Homework

7.4

e.) $\frac{3(K\cdot 3^{K+1}-(K+1)3^{K}+1)}{4}+(2K\cdot 3^{2K})\frac{4}{4}$

$=\frac{3(2K\cdot 3^{2K+1}-(2K+1)3^{2K}+1)}{4}$

$\Rightarrow 3(K\cdot 3^{K+1}-(K+1)3^{K}+1)+\cancel{4}(2K\cdot 3^{2K})4$

f.) $\frac{K+1}{2K}+1-\frac{1}{(K+1)^2} = \frac{(K+1)+1}{2(K+1)}\ \frac{K+2}{2K+2}$

$K^2+2K+1$

$K^2(K+1)+K^2+2K+1-1 = K^2(K+2)$

$K^3+K^2+K^2+2K+\cancel{1}-\cancel{1}$ $\qquad K^3+2K^2+2K$

3.) a.) $3^{K+1} > 2K^{+1}+(K+1)^2$

$3(2^{K+1}+K^2+2K+1)$

By assumption $3^K > 2K+K^2$

so $2K+K^2 < 3(2^{K+1}+K^2+2K+1)$

$\cancel{b.)}$ $3^{2K} \geq (2K)^3+K^3$

$3+3^K \geq 3K^3$

Homework

# 7.5

1.) a.) $4m+1 = 3^{2k}$

$3^{2(k+1)} - 1 = 4(9m+2)$

$9 \cdot 4m + 2$

$9(4m+2)$

b.) $6m+1 = 7^k$

$7^{k+1} - 1 = 6(7m+1)$

$= 7 \cdot 6m + 1$

$= 7(6m+6)$

c.) $4m = 11^k - 7^k$

$11^{k+1} - 7^{k+1} = 4(11m + 7m + 2)$

$4(\cancel{18}m^4 + 2)$

$= \cancel{18} \cdot 4m + 2$

$= \cancel{18}(4m+2)$ 4

d.) $7m = 9^k - 2^k$

$9^{k+1} 2^{k+1} = 7(9m - 2m + 1)$

$= 7(7m+1)$

$= 7(7m+1)$

e.) $2m - 2 = k^2 - 5k$

$(k+1)^2 - 5(k+1) = 2m - 2$

$k^2 + 2k + 1 - 5k - 5 = 2(m^2 - 5m - 2)$

$\cancel{= m^2 - 5m(2}$

$= -5m(2m^2 - 2)$

f.) $3m - 6 = n^3 - 4n$

$(k+1)^3 - 4(k+1) = 3(m^3 - 4m - 6)$

$-4m(3m^3 - 6)$

# Homework

## 7.5

3.) a.) base case $5^{2^0} = 5^1 = 5$ true

$C_{n+1} = 5^{2^{n+1}}$

$C_{n+1} = (5^{2^{n+1}})^2$

b.) base case $2^{0+1} = 2^1 = 2-1 = 1$ $b_0$ is true

$b_{n+1} = 2^{n+1} - 1$

$b_{n+1} = 2(2^{n+1} - 1) - 1$

c.) base case $-2(1) - 4 + 6(2^1)$

$a_{n+1} = -2n - 4 + 6 \cdot 2^n$ $-2 - 4 + 12$

$a_{n+1} = 2(-2n - 4 + 6 \cdot 2^n) + 2n$ $= 6$ so $a_1$ is true

d.) $g_0 = 0$ $\quad \frac{n(n+3)}{2} \Rightarrow \frac{0(0+3)}{2} = 0$

$g_{n+1} = \frac{n+1(n+1+3)}{3}$ true

$g_{n+1} = \frac{(n+1)(n+4)}{3} + n + 1$