2. (20 points) Use Laplace transforms to find the solution to the differential equation

$$\frac{dy(t)}{dt} + 2y(t) = e^{-3t}u(t), \text{ with } y(0^-) = 2$$

$$\mathcal{L}\left\{\frac{dy(t)}{dt}\right\} = sY(s) - y(0^-) = sY(s) - 2$$

$$\mathcal{L}\{2y(t)\} = 2Y(s)$$

$$\mathcal{L}\{e^{-3t}u(t)\} = \frac{1}{s+3}$$

$$\therefore \quad sY(s) - 2 + 2Y(s) = \frac{1}{s+3}$$

$$\therefore \quad Y(s)(s+2) = \frac{1}{s+3} + 2$$

$$\therefore \quad Y(s) = \frac{1}{(s+3)(s+2)} + \frac{2}{s+2}$$

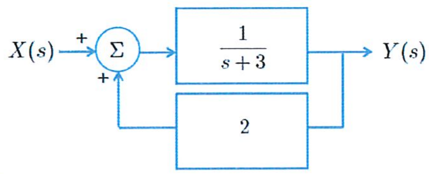
$$= \frac{1}{s+2} - \frac{1}{s+3} + \frac{2}{s+2}$$

$$= \frac{3}{s+2} - \frac{1}{s+3}$$

$$y(t) = 3e^{-2t}u(t) - e^{-3t}u(t)$$

3. (20 points) Find the transfer function $H(s) = \dfrac{Y(s)}{X(s)}$ of the following two feedback systems:

(a)



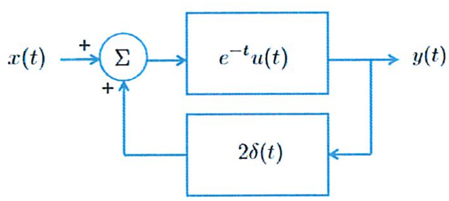
$$\left(X(s)+2Y(s)\right)\frac{1}{s+3} = Y(s)$$

$$\therefore \quad \frac{1}{s+3}X(s) = \left(1-\frac{2}{s+3}\right)Y(s) = \frac{s+1}{s+3}Y(s)$$

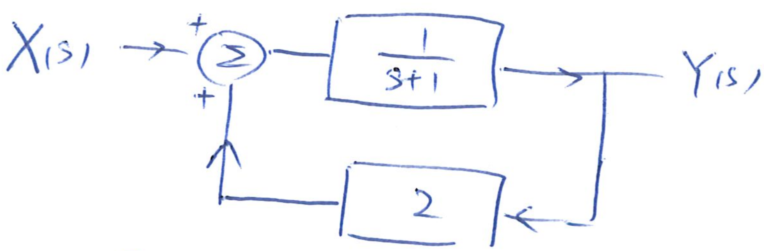
$$\therefore \quad X(s) = (s+1)Y(s)$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{1}{s+1}.$$

(b)



(Hint: consider the s-domain model of the feedback system)



$$\therefore \left[X(s) + 2Y(s)\right] \times \frac{1}{s+1} = Y(s)$$

similarly.

$$H(s) = \frac{Y(s)}{X(s)} = \frac{1}{s-1}.$$

4. (20 points) Given $x(t) = e^{-3t}u(t)$, find $\mathscr{L}\{t\,x(t)\}$.

$$X(s) = \mathscr{L}\{x(t)\} = \frac{1}{s+3}.$$

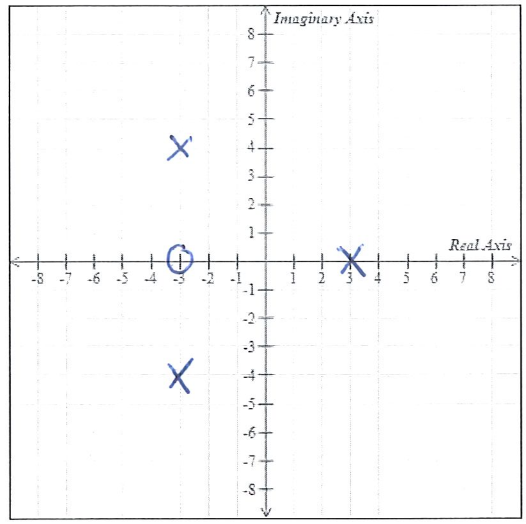
$$\mathscr{L}\{t\,x(t)\}$$

$$= -\left(\frac{1}{s+3}\right)'$$

$$= \frac{1}{(s+3)^2}.$$

5. (20 points) Consider $X(s) = \dfrac{s+3}{(s-3)(s^2+6s+25)}$ $= \dfrac{s+3}{(s-3)(s+3-4j)(s+3+4j)}$

(a) Indicate the poles and zeros for $X(s)$ on the graph provided



(b) Does the initial value $x(0^+)$ exist? Yes
If not, why not?

If so, calculate $x(0^+)$ and record it here:

$x(0^+) =$ 0

(c) Does the initial value $x(\infty)$ exist? No
If not, why not?
one pole in right half plane

If so, calculate $x(\infty)$ and record it here:

$x(\infty) =$ —

poles for $s^2+6s+25$:

$$s = \frac{-6 \pm \sqrt{36-4\times 25}}{2} = \frac{-6 \pm \sqrt{-64}}{2}$$

$$= -3 \pm 4j.$$

$$x(0^+) = \lim_{s\to\infty} sX(s) = \lim_{s\to\infty} \frac{s(s+3)}{(s-3)(s^2+6s+25)} = 0.$$

6. (20 points) Find the inverse Laplace transform.

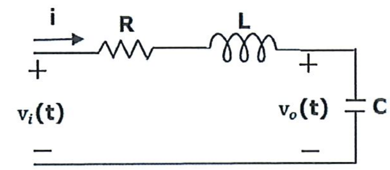
$$X(s) = \frac{1}{s+3} + \frac{s+8}{s^2+4s+10}$$

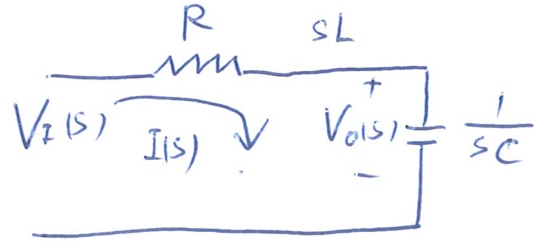
$$X(s) = \frac{1}{s+3} + \frac{s+2}{(s+2)^2+(\sqrt{6})^2} + \sqrt{6}\frac{\sqrt{6}}{(s+2)^2+(\sqrt{6})^2}$$

$$\downarrow$$

$$x(t) = e^{-3t}u(t) + e^{-2t}\cos(\sqrt{6}t)\,u(t) + \sqrt{6}\,e^{-2t}\sin(\sqrt{6}t)u(t)$$

7. (20 points) Determine $v_o(t)$ in the circuit below, given that $v_i(t) = 10u(t)$, $R = 3$ Ohms, $L = 0.5$ Henries, and $C = 0.25$ Farads.





$$V_i(s) = \frac{10}{s}\ V.$$

$$V_i(s) = I(s)\left(R + \frac{1}{sC} + sL\right)$$

$$V_o(s) = I(s) \cdot \frac{1}{sC}$$

$$\therefore V_o(s) = V_i(s)\,\frac{\frac{1}{sC}}{R + \frac{1}{sC} + sL}$$

$$= \frac{10}{s} \cdot \frac{4/s}{3 + \frac{4}{s} + 0.5s}$$

$$= \frac{40}{s(0.5s^2 + 3s + 4)}$$

$$= \frac{80}{s(s^2 + 6s + 8)}$$

$$= \frac{80}{s(s+2)(s+4)} = \frac{A}{s} + \frac{B}{s+2} + \frac{C}{s+4}.$$

$$A = \left.\frac{80}{(s+2)(s+4)}\right|_{s=0} = 10.$$

$$B = \left.\frac{80}{s(s+4)}\right|_{s=-2} = \frac{80}{-2 \cdot 2} = -20$$

$$C = \left.\frac{80}{s(s+2)}\right|_{s=-4} = \frac{80}{(-4)(-2)} = 10$$

$$\therefore V_o(s) = \frac{10}{s} + \frac{-20}{s+2} + \frac{10}{s+4}$$

$$V_o(t) = \left[10u(t) - 20e^{-2t}u(t) + 10e^{-4t}u(t)\right]\ V$$