Davis Mitchell
Grade: 92/100

+13/1/36+

AUBURN UNIVERSITY
Department of Electrical and Computer Engineering

ELEC 3800
Test 2

Thursday, November 14, 2019
75 minutes

General Instructions

1. Put your name in the name blank only. Test pages are numbered and can be associated together automatically.
2. This is a *closed book, closed notes* exam. However, one handwritten 3" × 5" notecard is allowed.
3. Show all work. Please put all of your work on the exam itself, preferably in the space provided. If you use the backs of the pages, please indicate that clearly so that you will receive appropriate credit. Partial credit will not ordinarily be given for multiple-choice questions, but free-response questions may receive partial credit.
4. If you do not find the exact numerical answer, mark the answer with the closest value.
5. All multiple-choice answers must be marked clearly where indicated with at least 50% filling. Black pen or pencil is acceptable. Marks must be ***dark***. You are responsible for points lost due to marks that are not dark enough. If you mark one answer and need to correct it, clearly indicate the answer you intended and leave a note for the instructor below your name on the cover page.
6. All multiple-choice questions are 3 points. Points for free-response questions are indicated by the maximum number in the box marked "reserved for instructor."

☐0	■0	☐0	☐0	☐0	■0	☐0	☐0	☐0	
☐1	☐1	☐1	☐1	☐1	☐1	☐1	☐1	☐1	
☐2	☐2	☐2	☐2	☐2	☐2	☐2	☐2	☐2	
☐3	☐3	■3	☐3	☐3	☐3	☐3	☐3	☐3	
☐4	☐4	☐4	☐4	☐4	☐4	☐4	☐4	☐4	
☐5	☐5	☐5	☐5	☐5	☐5	☐5	☐5	☐5	
☐6	☐6	☐6	■6	☐6	☐6	■6	■6	☐6	
☐7	☐7	☐7	☐7	☐7	☐7	☐7	☐7	☐7	
☐8	☐8	☐8	☐8	■8	☐8	☐8	☐8	☐8	
■9	☐9	☐9	☐9	☐9	☐9	☐9	☐9	■9	

⟵ please encode your student number here (leave blank if you don't know it or have your ID), and write your first and last names below.

Name: MITCHELL DAVIS

For the next four questions, use the following information. A parents group claims that college students spend at least 3 hours a day looking at their phones. A survey of 50 college students finds that the mean phone time is 2.7 hours with a sample variance of 2 hours.

Question 1 The null hypothesis is

3/3 ☐ $\overline{X} \leq 3$ ☐ $\overline{X} > 3$ ■ $\overline{X} \geq 3$ ☐ $\overline{X} \geq 2.8$ ☐ $\overline{X} = 3$

Question 2 What is the standard deviation of the sample mean?

3/3 ■ 0.20 ☐ 1.41 ☐ 0.30 ☐ 0.04

Question 3 Using an 88% confidence level, in what interval must the data fall to adopt the null hypothesis?

3/3 ☐ $[2.69, \infty)$ ☐ $[2.77, 3.23]$ ☐ $[2.69, 3.31]$ ☐ $[2.47, \infty)$ ■ $[2.77, \infty)$

Question 4 Suppose the samples are uniformly distributed. What distribution should be used to calculate probabilities for the sample mean?

3/3 ■ Gaussian ☐ Student's ☐ uniform ☐ none that are listed

Question 5

$$F(x,y) = \begin{cases} 0 & x < -1,\ y < 0 \\ \frac{1}{20}[y^2(x+1)^2 + 2y(x+1)^3] & -1 \leq x < 1, 0 \leq y < 1 \\ \frac{1}{20}[(x+1)^2 + 2(x+1)^3] & -1 \leq x < 1,\ y \geq 1 \\ \frac{1}{5}(y^2+4y) & x \geq 1,\ 0 \leq y < 1 \\ 1 & \text{otherwise} \end{cases}$$

Find the probability that $X \leq 0$ and $Y \leq 1$.

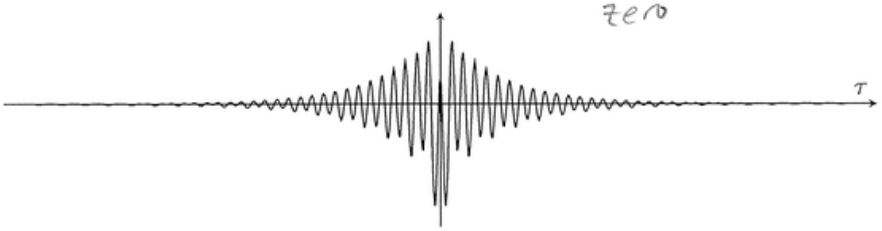
3/3 ☐ 1 ☐ 0.85 ■ 0.15 ☐ 0 ☐ 0.50

Question 6 A stationary random signal is sampled every 0.05 s, and the following samples are obtained (in order): 3, 0, -1, 1, 2, 1. Find the unbiased autocorrelation estimate for $\tau = 0.15$ s.

3/3 ☐ 0.60 ■ 0.67 ☐ 0.33 ☐ 2.67 ☐ 2.00

Question 7

peak not at zero

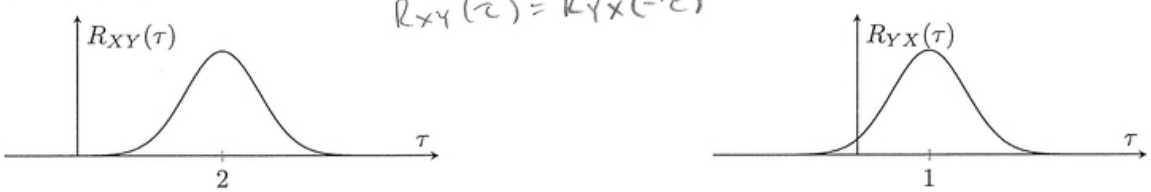


Indicate whether the function above is a valid autocorrelation function.

3/3 ☒ invalid ☐ valid

Question 8

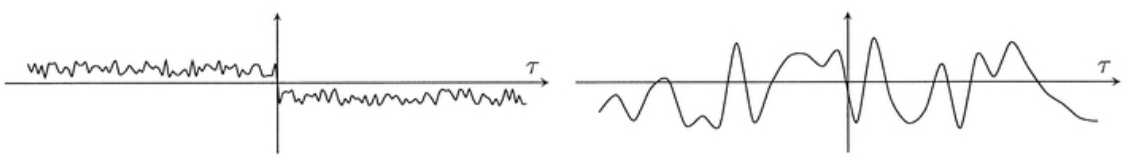
$R_{xy}(\tau) = R_{yx}(-\tau)$



Indicate whether the two functions above could be valid crosscorrelation functions for the same X and Y.

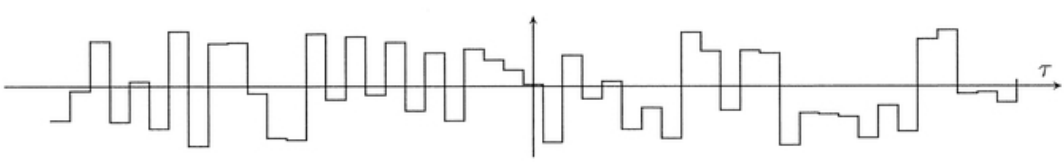
3/3 ☐ yes ☒ no ☐ insufficient information

Question 9 Which of the functions below appears more likely to be a sample function of a stationary random process?



3/3 ☐ left ☒ right

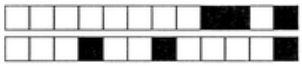
Question 10 Determine the category of this random process.



0/3 ☒ continuous ☐ discrete ☒ mixed

For the next four questions, consider gas mileage as a function of days since last fill-up. The following data is recorded:

days (d)	3	9	1	6
gas mileage in MPG (g)	22	19	25	17



+13/4/33+

Question 11 Find the unbiased sample mean of the gas mileage.

3/3

☐ 4.75 ☒ 20.75 ☐ 27.67 ☐ 3.50

$\frac{22+19+25+17}{4} = 20.75 = \bar{g}$

Question 12 Use linear regression to determine a model for gas mileage as a function of days since last fill-up. *From this model, what would be the predicted gas mileage if 4 days passed since last fill-up?*

10/10

☐0 ☐1 ☐2 ☐3 ☐4 ☐5 ☐6 ☐7 ☐8 ☐9 ☒10 *reserved for instructor*

$\bar{d} = \frac{1}{n}\sum d_i = \frac{3+9+1+6}{4} = 4.75 = \bar{d}$

$\overline{d^2} = \frac{1}{n}\sum d_i^2 = \frac{3^2+9^2+1^2+6^2}{4} = 31.75 = \overline{d^2}$

$\bar{g} = \frac{1}{n}\sum g_i = \frac{22+19+25+17}{4} = 20.75 = \bar{g}$

$\overline{g^2} = \frac{1}{n}\sum g_i^2 = \frac{22^2+19^2+25^2+17^2}{4} = 439.75 = \overline{g^2}$

$\overline{dg} = \frac{1}{n}\sum d_i g_i = \frac{3(22)+9(19)+1(25)+6(17)}{4} = 91 = \overline{dg}$

$m = \frac{\overline{dg} - \bar{d}\bar{g}}{\overline{d^2} - (\bar{d})^2} = \frac{91 - 4.75(20.75)}{31.75 - 4.75^2} = -0.823129 = m$ ✓

$b = \bar{g} - m\bar{d} = 20.75 - -0.8231(4.75)$

$b = 24.6597$

$b = 24.66$ ✓

$\bar{g} = b + m\bar{d}$

$\bar{g} = -0.8231(4) + 24.66$

$\bar{g} = 21.33$ mpg

Question 13 The correlation coefficient for this set of data tells us that

3/3

☒ ☐ Time between fill-ups has no relationship with gas mileage.

☒ ☐ More time between fill-ups is a good predictor of better gas mileage.

→ ☐ More time between fill-ups is a poor predictor of worse gas mileage.

→ ☒ More time between fill-ups is a good predictor of worse gas mileage.

$r = \frac{\overline{dg} - \bar{d}\bar{g}}{\sqrt{\overline{d^2} - (\bar{d})^2}\sqrt{\overline{g^2} - (\bar{g})^2}}$

$r = \frac{91 - 4.75(20.75)}{\sqrt{31.75 - 4.75^2}\sqrt{439.75 - 20.75^2}}$

$r = -0.8321$

$|r|$ is close to 1

highly correlated

negative → opposite

For the next four questions, consider the following joint density function.

$$f(x,y) = \begin{cases} \frac{3}{8}(x+y^2) & 0 \le x < 2,\ 0 \le y < 1 \\ 0 & \text{otherwise} \end{cases}$$

Question 14 Are X and Y independent?

3/3

☐ yes ☒ no ☐ insufficient information

$f_x(x) = \frac{3}{8}\int_0^1 x+y^2\,dy \qquad xy + \frac{y^3}{3}\Big|_0^1 = \left(x+\frac{1}{3}\right)\frac{3}{8}$

$f_y(y) = \frac{3}{8}\int_0^2 x+y^2\,dx = \frac{x^2}{2} + xy^2\Big|_0^2$

$(2+2y^2)\frac{3}{8}$

$f_x(x)\,f_y(y) = \frac{3}{8}(x+y^2)$

$(x+y^2)(2+2y^2)$

$2x + 2xy^2 + 2y^2 + y^4$

$\neq$ Not independent!

+13/5/32+

Question 15 $\Pr(X > 1, Y \leq 1)$ is

3/3

☐ 1 ☐ 0 ☐ 0.325 ☐ 0.31 ☒ 0.69

$\frac{3}{8}\int_1^2\int_0^1 (x+y^2)\,dy\,dx = xy + \frac{y^3}{3}\Big|_0^1 = \int_1^2 x + \frac{1}{3}\,dx = \frac{x^2}{2} + \frac{1}{3}x\Big|_1^2 \quad 2 + \frac{2}{3} - \left[\frac{1}{2} + \frac{1}{3}\right]$

$(11/6)\,3/8 = \boxed{\frac{11}{16}}$

Question 16 Find $F(x, y)$ in the interval $0 \leq x < 2,\ 0 \leq y < 1$.

10/10

☐0 ☐1 ☐2 ☐3 ☐4 ☐5 ☐6 ☐7 ☐8 ☐9 ■10 *reserved for instructor*

$\frac{3}{8}\int_0^Y\int_0^X a + b^2\,da\,db \quad \frac{a^2}{2} + ab^2\Big|_0^X = \int_0^Y \frac{x^2}{2} + xb^2\,db \quad \frac{bx^2}{2} + \frac{xb^3}{3}\Big|_0^Y = \boxed{\left(\frac{yx^2}{2} + \frac{xy^3}{3}\right)\frac{3}{8}}$

Question 17 Find $f_Y(y)$.

9/10

☐0 ☐1 ☐2 ☐3 ☐4 ☐5 ☐6 ☐7 ☐8 ■9 ☐10 *reserved for instructor*

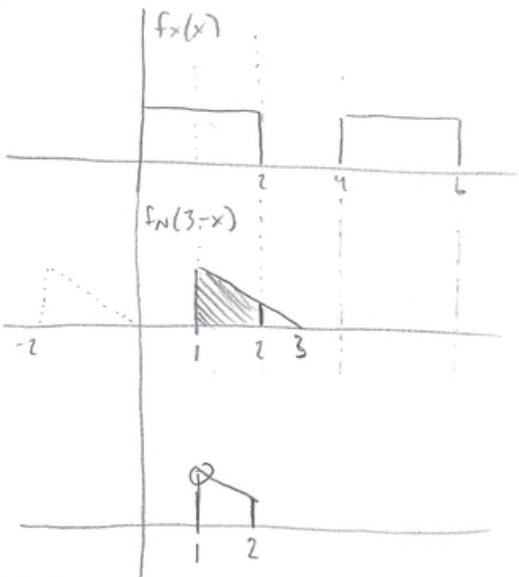
$f_Y(y) = \frac{3}{8}\int_0^2 x + y^2\,dx \rightarrow \frac{3}{8}\left[\frac{x^2}{2} + xy^2\right]\Big|_0^2 = \boxed{(2 + 2y^2)\frac{3}{8}} \quad 0 \leq y < 1$

$f(y|x) = f_N(y-x)$

Question 18 Let $Y = X + N$. For the density functions for X and N given below, sketch and label $f(x|y)$ for $y = 3$. (Height does not have to be labeled.) *What is the most likely value of X for $y = 3$?*

10/10 ☐0 ☐1 ☐2 ☐3 ☐4 ☐5 ☐6 ☐7 ☐8 ☐9 ■10 *reserved for instructor*

$f(x|y) = \frac{f(x,y)}{f_Y(y)} = \frac{f(y|x)f_X(x)}{f_Y(y)} = \frac{f_N(y-x)f_X(x)}{f_Y(y)} = \frac{f_N(3-x)f_X(x)}{f_Y(3)}$



Most likely value of $X = 1$

For the next two questions, let $Y(t) = \frac{1}{3}X(t-2) + N(t)$. $R_X(\tau) = 4e^{-|\tau|} + 2$, $R_N(\tau) = \delta(\tau)$ and $R_{XN}(\tau) = 2e^{-(\tau-1)^2}$.

Question 19 Find $R_{XY}(\tau)$.

6/10 ☐0 ☐1 ☐2 ☐3 ☐4 ☐5 ■6 ☐7 ☐8 ☐9 ☐10 *reserved for instructor*

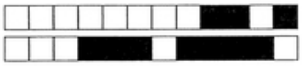
$R_{XY}(\tau) = E[X(t)Y(t+\tau)] = E\left[X(t)\cdot\left(\frac{1}{3}X(t-2+\tau) + N(t+\tau)\right)\right] =$

$= E\left[\frac{1}{3}X(t)X(t-2+\tau) + X(t)N(t+\tau)\right]$ → $R_X(\tau) = E[X(t)X(t+\tau)]$

$= \frac{1}{3}R_X(\tau) + R_N(\tau)$ ✗

$\tau - 2$

$\frac{1}{3}\cdot\left[4e^{-|\tau|} + 2\right] + \delta(\tau)$ → $R_{XY}(\tau) = \frac{4}{3}e^{-|\tau|} + \frac{2}{3}\delta(\tau)$



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Question 20 Find the variance of $X(t)$.

8/8

☐0 ☐1 ☐2 ☐3 ☐4 ☐5 ☐6 ☐7 ☒8 *reserved for instructor*

$\overline{X^2} = R_X(0) = 4e^0 + 2 = 6$

$(\overline{X})^2 = \lim_{\tau \to \infty} R_X(\tau) = 4e^{-\infty} + 2 = 2$

$\sigma^2 = \overline{X^2} - (\overline{X})^2 = 6 - 2 = \boxed{4}$