

21) a.)  $y(t) = 3x(t) + 1$

$$y_1(t) = 3x_1(t) + 1 \quad y_2(t) = 3x_2(t) + 1$$

$$z(t) = 3(a_1 x_1(t) + a_2 x_2(t)) + 1$$

$$= a_1 [3x_1(t) + 1] + a_2 [3x_2(t) + 1] \quad \boxed{\text{Linear}}$$

①  $z_1(t) = 3x(t) + 1 \rightarrow = 3x(t - \tau) + 1$

②  $z_2(t) = 3z(t) + 1 \rightarrow = 3x(t - \tau) + 1$

$$z_1(t) = z_2(t) \quad \boxed{\text{yes Time invariant}}$$

c.)  $\frac{dy}{dt} + t y(t) = x(t)$

$$\left[ \frac{d}{dt} y_1 + t y_1(t) \right] = x_1(t)$$

$$\left[ \frac{d}{dt} y_2 + t y_2(t) \right] = x_2(t)$$

$$z(t) = \left[ \frac{d}{dt} c y_1 + t \cdot c y_1(t) \right] + \left[ \frac{d}{dt} c y_2 + t c y_2(t) \right]$$

$$= a_1 \left[ \frac{d}{dt} y_1 + t y_1(t) \right] + a_2 \left[ \frac{d}{dt} y_2 + t y_2(t) \right]$$

$$\boxed{\text{Linear}}$$

$$C.) \quad z_1(t) = \frac{dy}{dt} + ty(t) = x(t) \rightarrow \frac{dy}{dt} + (t-\tau)(t-\tau)$$

$$z_2(t) = \frac{dy}{dt} + tz(t) = x(t) \rightarrow \frac{dy}{dt} + t(t-\tau)$$

Not Time invariant

$$22) a.) \quad y(t) = 3x(t-1)$$

$$y_1(t) = 3x_1(t-1) \quad y_2 = 3x_2(t-1)$$

$$z(t) = 3(a_1x_1(t-1) + a_2x_2(t-1))$$

$$= a_1(\underbrace{3x_1(t-1)}_{y_1}) + a_2(\underbrace{3x_2(t-1)}_{y_2})$$

Linear

$$\textcircled{1} \quad z_1(t) = 3x(t-1) \rightarrow 3x(t-\tau-1)$$

$$\textcircled{2} \quad z_2(t) = 3z(t) \rightarrow 3x([t-\tau]-1)$$

Yes Time invariant

$$C.) \quad \frac{dy}{dt} + y(t-1) = x(t)$$

$$x_1(t) = \frac{d}{dt} cy_1 + y_1(t-1) ; \quad x_2(t) = \frac{d}{dt} cy_2 + y_2(t-1)$$

$$z(t) = \frac{d}{dt} cy_1 + y_1(t-1) + \frac{d}{dt} cy_2 + y_2(t-1)$$

$$a_1 \left[ \frac{d}{dt} y_1 + y_1(t-1) \right] + a_2 \left[ \frac{d}{dt} y_2 + y_2(t-1) \right]$$

Linear

$$z_1(t) = \frac{dy}{dt} + y(t-1) \rightarrow \frac{dy}{dt} + [(t-\tau)-1]$$

$$z_2(t) = \frac{dy}{dt} + z(t) \rightarrow \frac{dy}{dt} + (t-\tau)$$

Not time invariant