Davis Mitchell
Grade: 87/100

+31/1/19+

AUBURN UNIVERSITY
Department of Electrical and Computer Engineering

ELEC 3800
Test 1

Thursday, October 3, 2019
75 minutes

General Instructions

1. Put your name in the name blank only. Test pages are numbered and can be associated together automatically.
2. This is a *closed book, closed notes* exam. However, one handwritten 3" × 5" notecard is allowed.
3. Show all work. Please put all of your work on the exam itself, preferably in the space provided. If you use the backs of the pages, please indicate that clearly so that you will receive appropriate credit. Partial credit will not ordinarily be given for multiple-choice questions, but free-response questions may receive partial credit.
4. If you do not find the exact numerical answer, mark the answer with the closest value.
5. All multiple-choice answers must be marked clearly where indicated with at least 50% filling. Black pen or pencil is acceptable. If you mark one answer and need to correct it, clearly indicate the answer you intended and leave a note for the instructor below your name on the cover page.
6. Free-response answers must appear in the box provided. Answers appearing outside the box may be ignored.
7. All multiple-choice questions are 3 points. Points for free-response questions are indicated by the maximum number in the box marked "reserved for instructor."

☐0	■0	☐0	☐0	☐0	■0	☐0	☐0	☐0
☐1	☐1	☐1	☐1	☐1	☐1	☐1	☐1	☐1
☐2	☐2	☐2	☐2	☐2	☐2	☐2	☐2	☐2
☐3	☐3	■3	☐3	☐3	☐3	☐3	☐3	☐3
☐4	☐4	☐4	☐4	☐4	☐4	☐4	☐4	☐4
☐5	☐5	☐5	☐5	☐5	☐5	☐5	☐5	☐5
☐6	☐6	☐6	■6	☐6	☐6	■6	■6	☐6
☐7	☐7	☐7	☐7	☐7	☐7	☐7	☐7	☐7
☐8	☐8	☐8	☐8	■8	☐8	☐8	☐8	☐8
■9	☐9	☐9	☐9	☐9	☐9	☐9	☐9	■9

⟵ please encode your student number here (leave blank if you don't know it or have your ID), and write your first and last names below.

Name: MITCHELL DAVIS

+31/2/18+

Question 1 Which of the following statements are true (in light of the Central Limit Theorem)?

I. the sum of 50 values drawn from a uniform distribution is Gaussian ✗

II. the sum of 5 values drawn from an exponential distribution is exponential ✓

III. the sum of 5 Gaussians is Gaussian ✓

0/3

☐ I only ☐ II only ☐ III only ☐ I and II ☒ I and III ■ II and III ☐ I, II, and III

Question 2 Seven employees are available to serve as waiters at your restaurant. You need at least two of them to work Thursday lunch. The probability that an individual employee is able to work at that time is 0.4. What is the probability that you will have enough waiters?

2/3

☐ 0.261 ☐ 0.012 ☒ 0.841 ☐ 0.159 ■ 0.739

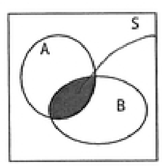
Question 3 Let $S = \{1,2,3,4,5,6,7,8,9,10,11,12\}$, $A = \{1,3,10,12\}$, and $B = \{3,6,10,11\}$. The set $\overline{A \cap B}$ is equal to: $\overline{A \cap B} = \bar{A} \cup \bar{B}$ $\bar{A} = 2,4,5,6,7,8,9,11$ $\bar{B} = 1,2,4,5,7,8,9,12$

$\bar{A} \cup \bar{B} = 1,2,4,5,6,7,8,9,11,12$

3/3

☐ $\{3,10\}$ ■ $\{1,2,4,5,6,7,8,9,11,12\}$ ☐ $\{2,4,5,7,8,9\}$ ☐ $\{1,3,6,10,11,12\}$ ☐ $\{2,4,5,7,8,9\}$

Question 4 The gray region below is



$= A \cap B$

3/3

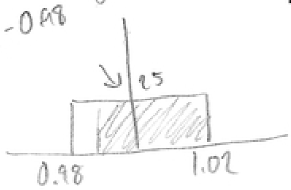
☐ $A \cup B$ ☐ $\bar{A} \cap \bar{B}$ ■ $\overline{\bar{A} \cup \bar{B}}$ ☐ $\Pr(A)\Pr(B)$

Question 5 The width of a semiconductor is uniformly distributed between 0.98 and 1.02 mm. What is the probability that the width will be greater than 0.99 mm?

3/3

$\frac{1}{1.02-0.98} = \frac{1}{0.04} = 25$

☐ 0.99 ☐ 0.25 ■ 0.75 ☐ 0.03 ☐ 0.97



$\int_{0.99}^{1.02} 25\,dx$ $25x\Big|_{0.99}^{1.02}$ $\boxed{0.75}$

For the next three questions, we know the following facts: $\Pr(A) = \frac{3}{7}$, $\Pr(B) = \frac{3}{7}$, $\Pr(C) = \frac{15}{28}$, $\Pr(A \cap B) = \frac{3}{28}$, $\Pr(B \cap C) = \frac{1}{7}$, and $\Pr(A \cap C) = \frac{5}{28}$.

Question 6 Find $\Pr(B|C)$.

3/3

☐ $\frac{1}{7}$ ☐ $\frac{4}{5}$ ☒ $\frac{4}{15}$ ☐ $\frac{1}{3}$ ☐ not enough information

$Pr(B|C) = \frac{Pr(B\cap C)}{Pr C} = \frac{1/7}{\frac{15}{28}} = \boxed{4/15}$

Question 7 Find $\Pr(A \cup C)$.

3/3

☒ $\frac{11}{14}$ ☐ $\frac{45}{196}$ ☐ $\frac{5}{28}$ ☐ $\frac{27}{28}$ ☐ not enough information

$Pr(A\cup C) = PrA + PrC - Pr(A\cap C) = 3/7 + \frac{15}{28} - 5/28 = \boxed{11/14}$

Question 8 Are B and C independent?

3/3

☐ yes ☒ no ☐ not enough information

$Pr(B\cap C) = Pr(B)Pr(C)$

$\frac{1}{7} \neq \frac{3}{7}\left(\frac{15}{28}\right) = \boxed{\frac{45}{196}}$

Question 9 A Gaussian random variable has mean 2 and variance 25. What is the probability that a random value from this variable is greater than 0?

3/3

☐ $Q(\frac{2}{5})$ ☐ $1 - Q(2)$ ☒ $1 - Q(\frac{2}{5})$ ☐ $1 - Q(\frac{2}{25})$ ☐ $Q(\frac{2}{25})$

$Pr(X>0) \to 1 - F(0) = 1 - \Phi\left(\frac{0-2}{5}\right) = 1 - \Phi\left(-\frac{2}{5}\right) = Q\left(-\frac{2}{5}\right) = \boxed{1 - Q\left(\frac{2}{5}\right)}$

For the next two questions, you are trying to get a time-sensitive message through to your mom. You decide to text your brother and ask him to relay the message and also text your mom directly. The probability that your brother will see the text in time is 0.95. The probability that he will relay a message in a timely manner is 0.6. The probability that your mom will see the text in time is 0.45.

Question 10 What is the probability R_{bro} that your brother will receive the message and relay it in a timely manner?

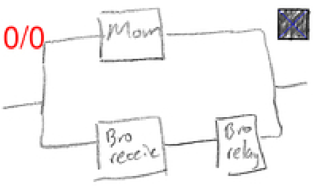
3/3

☒ 0.57 ☐ 0.02 ☐ 0.45 ☐ 0.98 ☐ 0.43

Series $R_{ser} = \prod_{i=1}^{n} R_i = (0.95)(0.6) = \boxed{0.57}$

Question 11 In terms of R_{bro} (if required), what is the probability that your mom will get the message in time?

0/0



☒ 1 - $0.55(1 - R_{\text{bro}})$ ☐ $0.55R_{\text{bro}}$ ☐ $0.55(1 - R_{\text{bro}})$ ☐ 0.45 ☐ $0.45R_{\text{bro}}$

Parallel => $F_p = \prod_{i=1}^{n} F_i$ $(1-0.57)(1-0.45) = 0.2365 = F_p$

$R = 1 - F_p = 0.7635$

$(1 - R_{BRO})(0.55) = F_p$ $1 - F_p = R$

$Pr(T_1) = 0.53$
$Pr(T_0) = 0.47$

For the next three questions, a noisy transmission system transmits 0s and 1s. The probability of a transmitted 1 is 0.53. The receiver detects the transmitted bits with error so that $\Pr(R_0|T_1) = 0.04$ and $\Pr(R_1|T_0) = 0.09$.

Question 12 Find the probability that a transmitted 0 will be detected as a 0 at the receiver.

$Pr(R_0|T_1) = 0.04$
$Pr(R_1|T_1) = 0.96$

3/3

☐ 0.953 ☒ 0.910 ☐ 0.040 ☐ 0.090 ☐ 0.960

$Pr(R_1|T_0) = 0.09$
$Pr(R_0|T_0) = 0.91$

$Pr(R_0|T_0) = 1 - Pr(R_1|T_0) = 1 - 0.09 = \boxed{0.91}$

Question 13 Find the probability that a 0 is detected at the receiver.

3/3

☐ 0.910 ☐ 0.500 ☒ 0.449 ☐ 0.040 ☐ 0.470

$Pr(R_0) = Pr(R_0|T_0)Pr(T_0) + Pr(R_0|T_1)Pr(T_1)$

$(0.91)(0.47) + (0.04)(0.53) = \boxed{0.4489}$

Question 14 Find the probability that a detected 0 at the receiver was actually transmitted as a 1, expressed in terms of available information. (Assume that $\Pr(R_0)$ has been calculated and is available.)

3/3

☐ $\frac{[1-\Pr(R_0|T_1)]\Pr(T_1)}{\Pr(R_0)}$ ☐ $\frac{\Pr(R_0|T_1)[1-\Pr(R_1)]}{1-\Pr(T_1)}$ ☒ $\frac{\Pr(R_0|T_1)\Pr(T_1)}{\Pr(R_0)}$ ☐ $\Pr(T_1)\Pr(R_0)$ ☐ $\frac{[1-\Pr(R_1|T_0)][1-\Pr(T_1)]}{\Pr(R_0)}$

A B

$Pr(T_1|R_0) = \frac{Pr(R_0|T_1)Pr(T_1)}{Pr(R_0)} =$

For the next three questions, consider the following distribution function.

$$F_X(x) = \begin{cases} 0 & x \le 0 \\ \frac{x}{2} & 0 < x \le 1 \\ \frac{1}{2} + \frac{1}{2}(1 - e^{-\frac{x-1}{3}}) & x > 1 \end{cases}$$

Question 15 Find $\Pr(\frac{1}{2} < X \le 2)$.

8/8

0 1 2 3 4 5 6 7 8 *reserved for instructor*

$F_X(2) - F_X(\frac{1}{2}) \rightarrow \frac{1}{2} + \frac{1}{2}(1 - e^{-\frac{x-1}{3}}) - \frac{x}{2}$

$\frac{1}{2} + \frac{1}{2}(1 - e^{-(\frac{2-1}{3})}) - \frac{1/2}{2}$

$\frac{1}{2} + \frac{1}{2}(1 - e^{-1/3}) - \frac{1}{4} = \boxed{0.3917}$

Question 16 Find an expression for the density function, and sketch and label.

7/8

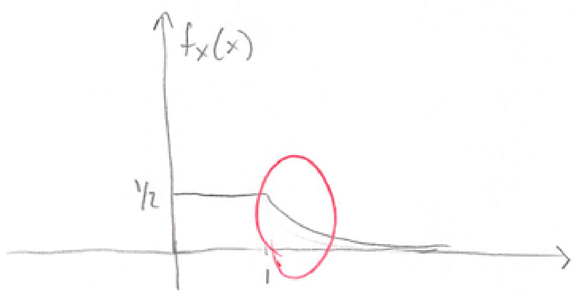
0 1 2 3 4 5 6 7 8 *reserved for instructor*

$f_X(x) = \frac{d}{dx} F_X(x)$

$$f_X(x) = \begin{cases} 0 & x \le 0 \\ 1/2 & 0 < x \le 1 \\ \frac{1}{6} e^{-(\frac{x-1}{3})} & x > 1 \end{cases}$$

$-\frac{1}{2} e^{-\frac{(x-1)}{3}}$

$\frac{1}{6} e^{-\frac{(x-1)}{3}}$



Question 17 Find the conditional distribution function for $X \le 2$.

5/8

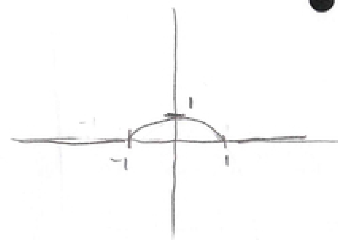
0 1 2 3 4 5 6 7 8 *reserved for instructor*

$F(x|M) = \frac{\Pr(X \le x, M)}{\Pr(M)} = \frac{\Pr(X \le x | X \le 2)}{\Pr(X \le 2)} = \frac{\Pr(X \le 2)}{\Pr(X \le 2)} = \boxed{1}$

$x > M = 1$

$x < M = \frac{F_X}{F_M}$

only $x > 2$



Use the following density function in the five problems below.

$$f_X(x) = \begin{cases} 0 & x \leq -1 \\ A(1-x^2) & -1 < x \leq 1 \\ 0 & x > 1 \end{cases}$$

Question 18 Find A that makes the density function above valid.

0/3

☒ $\frac{3}{4}$ ☐ $\frac{2}{3}$ ☐ $\frac{3}{2}$ ■ 1 ☐ $\frac{4}{3}$

$A(1-x^2) = 1$ $\quad -1 < x \leq 1$

$A(1-0^2) = 1$ $\quad x = 0$

$\boxed{A = 1}$

Question 19 Find $\Pr(0 < X \leq 0.5)$ in terms of A.

3/3

☐ $\frac{2}{3}A$ ☐ $2A$ ☐ $\frac{1}{2}A$ ☒ $\frac{11}{24}A$ ☐ A

$$\int_0^{0.5} A(1-x^2) \Rightarrow x - \frac{x^3}{3}\Big|_0^{0.5} = 0.4583\,A$$

$$= \boxed{\frac{11}{24}A}$$

Question 20 Find $\bar{X}$.

8/8

☐0 ☐1 ☐2 ☐3 ☐4 ☐5 ☐6 ☐7 ■8 *reserved for instructor*

$$\bar{X} = \int_{-\infty}^{\infty} x f_x(x)dx = \int_{-1}^{1} xA(1-x^2)dx = A\int_{-1}^{1} x - x^3 dx = \left(\frac{x^2}{2} - \frac{x^4}{4}\right)\Big|_{-1}^{1} \quad \left(\frac{1}{2} - \frac{1}{4}\right) - \left(\frac{1}{2} - \frac{1}{4}\right)$$

$$\frac{1}{2} - \frac{1}{4} - \frac{1}{2} + \frac{1}{4} = 0$$

$$\boxed{\bar{X} = 0}$$

Question 21 For the random variable whose density is defined above, $\int_{-\infty}^{\infty} x^2 f_X(x) = \frac{4}{15}A$. Find the standard deviation of X in terms of A.

☐0 ☐1 ☐2 ☐3 ■4 *reserved for instructor* 4/4

$\overline{X^2} = \frac{4}{15}A$ $\quad$ Variance $= \sigma^2 = \overline{X^2} - (\bar{X})^2$

Standard Deviation $= \sigma = \sqrt{\sigma^2}$

$$\sigma^2 = \overline{x^2} - (\bar{x})^2$$

$$\boxed{\sigma = \sqrt{\overline{x^2} - (\bar{x})^2}}$$

$$\sigma = \sqrt{\frac{4}{15}A - 0^2} = \sqrt{\frac{4}{15}A} = \sqrt{\frac{4}{15}} = \boxed{\sigma = 0.5164}$$

Question 22 If $y = \sqrt{x+2}$, find $f_Y(y)$.

☐0 ☐1 ☐2 ☐3 ☐4 ☐5 ☐6 ■7 ☐8 *reserved for instructor*

7/8

$f_Y(y) = \left|\frac{dx}{dy}\right| f_X(x)$

$f_Y(y) = 2\sqrt{x+2}\,(1-x^2) \quad -1 < x \leq 1$

$y = \sqrt{x+2}$

$x = y^2 - 2$

$\left|\frac{dx}{dy}\right| = \frac{1}{\frac{dy}{dx}} = \frac{1}{\frac{1}{2\sqrt{x+2}}} = 2\sqrt{x+2}$

$f_Y(y) = 2\sqrt{y^2-2+2}\,(1-(y^2-2)^2)$

$2\sqrt{y^2}\,(1-(y^2-2)^2$

$\frac{dy}{dx} = \frac{d}{dx}(x+2)^{1/2}$

$= \frac{1}{2}(x+2)^{-1/2}(1)$

$= \frac{1}{2\sqrt{x+2}}$

limits

$y = \sqrt{-1+2} = \sqrt{1} = 1$

$y = \sqrt{1+2} = \sqrt{3}$

$f_Y(y) = 2y(1-(y^2-2)^2, \quad 1 < y \leq \sqrt{3}$

Question 23 A space probe transmitter must last at least 5 years to justify the cost. Immediately after launch it is discovered that the space probe transmitter will interfere with another probe scheduled to be launched in 15 years. If the mean time to failure of the transmitter is 10 years, what is the probability that it will transmit at least 5 years but no more than 15 years?

☐0 ☐1 ☐2 ☐3 ☐4 ☐5 ☐6 ☐7 ■8 *reserved for instructor*

8/8

MTF = 10yrs

$Pr(5 < T < 15) \rightarrow \int_5^{15} \frac{1}{10} e^{\frac{-t}{10}}\,dt = -e^{\frac{-t}{10}}\Big|_5^{15} \quad -e^{\frac{-15}{10}} - (-e^{\frac{-5}{10}})$

$= -e^{-\frac{3}{2}} + e^{-1/2} = \boxed{0.3834}$