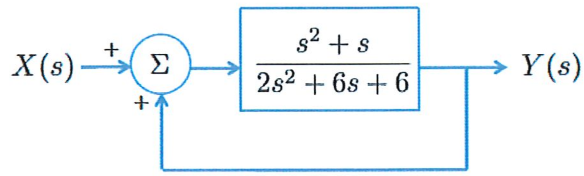
ELEC 2120 Name:________________________________

Test #2: 10/26/20, 9:50 am – 11:00 am

Open-book test. Formula sheet provided on Canvas.

1. (20 points) (a) Find the transfer function $H(s) = \dfrac{Y(s)}{X(s)}$ of the following feedback system:



$$[X(s) + Y(s)] \frac{s^2+s}{2s^2+6s+6} = Y(s)$$

$$\therefore \quad X(s) \frac{s^2+s}{2s^2+6s+6} = Y(s) \left[1 - \frac{s^2+s}{2s^2+6s+6}\right].$$

$$\therefore \quad X(s) \frac{s^2+s}{2s^2+6s+6} = Y(s) \frac{s^2+5s+6}{2s^2+6s+6}$$

$$\therefore \quad X(s)\,(s^2+s) = Y(s)\,(s^2+5s+6)$$

$$\therefore \quad H(s) = \frac{Y(s)}{X(s)} = \frac{s^2+s}{s^2+6s+6}$$

(b) Find the zeros and poles of $H(s)$

$$H(s) = \frac{s(s+1)}{(s+2)(s+3)}.$$

zeros: $s = 0$, $s = -1$.

poles: $s = -2$ $s = -3$.

2. (20 points) Given $x(t) = e^{-t}u(t)$, find $\mathscr{L}\{x(t-2)\}$ and $\mathscr{L}\{t\,x(t-2)\}$.

Hint: Because $u(t-T)\,u(t-T) = u(t-T)$, we can get $x(t-2)u(t-2) = x(t-2)$.

$$X(s) = \mathscr{L}\{x(t)\} = \frac{1}{s+1}$$

$$\mathscr{L}\{x(t-2)\} = e^{-2s} X(s) = e^{-2s}\frac{1}{s+1}.$$

$$\mathscr{L}\{t\,x(t-2)\} = -\frac{d\,\cancel{X(s)}\,\mathscr{L}\{x(t-2)\}}{ds}$$

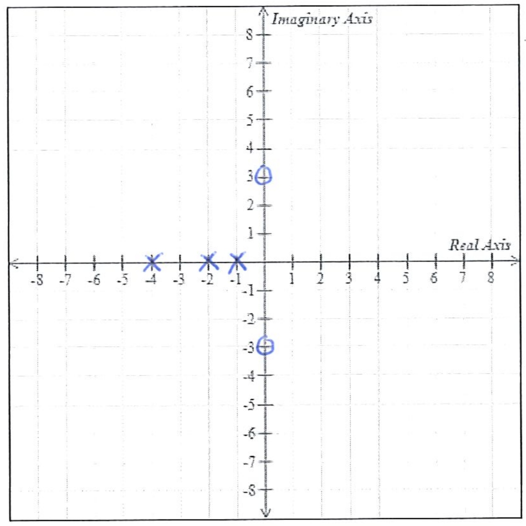
$$= -\frac{d}{ds}\left(e^{-2s}\frac{1}{s+1}\right)$$

$$= -\left[-2e^{-2s}\frac{1}{s+1} + e^{-2s}\cdot -\frac{1}{(s+1)^2}\right]$$

$$= 2e^{-2s}\frac{1}{s+1} + \frac{e^{-2s}}{(s+1)^2}$$

3. (20 points) Consider $X(s) = \dfrac{s^2+9}{2(s+1)(s^2+6s+8)}$

(a) Indicate the poles and zeros for $X(s)$ on the graph provided



(b) Does the initial value $x(0^+)$ exist? ________
If not, why not?

If so, calculate $x(0^+)$ and record it here:

$x(0^+) =$ ______________

(c) Does the final value $x(\infty)$ exist? ________
If not, why not?

If so, calculate $x(\infty)$ and record it here:

$x(\infty) =$ ______________

a) poles: $s=-1$, $s=-2$, $s=-4$

zeros: $s=\pm 3j$.

b) $x(0^+) = \lim_{s\to\infty} sX(s)$

$= \lim_{s\to\infty} \dfrac{s(s^2+9)}{2(s+1)(s^2+6s+8)} = \dfrac{1}{2}$.

c) $x(\infty)$ exists

$x(\infty) = \lim_{s\to 0} sX(s)$

$= \lim_{s\to 0} \dfrac{s(s^2+9)}{2(s+1)(s^2+6s+8)} = 0$.

4. (20 points) The transfer function of a linear and time-invariant system is

$$H(s) = \frac{2s+6}{s^2+4s+8}$$

(a) Compute the impulse response $h(t)$ of this system.

$$h(t) = \mathcal{L}^{-1}\{H(s)\}.$$

$$H(s) = 2\frac{s+2}{(s+2)^2+2^2} + \frac{2}{(s+2)^2+2^2}$$

$$\therefore\ h(t) = 2e^{-2t}\cos(2t)u(t) + e^{-2t}\sin(2t)u(t).$$

(b) If the input of this system is $x(t) = e^{-3t}u(t)$, compute the output $y(t)$ of this system.

$$X(s) = \frac{1}{s+3}.$$

$$Y(s) = H(s)X(s)$$

$$= \frac{2s+6}{s^2+4s+8} \cdot X(s)$$

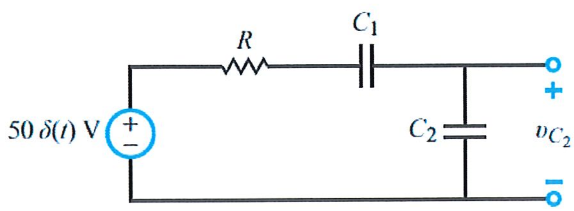
$$= \frac{2s+6}{s^2+4s+8} \quad \frac{1}{s+3}$$

$$= \frac{2}{s^2+4s+8}$$

$$= \frac{2}{(s+2)^2+2^2}$$

$$y(t) = \mathcal{L}^{-1}\{Y(s)\} = e^{-2t}\sin(2t)u(t).$$

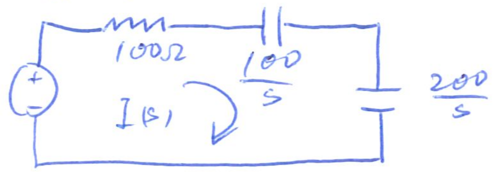
5. (20 points) Determine $v_{C_2}(t)$ in the circuit below, given that $R = 100\ \Omega,\ C_1 = 0.01\ F$, and $C_2 = 0.005\ F$.



$V_{C_1}(0^-) = 0.$

$V_{C_2}(0^-) = 0$.

$V_{in}(s) = 50$



$V_{C_2}(s)$.

$$V_{in}(s) = I(s)\left(100 + \frac{100}{s} + \frac{200}{s}\right)$$

$$V_{C_2}(s) = I(s)\,\frac{200}{s}$$

$$\therefore V_{C_2}(s) = \frac{V_{in}(s)\,\frac{200}{s}}{100 + \frac{100}{s} + \frac{200}{s}}$$

$$= \frac{2V_{in}(s)}{s+3}$$

$$= \frac{100}{s+3}$$

$$\therefore V_{C_2}(t) = \mathcal{L}^{-1}\{V_{C_2}(s)\} = 100\,e^{-3t}\,u(t) \quad V$$