ORDERED PHASE SEQUENCE TESTING IN SLM FOR IMPROVED BLIND DETECTION

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ABSTRACT

Selected mapping (SLM) is a promising technique to reduce the high peak-to-average power ratio (PAR) of orthogonal frequency division multiplexing (OFDM) signals. In this paper we present an altered form of SLM that tests phase sequences in an ordered manner and ceases testing when the PAR is below a threshold. Aside from having a reduced computational complexity compared to traditional SLM, our modification allows for maximum *a posteriori* (MAP) blind detection of SLM phase mappings in the receiver. MAP detection is possible because, by testing the phase sequences in an ordered way, different phase mappings have different probabilities of occurrence. We derive the MAP metric as well as a lower-complexity approximation to the MAP metric.

1. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) is a popular transmission method in digital communications today. It is the method of choice in many standards including 802.11a, 802.11g, 802.16, HIPERLAN 2, Digital Audio Broadcast (DAB), and Digital Video Broadcast (DVB). By partitioning a wideband fading channel into flat narrowband channels, OFDM is able to mitigate the detrimental effects of multipath interference, while maintaining a high spectral efficiency. However, the prices paid for these advantages are low power efficiencies and occasional clipping errors, which are due to the high peak-to-average power ratio (PAR) exhibited by OFDM signals.

A discrete time OFDM symbol is given by the inverse discrete Fourier transform (IDFT) of the frequency domain signal, or

$$x[n] = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X[k]\, e^{j\frac{2\pi k n}{N}}, \quad 0 \le n \le N-1, \quad (1)$$

where $\{X[k]\}_{k=0}^{N-1} \triangleq \boldsymbol{X}$ is the frequency domain sequence drawn from a known constellation, $\{x[n]\}_{n=0}^{N-1} \triangleq \boldsymbol{x}$ is the discrete-time-domain sequence, and N is the number of subcarriers. For the analysis in this paper we ignore the cyclic prefix that is present in practical OFDM systems because it does not contribute to the PAR. The PAR of a discrete-time OFDM symbol is defined as

$$\mathrm{PAR}\{x[n]\} = \frac{\max\limits_{0\le n\le N-1} |x[n]|^2}{E[|x[n]|^2]}. \quad (2)$$

There have been many PAR reduction schemes proposed. One type of approach is to modify the time-domain signal with a distortion method such as clipping [1] or filtered clipping [2]. However, these distortion techniques are of limited use because they can induce distortion errors and/or spectral regrowth. Another type of approach uses a distortionless reversible transform to lower the PAR. Coding approaches that only transmit low-PAR OFDM symbols fall into this category [3]. Another more popular distortionless approach is to transform frequency domain signal in a way that reduces the PAR of the time-domain signal. SLM [4], tone injection [5], tone reservation [5], constellation extension [6] and partial transmit sequence [7] are all distortionless PAR reduction techniques that manipulate the signal in the frequency domain prior to the IDFT. Some require side information to be transmitted, others have a lower data rate compared to traditional OFDM and all of these methods require additional computational resources to execute.

In Section 2, we describe SLM. In Section 3, we discuss how a phase sequence mapping in an SLM system can be detected without side information. In Section 4, we describe a practical system and how such a system is affected by PAR reduction. We then use this perspective to show that it is not necessary to test SLM phase sequence mappings past a certain point. As a result, different phase sequence mappings have different probabilities of occurrence. In Section 5, we derive the MAP criterion for blindly (without side information) determining which phase sequence mapping was used in the receiver. In Section 6, we so how the MAP criterion affects the bit error rate. Finally, conclusions are drawn in Section 7.

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2. SELECTED MAPPING

Selected mapping (SLM) [4] is a distortionless, low complexity PAR reduction technique that works by randomly phasing the constellation points in order to lower the time domain peaks. By only adjusting the phase of the constellation points, the mean power of the signal is unaffected.

Specifically, in a SLM system, D phase rotation sequences $\{\varphi^{(d)}[k]\}_{k=0}^{N-1} \triangleq \boldsymbol{\varphi}^{(d)}$ are applied to the frequency-domain symbols $\boldsymbol{X}$ to create D new sets of frequency-domain symbols $\boldsymbol{X}^{(d)} \triangleq \boldsymbol{X} \cdot e^{j\boldsymbol{\varphi}^{(d)}}$, where $\cdot$ denotes element-wise multiplication. Each of the D rotation sequences are independent of each other and the phases are all uniformly distributed on $[0, 2\pi)$. Define $\boldsymbol{x}^{(d)} \triangleq \mathrm{IDFT}\{\boldsymbol{X}^{(d)}\}$. For transmission, the PAR of each $\{\boldsymbol{x}^{(d)}\}_{d=1}^{D}$ is computed and the lowest PAR signal is transmitted. We denote the transmitted signal by $\boldsymbol{x}^{(\tilde{d})}$ where

$$\tilde{d} = \underset{1 \le d \le D}{\arg\min}\ \mathrm{PAR}\{\boldsymbol{x}^{(d)}\}.$$

The D phase sequences are assumed to be known by the receiver. The receiver must determine which of the D sequences was used in the transmitter. This can either be done with side information or, as we investigate in this paper, blindly. Once $\tilde{d}$ is detected by the receiver, the transmitted frequency domain data can be recovered by simply derotating the received frequency-domain signal phases by $-\boldsymbol{\varphi}^{(\tilde{d})}$.

As was mentioned in the last section, SLM does require additional computational resources, which means more power usage. This was studied in [8] and it was shown that the additional power consumption is negligible compared to the power savings through PAR reduction. Also, in [9], the effect of latency in a SLM system was studied and a queuing architecture was proposed that mitigates latency problems.

3. BLIND SEQUENCE DETECTION

Blind selected mapping (BSLM) was first mentioned in [4] and studied in [10]. Additionally, a novel method was proposed in [11] that integrates channel sounding and blind phase sequence detection in OFDM. That method is very effective at blindly detecting a small number of BSLM phase sequences. However, when the channel is stationary (DSL, immobile wireless links, etc) and channel sounding is not necessary for every symbol period, it is desirable to leave out the pilot tones in order to increase data rate. This stationary channel case is what the analysis in this paper applies to.

For sequence detection in BSLM it is assumed that both the transmitter and receiver have the set of possible phase sequences. The basic idea is that the receiver takes an IDFT of the received baseband signal and then uses its set of phase sequences along with some sort of metric to determine which sequence the transmitter used. Traditionally, it is assumed that the receiver "derotates" the received symbols and compares each derotation to the symbol constellation $\mathcal{C}$. However, it is convenient in the derivation of the detection metric to use a different, but equivalent, method.

Instead of derotating the received frequency-domain signal, assume that the received signal is compared to a rotated constellation. We will denote a rotated constellation sequence by $\boldsymbol{\mathcal{C}}^{(d)} = \{\mathcal{C}^{(d)}[k]\}_{k=0}^{N-1} \triangleq \mathcal{C} \cdot e^{j\boldsymbol{\varphi}^{(d)}}$. Notice that the constellation varies with k because the phase sequence varies with k. In other words, the receiver has to determine which of the D constellation *sequences* was used for transmission. Once the constellation is known for each subcarrier (i.e. the constellation sequence is known), then classic decoding methods can be used to determine what data was sent.

4. ORDERED PHASE-SEQUENCE TESTING

Let us consider a practical transmission system and the role that PAR reduction plays in it. We consider the case where the power amplifier (PA) is not adaptively biased, which means that the system is designed with a certain constant clipping threshold. For example, class A amplifiers require a bias power that is twice the clipping threshold [12].

The goal of PAR reduction is to increase power efficiency while keeping the probability of clipping at an acceptably low level. Clipping needs to be minimized because it is a distorting operation that increases the error rate. Accordingly, let us quantify clipping in a transmission system by two parameters: the (power) clipping level, γ_o, and the probability of clipping, Pr(PAR$>\gamma_o$) (assume the average power is normalized to one). For instance, a system with $\gamma_o = 7$dB that is able to tolerate at most 1 clipped OFDM block in 10,000 would have Pr(PAR$>\gamma_o$) $\triangleq p = 10^{-4}$.

Because the PA is not adaptively biased the power efficiency of the system is determined exclusively by the average symbol power ($E[|x[n]|^2]$) and the bias point of the PA, neither of which is changed dynamically. The point is that once the system is designed for a certain PAR, any further PAR reduction is pointless.

Figure 1 is an illustration of this concept through the complementary cumulative distribution function (CCDF) curves. Notice that all of the curves pass through the point (7dB, 10^{-4}), but they all correspond to different distributions. For a system designed around these parameters ($\gamma_o = 7$dB, $p = 10^{-4}$), each of the curves has the same power efficiency and clipping probability. Therefore, if some modification that increases the blind detection rate also changes the shape of the CCDF from one curve to another in Fig. 1, the system will not be sacrificing any PAR reduction performance.

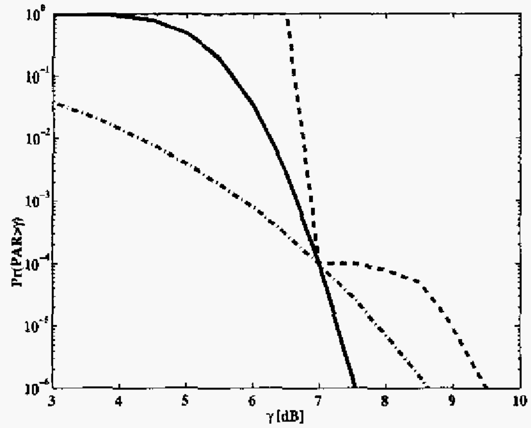

Figure 1: Different CCDFs that all pass through the point (7dB, 10^{-4}). We show that in a practical system designed for a probability of clipping of 10^{-4} and clipping level of 7dB, these curves all have the same power efficiency performance.

Thus, a SLM system only has to test phase sequences until a signal with a PAR$<\gamma_o$ is found. It is possible that further phase mappings could produce an even lower PAR signal, but we just established that any further PAR reduction will not further improve the power efficiency for a given PA with a fixed bias and probability of clipping. It is also possible that after $D_{\max}$ mappings the PAR is still larger than the clipping level. In this case the signal is simply sent despite the fact that it will be clipped in the PA. Note, however, that $D_{\max}$ is chosen to ensure that the probability of clipping is kept to the specified level.

It has been shown in [4] that the CCDF of the PAR in a Nyquist sampled OFDM signal is

$$\Pr(PAR_{SLM} > \gamma) = \left(1 - (1 - e^{-\gamma})^N\right)^D, \qquad (3)$$

where N is the number of subcarriers and D is the number of independent phase mappings. For future brevity we will define $\Gamma^N \triangleq (1 - e^{-\gamma})^N$ and $\Gamma_o^N \triangleq (1 - e^{-\gamma_o})^N$.

If the SLM process stops when PAR$<\gamma_o$ then we can solve for $D_{\max}$ to guarantee that Pr(PAR$>\gamma_o) \leq p$. In general, there are three system parameters, $(p, \gamma_o, D_{\max})$, any two of which determine the third.

With $D_{\max}$ and γ_o determined, the probability of selecting the d^{th} mapping becomes

$$\begin{aligned}\Pr(d) &= \Pr(\text{PAR} > \gamma_o)^{d-1} - \Pr(\text{PAR} > \gamma_o)^d \\ &= \begin{cases} \Gamma_o^N\left(1 - \Gamma_o^N\right)^{d-1}, & 0 < d < D_{\max}, \\ \left(1 - \Gamma_o^N\right)^{D_{\max}}, & d = D_{\max}. \end{cases} \qquad (4)\end{aligned}$$

Because the mappings are performed in the same order every time, the expression in (4) is the probability that the d^{th} phase sequence was used in transmission [13]. We will show in the next section how this analysis leads to an improved sequence detection performance.

5. MAXIMUM A POSTERIORI PROBABILITY OF DETECTION

As opposed to maximum likelihood (ML) detection, maximum *a posteriori* (MAP) detection does not assume that all the phase sequences have the same probability of occurrence. In a MAP receiver, the correct phase sequence index, $\tilde{d}$ is the one that maximizes the probability that the d^{th} constellation sequence, $\mathcal{C}^{(d)}$, was used in transmission given the received sequence, $\mathbf{R}$, or

$$\begin{aligned}\tilde{d}^{\,(\text{MAP})} &= \underset{1 \leq d \leq D_{\max}}{\arg\max} \Pr[\mathcal{C}^{(d)}|\mathbf{R}] \\ &= \underset{1 \leq d \leq D_{\max}}{\arg\max} \frac{\Pr[\mathbf{R}|\mathcal{C}^{(d)}]\Pr[\mathcal{C}^{(d)}]}{\sum_{m=1}^{D_{\max}} \Pr[\mathbf{R}|\mathcal{C}^{(m)}]\Pr[\mathcal{C}^{(m)}]} \qquad (5) \\ &= \underset{1 \leq d \leq D_{\max}}{\arg\max} \Pr[\mathbf{R}|\mathcal{C}^{(d)}]\Pr[\mathcal{C}^{(d)}]. \qquad (6)\end{aligned}$$

Note that the denominator in (5) is irrelevant because it does not depend on d. If we had used the conventional derotation receiver, then the denominator would have a d dependence and would have to be included in the formulation of the metric.

In an AWGN channel the joint conditional probability density function (pdf), $f_{R[0],\ldots,R[N-1]|\mathcal{C}^{(d)}}(r_0, \ldots, r_{N-1})$, is

$$f_{\mathbf{R}|\mathcal{C}^{(d)}}(\mathbf{r}) = \frac{1}{Q^N} \prod_{k=0}^{N-1} \sum_{q=0}^{Q-1} \frac{1}{\pi N_o} e^{-\|r[k] - \hat{c}_q^{(d)}[k]\|^2/N_o}, \quad (7)$$

where $N_o/2$ is the noise power, $\{\hat{c}_q^{(d)}[k] \in \mathcal{C}^{(d)}[k]\}_{k=0}^{N-1}$, and $\|\cdot\|$ represents the Euclidean distance in the complex plane. We can substitute $\Pr(d)$ for $\Pr[\mathcal{C}^{(d)}]$ and $f_{\mathbf{R}|\mathcal{C}^{(d)}}(\mathbf{r})$ for $\Pr[\mathbf{R}|\mathcal{C}^{(d)}]$ in (6) to obtain

$$\tilde{d}^{\,(\text{MAP})} = \underset{1 \leq d \leq D_{\max}}{\arg\max} \Pr(d) \prod_{k=0}^{N-1} \sum_{q=0}^{Q-1} e^{-\|r[k] - \hat{c}_q^{(d)}[k]\|^2/N_o}, \qquad (8)$$

where $\Pr(d)$ is given in (4). Notice we were able to leave out the constant terms in $f_{\mathbf{R}|\mathcal{C}^{(d)}}(\mathbf{r})$ because of the arg max operation.

Observe that

$$\underset{1 \leq d \leq D_{\max}}{\arg\max} \ln\left[\sum_{q=0}^{Q-1} e^{-\|r[k] - \hat{c}_q^{(d)}[k]\|^2/N_o}\right] \approx \underset{\substack{1 \leq d \leq D_{\max} \\ 0 \leq q \leq Q-1}}{\arg\min} \|r[k] - \hat{c}_q^{(d)}[k]\|, \quad (9)$$

which is true because the constellation point used in the transmitter, $\hat{c}_{\tilde{q}}^{(\tilde{d})}[k]$, dominates the sum over Q on the left hand side of (9). This approximation is very convenient because it significantly reduces the computational complexity of the metric. Using this approximation, the suboptimal

MAP (AMAP) criterion becomes

$$\tilde{d}^{(\mathrm{AMAP})} = \arg\max_{1 \le d \le D_{\max}} \left[\ln[\Pr(d)] - \frac{1}{N_o} \sum_{k=0}^{N-1} \min_{0 \le q \le Q-1} \|r[k] - \hat{c}_q^{(d)}[k]\|^2 \right], \quad (10)$$

where $\Pr(d)$ is given in (4).

The expression in (10) requires D times as many $\|\cdot\|^2$ operations as non-SLM ML decoding. Also, unlike standard ML decoding, a minimum operation is necessary instead of an argument minimum operation. Finally, it is necessary to estimate the noise power because a factor of $1/N_o$ is used in the criterion.

6. SIMULATION

Figure 2 is a plot of the bit error rate (BER) in an AWGN channel with a hard limiter 10 dB above the average signal power. The plot illustrates how different coding gains can affect the BER in a BSLM system. For the uncoded case the BER is dominated by errors in constellation point detection which means that the ML and MAP curves coincide. The OFDM without SLM curve is higher because of clipping errors incurred by the hard limiter. When a coding gain of 6 dB is used we can see a distinct difference between the three curves. Specifically, we can see that the MAP criterion has about a 0.8 dB advantage over the ML criterion at the 10^{-6} BER level.

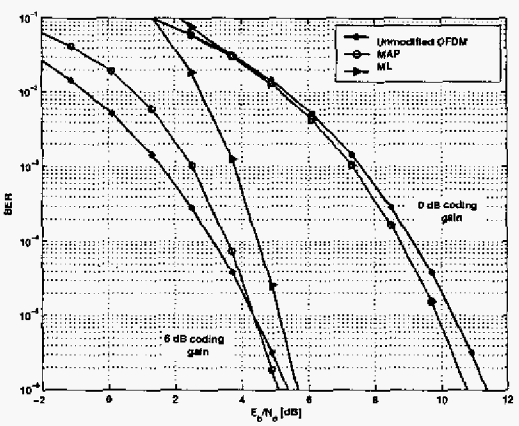

Figure 2: Bit error rate plot for BSLM in an AWGN channel with a hard limiter and 4-QAM modulation. The leftmost set of lines correspond to a system with a 6 dB coding gain, while the lines on the right correspond to the uncoded case. For each case there is a plot of BER for the unmodified OFDM, detection with the ML criterion, and detection with the MAP criterion. Plot parameters: $N = 64$, $\gamma_o = 10$ dB, $p = 10^{-7}$ and $D_{\max} = 3$.

7. CONCLUSIONS

In this paper we have presented the MAP criterion for detecting phase sequences in BSLM. In deriving this MAP criterion it was necessary to define several details about how the transmitter and receiver operate. Specifically, the transmitter tests phase mappings in an ordered way so that it only uses as many mappings as necessary to get below the clipping level, and the receiver compares the received symbols to a rotated constellation. The first specification ensures that each mapping has a different probability, while the second allows for a simplified MAP metric.

8. REFERENCES

[1] D. Kim and G. L. Stuber, "Clipping noise mitigation for OFDM by decision-aided reconstruction," *IEEE Communications Letters*, vol. 3, no. 1, pp. 4-6, Jan. 1999.

[2] J. Armstrong, "Peak-to-average reduction for OFDM by repeated clipping and frequency domain filtering," *IEE Electronics Letters*, vol. 38, pp. 246-247, May 2002.

[3] A. E. Jones, T. A. Wilkinson, and S. K. Barton, "Block coding scheme for reduction of peak to mean envelope power ratio of multicarrier transmission scheme," *IEE Electronics Letters*, vol. 30, no. 25, pp. 2098-2099, Dec. 1994.

[4] R. W. Bauml, R. F. H. Fischer and J. B. Huber, "Reducing the peak-to-average power ratio of multicarrier modulation by selected mapping," *IEE Electronics Letters*, vol. 32, no. 22, pp. 2056-2057, Oct. 1996.

[5] J. Tellado, *Multicarrier Modulation with Low PAR: Applications to DSL and Wireless*, Kluwer Academic Publishers, 2000.

[6] B. S. Krongold and D. L. Jones, "PAR reduction in OFDM via active constellation extension," *IEEE Transactions on Broadcasting*, vol. 49, no. 3, pp. 258-268, Sept. 2003.

[7] S. H. Mller and J. B. Huber, "OFDM with reduced peak-to-average power ratio by optimum combination of partial transmit sequences," *IEE Electronics Letters*, vol. 33, no. 5, pp. 368-369, Feb. 1997.

[8] R. J. Baxley and G. T. Zhou, "Power savings analysis of peak-to-average power ratio reduction in OFDM," *IEEE Trans. on Consumer Electronics*, vol. 50, no. 3, pp. 792-798, Aug. 2004.

[9] H. Qian, C. Xiao, N. Chen, and G. T. Zhou, "Dynamic selected mapping for OFDM," *Proc. IEEE Intl. Conference on Acoustics, Speech, and Signal Processing*, Philadelphia, PA, March 2005.

[10] A. D. S. Jayalath and C. Tellambura, "A blind SLM receiver for PAR-reduced OFDM," *Proc. IEEE Vehicular Technology Conference*, pp. 219-222, Sept. 2002.

[11] N. Chen and G. T. Zhou, "Peak-to-average power ratio reduction in OFDM with blind selected pilot tone modulation," *Proc. IEEE Intl. Conference on Acoustics, Speech, and Signal Processing*, Philadelphia, PA, March 2005.

[12] S. C. Cripps, *RF Power Amplifiers for Wireless Communications*. Norwood, MA: Artech House, 1999.

[13] R. J. Baxley and G. T. Zhou, "Assessing peak-to-average power ratios for communications applications," *Proc. IEEE MILCOM Conference*, Monterey, CA, October 31 - November 3, 2004.