TIME-DOMAIN IMPULSE INJECTION METHOD FOR CREST FACTOR REDUCTION OF OFDM SIGNALS

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ABSTRACT

This paper treats the problem of crest factor reduction (CFR) for orthogonal frequency division multiplexing (OFDM) signals. The proposed impulse injection method reduces the crest factor of the time-domain signal by injecting judiciously designed complex-valued impulse in selected discrete-time locations while keeping both the in-band error vector magnitude (EVM) and out-of-band spectral emission under control. Numerical examples are provided to demonstrate the effectiveness of the proposed method.

Index Terms— Crest factor reduction (CFR), orthogonal frequency division multiplexing (OFDM), error vector magnitude (EVM), spectral emission mask

1. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) is a widely adopted modulation technique for wireless communications because of its high spectral efficiency and robustness against frequency-selective fading channels. However, the OFDM waveform usually suffers from high crest factor (CF). The high CF nature of OFDM often calls for oversizing of RF front-end components such as mixer, digital-to-analog converter (DAC), input power amplifier (PA), etc, in order to avoid non-linear distortion. For example, the PA has to operate with a large input back-off to avoid non-linear distortion, yielding low power efficiency and low transmitted signal power.

In order to avoid modifications at the receiver side, various distortion-based crest factor reduction (CFR) algorithms have been developed to reduce the CF of OFDM waveforms at the transmitter side. Among distortion-based methods, the repeated clipping and filtering algorithm [1] first clips the time-domain signal, and then filters the clipped signal to control out-of-band emission; however, time-domain peak regrowth occurs during the filtering stage which is uncontrolled. In [2, 3], spectral shaped waveforms are generated to cancel the pre-existing peak while paying attention to the spectral mask. However, in-band error vector magnitude (EVM) is left uncontrolled in [1, 2, 3]. The works in [4, 5] adopt convex optimization to minimize the symbol-wise CF or symbol-wise EVM, but their computational complexity is high.

In this paper, we propose a novel OFDM CFR algorithm that has lower complexity than [4, 5], while still achieving near optimal CFR. Our proposed time-domain impulse injection method aims to reduce the CF of the time-domain input signal while ensuring that both in-band distortion and out-of-band spectral emission are kept below the specified limits.

2. SYSTEM MODEL

In an OFDM system, a discrete time-domain signal $\mathbf{y} = [y[0], y[1], \ldots, y[N-1]]$ is generated by applying inverse FFT (IFFT) operation to the frequency-domain signal:

$$y[n] = \text{IFFT}(Y_k) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} Y_k e^{j2\pi kn/N}, \qquad (1)$$

where $j = \sqrt{-1}$. To approximate the peak amplitude of the continuous time-domain signal, the IFFT is applied to the zero-padded frequency-domain signal to produce an over-sampled time-domain signal. The frequency domain signal $\mathbf{Y} = [Y_0, Y_1, \ldots, Y_{N-1}]$ is obtained by zero-padding the actual information bearing OFDM symbol $\mathbf{Y}^{(d)} = [Y^{(d)}_{-d/2}, \ldots, Y^{(d)}_{d/2-1}]$ with $(N-d)$ zeros. By zero-padding a length d vector to length N and then taking the IFFT, the effective over-sampling rate of the resulting time-domain signal is N/d. The crest factor of $\mathbf{y}$ is defined as

$$\text{CF}(\mathbf{y}) = \frac{\|\mathbf{y}\|_\infty}{\sqrt{\|\mathbf{y}\|_2^2/N}}, \qquad (2)$$

where $\|\cdot\|_q$ denotes the q-norm of the subject vector.

Distortion-based CFR techniques aim to reduce the CF of transmitted signals by deliberately introducing distortion into the signal, with the hope that the in-band EVM and out-of-band spectrum emission will meet the limits dictated by the communication standard. Let $\mathbf{Y}^\dagger = [Y_0^\dagger, Y_1^\dagger, \ldots, Y_{N-1}^\dagger]$ denote the N-length FFT of the modified time-domain signal $\mathbf{y}^\dagger$ whose CF will be smaller than that of $\mathbf{y}$. Block-wise EVM can be defined as

$$\epsilon(\mathbf{Y}, \mathbf{Y}^\dagger) = \sqrt{\frac{\sum_{k\in\Omega_i} |Y_k - Y_k^\dagger|^2}{dP_i}}, \qquad (3)$$

where Ω_i is the set of in-band subcarriers and has cardinality $|\Omega_i| = d$. P_i is the average power of the in-band subcarriers. The RMS EVM is defined as

$$\text{RMS EVM} = \sqrt{\mathcal{E}[\epsilon(\mathbf{Y}, \mathbf{Y}^\dagger)^2]}, \tag{4}$$

where $\mathcal{E}[\cdot]$ denotes statistical expectation. In order for $\mathbf{y}^\dagger$ to be transmitted, the above RMS EVM should be smaller than some prescribed threshold.

Let Ω_o denote the set of out-of-band frequencies with cardinality $|\Omega_o| = N - d$. The modified signal $\mathbf{y}^\dagger$ can be transmitted subject to the spectral emission mask constraint $\mathcal{M}_k$

$$E[|Y_k^\dagger|^2] \leq \mathcal{M}_k, \; k \in \Omega_o. \tag{5}$$

3. TIME-DOMAIN IMPULSE INJECTION METHOD

The most straightforward way to reduce the CF is by clipping. The envelope of the complex-valued time domain signal $y[n]$ is clipped to a maximum permissible amplitude threshold $A_{\max}$ so that the clipped signal

$$\bar{y}[n] = \begin{cases} y[n], & n \notin \mathcal{H}, \text{ i.e., } |y[n]| \leq A_{\max}, \\ \frac{A_{max}}{|y[n]|} y[n], & n \in \mathcal{H}, \text{ i.e., } |y[n]| > A_{\max}. \end{cases} \tag{6}$$

In (6), $\mathcal{H}$ is a set of $|\mathcal{H}| = H$ "high points" where $|y[n]| \geq A_{\max}$. If we write

$$\bar{y}[n] = y[n] + e[n], \tag{7}$$

we can also say that we have injected time-domain impulse sequence $e[n]$ into $y[n]$ to produce a distorted signal $\bar{y}[n]$ where $e[n] = \sum_{l \in \mathcal{H}} \rho_l \delta[n-l]$. $\delta[\cdot]$ is the Kronecker delta function and $\rho_l = \bar{y}[l] - y[l] = (A_{\max}/|y[l]| - 1)\, y[l]$. Obviously, the clipped signal $\bar{y}[n]$ satisfies $|\bar{y}[n]| \leq A_{\max}$ for all n. By taking N-length FFT on both sides of (7), we can obtain in the frequency-domain, $\bar{\mathbf{Y}} = \mathbf{Y} + \mathbf{E}$, where E_k, the kth subcarrier of $\mathbf{E}$, can be expressed in term of ρ_l as

$$E_k = \frac{1}{\sqrt{N}} \sum_{l \in \mathcal{H}} \rho_l e^{-j2\pi kl/N}, \tag{8}$$

We can see that any time-domain impulse value ρ_l affects all subcarriers in the frequency domain. The simple clipping will generate both in-band distortion and out-of-band spectral emission that are left uncontrolled.

In the simple clipping framework, both the impulse locations $l \in \mathcal{H}$ and the impulse values ρ_l of the distortion signal $e[n]$ are fixed per given signal $y[n]$. Instead of that, we design a distortion waveform

$$d[n] = \sum_{l \in \mathcal{H} \cup \mathcal{L} \cup \mathcal{S}} \gamma_l \delta[n-l], \tag{9}$$

and the corresponding modified signal

$$\tilde{y}[n] = y[n] + d[n]. \tag{10}$$

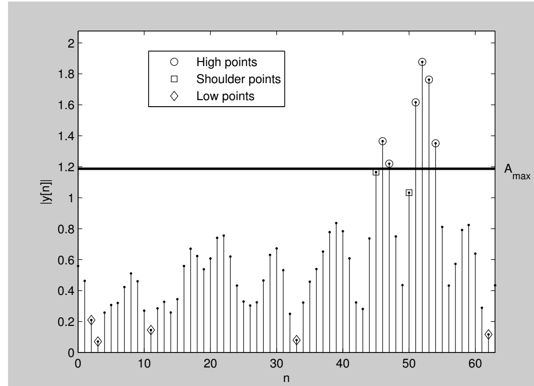

Fig. 1. Illustration of the concept of high, shoulder and low points, for $N = 64$, $A_{\max} = 1.19$ $(4.5dB)$, $L = 5$, and $\alpha = 0.85$.

In (9), the impulse values γ_l are complex-valued parameters to be optimized, and we allow additional degrees of freedom by including more impulse locations $l \in \mathcal{L}$ and $l \in \mathcal{S}$. Denote $L = |\mathcal{L}|$. We refer to the smallest L values of $|y[n]|$ as "low points", $n \in \mathcal{L}$. On the other hand, $l \in \mathcal{S}$ designate "shoulder points" where $|y[n]|_{n \in \mathcal{S}}$ have values that are between $\alpha A_{\max}$ and $A_{\max}$, with $0 < \alpha < 1$. The rationale for including the low points $\mathcal{L}$ is that they offer the most headroom or space to maneuver for optimization, so $|\tilde{y}[n]|_{n \in \mathcal{L}}$ will not exceed $A_{\max}$ easily. Next, we would like to give the rationale for including the shoulder points. Recall that in the clipping and filtering method [1], each time we lowpass filter the clipped signal to remove out-of-band emission, peak regrowth occurs and the most vulnerable for subsequent clipping are places like the shoulder points. Thus it seems intuitive that manipulation of the shoulder points can have direct consequence for out-of-band emission. The number of low points L and the value of α (which influences the size of shoulder points) are design parameters. The larger the L and/or the smaller the α, the more degrees of freedom and thus the better performance but the higher the computational load. Fig. 1 illustrates the concept of high points, shoulder points and low points for a 64-point sequence.

The objective of the impulse injection method is to find $N_s = H + L + S$ complex values for $d[l], l \in \mathcal{H} \cup \mathcal{L} \cup \mathcal{S}$ to ensure that $|\tilde{y}[n]| \leq A_{\max}, \forall n$, while still satisfying the other transmission requirements. Let $\mathbf{D}$ denote the N-length FFT of $\mathbf{d}$ and D_k denote the kth subcarrier, i.e.,

$$D_k = \frac{1}{\sqrt{N}} \sum_{l \in \mathcal{H} \cup \mathcal{L} \cup \mathcal{S}} \gamma_l e^{-j2\pi kl/N}. \tag{11}$$

Thus, $D_k, k \in \Omega_i$ represents in-band distortion. It has been studied in [5] that minimizing the block-wise EVM is better suited for the objectives laid out in the communications standards. Accordingly, in this paper, the block-wise in-band distortion is minimized. Assume the original signal $\mathbf{y}$ has no energy in the out-of-band subcarriers, then $D_k, k \in \Omega_o$ represents the out-of-band content. To satisfy the spectral emission mask constraint in (5), we have

$$|D_k| \leq \sqrt{\mathcal{M}_k}, \quad k \in \Omega_o. \tag{12}$$

We formulate the block-wise in-band distortion minimization problem as follows:

$$\begin{aligned}
\underset{\mathbf{d}}{\text{minimize}} \quad & g \\
\text{subject to} \quad & \text{Amplitude constraint} \\
& |y[l] + d[l]| \leq A_{\max}, \quad l \in \mathcal{H} \cup \mathcal{L} \cup \mathcal{S} \\
& \text{In-band error constraint} \\
& \sum_{k \in \Omega_i} |D_k|^2 \leq g \\
& \text{Out-of-band spectral constraint} \\
& |D_k| \leq \sqrt{\mathcal{M}_k}, \quad k \in \Omega_o
\end{aligned} \tag{13}$$

Note that in (13), we do not include explicitly, the EVM constraint from the communication standard. Rather, we seek a minimal EVM bound g and strive to reach a value for g that is below the limit dictated by the communication standard. In general, the larger the $A_{\max}$, and/or the larger the L and/or the smaller the shoulder parameter α, the smaller the g.

To efficiently solve the optimization problem (13), we utilize the interior-point method (IPM) as in [4, 5]. The goal of the IPM is to compute the search direction $\mathbf{v}$ for the N_s variables in the distortion vector $\mathbf{d}$ per iteration. Solving the direction $\mathbf{v}$ is equivalent to solving the following linear equation:

$$\frac{\partial^2 f}{\partial \mathbf{d}^2} \mathbf{v} = \frac{\partial^2 f}{\partial \mathbf{d} \partial g} - \frac{\partial f}{\partial \mathbf{d}}, \tag{14}$$

where $f(\cdot)$ is the log-barrier function. For the task in (13), f can be written as

$$\begin{aligned}
f \quad = \quad & tg - \log(g) - \log\left(g - \sum_{k \in \Omega_i} |D_k|^2\right) \\
& - \sum_{l \in \mathcal{H} \cup \mathcal{L} \cup \mathcal{S}} \log\left(A_{\max}^2 - |y[l] + d[l]|^2\right) \\
& - \sum_{k \in \Omega_o} \log(\mathcal{M}_k - |D_k|^2),
\end{aligned} \tag{15}$$

where t is the barrier parameter which determines the precision of optimization to be within $\mathcal{O}(1/t)$. The IPM complexity per iteration is about 4 FFTs plus solving the N_s linear equations in (14). According to [6], for the $N/4$ variables IPM employed in [4, 5], the computational complexity can be reduced to $\mathcal{O}(M(N/2 + N^2/16) + N^2/8 + 4N \log N)$ if the M iterations of conjugate gradient methods are employed. In this paper, since we have only N_s variables, the complexity can be further reduced to $\mathcal{O}(M(2N_s + N_s^2) + 2N_s^2 + Pr(N_s/N)4N \log N)$, where the $Pr(\cdot)$ stands for the

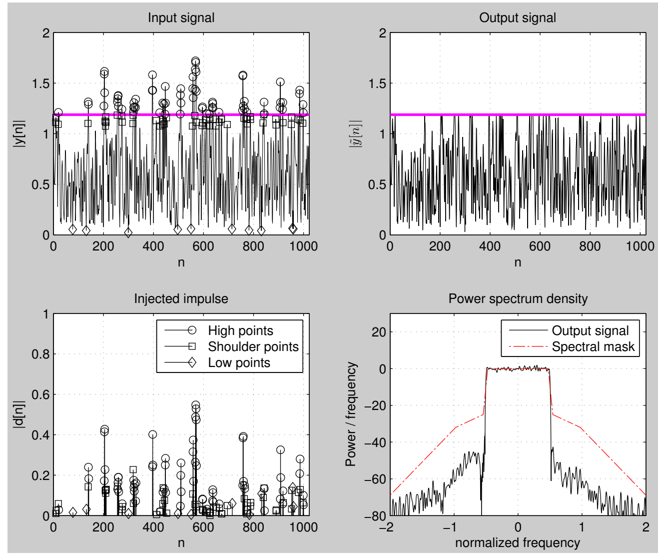

Fig. 2. One realization of the impulse injection method for an OFDM signal, $A_{\max} = 1.19$ $(4.5dB)$, $L = 10$, and $\alpha = 0.9$.

proportional function given in [7] which depends on the FFT input vector density.

We wish to point out a practical consideration for the implementation of (12). It will entail a lot of computation if we calculate $|D_k|$ and apply the spectral mask constraint for each $k \in \Omega_o$. Since D_k is smooth as a function of the subcarrier index k, instead of calculating D_k for all out-of-band locations Ω_o, we can take a subset Ω_{os}, for example, equi-space sub-sampled values of Ω_o, and only evaluate $|D_k|$ over Ω_{os} to see if out-of-band emission requirement is met. The rationale is that if $|D_k|$ behaves well at sub-sampled values of Ω_o and $|D_k|$ is a smooth function, it will not likely shoot up in between sample points.

4. NUMERICAL RESULTS

In this example, we adopted the spectral mask from the 802.16 standard [8]. Length $N = 1024$ input blocks were generated by taking the 4 times over-sampled 1024-length FFT of the 256 QPSK symbols. The subset Ω_{os} consisted of 66 equi-spaced points of the out-of-band frequency set Ω_o. One sample realization is shown in Fig. 2. The magnitude threshold was $A_{\max} = 1.19$ $(4.5dB)$. The number of low points was $L = N \times 1\% = 10$ and the parameter $\alpha = 0.9$ in this case. The optimized in-band EVM for this block was 6%.

Fig. 3 shows the spectrum emission performance for different selections of low points and shoulder points. Monte-Carlo experiments were run in the simulation and the magnitude threshold $A_{\max} = 6dB$. We can see that the shoulder points performed better for shaping the out-of-band spectrum emission than the low points. Fig. 4 shows the RMS EVM

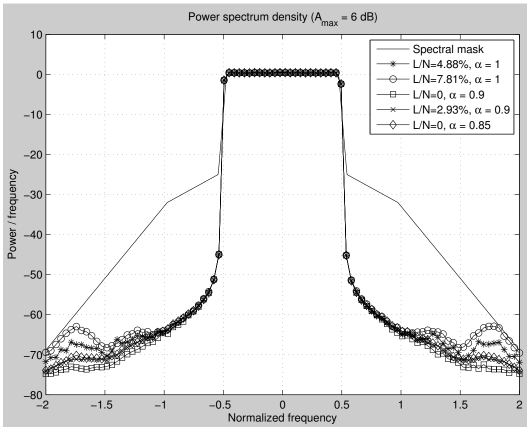

Fig. 3. Spectrum emission performance for varying L and α parameters.

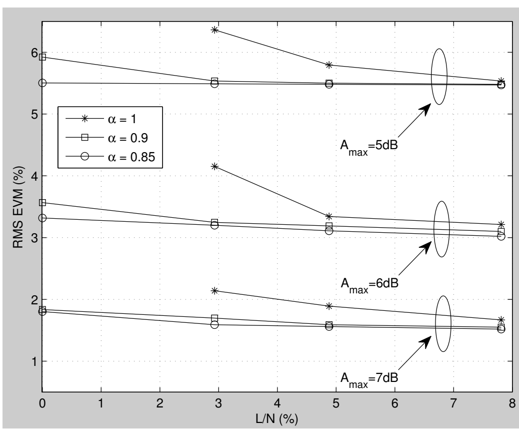

Fig. 4. RMS EVM for different α values.

for different α values. Different selections of low points and shoulder points were simulated for comparison. 1024 times Monte-Carlo experiments were conducted for each selection. We can see that the more the number of low points and shoulder points, the lower the RMS EVM. Very little improvement was observed if $L/N > 4.88\%$ or $\alpha < 0.9$.

To compare computational complexity, we simulated six different selections of low points and shoulder points and compared the results with the technique in [5] and the clipping and filtering method in [1]. The average numbers of variables per block N_s was 30, 50, 80, 40, 70, and 58 for the six cases, respectively. The RMS EVM versus the computational complexity is shown in Fig. 5. The repeated clipping and filtering method had much lower complexity but very high RMS EVM, which failed to satisfy the 6% requirement in the standard [8]. The proposed method outperformed the work [5] in computational complexity with very close RMS EVM performance.

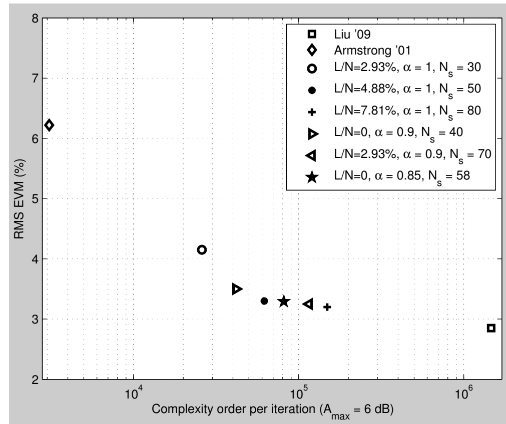

Fig. 5. RMS EVM versus computational complexity for the techniques in [1, 5] and the proposed method with varying parameters.

5. CONCLUSIONS

In this paper, we proposed a novel time-domain impulse injection method to achieve CFR for OFDM signals. The block-wise in-band EVM is minimized subject to deterministic (as opposed to statistical) crest factor and out-of-band spectrum mask constraints. This is an effective CFR method since the crest factor can be guaranteed to not exceed a prescribed threshold. It is advantageous over [1, 2, 3] because it takes in-band EVM into consideration. Simulation results show that the complexity of the proposed method is an order of magnitude lower than that of the optimal technique presented in [5], at the expense of only a slight performance degradation.

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