Constrained Clipping for Crest Factor Reduction in Multiple-user OFDM*

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***Abstract* — Crest factor reduction (CFR) techniques aim to solve the high peak-to-average power ratio (PAR) problem encountered in the transmission of modern communication signals such as orthogonal frequency division multiplexing (OFDM). Constrained clipping is a promising CFR technique that is capable of large PAR reductions while guaranteeing that EVM and spectral mask constraints are met. In this paper, we analyze the computational complexity of constrained clipping and show that it is a practical technique that can be readily implemented. We then demonstrate its performance in a multiple-user OFDM environment such as the IEEE 802.16 OFDMA forward link. More than 5dB of PAR reduction can be achieved in a four-user scenario.**

***Index Terms* — clipping, crest factor reduction, error vector magnitude, OFDM, peak-to-average power ratio, spectral mask.**

I. Introduction

OFDM has been widely adopted as the physical layer transmission technique by various communications systems, including digital audio broadcasting (DAB), digital video broadcasting (DVB), wireless LAN and wireless MAN. However, one major disadvantage of OFDM is that its time-domain envelope exhibits large variations which can drive the transmitting power amplifier (PA) into its nonlinear operation. The envelope variation can be quantified by the peak-to-average power ratio (PAR) or the crest factor (CF)[1].

We have recently proposed a constrained clipping crest factor reduction (CFR) technique that utilizes in-band error vector magnitude (EVM) control and out-of-band spectrum shaping [3] to achieve CFR for OFDM. Constrained clipping has the following characteristics: (i) large PAR reducing capability; (ii) EVM and spectrum performances that are guaranteed to meet regulatory requirements; (iii) simple structure - no receiver side processing or feedback, no side information transmission.

In this paper, we will investigate the computational complexity issue of the constrained clipping algorithm as well as its performance in the multiple-user OFDM environment.

II. The Constrained Clipping Method

When considering OFDM for CFR purposes, the continuous-time representation is of interest since it is the continuous-time signal that passes through the PA. Accordingly, it is necessary to up-sample the Nyquist-rate sampled signal by a factor of at least four ($L \geq 4$) so that the continuous-time peaks can be accurately approximated by those of the discrete-time samples [5]. The nth sample of the over-sampled time-domain signal is

$$x_n^{(L)} = \frac{1}{\sqrt{LN}} \sum_{k=-LN/2}^{LN/2-1} X_k^{(L)} \, e^{j\frac{2\pi kn}{LN}}, \;\; 0 \leq n \leq LN-1, \quad (1)$$

where N is the number of subcarriers, L is the over-sampling rate and $X_k^{(L)}$ is the zero-padded frequency domain symbol. Define the out-of-band subcarriers to be $\mathcal{O}$: $[-LN/2, -N/2-1] \cup [N/2, LN/2-1]$ and the in-band subcarriers to be $\mathcal{I}$: $[-N/2, N/2-1]$. The zero-padded $X_k^{(L)}$ is generated according to $X_k^{(L)} = X_k, k \in \mathcal{I}$ and $X_k^{(L)} = 0, k \in \mathcal{O}$.

A popular CFR technique is to simply clip $x_n^{(L)}$ to some level A_{max} to create

$$\bar{x}_n^{(L)} = \begin{cases} x_n^{(L)}, & |x_n^{(L)}| \leq A_{max} \\ A_{max} \, e^{j\angle x_n^{(L)}}, & |x_n^{(L)}| > A_{max}, \end{cases} \quad (2)$$

where $|x_n^{(L)}|$ is the amplitude of the nth sample before clipping and A_{max} is the maximum permissible amplitude. Clearly, clipping can effectively perform CFR, however, it creates distortions to the signal both in-band and out-of-band. In most communications standards such as IEEE 802.16 [1], limits in the form of an EVM threshold Th and a spectral mask $P(\omega)$ are placed on transmitted signals so that the distortions are kept low. If the clipped signal is left unprocessed, then the in-band distortion may exceed the EVM threshold and/or the out-of-band distortion may not meet the spectral mask. Band-limited clipping techniques have been employed to reduce adjacent channel distortion [6]; however, they cause degradation in the EVM.

In [3], constrained clipping was proposed to reduce the PAR while still meeting the EVM and spectral requirements; it is shown to be superior to existing clipping distortion control methods [2], [7], [4] . The constrained clipping algorithm is illustrated in Fig. 1 and is briefly described next.

Denote the FFT of $\bar{x}_n^{(L)}$ by $\bar{X}_k^{(L)}$. EVM for the single-user case is defined as [1]

$$EVM\{\bar{x}_n^{(L)}\} = \frac{1}{S_{max}} \sqrt{\frac{1}{N} \sum_{k \in \mathcal{I}} |E_k|^2}, \quad (3)$$

*This work was supported in part by the US Army Research Laboratory Communication and Networks Collaborative Technology Alliance program, the US National Science Foundation under the Graduate Research Fellowship Program, and Altera Corporation.

[1] $PAR = CF^2$, so PAR and CF are the same on the dB scale.

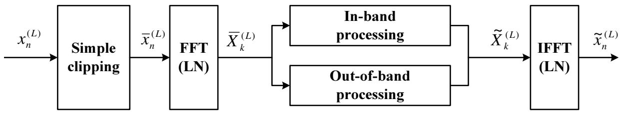

Fig. 1. Block diagram of the constrained clipping algorithm.

where S_{max} is the maximum amplitude of the symbol constellation, $|E_k|^2$ is the error vector power on the kth subcarrier and $E_k = \bar{X}_k^{(L)} - X_k^{(L)}$ $(k \in \mathcal{I})$.

The in-band processing unit is only needed when $EVM\{\bar{x}_n^{(L)}\} > Th$. When called for, the in-band processing unit first sorts $|E_k|$ in ascending order. If the rms average of the M smallest $|E_k|$ values is less than or equal to $Th \cdot S_{max}$, but the rms average of the $M+1$ smallest $|E_k|$ values is greater than $Th \cdot S_{max}$, we record the value M and collect the subcarrier indices k that correspond to the M smallest $|E_k|$ values in a set $\mathcal{M}$. We obtain the in-band modified vector $\widetilde{X}_k^{(L)}$ according to

$$\widetilde{X}_k^{(L)} = \begin{cases} \bar{X}_k^{(L)}, & k \in \mathcal{M} \subseteq \mathcal{I}, \\ X_k + Th \cdot S_{max}\ e^{j\angle E_k}, & k \in (\mathcal{I} \setminus \mathcal{M}). \end{cases} \tag{4}$$

The above procedure guarantees that $EVM\{\widetilde{x}_n^{(L)}\} \leq Th$ and that PAR$\{\widetilde{x}_n^{(L)}\} \ll$ PAR$\{x_n^{(L)}\}$ as shown in [3].

In order to guarantee that the spectral mask is not violated, the out-of-band processing unit performs a spectral clipping operation to create

$$\widetilde{X}_k^{(L)} = \begin{cases} \bar{X}_k^{(L)}, & |\bar{X}_k^{(L)}|^2 \leq P_k, k \in \mathcal{O}, \\ \sqrt{P_k}\ e^{j\angle \bar{X}_k^{(L)}}, & |\bar{X}_k^{(L)}|^2 > P_k, k \in \mathcal{O}, \end{cases} \tag{5}$$

where P_k is the spectral mask $P(\omega)$ sampled at $\omega = 2\pi k/(LN)$. By the out-of-band processing, PSD$\{\widetilde{x}_n^{(L)}\} \leq P(\omega)$ can be guaranteed for each $\omega = 2\pi k/(LN)$, $k \in \mathcal{O}$.

III. Computational Complexity Analysis for the Constrained Clipping Algorithm

In this section, we will investigate the complexity of the proposed constrained clipping algorithm. From Fig. 1, we find that the main components of the CFR algorithm include:

- One IFFT and one FFT
- Time-domain clipping
- In-band and out-of-band processing units

For each component in the algorithm, the computational complexity will be quantified by the number of instructions per OFDM symbol. Percentage of contribution of each component to the complexity will also be given in order to provide a clear understanding about the relative complexity.

In the implementation of constrained clipping, A_{max} is chosen to be a small value relative to the signal peaks. For this reason, the probabilities of not using time-domain clipping and spectral clipping are negligible. Hence, we assume these two components are called for by every OFDM symbol in the computational complexity analysis.

A. Complexity of the FFT/IFFT Units

It is known that an LN-point FFT or IFFT requires $(LN/2)\log_2(LN)$ multiplications and $LN\log_2(LN)$ additions. Assuming that the multiplication operation requires α_M instructions and the addition operation requires α_A instructions, the total number of instructions for carrying out the IFFT and FFT operations is $I_{IFFT+FFT} = (\alpha_M + 2\alpha_A)(LN)\log_2(LN)$.

B. Complexity of Time-domain Clipping

There are LN samples of $x_n^{(L)}$ in the time-domain that pass through the clipping unit. If $|x_n^{(L)}|^2$ is larger than A_{max}^2, the nth sample is clipped and $\bar{x}_n^{(L)} = A_{max}\ x_n^{(L)}/|x_n^{(L)}|$ is the nth sample after clipping.

In order to calculate $|x_n^{(L)}|^2$, we need $2LN$ multiplications and LN additions. LN comparisons are to be made in order to decide which samples need to be clipped. Assuming that the number of clipped samples is K_t, there are K_t multiplications, K_t divisions and K_t square root operations[2] to obtain the final clipped time-domain signal $\bar{x}_n^{(L)}$. Therefore, the number of instructions required to implement time-domain clipping is $I_{clipping} = 2(\alpha_M + \alpha_A)LN + 7\alpha_M K_t$.

C. Complexity of In-band Processing

EVM$\{\bar{x}_n^{(L)}\}$ in (3) is calculated at first in the in-band processing unit which involves $3N$ additions and $2N$ multiplications. In the next step, two cases should be studied separately: with sorting and without it. When EVM$\{\bar{x}_n^{(L)}\} \leq Th$, sorting is not needed, i.e., the in-band processing step is by-passed. The number of instructions without sorting is $I_{ib\ (w/o\ s)} = 3\alpha_A N + 2\alpha_M N$.

When EVM$\{\bar{x}_n^{(L)}\} > Th$ occurs, we choose a sorting algorithm, with complexity $O(N\log_2 N)$, to arrange $|E_k|^2$ $(k \in \mathcal{I})$ in ascending order. Hence, we will need about $N\log_2 N$ comparisons which correspond to $\alpha_A N \log_2 N$ instructions. With sorted $|E_k|^2$ $(k \in \mathcal{I})$, we need $M+1$ additions, $2M+2$ multiplications and $M+1$ comparisons to construct the set $\mathcal{M}$. Once $\mathcal{M}$ is determined, $(N-M)$ $\bar{X}_k^{(L)}$ values should be changed to $\widetilde{X}_k^{(L)}$ as described in (4) which involves $5(N-M)$ multiplications and $(N-M)$ additions. The number of instructions required to implement in-band processing with sorting is $I_{ib\ (w/\ s)} = 3\alpha_A N + 2\alpha_M N + \alpha_A N\log_2 N + (N+M+2)\alpha_A + (5N-3M+2)\alpha_M$.

Summarizing, the number of instructions needed to implement in-band processing is $I_{ib} = p_i I_{ib\ (w/\ s)} + (1-p_i) I_{ib\ (w/o\ s)}$ where p_i is the probability of triggering the sorting algorithm (i.e. the probability that EVM$\{\bar{x}_n^{(L)}\} > Th$).

[2]We assume that the complexity of a square root operation is equivalent to that of five multiplications.

D. Complexity of Out-of-band Processing

Similar to the time-domain clipping, spectral clipping needs $2N(L-1)$ multiplications and $N(L-1)$ additions to calculate $|\bar{X}_k^{(L)}|^2$ ($k \in \mathcal{O}$) and $N(L-1)$ comparisons to determine whether clipping should be done. Suppose the number of clipped samples is K_f in the spectral clipping, then there are K_f multiplications, K_f divisions and K_f square root operations. Summarizing, the number of instructions required to implement out-of-band processing is $I_{oob} = 2(\alpha_M + \alpha_A)(L-1)N + 7\alpha_M K_f$.

E. Complexity of the Constrained Clipping Algorithm

The overall computational complexity of the constrained clipping algorithm is $I_{CFR} = I_{IFFT+FFT} + I_{clipping} + I_{ib} + I_{oob}$ which depends on various parameters, K_t, p_i, M and K_f. These parameters will change with different A_{max}. From [3], we know that the CFR performance is also closely related to A_{max}. Hence, the computational complexity varies with different CFR goals.

Typical parameters for the complexity of constrained clipping are $L = 4$, $\alpha_A = 1$, $\alpha_M = 2$, $K_t = 0.1LN = 0.4N$, $p_i = 0.1$, $K_f = N(L-1)2/3 = 2N$, and $M = 0.75N$. Using these parameters, we infer that with $N = 2048$, the IFFT+FFT units account for 71% of the overall computational complexity. Complexity contributions from the other units are: out-of-band processing 16%, time-domain clipping 10%, in-band processing 3%.

The computational complexity of selective mapping (SLM), a popular distortionless CFR method, is proportional to the number of used IFFT units. To achieve similar PAR reduction as the constrained clipping [3], SLM requires approximately 8 IFFT units. So we can save nearly 2/3 computational complexity by applying the constrained clipping.

IV. Performance of the Constrained Clipping Technique in Multiple-user Environment

A. EVM Definition in Multiple-user Environment

In the IEEE 802.16 system [1], different users can select appropriate modulation schemes to achieve the optimal transmission performance. Supported modulation schemes include BPSK, QPSK, 16QAM and 64QAM. The constellations shall be normalized by multiplying the constellation points with a scale factor c to achieve equal average power; for example, $c = 1/\sqrt{2}$ for QPSK, $c = 1/\sqrt{10}$ for 16QAM and $c = 1/\sqrt{42}$ for 64QAM [1]. For the normalized constellations, $S_{max} = 1$ for BPSK and QPSK, $S_{max} = \sqrt{18/10}$ for 16QAM and $S_{max} = \sqrt{98/42}$ for 64QAM.

We can divide the data subcarriers into several groups according to the chosen modulation schemes and calculate the corresponding EVMs. Let EVM_{BPSK}, EVM_{QPSK}, EVM_{16QAM} and EVM_{64QAM} denote the calculated EVM values for the respective constellation. For example, if only QPSK and 64QAM are used, we have $EVM_{QPSK} = \sqrt{\frac{1}{|\mathcal{N}_{QPSK}|}\sum_{k\in\mathcal{N}_{QPSK}} |E_k|^2}$ and $EVM_{64QAM} = \sqrt{\frac{42}{98}}\sqrt{\frac{1}{|\mathcal{N}_{64QAM}|}\sum_{k\in\mathcal{N}_{64QAM}} |E_k|^2}$ where $\mathcal{N}_{QPSK}$ is the set of subcarrier indices with the normalized QPSK constellation and $\mathcal{N}_{64QAM}$ is the set of subcarrier indices for the 64QAM constellation, $|\cdot|$ denotes cardinality of the set and $|\mathcal{N}_{QPSK}| + |\mathcal{N}_{64QAM}| = N$.

The calculated EVM values are compared with the maximum permissible EVM values specified in the standard. We will first compare EVM_{BPSK} to the threshold Th_{BPSK} for BPSK if BPSK is adopted by some users. If $EVM_{BPSK} > Th_{BPSK}$, in-band processing is employed to make $EVM_{BPSK} \le Th_{BPSK}$. Similar procedures will be used for QPSK, 16QAM, 64QAM if they are used by some other users.

B. Performance Results in Multiple-user Environment

In [3], we have found that the output PAR complementary cumulative distribution function (CCDF) is convex in the clipping level A_{max}. Naturally, we want to find the A_{max} that minimizes the output PAR; but because it is very difficult to theoretically analyze our algorithm, we perform the minimization empirically.

Often times, it is convenient to normalize the clipping level so that it is independent of the signal power. This is commonly done with the clipping ratio (CR) parameter, which is defined as $CR = 20\ \log_{10} A_{max}/\sigma_x$ [dB] where σ_x is the standard deviation of the signal of interest $x_n^{(L)}$.

In the following simulations, we first determine the optimal clipping level. Based on the selected CR, the PAR reduction capability of the constrained clipping method is compared with Armstrong's method described in [2]. With Armstrong's method, nothing is done to control the in-band EVM; the out-of-band spectral regrowth stays far below the spectrum mask which essentially wastes energy that is allotted by the standard that can be used for CFR.

We apply the adaptive modulation and coding (AMC) subcarrier allocation scheme[3] [1]. We assume that there were four users in one OFDMA symbol and every user occupied 8 subchannels; the user's modulation was BPSK, QPSK, 16QAM and 64QAM, respectively, for user 1 through 4.

1) Choosing the optimal clipping ratio: We examined the PAR reduction of different clipping ratios in order to choose the optimal one for the respective PAR goals. The result is shown in Table I. We can see that CRs between 4.0dB and 4.8dB all achieved similar PAR reduction results. $CR = 4.2$dB is chosen as the optimal clipping ratio for 6-7dB PAR goals at the 10^{-3} CCDF level.

2) Application of optimized clipping ratios: Here, we show the PAR reduction capability using the optimal clipping ratio for the multiple-user case. Armstrong's method with $CR = 4.2$dB was also included for comparison. We can see from

[3]The total number of subcarriers is $N = 2048$, consisting of 48 data subcarriers per subchannel $\times$ 32 subchannels, 192 pilot subcarriers, 319 null subcarriers at the band edges, and 1 null subcarrier at the DC.

TABLE I

PARs AT THE 10^{-3} CCDF LEVEL WITH VARIOUS CRs.

CR (dB)	3.6	3.8	4.0	4.2	4.4	4.8	5.2
PAR (dB)	6.95	6.90	6.81	6.79	6.82	6.84	6.91

Fig. 2 that the constrained clipping method outperformed Armstrong's method by about 0.6dB at the 10^{-3} CCDF level and achieved more than 5dB of PAR reduction at the 10^{-3} CCDF level relative to the original OFDM signal.

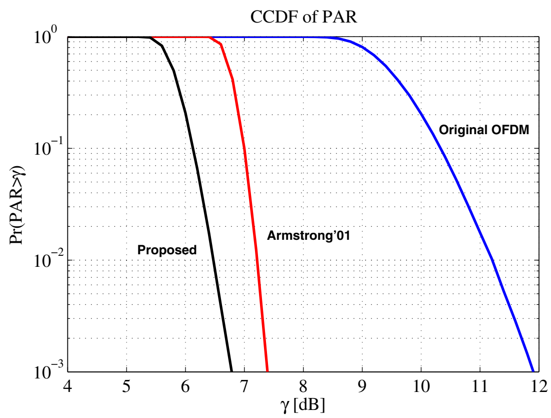

Fig. 2. Comparison between Armstrong's method and the proposed method, $N = 2048$ and four-user case.

3) Spectral characteristics: By using the normalized constellations, the different modulations all have unit average power. Afterwards, we perform out-of-band processing to deal with out-of-band radiation by applying (5). Shown in Fig. 3 is the power spectral density (PSD) with the constrained clipping method. Also shown are PSDs of the spectral mask, Armstrong's method and the simple clipping method. We can clearly see that the spectrum of the constrained clipping method closely follows the spectral mask where simple clipping violates the spectral mask. Also, we observe from Fig. 3 that Armstrong's method exceeds the spectral mask by a considerable margin. In contrast, constrained clipping mitigates the out-of-band signal energy to ensure tight spectral compliance without unnecessary use of energy.

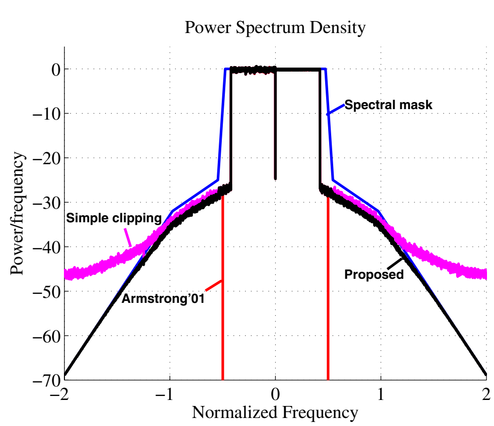

Fig. 3. PSD plot for four-user case, $N = 2048$ and $CR = 4.2$ dB.

4) EVM characteristics: Fig. 4 is a plot of the probability that an OFDM symbol processed by Armstrong's method will exceed the allowed EVM threshold for three different modulations. These curves also represent the probability that constrained clipping will perform in-band processing (i.e. p_i in the in-band processing). For Armstrong's method, even with a high clipping ratio, there is still some probability that the in-band signal will violate the EVM constraint. Constrained clipping guarantees that the in-band signal will never violate the EVM threshold for any symbol.

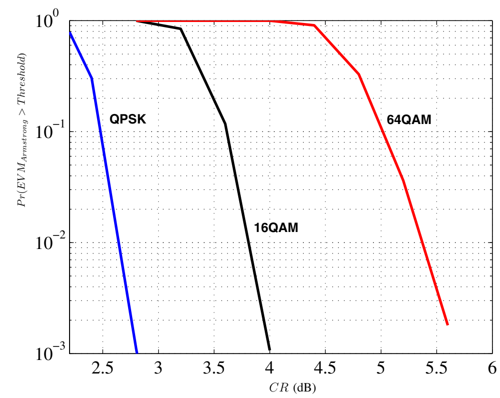

Fig. 4. Probabilities that Armstrong's method violates the EVM thresholds for different modulation schemes. $Th_{QPSK} = 12\%$, $Th_{16QAM} = 6\%$ and $Th_{64QAM} = 3\%$ [1].

V. CONCLUSIONS

In this paper, we analyzed the computational complexity of constrained clipping and showed that it is a practical and promising CFR technique. We demonstrated that constrained clipping can yield excellent CFR results in the multiple-user OFDM environment such as 802.16. More than 5dB of PAR reduction was achieved at the 10^{-3} CCDF level relative to the original OFDM signal. The EVM and spectrum of the transmitted signals processed by constrained clipping are guaranteed to meet the requirements of the standard.

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