A METHOD FOR JOINT PEAK-TO-AVERAGE POWER RATIO REDUCTION AND SYNCHRONIZATION IN OFDM

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ABSTRACT

Peak-to-average power ratio (PAR) reduction is an effective way to increase the power efficiency and decrease distortion noise in orthogonal frequency division multiplexing (OFDM) systems. Many PAR reduction schemes have been proposed, but few incorporate symbol-wise channel estimation and none accommodate per symbol synchronization. In this paper we present joint synchronization pilot sequence (JSPS) selected mapping (SLM), which is a joint PAR reduction and synchronization scheme. In harsh peak-limited channels we show that JSPS-SLM can achieve large bit error rate (BER) performance gains.

1. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) is an effective high speed communications technique that allows for simple multipath channel equalization. However, OFDM suffers from large peak-to-average power ratios (PARs) and sensitivity to carrier frequency offset (CFO). High PAR can lead to distortion noise and low power efficiency in peak limited channels, while CFO sensitivity can lead to inter-carrier interference (ICI).

In this paper we develop a scheme that addresses both of these problems. The scheme is called joint synchronization pilot sequence (JSPS) selected mapping (SLM). The goal of our JSPS-SLM design is to simultaneously consider several channel impairments in a practical OFDM system and generate an unified approach to correcting for all of the impairments. The hope is that by simultaneously addressing the channel impairments along with the PAR, bandwidth overhead devoted to channel impairment correction can also be used for PAR reduction, which will lead to greater bandwidth efficiencies.

JSPS-SLM is designed as a combined PAR reduction and embedded synchronization technique. Many PAR reduction schemes are designed without full consideration of the overall OFDM system. Specifically, other PAR reduction schemes generally assume perfect channel state information (CSI) and that the transmitter and receiver are perfectly synchronized in frequency and time. These assumptions may be realistic for static channels where a preamble-type synchronization sequence is used periodically. But for mobile communications applications, the channel is rarely static. Furthermore, it was shown in [1] that frequency-hopped ODFM is not bandwidth efficient when preamble synchronization is used.

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2. SYSTEM MODEL

To make a realistic characterization of the channel model that exists in mobile communications many different channel effects have to be included. Obviously, multipath fading and some sort of additive noise will be present. In addition to these two standard channel model components, a realistic, unsynchronized channel will have carrier frequency offset (CFO) and timing offset (TO) effects, which are often ignored in OFDM PAR analyses. Furthermore, in a peak power-constrained system, it is necessary to include the clipping characteristic of the power amplifier (PA) in the channel model. By including the PA in the channel model, the signal dynamic range becomes a design consideration making PAR re-

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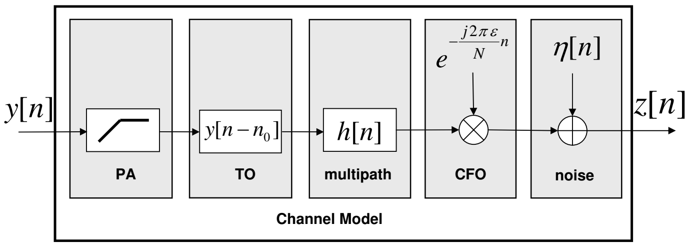

Fig. 1. Channel model of the proposed system.

duction an attractive possibility.

In this paper, the channel model is characterized by

$$z[n] = \left(f_{PA}\left(y[n-n_0]\right) \star h[n]\right) e^{-j2\pi\epsilon/N} + \eta[n]. \quad (1)$$

where $f_{PA}(\cdot)$ is the power amplifier input-to-output characteristic, which is assumed to be time-invariant with respect to each OFDM symbol and $\star$ is the convolution operator. A block diagram of the channel is displayed in Fig. 1.

The baseband data-bearing part of the transmitted signal prior to cyclic extension can be expressed as

$$\begin{aligned} x[n] &= \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k \, e^{j2\pi kn/N} && (2) \\ &= IDFT\{X_k\}. && (3) \end{aligned}$$

where $\{X_k\}_{k=0}^{N-1}$ are drawn from a finite constellation and $n \in \{0, 1, ..., N-1\}$.

3. PAR, SELECTED MAPPING AND SYNCHRONIZATION

PAR is one metric for assessing the dynamic range of a signal. Signals with a low PAR are preferable because they allow the PA to operate at higher power efficiencies [2]. The PAR is defined

$$PAR\{y[n]\} = \frac{\max_n |y[n]|^2}{\mathrm{E}\left[|y[n]|^2\right]}. \quad (4)$$

Selected mapping (SLM) is a popular PAR reduction tool that can reduce the PAR of OFDM symbols by several dBs [3]. In SLM, D candidate OFDM signals are generated by

$$x^{(d)}[n] = IDFT\{X_k e^{j\phi_k^{(d)}}\} \quad (5)$$

where $d \in \{1, 2, ..., D\}$. The index of the candidate signal with the lowest PAR is

$$\bar{d} = \min_{d \in \{1,2,...,D\}} PAR\{x^{(d)}[n]\}, \quad (6)$$

thus the transmitted signal $x^{(\bar{d})}[n]$ has the lowest PAR among the candidates. In order for the receiver to recover X_k, it must know $\phi_k^{(\bar{d})}$. It is reasonable to assume that the table of all values of $\phi_k^{(d)}$ is known at both the transmitter and receiver. With this assumption, the receiver only needs knowledge of $\bar{d}$ in order to recover X_k.

There have been many proposals for $\bar{d}$ recovery; some involve the transmission of side information (SI) and others work blindly without SI. The SI-based methods are not desirable because they reduce the bandwidth efficiency by utilizing bits that could otherwise be used for information transmission. Several promising blind detection methods have been proposed [4–8]. However, all of these methods assume that the receiver is perfectly synchronized with the transmitter. This may be the case in static channels, however, in mobile or frequency hopping environments it may be necessary to estimate the channel every symbol.

The purpose of this paper is to outline a blind phase sequence detection method that does not require time or frequency synchronization or knowledge of the channel. We will show that the resulting scheme is robust in these unsynchronized, multipath, peak-limited channels.

4. JSPS-SLM OFDM

As we will explain, JSPS-SLM OFDM is a PAR reduction technique that is robust in harsh channels. The basic idea is to combine the embedded synchronization technique from [1] with SLM. However, the combination is not trivial as many design parameters need to be considered.

The transmitted signal is made up of two parts, the data part X_k and the JSPS part S_k, that are combined through the parameter ρ. The resulting signal in the frequency domain is

$$Y_k = \sqrt{\rho} S_k + \sqrt{1-\rho} X_k, \quad (7)$$

where ρ is the embedding factor that dictates how much signal power is allocated to S_k. Here S_k and, equivalently $s[n] = IDFT\{S_k\}$, is a sequence known to the transmitter and the receiver that can be used for CFO estimation and TO estimation according to the procedures in [9]. In [1], the authors extended the method in [9] so that *embedded* sequences could be used for multipath channel estimation in addition to CFO and TO estimation.

In the proposed JSPS-SLM method, D candidate signals $\{y^{(d)}[n]\}_{d=1}^{D}$ are generated according to

$$Y_k^{(d)} = \sqrt{\rho} S_k^{(d)} + \sqrt{1-\rho} X_k e^{j\phi_k^{(d)}}, \tag{8}$$

where $y^{(d)}[n] = IDFT\{Y_k^{(d)}\}$. In Section 5, we will outline the design criteria for generating $S_k^{(d)}$. The candidate with the lowest PAR will be selected for transmission. Thus, if

$$\tilde{d} = \min_{d \in \{1,2,\ldots,D\}} PAR\{y^{(d)}[n]\}, \tag{9}$$

then the transmitted signal is $y^{(\tilde{d})}[n]$. The goal is for the receiver to be able to recover X_k, but the problem is that $\tilde{d}$ is unknown in the receiver. It is assumed that the receiver has knowledge of $S_k^{(d)}$ and $\phi_k^{(d)}$. So, in order for the receiver to determine $S_k^{(\tilde{d})}$ and $\phi_k^{(\tilde{d})}$, $\tilde{d}$ must be recovered. Fortunately, the conjugate correlation receiver can be used to do this. The received signal can now be expressed as

$$z^{(\tilde{d})}[n] = \left(f_{PA}\left(y^{(\tilde{d})}[n-n_0] \right) \star h[n] \right) e^{-j2\pi\epsilon/N} + \eta[n]. \tag{10}$$

Define the conjugate correlation (CC) between two length-N sequences to be

$$\mathrm{CC}\{a[n], b[n]\} = \left(\sum_{n=0}^{N/2-1} a^*[n] b[n-u] \right) \cdot \left(\sum_{n=N/2}^{N-1} a^*[n] b[n-u] \right)^*, \tag{11}$$

where $(\cdot)^*$ is the conjugate operation. In the receiver, D conjugate correlation outputs are generated using

$$r^{(d)}[u] = \mathrm{CC}\{s^{(d)}[n], z^{(\tilde{d})}[n-u]\}. \tag{12}$$

From $r^{(d)}[u]$, the SLM index, the TO and the CFO can be estimated by

$$\hat{\tilde{d}} = \arg\max_d |r^{(d)}[u]| \tag{13}$$

$$\hat{n}_0 = \arg\max_u |r^{(\hat{\tilde{d}})}[u]| \tag{14}$$

$$\hat{\epsilon} = \arg\left(r^{(\hat{\tilde{d}})}[\hat{n}_0] \right). \tag{15}$$

Hence, JSPS-SLM uses a robust and completely novel blind phase sequence detection criterion as defined in (13). As opposed to the other methods, the phase sequence index is detected based on the conjugate correlation with the candidate synchronization sequences, $s^{(d)}[n]$. Designing these sequences to optimize performance is a difficult problem that we will address in Section 5.

To illustrate how X_k can be recovered from $z^{(\tilde{d})}[n]$ using the estimators in (13-15), let us decompose the frequency-domain data signal into three non-overlapping parts: 1) data subcarrier denoted by the set of indices $\mathcal{K}_d$, 2) pilot subcarriers denoted by the set of indices $\mathcal{K}_p$, and 3) null subcarriers denoted by the set of indices $\mathcal{K}_n$. In this system configuration, $X_{k \notin \mathcal{K}_d} = 0$, so X_k only contains energy in the data subcarriers. The null subcarriers are constrained to zero to limit the amount of out-of-band spectral energy that encroaches on neighboring channels. The pilots will be defined as part of S_k. Just as with X_k, S_k can also be decomposed using the same three set of subcarrier indices: 1) synchronization subcarriers, $\mathcal{K}_d$, 2) pilot subcarriers, $\mathcal{K}_p$, and 3) null subcarriers, $\mathcal{K}_n$.

To recover X_k, first the receiver generates the quantity

$$W_k^{(\tilde{d})} = IDFT\left\{ z^{(\tilde{d})}[n+\hat{n}_0] e^{j2\pi n\hat{\epsilon}/N} \right\}, \tag{16}$$

which gives $W_k^{(\tilde{d})} = Y_k^{(\tilde{d})} H_k + \eta_k + \delta_k + \iota_k$, where δ_k is the frequency domain noise caused by the power amplifier, ι_k is the inter-carrier interference (ICI) and where H_k and η_k are the IDFTs of $h[n]$ and $\eta[n]$, respectively. From $W_k^{(\tilde{d})}$ the channel in the pilot subcarriers can be estimated using

$$\hat{H}_k = \frac{W_k^{(\tilde{d})}}{S_k^{(\hat{\tilde{d}})} \sqrt{\rho}}, \quad k \in \mathcal{K}_p. \tag{17}$$

These pilot subcarrier channel estimates can be easily interpolated to the data-bearing subcarriers, $k \in \mathcal{K}_d$, using the techniques described in [10] so that $\hat{H}_k$ is defined for $k \in \mathcal{K}_d \bigcup \mathcal{K}_p$. Finally, the data symbol can be estimated via

$$\hat{X}_k = \frac{e^{-j\phi_k^{(\hat{\tilde{d}})}}}{\sqrt{1-\rho}} \left(\frac{W_k^{(\tilde{d})}}{\hat{H}_k} - \sqrt{\rho} S_k^{(\hat{\tilde{d}})} \right), \quad k \in \mathcal{K}_d \quad (18)$$

It is obvious from (18) that the calculation of $\hat{X}_k$ is highly sensitive to the estimated parameter $\hat{\tilde{d}}$. If $\hat{\tilde{d}} \neq \tilde{d}$, then $\hat{X}_k$ will be a phase scrambled version of X_k, making data recovery impossible, thus it is imperative that $\tilde{d}$ is estimated correctly.

5. SYNCHRONIZATION SEQUENCE DESIGN

So far, we have introduced the transmitter and receiver structure for JSPS-SLM OFDM. In this section, we consider the design issues involved in generating $S_k^{(d)}$. The JSPS-SLM system can be broken into three major sections: i) PAR reduction in (9), ii) synchronization in (13-15), and iii) channel estimation in (17). This Section will outline the various design tradeoffs and considerations in each of these major parts.

5.1. PAR Reduction

In JSPS-SLM, there are two main sources of PAR reduction. One source is the fact that we are using D candidate signals and selecting the candidate with the lowest PAR. A more subtle source of PAR reduction in JSPS-SLM is in the design of $S_k^{(d)}$.

It was shown in [11], that by cleverly designing $S_k^{(d)}$, large PAR reductions are possible. Specifically, when $IDFT\{S_k^{(d)}\} = s^{(d)}[n]$, has low PAR, the combined sequence, $y^{(d)}[n] = \sqrt{\rho} s^{(d)}[n] + \sqrt{1-\rho} x^{(d)}[n]$, will, on average, have lower PAR than $x^{(d)}[n]$. The extent of the PAR reduction is largely dictated by the size of ρ. Larger values for ρ lead to larger PAR reductions. Also, in [11], it was shown that by improperly choosing the embedding sequences (e.g. high PAR sequences) will actually lead to a PAR increase. So it is vital that the set of sequences $\{s^{(d)}[n]\}_{d=1}^D$ all have low PAR.

However, designing D sequences $\{s^{(d)}[n]\}_{d=1}^D$, that all have low PAR and that meet the desired spectral constraints is not a trivial problem. In [12], we developed a flexible framework for generating a set of low-PAR sequences with an arbitrary power spectral density (PSD) that involved convex optimization techniques. Using this method it is possible to generate a set of sequences that all have $PAR < 0.5$dB. Deploying the resulting sequences $\{s^{(d)}[n]\}_{d=1}^D$ in JSPS-SLM results in large PAR reductions as demonstrated in Fig. 2. For instance, the PAR reduction at the 10^{-3} probability level is 4dB from the $\rho = 0,\ D = 1$ case to the $\rho = 0.3, D = 8$.

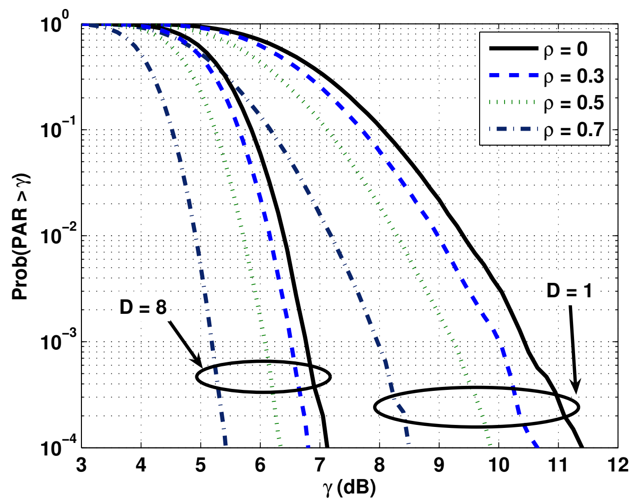

Fig. 2. PAR CCDF for JSPS-SLM for different embedding factors ρ.

5.2. Synchronization

In the context of JSPS-SLM, synchronization includes $\tilde{d}$ detection, n_0 estimation and ϵ estimation. The first step in JSPS-SLM synchronization is determining which phase sequence index was transmitted, via the criterion in (13). From (13) it is apparent that correct detection of $\tilde{d}$ depends on the peaks of $|r^{(d)}[u]|$ for $d \neq \tilde{d}$. When these spurious peaks are all less than the peak in $|r^{(\tilde{d})}[u]|$, $\tilde{d}$ will be correctly detected. Thus, the set of $\{s^{(d)}[n]\}_{d=1}^D$ should be designed so that spurious peaks are low.

Once, $\tilde{d}$ is detected, the next step is to determine the TO, n_0 via (14). Equation (14), shows that n_0 is determined based on the maximum of the $\hat{\tilde{d}}$th conjugate correlation output. Assuming that $\hat{\tilde{d}} = \tilde{d}$, n_0, will be detected correctly as long as $|r^{(\tilde{d})}[n_0]| > |r^{(\tilde{d})}[n]|$ for $n \neq n_0$. Various performance simulations using different design parameters were provided in [12].

The final step in synchronization, is the estimation of the CFO parameter ϵ. There is no obvious relationship between the design of $\{s^{(d)}[n]\}_{d=1}^D$ and the estimation error $\mathrm{E}\left[|\epsilon - \epsilon|^2\right]$. Thus, for the purposes of this paper, it is assumed that if the other two synchronization design criteria are met, that the resulting set of sequences $\{s^{(d)}[n]\}_{d=1}^{D}$, will also achieve adequate performance.

5.3. Channel Estimation

The performance of the zero forcing channel estimator defined in (17) is sensitive to the positions of the pilots and to the power allocated to the pilot subcarriers, $S_k,\ k \in \mathcal{K}p$. There is a large amount of literature involving optimal pilot placement for OFDM. When the subcarrier spacing is less than the number of null subcarriers (i.e. $|\mathcal{K}_n| \leq N/|\mathcal{K}_p| - 1$), placing constant power pilots evenly throughout the N subcarriers minimizes the channel estimation MSE [13]. When this condition is not satisfied, other pilot positioning techniques have been proposed [14, 15].

The amount of power in the pilot subcarriers can be quantified through

$$\beta = \frac{\sum_{k \in \mathcal{K}_p} |S_k|^2}{\sum_{k \in \mathcal{K}_p \bigcup \mathcal{K}_d} |S_k|^2}, \tag{19}$$

which is the ratio of pilot power to the total JSPS power. Without loss of generality, set $\sum_{k \in \mathcal{K}_p \bigcup \mathcal{K}_d} |S_k|^2 = 1$ and $\mathrm{E}\left[\sum_{k \in \mathcal{K}_p \bigcup \mathcal{K}_d} |X_k|^2\right] = 1$. Now, to get an approximate relationship between the symbol estimation error and the parameters $\beta,\ \rho$, and the noise variance, assume that all of the synchronization works perfectly (i.e. $\hat{\tilde{d}} = \tilde{d}$, $\hat{n}_0 = n_0$ and $\hat{\epsilon} = \epsilon$) and that no distortion noise is introduced by the transmitter PA. Also, we assume that the interpolated channel estimate in each data subcarrier has the same error variance as the channel estimate in the pilot subcarriers (i.e. $\mathrm{E}[|\hat{H}_k - H_k|^2] = \sigma_\eta^2 \frac{|\mathcal{K}_p|}{\beta\rho}\ \forall\ k \in \mathcal{K}_p \bigcup \mathcal{K}_d$). This assumption is valid when the zero-forcing channel estimator is used, with evenly-spaced equal-power pilots. With these assumptions, we have $W_k^{(\tilde{d})} = Y_k^{(\tilde{d})} H_k + \eta_k$, where η_k is complex Gaussian distributed (i.e., $\eta_k \sim \mathcal{CN}(0, \sigma_\eta^2)$).

Using these assumptions, (18), and the first order approximation that $\mathrm{E}[|\eta_k|^2 |\hat{X}_k|^2] \approx \sigma_\eta^2$ for $k \in \mathcal{K}_d$, the symbol estimate MSE is

$$\mathrm{E}\left[|\hat{X}_k - X_k|^2\right] = \frac{\sigma_\eta^2}{\sigma_{H_k}^2}\left(\frac{(1-\beta)|\mathcal{K}_p|}{\beta(1-\rho)|\mathcal{K}_d|} + \frac{|\mathcal{K}_p|}{\beta\rho} + \frac{1}{1-\rho}\right) \tag{20}$$

for $k \in \mathcal{K}_d$. Notice that the minimizing β is $\beta = 1$ when perfect synchronization is assumed. However, in order to achieve acceptable synchronization performance it will be necessary to have $\beta < 1$. Some analysis on the effect of β is synchronization can be found in [12]. However, once β is chosen such that the synchronization performance is adequate, ρ should be chosen such that the symbol MSE in (20) is minimized.

6. SIMULATIONS

The plot in Fig. 3 demonstrates the BER performance of JSPS-SLM. To generate the plot, the convex optimization technique in [12] was used to generate the sequences $\{s^{(d)}[n]\}_{d=1}^{D}$ for the two JSPS-SLM designs. For the dot-dash non-JSPS-SLM curve, embedded synchronization was used without any PAR reduction considerations (i.e. $D = 1$ and $S_k^{(1)}$ was generated with the prescribed power profile but random phases). The embedded synchronization scheme was also plotted for the case when the CFO is not estimated to show how poor the performance of a PAR-reduction scheme that does not use CFO estimation could be.

For the plot, an ideal soft limiter channel was used with an input backoff (IBO) of 3dB. The CFO was set to a constant $\epsilon = 0.2$. The multipath channel, $h[n] \sim \mathcal{CN}(0, Ae^{-n})$, was set to length 16 with an exponential delay spread such that $A\sum_{n=0}^{15} e^{-n} = 1$. Also, $N = 256$, $|\mathcal{K}_p| = 16$, $|\mathcal{K}_d| = 240$ and $|\mathcal{K}_n| = 0$. The pilot tones were evenly spaced with equal power. The embedding factors were chosen to be $\beta = 0.25$ and $\rho = 0.35$.

The plot shows that JSPS-SLM outperforms the embedded synchronization schemes. Also, at high SNRs the $D = 8$ JSPS-SLM case performs more than 5dB better than the $D = 1$ case.

7. CONCLUSIONS

In this paper we have outlined the JSPS-SLM technique for OFDM, which is a joint PAR reduction synchronization channel estimation scheme. Also, we have outlined the major design decisions that must be made for JSPS-SLM. In addition to a high bandwidth efficiency, with

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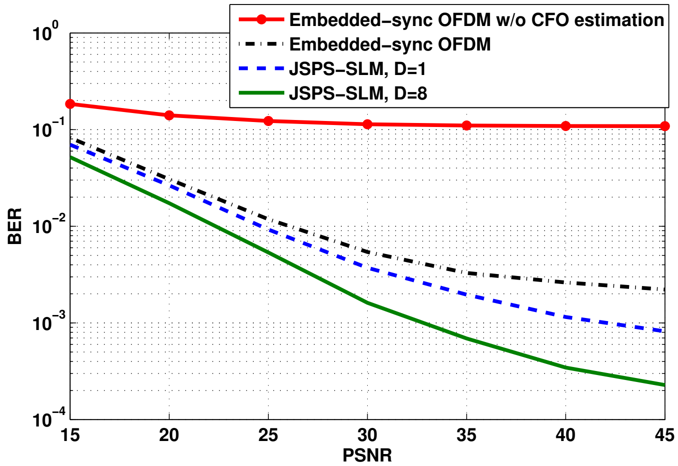

Fig. 3. BER for JSPS-SLM and regular OFDM where $\epsilon = 0.2$ and $IBO = 3$dB.

proper design, we have shown that large PAR reductions are possible using JSPS-SLM. Finally, we demonstrated that large BER improvements are also possible using JSPS-SLM in harsh channel environments.

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