SELECTED MAPPING WITH MONOMIAL PHASE ROTATIONS FOR PEAK-TO-AVERAGE POWER RATIO REDUCTION IN OFDM

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ABSTRACT

Selected mapping (SLM) is a promising technique to reduce the high peak-to-average power ratio (PAR) of OFDM signals. In this paper, we propose a novel concept of using monomial phase sequences for SLM. The selected monomial phase parameter is not transmitted as side information, but can be detected at the receiver. We demonstrate that it is possible to achieve significant PAR reduction with no loss of information rate and little degradation in the symbol error rate. The cubic phase sequence is found particularly suitable for this application.

Keywords: OFDM, Peak-to-average power ratio, selected mapping

1. INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) is well known for its robust performance over frequency selective channels and its spectrum efficiency. However, the price paid for the high spectral efficiency in OFDM is its low power efficiency. Handling very large peak-to-average power ratios (PARs) is a challenging issue for the RF portion of the transmitter.

Denote by $\{X_k\}_{k=0}^{N-1}$ the frequency domain OFDM signal drawn from a known constellation, where N is the number of sub-carriers. The complex baseband OFDM signal defined over the time interval $t \in [0, T_s]$ is given by

$$x(t) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k \, e^{j\frac{2\pi kt}{T_s}}, \qquad (1)$$

where T_s is the OFDM symbol period and $j = \sqrt{-1}$. Assume that X_k has zero-mean and variance $\sigma^2 = E[|X_k|^2]$. It follows easily that the average power in $x(t)$ is $E[|x(t)|^2] = \sigma^2$.

The peak-to-average power ratio (PAR) of the continuous time OFDM signal is defined as (see e.g., [9])

$$\mathrm{PAR}\{x(t)\} = \frac{\max_{0 \le t \le T_s} |x(t)|^2}{E[|x(t)|^2]}. \qquad (2)$$

Critically (Nyquist-rate) sampled OFDM symbol is represented as

$$x_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k \, e^{j\frac{2\pi kn}{N}}, \quad 0 \le n \le N-1, \qquad (3)$$

and the corresponding discrete-time PAR is

$$\mathrm{PAR}\{x_n\} = \frac{\max_{0 \le n \le N-1} |x_n|^2}{E[|x_n|^2]}. \qquad (4)$$

For a reasonably large N, the complementary cumulative distribution function (CCDF) of the discrete-time PAR in (4) is given by [1]

$$\Pr(\mathrm{PAR}\{x_n\} > \gamma) = 1 - \left(1 - e^{-\gamma}\right)^N. \qquad (5)$$

From the above formula, we infer for example that with $N = 256$ subcarriers, the median value of $\mathrm{PAR}\{x_n\}$ is 7.72 dB, and with 99.6% chance, $\mathrm{PAR}\{x_n\}$ lies within the range 5.5 dB to 10.5 dB. These are large PARs as compared to constant modulus (PAR = 0 dB) signals such as FM or unfiltered PSK. A large PAR means that the power amplifier (PA) must be oversized to handle the peak power for a given average power. Since the PA is peak power limited and its efficiency is determined by the average output power, a high PAR also means that the PA efficiency will suffer greatly. PAR reduction leads to improved power efficiency which translates into reduced power consumption and longer battery life.

Many PAR reduction techniques have been proposed in the past decade. They include clipping, coding, tone reservation, tone injection, partial transmit sequence, selected mapping, companding approaches etc and various combinations of the above; see [1], [2], [5], [6], [8], [9], [10] and references therein. These methods entail various performance - complexity - information rate tradeoffs.

We are interested in the selected mapping (SLM) approach described in [1] (see also [8]), as it is a low complexity, distortionless PAR reduction method. Let

$$U_k^{(m)} = X_k \, b_k^{(m)} = X_k \, e^{j\phi_k^{(m)}} \qquad (6)$$

be a phase rotated version of X_k, and form the corresponding time-domain OFDM symbol $u^{(m)}(t)$ according to (1). In SLM, the transmitted signal, $u^{(\bar{m})}(t)$, has the lowest PAR among the M candidate OFDM signals (including the original $x(t)$); i.e.,

$$\bar{m} = \arg\min_{1 \le m \le M} \mathrm{PAR}\{u^{(m)}(t)\}.$$

Note that $\{u^{(m)}(t)\}_{m=1}^{M}$ all contain the same information about X_k, and $E[|u^{(m)}(t)|^2] = E[|x(t)|^2]$. Any reduction in PAR is due to $U_k^{(\tilde{m})}$ properly "de-phased" resulting in a low peak value in $|u^{(\tilde{m})}(t)|$. Obviously, the larger the M, the better the PAR reducing capability of SLM, with accompanying increase in computational cost.

In SLM, the receiver must have knowledge of the index $\tilde{m}$ of the optimum phase sequence $\phi_k^{(\tilde{m})}$. One straightforward method is to transmit $\tilde{m}$ as side information, requiring $\log_2 M$ bits. As this side information is most important for decoding, it should be protected by channel coding [1]. The price paid for this approach is some loss of information rate due to the passing of the side information, and up to $M-1$ extra IFFTs that are performed at the transmitter.

In [2], a method for SLM without explicit side information was described. Labels representing the side information are inserted as prefix to the input data and passed through a scrambler. There is a small rate loss due to label insertion (implicit side information), as well as 0.2dB performance loss. In [5], a blind SLM receiver was proposed which avoids the transmission of any side information, demonstrating that it is possible to achieve distortionless PAR reduction with no loss of information rate. However, additional FFTs at the receiver do incur computational cost. Reference [5] did not provide specifics for the generation of $\phi_k^{(m)}$ nor results on PAR reduction.

The present paper builds upon the approach of [5]. We show that monomial phase sequences $\phi_k = ak^p/N$ with $p \geq 2$ can yield near optimum performance for SLM, and the transmission of side information (monomial phase parameter a) is not required. This is in contrast with [1], [6] where phases from discrete constellations were used. It is hopeful that the simple parametric form of the phase sequence may also lead to computational savings in terms of avoiding some of the IFFTs that are ordinarily needed for SLM.

2. SLM WITH PARAMETRIC PHASE ROTATIONS

2.1. SLM and PAR calculation

Suppose that $\{z_n^{(m)}\}_{m=1}^{M}$ are M *independent, Nyquist rate sampled* OFDM signals of N sub-carriers each, and let

$$\mathrm{PAR}_{low} = \min_{1\leq m\leq M} \mathrm{PAR}\{z_n^{(m)}\}.$$

Assuming that N is reasonably large, it is shown in [1] that the CCDF of PAR_{low} is

$$\Pr(\mathrm{PAR}_{low} > \gamma) = (1-(1-e^{-\gamma})^N)^M. \qquad (7)$$

In SLM, since $U_k^{(m)}$ all relate to the same X_k (c.f. (6)),

$$\mathrm{PAR}_d = \min_{1\leq m\leq M} \mathrm{PAR}\{u_n^{(m)}\} \qquad (8)$$

will not achieve the performance given in (7). However, well designed $\phi_k^{(m)}$ sequences can yield PAR_d whose performance approaches that of (7).

In reality, what matters is the PAR of the continuous-time signal. Let

$$\mathrm{PAR}_c = \min_{1\leq m\leq M} \mathrm{PAR}\{u^{(m)}(t)\} \qquad (9)$$

be the continuous-time PAR as the result of SLM. Since in general, $\max_{0\leq t\leq T_s}|x(t)| \geq \max_{0\leq n\leq N-1}|x_n|$ [10], we expect the continuous-time PAR to be no smaller than the discrete-time PAR; i.e., $\mathrm{PAR}_c \geq \mathrm{PAR}_d$. In practice, we evaluate the continuous-time PAR by oversampling the time-domain waveform. Denote the L-time oversampled OFDM signal by

$$x_{n/L} = \frac{1}{\sqrt{NL}} \sum_{k=0}^{N-1} X_k\, e^{j\frac{2\pi kn}{NL}}, \qquad 0 \leq n \leq NL-1. \qquad (10)$$

Oversampling is equivalent to zero-padding $\{X_k\}_{k=0}^{N-1}$ to length NL prior to taking the IFFT. It is shown in [10] that $\mathrm{PAR}\{x(t)\} \approx \mathrm{PAR}\{x_{n/L}\}$ for $L \geq 4$. For SLM, let

$$\mathrm{PAR}_o = \min_{1\leq m\leq M} \mathrm{PAR}\{u_{n/L}^{(m)}\}. \qquad (11)$$

It follows that $\mathrm{PAR}_o \approx \mathrm{PAR}_c \geq \mathrm{PAR}_d$. The subscripts in (8), (9) and (11) signify discrete-time, continuous-time, and oversampled PAR, respectively. In our PAR calculations, we use (11) with $L = 4$ and compare it with (7) (which serves as a lower bound).

2.2. Monomial phase rotations

Let $\phi_k = ak^p/N$, $0 \leq k \leq N-1$, where p is a positive integer. Thus, $b_k = e^{j\phi_k}$ is a monomial phase signal. When $p = 1$, u_n is a time-shifted version of x_n, thus $p = 1$ cannot be used for PAR reduction. When $p = 2$, b_k is a chirp signal; when $p = 3$, b_k is a cubic phase signal, and so forth. Figure 1 shows the principal argument of b_k; i.e., $ARG[b_k]$, for $N = 256$, $a = 11/16$, and $p = 3$. We see that although ϕ_k is a simple deterministic sequence, $ARG[b_k]$ appears random.

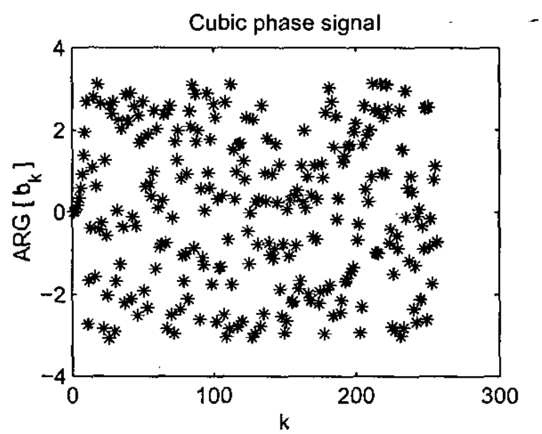

Figure 1. Principal argument of $\exp(jak^3/N)$ with $a = 11/16$ and $N = 256$.

In OFDM transmission, a cyclic prefix (CP) is inserted into the time-domain signal x_n, prior to digital to analog conversion. Since the CP does not change the peak or the average power of the transmitted signal, we do not consider the CP in the PAR analysis. At the receiver, the CP is removed, and FFT is applied to the received data. After the one-tap equalization, we obtain

$$W_k = X_k \, e^{j\bar{a}k^p/N} + V_k, \quad 0 \le k \le N-1, \qquad (12)$$

where $\bar{a}$ is the optimum monomial phase parameter that was determined by the transmitter and V_k is attributed to channel noise. We restrict a to belong to a finite set $\mathcal{A}$, and denote by C the set of constellation points for X_k.

We need to detect $\bar{a}$ from $\{W_k\}_{k=0}^{N-1}$ before detecting X_k from the same data. For a given $a \in \mathcal{A}$, we first calculate the minimum squared distance d_k between $W_k \, e^{-jak^p/N}$ and any of the constellation points in C, and then sum d_k over all $0 \le k \le N-1$ to yield the cost function $J(a)$. The estimate for $\bar{a}$ is taken as the $a \in \mathcal{A}$ that minimizes $J(a)$. Since $J(a)$ generally increases with N, blind detection of $\bar{a}$ is easier when N is larger.

By restricting a to be a rational number (instead of $\pi\times$ rational), we ensure that $e^{jak^p/N}$ will not rotate one symbol in C to a different symbol in C.

3. SIMULATIONS

In our simulations, the sub-carriers are QPSK modulated. The set of a values are $\mathcal{A} = \{1/16, 2/16, \ldots, 16/16\}$; thus $M = 17$ (including the original $x(t)$).

3.1. The effect of oversampling and comparison with eq. (7)

First, we would like to show that with the cubic phase sequence $\phi_k = ak^3/N$, we can achieve near optimum PAR reduction within the framework of SLM, by comparing the CCDF of $\text{PAR}_d = \text{PAR}\{u_n^{(\bar{m})}\}$ with (7). We generated 45,000 blocks of $\{X_k\}_{k=0}^{N-1}$, each with $N = 256$ sub-carriers. Figure 2 shows the CCDF of the PARs before and after SLM, with oversampling ($L = 4$) or with Nyquist rate sampling ($L = 1$). From right to left are the CCDF of $\text{PAR}\{x_{n/L}\}$ (solid line), $\text{PAR}\{x_n\}$ (dashed line), $\text{PAR}_o = \text{PAR}\{u_{n/L}^{(\bar{m})}\}$ (dash dotted line), $\text{PAR}_d = \text{PAR}\{u_n^{(\bar{m})}\}$ (dotted line), and eq. (7) (solid line). With Nyquist rate sampling, the performance of SLM with $\phi_k = ak^3/N$ is very close to that given by (7) which is achievable if the $U_k^{(m)}$ sequences were truly independent for different m's. With SLM, the PAR with 4-time oversampling was approximately 0.7 dB higher than that with Nyquist rate sampling (the dash-dotted and dotted lines are separated by approximately 0.7 dB). Without SLM, the difference between oversampling or not (the separation between the two right most curves) was approximately 0.5 dB. However, the PAR reduction capability should be assessed using the CCDF curves with oversampling. From Figure 2, we see that at the 10^{-3} CCDF level, SLM with cubic phase rotation gave rise to a PAR reduction performance of approximately 3.2 dB (11.2 minus 8 dB) for $N = 256$. Over an AWGN channel with SNR ranging from 5 to 13 dB, the receiver was able to detect blindly, the $\bar{a}$ values selected by the transmitter (zero error out of 45,000 blocks), and there was no degradation in the symbol error rate (SER) performance as compared to the case when SLM was not used.

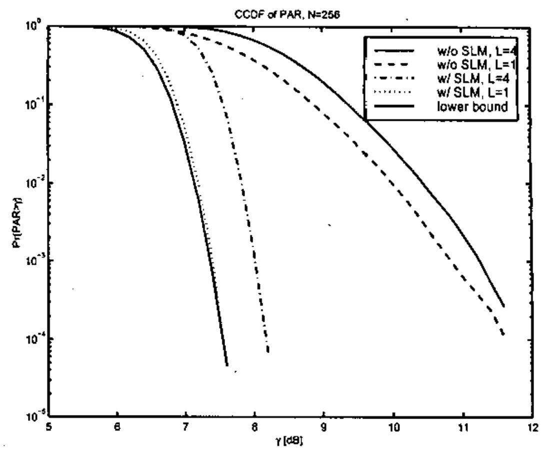

Figure 2. CCDF of PAR before and after SLM, with ($L = 4$) or without ($L = 1$) oversampling. Number of sub-carriers $N = 256$; phase sequence $\phi_k = ak^3/N$.

3.2. The effect of the monomial phase order p

N = 256. We generated 45,000 blocks of OFDM symbols with $N = 256$ sub-carriers each. Figure 3 shows from right to left, the CCDF of $\text{PAR}\{x_{n/L}\}$ (without SLM; solid line), of $\text{PAR}\{u_{n/L}^{(\bar{m})}\}$ (with SLM; $L = 4$) for $p = 2$ (dashed line), $p = 3$ (dash-dotted line), $p = 4$ (dotted line), and the lower bound (7) (solid line). We observe that the cubic and the 4th-order phase functions performed almost identically and both gave better PAR reduction than the chirp signal. Over an AWGN channel with SNR ranging from 5 to 13 dB, the receiver was able to detect perfectly (zero error out of 45,000 blocks), the $\bar{a}$ values selected by the transmitter, for all three p values. There was no degradation in SER as compared to the case when SLM was not used.

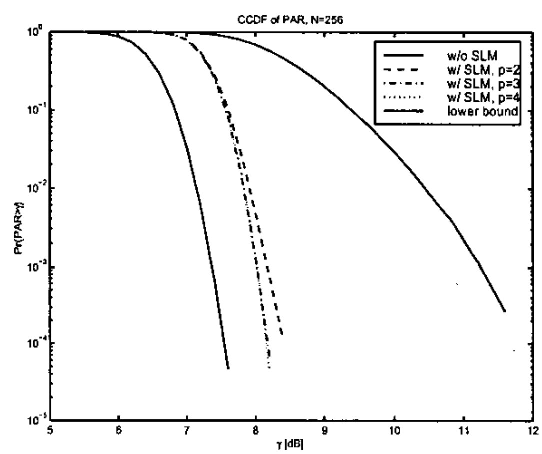

Figure 3. SLM with phase sequence $\phi_k = ak^p/N$ of orders $p = 2$, $p = 3$ and $p = 4$. Number of sub-carriers $N = 256$.

N = 64. We generated 180,000 blocks of OFDM symbols with $N = 64$ sub-carriers each. Similar to Figure 3, Fig-

SNR	5 dB	6 dB	7 dB	8 dB
$p = 2$	6.545%	0.875%	0.075%	0%
$p = 3$	5.440%	0.910%	0.040%	0%
$p = 4$	5.290%	0.720%	0.025%	0.005%

Table 1. Error rates in detecting $\bar{a}$ at various SNR levels at the receiver (AWGN channel). Error rates were zero for SNR higher than 8 dB. Number of sub-carriers $N = 64$.

ure 4 shows the performance of SLM with monomial phase orders $p = 2, 3, 4$. Again, $p = 3$ and $p = 4$ performed similarly, and their superiority to $p = 2$ is evident (more so than the $N = 256$ case). With $p = 3$ or $p = 4$, SLM resulted in approximately 3.8 dB (10.7 minus 6.9 dB) of PAR reduction at the 10^{-3} CCDF level. Although there is a gap between the CCDF curve corresponding to $p = 3$ (or $p = 4$) and that given by (7), keep in mind that the PAR values were calculated with oversampling, whereas (7) corresponds to Nyquist rate sampling. As we explained earlier, with oversampling, a few tenths of a dB increase in PAR values is expected.

Over an AWGN channel with SNR ranging from 5 to 13 dB with 20,000 blocks at each SNR, the error rates in detecting $\bar{a}$ are given in Table 1. The receiver was able to detect $\bar{a}$ perfectly (zero error out of 20,000 blocks) at SNR levels higher than 8 dB for any of the $p = 2, 3, 4$ used. Due to decoding errors for $\bar{a}$ at SNR= 5 dB and SNR= 6 dB, the SER was higher than the case without SLM. However, there was virtually no SER degradation at SNR$\geq$ 7 dB.

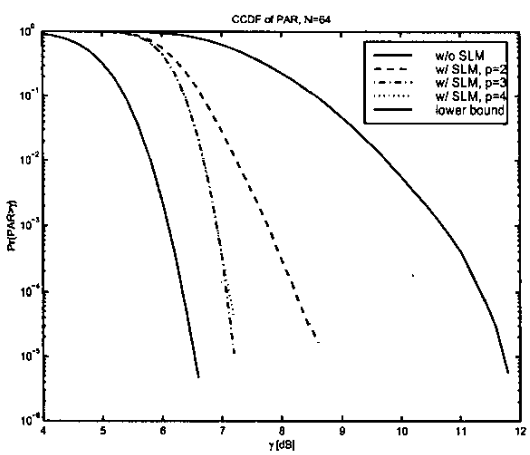

Figure 4. SLM with phase sequence $\phi_k = ak^p/N$ of orders $p = 2$, $p = 3$ and $p = 4$. Number of sub-carriers $N = 64$.

3.3. Performance in the presence of multipath fading

In this example, we demonstrate the performance of SLM with monomial phase rotations in the presence of multipath fading. The frequency selective block fading channel is modeled by an FIR filter with impulse response $h[n] = [0.15; 0.65; 0.15; 0.05]^T$, which is taken from [7]. The number of subcarriers is $N = 64$. The monomial phase parameter $\bar{a}$ is blindly detected as described in Section 2.2. A one-tap frequency domain equalizer and a subsequent symbol classifier provide the estimates for the source symbols X_k. We experimented with monomial phase orders $p = \{2, 3, 4\}$. 20,000 independent Monte Carlo runs were carried out.

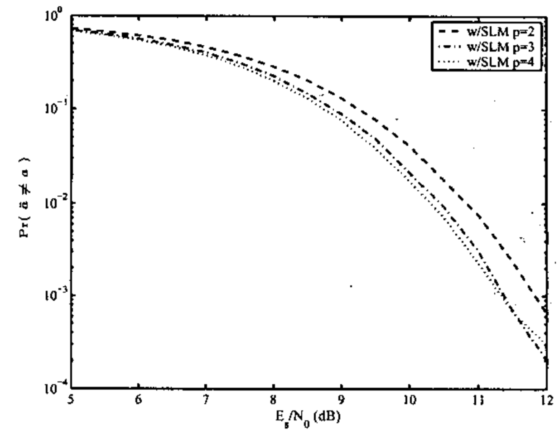

Figure 5. Error rates in detecting $\bar{a}$; $N = 64$, $M = 17$.

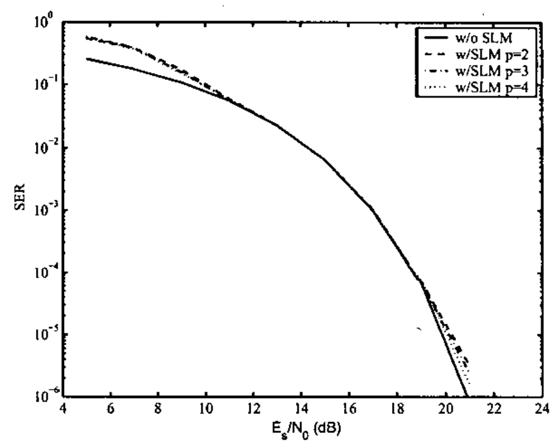

Figure 6. SER performance for $N = 64$, $M = 17$.

Fig. 5 shows the error rates in the detection of $\bar{a}$. We can see that $p = 3$ and $p = 4$ resulted in similar error rates in the detection of $\bar{a}$ and both were better than the case with $p = 2$ (there were no errors in the $\bar{a}$ estimates when SNR> 12 dB for the number of trials carried out). Fig. 6 compares the SER performance for SLM with monomial phase rotations and that without SLM. We see that in terms of the SER, monomial phase rotations with $p = 2, 3, 4$ all performed similarly; the performance was close to the case without SLM.

4. DISCUSSIONS

The so-called Newman phases, $\phi_k = \pi k^2/N$ (i.e., $p = 2$, $a = \pi$) are known to yield low PAR values for *unmodulated* multitone signals [3], [4]. However, they do not necessarily offer PAR reduction for OFDM. Let $U_k = X_k \, e^{jak^2/N}$ and a vary from 0 to 2π with step size 0.01. Note that $a = 0$ corresponds to the case without phase rotation; i.e., $U_k = X_k$ and thus the PAR value at $a = 0$ indicates the initial PAR. Figure 7 shows $\text{PAR}\{u_{n/L}\}$ as a function of a for $N = 64$ and $N = 256$, respectively. In the top figure ($N = 64$), the initial PAR was 8.85 dB; the lowest PAR after phase rotations was 5.50 dB which was achieved with $a = 4.04$. In comparison, Newman phase rotation (with $a = \pi$) yielded a PAR of 7.41 dB. For the example with $N = 256$ (bottom figure), the initial PAR was 8.12 dB; the lowest PAR after phase rotations was 6.79 dB which occurred at $a = 1.42$. Newman phase rotation yielded a PAR of 9.24 dB, worse than the initial PAR. With this experiment, we wish to clarify that although the Newman phase sequence itself has a very low PAR, it may not be useful to lower the PAR of an OFDM signal.

The fact that the monomial phase sequences are effective in reducing the PAR for SLM may be attributed to the fact that $ARG[b_k]$, or equivalently ak^p/N modulo 2π, exhibits certain pseudo-randomness behavior. Moreover, $ARG\ [b_k]$ is fairly sensitive to the a value used, making blind detection of $\tilde{a}$ possible and reliable, especially for large N's.

With a parametric form for ϕ_k, an interesting question is, given $\{X_k\}_{k=0}^{N-1}$, whether it is possible to predict which $a \in \mathcal{A}$ values are likely to yield low PAR values $\text{PAR}\{u_{n/L}\}$ without performing all the IFFTs. We are currently examining this topic.

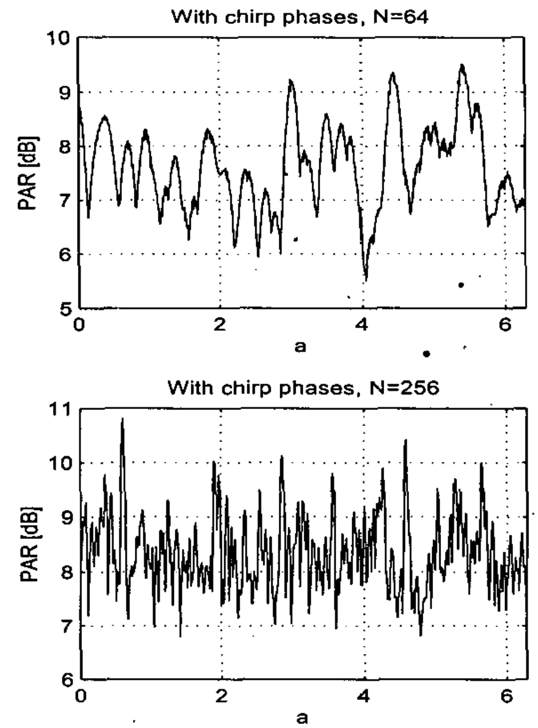

Figure 7. PAR as a function of a for chirp phase rotations $\phi_k = ak^2/N$.

5. CONCLUSIONS

SLM is a low complexity, distortionless PAR reduction technique for OFDM. It is even more attractive with the recent discovery in [5] that it is possible to communicate with SLM-OFDM without the need to transmit side information. We presented the novel concept of using monomial phase sequences for SLM and found that the cubic phase sequences offer near optimum PAR reducing capability. As compared to conventional "non-parametric" SLM, the simple parametric form of the monomial phase sequences may also offer potential for computational savings at the transmitter.

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