Polarization-Based Zero Forcing Suppression with Multiple Degrees of Freedom

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***Abstract* — Polarization-based suppression with zero forcing (ZF) is limited by similarities in the polarization-frequency response of the desired and interference signals. At subcarriers where the responses are similar, suppression of the interference also leads to suppression of the desired signal, and achievable cancellation ratios are correspondingly limited. A measure called the polarization power coupling (PPC) function is introduced for determining the impact of ZF on desired signal suppression. The PPC function provides a useful measure for predicting the magnitude of the suppression of the desired signal as a function of the subcarrier frequency. We consider the use of an additional degree of freedom (DOF), e.g., with a partially correlated or uncorrelated PPC response, to provide diversity detection for improving symbol error rate (SER) performance associated with recovery of the desired signal. The approach employs suppression diversity on a subcarrier-by-subcarrier basis and uses the PPC function to identify the ZF filter response leading to a better estimate of the desired signal symbols in each subcarrier. The method is shown to provide reduced suppression of the desired signal and to improved SER performance. An alternative use of the additional DOF is considered for suppression of a second source in a two-stage processing scheme, and a receiver architecture is proposed that exploits the diversity and suppression extensions enabled by an additional degree of freedom.**

***Index Terms* — Polarization, Zero Forcing, Suppression, Degree of Freedom, Diversity.**

I. INTRODUCTION

Wireless systems are increasingly challenged to operate in environments with interference, and technologies enabling robust interference mitigation continue to be of great interest to the wireless community. A rich body of literature exists associated with interference suppression techniques, and the techniques that may apply to any given situation will be dependent upon a number of factors including synchronicity of the sources, signal bandwidths, directions of arrival, channel responses, waveform designs, channel knowledge, receiver architecture, and many other salient factors.

The particular problem that we consider involves the recovery of a wideband source in rich multipath and in the presence of a co-directional, asynchronous, bursty co-channel interference signal, where the co-channel signal could take the form of multi-user interference or of an unknown wideband interference source. The problem poses certain challenges to processors such narrowband space- and wideband space-time processors since space-based processors are generally incapable of suppressing co-channel interference when the signal components are not resolvable in the angle dimension [1]. Techniques that exploit bandwidth differences between the signals [2] are also not tractable in our case since the co-channel signals are assumed to possess similar bandwidths. Methods that exploit signal cyclostationarities [3] have limited application since the signals in our case may be homogeneous. The fact that the signals can be asynchronous, such as in bursty operation, limits the applicability of methods that rely on time-aligned waveforms or approaches that rely on time-varying adaptive weights [4] that have to be changed when interference source transmissions toggle between off and on states. Techniques that presume certain signal knowledge of homogeneous signals [5] also have limited application to address unknown interference sources.

We consider a polarization–based zero-forcing architecture to achieve interference suppression under these challenging constraints. Past work has shown that polarization-based architectures have the ability to separate two narrowband signals arriving at an array from very closely spaced incidence angles when the signals exhibit different polarizations [6, 7]. For wider-band signals it has also been shown that the architecture can be extended to channels exhibiting frequency selectivity through use of a subbanded architecture [8]. In this case, the ZF filtering approach enables suppression of co-directional wideband interference by exploiting differences in the polarization-frequency responses associated with the sources. The architecture requires estimation of the subband polarimetric channel states associated with each interferer, which can be obtained when the interference signals are received "in the clear".

Zero forcing in the polarization domain also enables low-complexity suppression of asynchronous interference and burst interference, provided that the channel estimates of the interference are not stale, e.g., they have been estimated within the last T_c seconds, where T_c is the channel coherence time associated with each *subband*. Channel estimates for an interferer can be obtained in a conventional fashion when the interference signal can be demodulated, whereas for unknown interference, a *relative* channel estimate useful for suppression can be derived from the Wolf coherency matrix [9].

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In this work we consider a notional polarimetric ZF suppression receiver with an architecture consisting of three degrees of freedom, including two dual polarized outputs and a third antenna port that could represent either a space DOF, an orthogonal polarization DOF, or an antenna beam DOF. Using the two dual-polarized antenna outputs, ZF is applied on a subband-by-subband basis for interference suppression. The extra degree of freedom is also employed with one of the original DOFs in a separate ZF filtering operation and both ZF filter outputs are combined in a suppression diversity scheme that minimizes suppression of the desired signal in each subband.

The existence of an additional DOF presents other opportunities to augment the capabilities of a suppression receiver, for example to enable operation in an environment with an additional interference source. To underscore these benefits, we consider use of the DOF in a ZF two-stage suppression processor to achieve suppression of multiple interference signals and provide results for an example with two unknown interferers.

Finally, a full receiver architecture supporting these and other processing modes to accommodate various signal/interference contingencies is also presented. A feature of the receiver is its need for channel state information (or relative channel state information) for each interference source, information that is usually accessible in bursty communications environments. These up-to-date channel estimates (or relative channel estimates) provide the needed side information to accommodate the various signal processing modes.

In the next section, we introduce the signal models and the ZF algorithm that forms the basis of the suppression processing. Suppression performance measures, particularly those based on the cancellation ratio, are also discussed. Section III introduces the polarization power coupling function, which provides a measure for predicting the relative suppression of the desired signal induced by ZF processing. In Section IV, interference suppression performance results are presented, where the performance is quantified in terms of symbol error occurrences for various SIR values and also in terms of the cancellation ratio. A suppression diversity strategy is presented in Section V, which assumes an additional degree of freedom is available to enhance the achieved SER performance. In Section VI we discuss application of the degree of freedom for the suppression of an additional interferer and a receiver processing architecture is proposed that exploits the diversity and suppression extensions enabled by the additional degree of freedom. The conclusions are presented in the final section of the paper.

II. SIGNAL MODELS

Assume two OFDM-based co-channel signals [10]

$$s_D(t) = CQ^H x_D \in \mathbf{C}^{N+L} \tag{1}$$

and

$$s_I(t) = CQ^H x_I \in \mathbf{C}^{N+L} \tag{2}$$

where x_D and x_I are the frequency domain OFDM payloads, Q is the DFT matrix, C adds the cyclic prefix, N is the DFT matrix size and L is the length of cyclic prefix. The signals $s_D(t)$ and $s_I(t)$ propagate through polarimetric vector channels $\underline{h}_D$ and $\underline{h}_I$, respectively, to yield the received signal vectors $\underline{y}_D(t)$ and $\underline{y}_I(t)$ at the receiver, where the signals, in general, will be asynchronous. We assume that $\underline{Y}_D(k)$ and $\underline{Y}_I(k)$ are not time coincident at the receiver at all times, and hence there is opportunity to form estimates of their respective channel vector states when the signals do not overlap. At the receiver, after synchronization, removal of the cyclic prefix, and application of the FFT, we may represent the received vector for each signal as a function of the subcarrier index

$$\underline{Y}_D(k) = \begin{bmatrix} Y_D^v(k) & Y_D^h(k) \end{bmatrix}^T = \underline{H}_D(k) x_D(k) \tag{3}$$

and

$$\underline{Y}_I(k) = \begin{bmatrix} Y_I^v(k) & Y_I^h(k) \end{bmatrix}^T = \underline{H}_I(k) x_I(k) \tag{4}$$

where the superscripts v and h represent orthogonally polarized signal components. $\underline{H}_D$ and $\underline{H}_I$ denote the true vector channel gains associated with subcarrier k represented as:

$$\underline{H}_D(k) = \begin{bmatrix} H_D^V(k) & H_D^h(k) \end{bmatrix}^T \tag{5}$$

and

$$\underline{H}_I(k) = \begin{bmatrix} H_I^V(k) & H_I^h(k) \end{bmatrix}^T \tag{6}$$

The corresponding estimates formed by the receiver are denoted by $\widetilde{H}_D(f)$ and $\widetilde{H_I}(f)$. Using known preambles, the channel states for both signals may be estimated for each subcarrier and will remain valid over the coherence time of the subband channel, T_c, which is assumed to be sufficiently large so that the channel for each subband is approximately stationary over the duration of several transmitted packets.

When both signals are time-coincident at the receiver, zero-forcing of each signal may be readily applied on a subcarrier-by-subcarrier basis to recover the desired signal. The output of the ZF suppression filter is given by

$$Z(k) = \underline{W}^T(k)\left(\underline{Y}_D(k) + \underline{Y}_I(k)\right) \tag{7}$$

where

$$W(k) = \left[\widetilde{H}_I^h(k) \quad -\widetilde{H}_I^v(k)\right]^T \tag{8}$$

The interference power at the output of the ZF filter is given by

$$\begin{aligned} P_I^{out}(k) &= \underline{W}^T(k) Y_I(k) \left(\underline{W}^T(k) Y_I(k)\right)^* \\ &= \left|H_I^h(k) Y_D^v(k) - H_I^v(k) Y_D^h(k)\right|^2 \end{aligned} \tag{9}$$

We note that when $\widetilde{H}_I(f) = H_I(f)$ this reduces to $P_I^{out}(k) \approx 0$. The null is only approximate due to limitations from noise and depolarization that may exist within the subcarrier band. The desired signal power at the output of the ZF filter is given by

$$\begin{aligned} P_D^{out}(k) &= \underline{W}^T(k) \underline{Y}_D(k) \left(\underline{W}^T(k) \underline{Y}_D(k)\right)^* \\ &= \left|H_I^h(k) Y_D^v(k) - H_I^v(k) Y_D^h(k)\right|^2 \end{aligned} \tag{10}$$

The corresponding cancellation ratios for each subcarrier k may be estimated through

$$\rho_{dB}(k) = 10\log_{10}\left(\frac{P_D^{out}(k)/P_I^{out}(k)}{P_D^{in}(k)/P_I^{in}(k)}\right), \tag{11}$$

where

$$P_I^{in}(k) = \underline{Y}_I^T(k)\underline{Y}_I^*(k) \tag{12}$$

and

$$P_D^{in}(k) = \underline{Y}_D^T(k)\underline{Y}_D^*(k) \tag{13}$$

and the total effective cancellation ratio is determined from

$$\rho_{dB}^{Tot} = 10\log_{10}\left(\frac{\sum_{k=1}^{N} P_D^{out}(k)/\sum_{k=1}^{N} P_I^{out}(k)}{\sum_{k=1}^{N} P_D^{in}(k)/\sum_{k=1}^{N} P_I^{in}(k)}\right) \tag{14}$$

III. POLARIZATION COUPLING FUNCTION

The following theorem quantifies the extent of desired signal suppression that occurs with ideal ZF of the interference (i.e., when $\widetilde{H}_I(f) = H_I(f)$), which depends on the polarization difference between the desired signal and the interference signal at each subcarrier frequency.

Theorem: *Given two signals with arbitrary polarizations represented by Stokes vectors $S_1(f)$ and $S_2(f)$, then ideal ZF filtering of either signal will yield a suppression factor, $\zeta(f)$, that describes the extent to which the other signal is attenuated by the ZF filter, where*

$$\zeta(f) = |S_1(f) - S_2(f)|^2/4 \tag{15}$$

***Proof**: The suppression is governed by $\zeta(f) = \sin^2(\phi/2)$, where ϕ is the central angle between the two Stokes vectors. By geometry, $\sin(\phi/2) = d_h/2$, where d_h = $|S_1(f)-S_2(f)|$. Therefore, the suppression may be rewritten as*

$$\zeta(f) = |S_1(f) - S_2(f)|^2/4$$

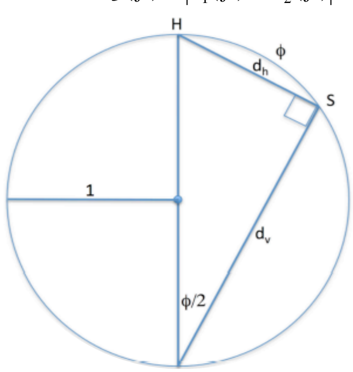

Figure 1. Geometry Associated with a Cross Section of the Poincare Sphere

The resulting function $\zeta(f)$ is referred to as the polarization power coupling (PPC) function. Examples of polarization state variation w.r.t frequency from experimentation [11] are shown in Figure 2. Using these as specific channel responses for the desired and interference signals, the corresponding PPC function is plotted in the top panel of Figure 3. In the middle panel, we also plot a second PPC, corresponding to one of the original antenna outputs and a speculative third DOF.

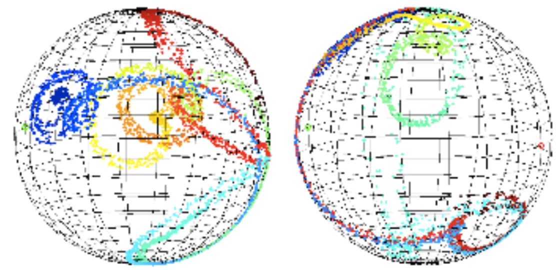

Figure 2. Channel Responses Derived from Experimental Impulse Response Measurements

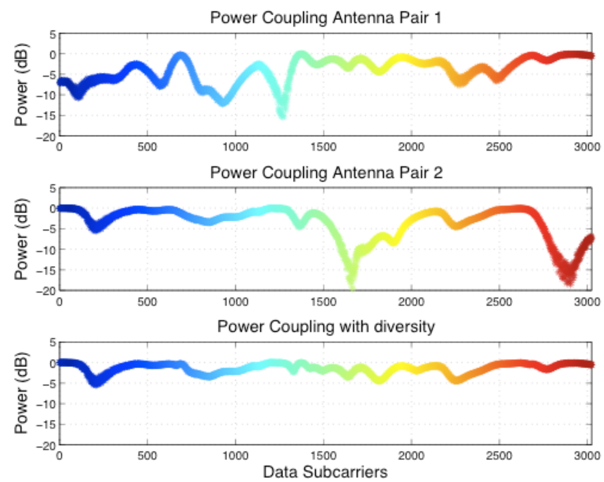

Figure 3. Polarization Power Coupling Function for the Channel Responses Shown in Figure 2

The PPC function quantifies the polarization-based power suppression of the desired signal based on ideal suppression of the interference signal (where source 1 and source 2 take opposite roles as the desired and interfering signals if we are trying to recover both signals). The function yields notches where the polarization-frequency responses of the two sources are nearly the same. Interestingly, the PPC applies to source 1 for a ZF filter designed to suppress source 2 (ZF1), and also applies to source 2 at the output of a ZF filter designed to suppress source 1 (ZF2). This is evident in the example shown in Figures 4 and 5. The power spectra of the V and H components of source 1 and source 2 are shown in Figure 4. The power spectra of source 1 and source 2 at the output of ZF1 and ZF2 are shown in Figure 5. Note that source 2 has been suppressed by roughly 30 dB, and that the power spectrum of the desired signal (source 1 in this case) evidences scaling by the PPC function. Similarly, referring to the output of ZF2, source 1 has been suppressed by roughly 30 dB and the power spectrum of the desired signal (source 2 in this case) also evidences scaling by the PPC function.

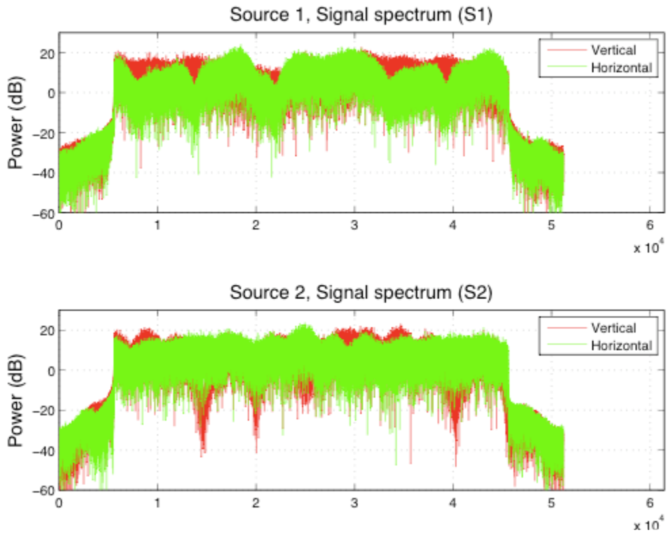

Figure 4. Power Spectra of Source 1 V and H components and the Signal Outputs Associated with ZF1

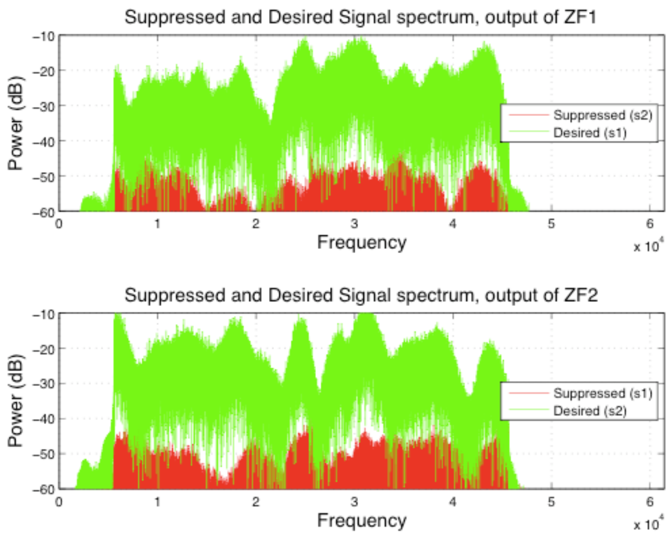

Figure 5. Power Spectra of Source 2 V and H components and the signal outputs Associated with ZF2

IV. Interference Mitigation Processing

The suppression performance of the ZF filtering approach may be expressed in terms of the cancellation ratio as a function of the subcarrier or as an average over all subcarriers. Figure 6 shows the cancellation ratio (CR) of both ZF filters as a function of subcarrier index. It can be seen that the CR fades at subcarrier indices for which the polarization-frequency responses of both responses are similar, as would be expected. The average cancellation ratio for both ZF filters in the example is approximately 32 dB.

Signal recovery performance after suppression is qualitatively expressed in Figures 7 and 8 for each ZF filter. Each figure shows symbol error occurrences associated with demodulation of a weak desired signal as a function of SIR, where 10 OFDM symbols were employed in the analysis. Figure 7 corresponds to the demodulation of source 1 at the output of the ZF1 and Figure 8 to the demodulation of source 2 at the output of the ZF2. Note that subcarriers at which cluster of symbol errors occur as the SIR decreases are closely tied to the notches in the PPC function. This is to be anticipated since these notches indicate where the desired signal has been suppressed along with the interference. An approach to limit the suppression of the desired signal is considered that exploits an additional degree of freedom for diversity on a symbol-by-symbol basis.

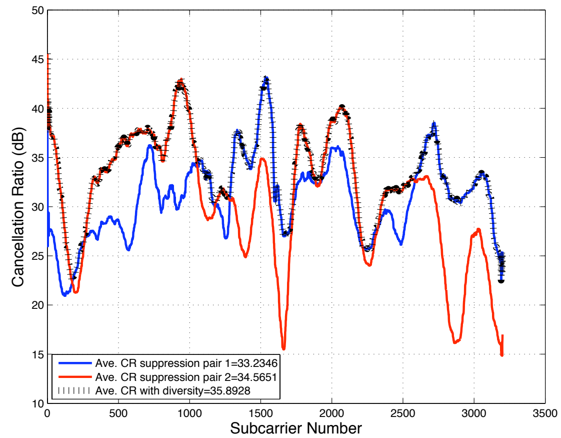

Figure 6. Cancellation Ratio versus Subcarrier for the ZF for Pair 1 (blue), the ZF for Pair 2 (red) and with Diversity (black)

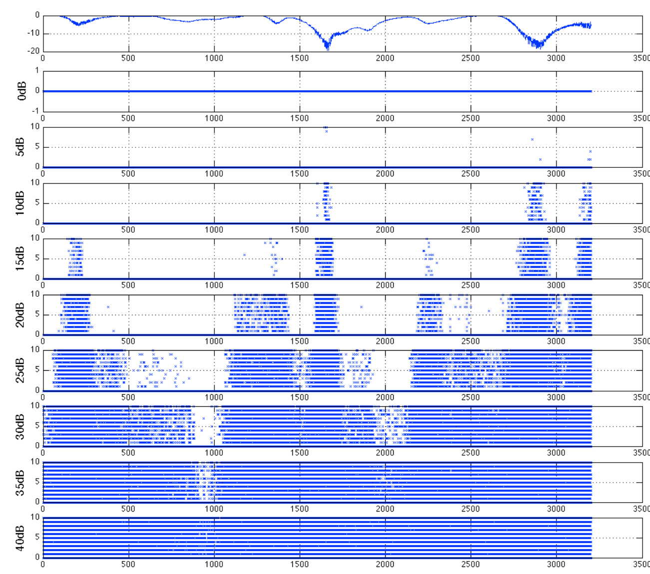

Figure 7. Symbol Error Occurrences as a Function of Subcarrier for Different SIR Levels for the original pair of antenna ports

Notionally, we assume that another antenna port is available to provide an additional degree of freedom. For example, the three ports could correspond to the outputs of a tripole antenna [12] with a vertical dimension V, and two horizontal dimensions H1and H2, where the H1 and H2 dimensions are rotated 90 degrees relative to each other in the azimuth plane. Alternatively, the antennas could include a dual polarized antenna and a spatially separated element. Other configurations are also possible. We proceed with the assumption that the PPC response associated with the new degree of freedom is distinct from the original PPC response. For example, consider the case where the new PPC response is as shown in the middle panel of Figure 3. A similar performance evaluation may be conducted using the new PPC response, and is shown in Figure 8. Similar trends occur, such as the initial clustering of symbol errors at the frequencies where the PPC is notched. Since the PPC function for the DOF will in general be different from the original PPC function, joint processing can be employed to effect gain from the diversity. In the next section we provide a simple method for achieving diversity gains based on the PPC functions.

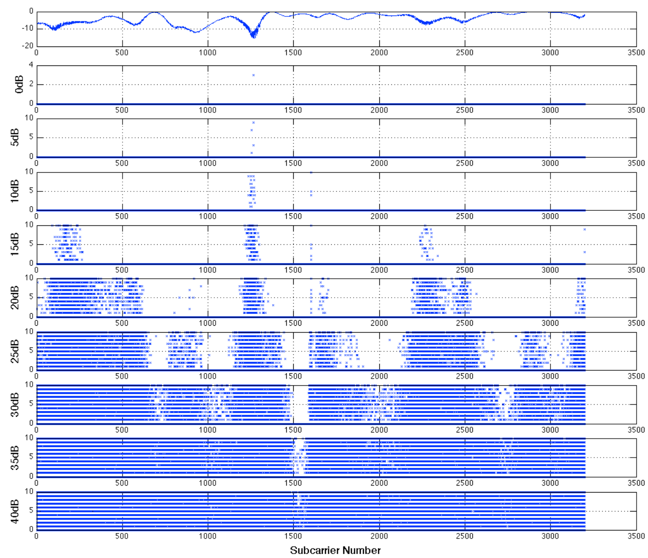

Figure 8. Symbol Error Occurrences as a Function of Subcarrier for Different SIR Levels when a second degree of freedom is used in place of one of the original antennas

V. Diversity Processing

The addition of a second degree of freedom provides the possibility of diversity processing to improve the SER performance associated with the recovery of the desired signal. The particular approach involves the comparison of the two PPC response values on a subcarrier-by-subcarrier basis, and selecting the demodulated symbols from the antenna pair with the highest PPC value. This processing strategy leads to a modified PPC response defined by

$$PPC_{div} = \max(PPC_1, PPC_2) \qquad (16)$$

Using the PPC responses from Figure 3 (subplots 1 and 2), PPC_{div} is computed and plotted in figure 3 (subplot 3), and the resulting SER performance is also shown in Figure 9. We note that the use of this form of suppression diversity leads to an improvement in the performance, enabling recovery of uncoded symbols with few errors at SIR values of -15 dB. Both coding and interleaving could be applied to improve upon these results.

VI. Extension to an Additional Interference Signal

The additional degree of freedom can also be utilized to suppress a second interference source. An architecture that we propose for accomplishing this is a two-stage suppression filter shown at the bottom of Figure 10, which is an extension of the architecture described in [10]. In this design, the first stage filters employ both pairings of the antennas (with their respective PPC functions) to suppress one of the interfering signals using available channel estimates. The residues at the output of these filters will ideally exhibit contributions only from the remaining two sources. The second stage filter is then applied to suppress the remaining interferer. The residue at the output of the second filter may then be processed in a conventional manner to recover the desired signal. The order of the filters is a design consideration, and it would seem logical to assign the first stage to the more stationary interference source, but otherwise could be handled in the receiver architecture by including processing paths to accommodate both combinations. Figure 11 demonstrates the efficacy of the approach for two "unknown" equal-power filtered complex noise signals, assuming a 5-tap channel with uncorrelated V and H responses. The achieved suppression of the interference power are reported as a function of the number of subbands assumed for the architecture.

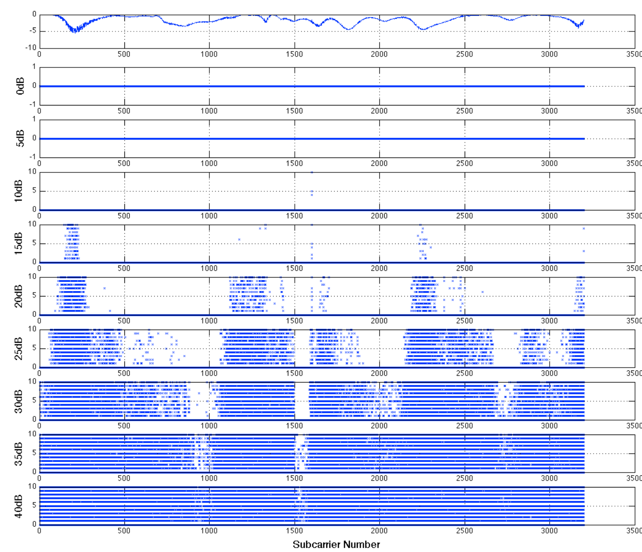

Figure 9. PPC Diversity Response and the Corresponding Symbol Error Occurrences as a Function of Subcarrier for Different SIR Levels

The receiver architecture proposed in Figure 10 provides processing paths to accommodate signal processing strategies including suppression, to enhance the likelihood of signal recovery for a number of signal/interference contingencies. Processing includes direct demodulation paths, one-stage suppression processors for the cancellation of a single interferer, suppression with diversity if an additional DOF is present, and two-stage suppression processing to suppress multiple interference sources. The entire architecture is channel driven, and hence, when equipped with recent channel estimates for each of the signals in the environment, the notional receiver has capability to accommodate a wide range of signal/interference contingencies.

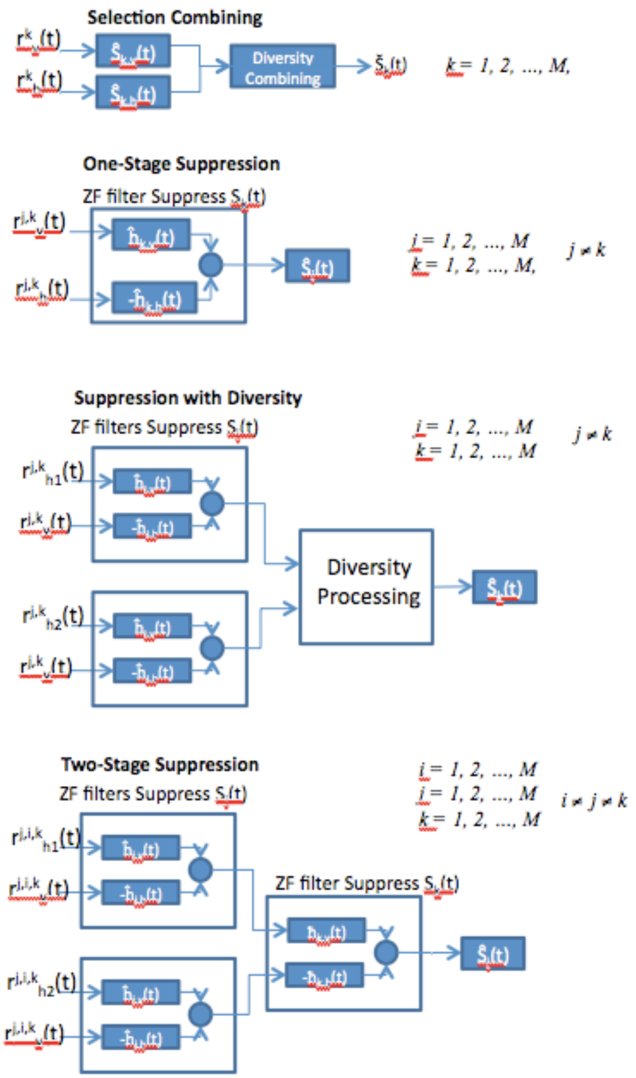

Figure 10. Receiver Block Diagrams Supporting Different Forms of Processing for various Signal Contingencies

VII. Conclusion

The impact of ZF on desired signal recovery is well characterized by a power coupling function that describes the suppression of the desired signal as a function of subcarrier. Resulting CR and SER performances are illustrated demonstrating an ability to recover weak signals in stronger interference. Incorporating an additional degree of freedom provides additional capability for reducing errors in desired signal recovery. This is accomplished through application of two ZF filters, each with distinct PPC responses, and selecting the symbol decisions from the filter with the highest PPC value on a subcarrier-by-subcarrier basis. The method is seen to provide improved SER performance. The additional DOF can also be employed to suppress multiple interference sources. A receiver architecture with multiple processing paths has been postulated that requires up-to-date channel estimates of the interference signals to enable a wide range of processing schemes.

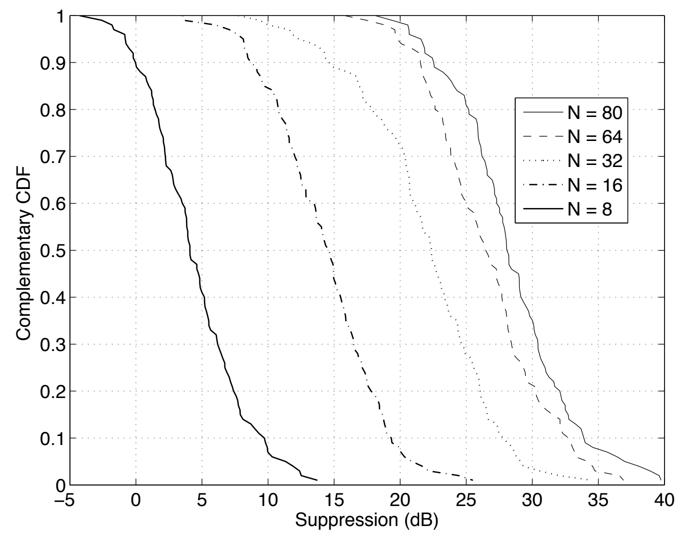

Figure 11. Performance of a ZF Two-Stage Architecture for the Suppression of Two Unknown Interferers.

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