Peak-to-Average Power Ratio and Power Efficiency Considerations in MIMO-OFDM Systems

Chunming Zhao, Robert J. Baxley, and G. Tong Zhou

***Abstract*— Peak-to-average power ratio (PAR) reduction is a powerful tool that can increase power efficiency and reduce distortion noise in MIMO-OFDM systems. Various PAR metrics have been used in the MIMO-OFDM literature without justifications regarding the physical mechanisms that support those PAR definitions. In this paper, we show that in order to deliver high power efficiency, two MIMO-OFDM linear-scaling schemes are possible: one that scales all MIMO branches with the same factor and another that uses a different scaling factor for each branch. For each system, we derive a meaningful PAR metric that should be used.**

***Index Terms*— multiple-input multiple-output (MIMO), orthogonal frequency division multiplexing (OFDM), peak-to-average power ratio (PAR), power efficiency.**

I. Introduction

In frequency-selective fading channels, a popular mechanism to achieve high data-rate transmission is multiple-input multiple-output (MIMO) orthogonal frequency division multiplexing (OFDM). High peak-to-average power ratio (PAR) is a major disadvantage of OFDM. Some PAR reduction methods for single-input single-output (SISO) OFDM systems have been extended to MIMO-OFDM; for example, selected mapping [1], partial transmit sequence [2], convex optimization [3], and cross-antenna rotation and inversion [4]. Several novel PAR reduction methods that are specifically for MIMO-OFDM have also been proposed, for example, unitary rotation [5], and spatial shifting [6].

The goal of PAR reduction here is to improve the overall system power efficiency. For SISO-OFDM systems, there is a straightforward relationship between PAR and efficiency. For MIMO-OFDM, however, it is not obvious how the multiple branch PARs should be combined into a single metric that reflects the overall system power efficiency. In fact for MIMO-OFDM, different PAR metrics have been used by different authors [1], [2], [5], [6], and it is not clear why those are meaningful metrics. The objectives of this paper are three-fold: 1) to clarify how PAR is related to power efficiency in a MIMO-OFDM system, 2) to analyze the mean power efficiency realized by different MIMO-OFDM systems, and 3) to define PAR metrics that maximize the power efficiency for MIMO-OFDM.

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II. Review of PAR Definition and Linear Scaling in SISO-OFDM Systems

In SISO-OFDM systems, the time-domain signal is generated by the inverse Fourier transform as

$$x(t) = \frac{1}{\sqrt{N}} \sum_{k=-N/2}^{N/2-1} X_k \, e^{\frac{j2\pi kt}{T}}, \qquad t \in (0, T], \tag{1}$$

where X_k is the frequency-domain symbol, N is the number of subcarriers, and T is the symbol duration. It is well known that $x(t)$ is approximately complex Gaussian distributed with significant envelope variations. If $x(t)$ goes through a power amplifier (PA), high peaks in $x(t)$ will be clipped, thus increasing the bit error rate (BER) and causing interference to other users. To avoid such nonlinear distortions, $x(t)$ has to be backed-off to the linear region of the PA, reducing the power efficiency of the system. A popular metric that characterizes the dynamic range of $x(t)$ is the peak-to-average power ratio (PAR) defined as $\mathrm{PAR} = \max_{t\in(0,T]} |x(t)|^2 / E[|x(t)|^2]$, where $E[\cdot]$ denotes statistical expectation.

Suppose there are M transmit antennas in a MIMO-OFDM system. Thus, M high power amplifiers will be needed, one in front of each antenna. The mth branch signal $x_m(t)$ has a PAR value defined as

$$\mathrm{PAR}_m = \frac{\max_{t\in(0,T]} |x_m(t)|^2}{E[|x_m(t)|^2]}. \tag{2}$$

In this paper, we will explore the relationship between the MIMO-OFDM power efficiency and branch PAR values $\{\mathrm{PAR}_m\}_{m=1}^{M}$. We assume that identical PAs are used in the M transmit branches.

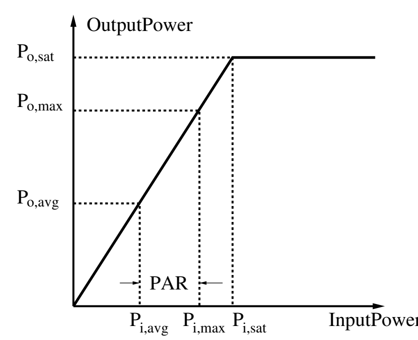

Fig. 1. An ideal linear PA characteristic.

It is well known that the PA is a peak power limited device. Figure 1 shows the input-output characteristic of an ideal linear

PA, achievable by predistorting a nonlinear PA. The DC to RF power conversion efficiency is defined as $\eta = P_{\rm o,avg}/P_{\rm dc}$, where $P_{\rm o,avg}$ is the average output power, and $P_{\rm dc}$ is the power drawn from the DC source. We can linearly scale the peak power of the input signal $P_{\rm i,max}$ to the saturation level $P_{\rm i,sat}$ to deliver the maximum power efficiency; the scaling factor can be obtained as $\zeta = P_{\rm i,sat}/\max_{t\in(0,T]}|x(t)|^2$ [7]. Thus, $\sqrt{\zeta}\, x(t)$ is the PA input signal. Note that this "linear scaling" approach is one type of the "back-off" technique. During each OFDM symbol period, ζ remains constant, but will vary from symbol to symbol. As such, the efficiency of a class A PA is

$$\eta = \frac{P_{\rm o,avg}}{P_{\rm dc}} = \frac{P_{\rm o,avg}}{2P_{\rm o,max}} = \frac{P_{\rm i,avg}}{2P_{\rm i,max}} = \frac{0.5}{\rm PAR}. \tag{3}$$

The above relationship clearly indicates that the power efficiency can be increased by reducing the PAR of the input signal.

III. MIMO Linear Scaling

In this section, the linear scaling scheme of [7] is extended to MIMO-OFDM. The following two scenarios are possible: 1) a branch-independent scaling factor c is used for all the M branches, 2) branch-dependent scaling factors $\{d_m\}_{m=1}^{M}$ are used so that each branch has a unique scaling factor. We will show that these scenarios will justify different PAR metrics that are meaningful from the power efficiency point of view.

The power efficiency of the mth PA in the MIMO-OFDM system can be written as $\eta_m = P_{\rm o,avg}^{(m)}/P_{\rm dc}$, where $P_{\rm o,avg}^{(m)}$ is the average output power of the mth PA. The overall DC to RF power conversion efficiency for the M PAs is

$$\bar\eta = \frac{\sum_{m=1}^{M} P_{\rm o,avg}^{(m)}}{MP_{\rm dc}} = \frac{1}{M}\sum_{m=1}^{M}\eta_m, \tag{4}$$

which is the average of the power efficiencies at the M branches.

A. Identical Scaling Factor (ISF) Case

When a MIMO-OFDM system uses channel state information (CSI) at the transmitter to beam form the transmitted signal, it is only possible to apply an identical scaling factor across all transmit branches without disrupting data transmission. By placing the largest of all M signal peaks at the saturation point of the PA, the scaling factor is obtained as $c = P_{\rm i,sat}/\max_m\{\max_{t\in(0,T]}|x_m(t)|^2\}$. Thus, the mth branch PA input signal is $\sqrt{c}\, x_m(t)$. Assume that $E[|x_m(t)|^2]$ is the same $\forall m$, then η_m is the same $\forall m$. Define

$$\bar m = \arg\max_{1\le m\le M}\{{\rm PAR}_m\}. \tag{5}$$

The $\bar m$th branch has ${\rm PAR}_{\bar m} = \max_{1\le m\le M}{\rm PAR}_m$, and the corresponding efficiency is $\eta_{\bar m} = 0.5/{\rm PAR}_{\bar m}$. Therefore, the overall efficiency is $\bar\eta^{(\rm is)} = \eta_{\bar m}$ according to (4). In other words, the power efficiency in the ISF case is determined by the worst-case branch PAR. Accordingly, the PAR metric in an ISF MIMO-OFDM system is the *worst branch* PAR; i.e.,

$${\rm PAR}^{(\rm is)} = {\rm PAR}_{\bar m}. \tag{6}$$

This is the metric that should be minimized in order to achieve maximum transmit power efficiency for the overall MIMO system.

If the signal modification is linear, as is the case in beamforming, the PAR distribution in each branch is the same. Let us denote the complementary cumulative distribution function (CCDF) of the PAR of $x_m(t)$ by $F(\gamma)$ and the corresponding probability density function (PDF) by $f(\gamma)$. Assuming that the M PARs have the same distribution and are mutually independent, the CCDF of the PAR in (6) is

$$\Pr\{{\rm PAR}_{\bar m} > \gamma\} = 1 - [1-F(\gamma)]^M. \tag{7}$$

With this, we can write

$$\begin{aligned} E\left[\bar\eta^{(\rm is)}\right] &= E[\eta_{\bar m}] = 0.5E\left[\frac{1}{{\rm PAR}_{\bar m}}\right] \\ &= \int M[1-F(\gamma)]^{M-1} f(\gamma)\frac{0.5}{\gamma}d\gamma. \end{aligned} \tag{8}$$

B. Multiple Scaling Factors (MSF) Case

In MIMO systems where no *a priori* information about the channel is used in the transmitter, (e.g. space-time block codes), it is possible to apply a scaling factor to each branch independently. Similar to the SISO case, each of these scaling factors can be viewed as part of the MIMO channel and can be equalized at the receiver. Denote the scaling factor on the mth branch by $d_m = P_{\rm i,sat}/\max_{t\in(0,T]}|x_m(t)|^2$. Then, the mth branch PA input signal becomes $\sqrt{d_m}\, x_m(t)$. After the linear scaling, the peak powers of all branches are positioned at $P_{\rm i,sat}$ and the branch power efficiency can be written as $\eta_m = 0.5/{\rm PAR}_m$, $1\le m\le M$. Substituting η_m into (4), we can obtain the overall MIMO system power efficiency as

$$\bar\eta^{(\rm ms)} = \frac{0.5}{M}\sum_{m=1}^{M}\frac{1}{{\rm PAR}_m}. \tag{9}$$

In the spirit of (3), if we define a MIMO PAR metric ${\rm PAR}^{(\rm ms)}$ such that $\bar\eta^{(\rm ms)} = 0.5/{\rm PAR}^{(\rm ms)}$, then

$${\rm PAR}^{(\rm ms)} = \left(\frac{1}{M}\sum_{m=1}^{M}\frac{1}{{\rm PAR}_m}\right)^{-1}. \tag{10}$$

Thus, for a MIMO system that allows for independent scaling on each branch, the *harmonic average* of the branch PARs is the proper PAR metric. Note that in this configuration, each branch delivers the same peak power but different average powers on a per-block basis.

With the help of (4), we can express the expected value of $\bar\eta^{(\rm ms)}$ as

$$E\left[\bar\eta^{(\rm ms)}\right] = \frac{0.5}{M}\sum_{m=1}^{M}E\left[\frac{1}{{\rm PAR}_m}\right] = \int f(\gamma)\frac{0.5}{\gamma}d\gamma. \tag{11}$$

According to [8], the CCDF of the PAR of $x_m(t)$ can be approximated by $F(\gamma) = 1-\exp\{-b(N)e^{-\gamma}\}$, where $b(N) = N\sqrt{\frac{\pi}{3}\ln N}$. In this case, equation (8) yields for the ISF configuration

$$E\left[\bar\eta^{(\rm is)}\right] = \int_{e^{-1}}^{0}\frac{Mb(N)e^{-Mb(N)u}}{2\log(u)}du, \tag{12}$$

and equation (11) yields for the MSF configuration

$$E\left[\bar{\eta}^{(\mathrm{ms})}\right] = \int_{e^{-1}}^{0} \frac{b(N)e^{-b(N)u}}{2\log(u)} du, \tag{13}$$

which can be evaluated numerically.

Figure 2 is a plot of the mean power efficiency for several different MIMO-OFDM configurations. The figure shows that the MSF configuration performs the best regardless of the number of transmit antennas. On the other hand, the ISF system power efficiency degrades as the number of transmit antennas increases. This makes intuitive sense since under the MSF configuration it is possible to extract the most power efficiency out of each branch, whereas the ISF system efficiency is heavily influenced by the branch with the largest PAR.

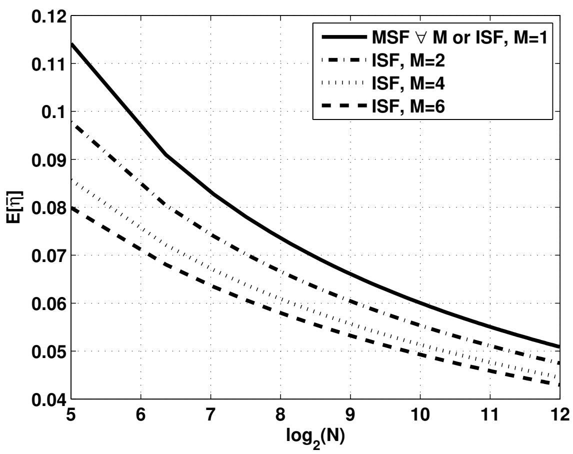

Fig. 2. MIMO power efficiency under the MSF and ISF configurations. N is the number of subcarriers.

C. Efficiency Improvement Example

It is worthwhile to investigate the efficiency improvement that can be realized when the proper metric is used in MIMO PAR reduction schemes. As an example, consider concurrent selected mapping (cSLM) [1], where each mapping is generated by applying the same phase sequence across all M antenna branches. The mapping that produces the minimum arithmetic-average PAR is then selected for transmission.

cSLM is used as an example here to illustrate the importance of using proper PAR metrics in achieving the maximum overall power efficiency. In Fig. 3, $N = 32$ or 1024 and $D = 2$ or 8 or 64, where D is the number of signal mappings in cSLM. The solid curves show the percent power efficiency improvement when the metric given in (6) is used instead of the arithmetic-average branch PAR [1] under the ISF scenario. Similarly, the dashed curves demonstrate the percent power efficiency improvement in the MSF scenerio when the harmonic average metric in (10) is used as the selection metric instead of the worst branch PAR [4].

IV. Conclusions

In this paper, we derived the power efficiency maximizing PAR metrics for MIMO-OFDM systems. Under two different system setups, we obtained the relationship between the branch PARs and the overall power efficiency. For each case we have provided the metric that can be used by PAR reduction system designers in order to maximize power efficiency. For the ISF case, the PAR metric is defined as the maximum of the branch PARs as in (6). For the MSF case, the proper PAR metric should be the harmonic average of the individual branch PARs as defined in (10). In some papers, for example, [1], [2], and [6], the *arithmetic average* of the branch PARs was used as the metric for PAR reduction – we fail to relate this PAR metric to meaningful physical mechanisms. Moreover, we demonstrated through a cSLM example, that by using the proper PAR metric, the power efficiency can be improved.

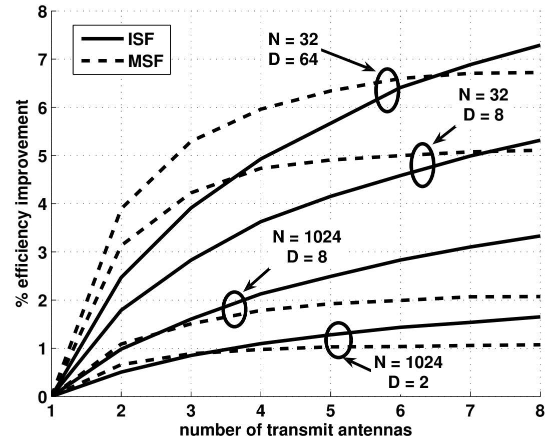

Fig. 3. Percent power efficiency improvement when proper PAR metrics are applied in the cSLM technique.

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