SNDR ANALYSIS FOR TRANSCEIVER NONLINEARITIES IN AWGN CHANNELS

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ABSTRACT

Nonlinear effects can significantly degrade the performance of communication systems. We propose a signal-to-noise-and-distortion ratio (SNDR) analysis framework for the general case when nonlinearities exist at both the transmitter and the receiver. In some cases, the nonlinearities can be modeled by polynomials, and closed-form expressions for the SNDR are possible. We provide examples to illustrate the usefulness of the analysis and the insight that closed-form SNDR analysis provides.

Index Terms— signal-to-noise-and-distortion ratio (SNDR), nonlinear system, companding

1. INTRODUCTION

Electronic devices in modern communication systems may exhibit nonlinear effects under certain operating conditions. At the transmitter, when a signal with a large peak-to-average power ratio (PAR) passes through a power amplifier, it may experience nonlinear distortion. The analog-to-digital converter (ADC) or the mixer at the receiver can also induce nonlinear distortions. Moreover, nonlinear transformations may be introduced deliberately, as in the case of companding [1–3] for PAR reduction. Hence, we encounter situations where nonlinearity exists at both the transmitter and the receiver.

Signal-to-noise-and-distortion ratio (SNDR) is a useful metric to evaluate the error performance when nonlinearities are present [4, 5]. SNDR analysis for amplitude-limited nonlinearities has been investigated in [5]; it is shown that among all amplitude-limited nonlinearities, clipping with a specific gain is optimal in terms of maximizing the SNDR. However, the analysis in [4, 5] assumed that the receiver is linear, which is not true in scenarios described above.

The objective of the present study is to propose an SNDR analysis framework for the case where the transmitter and the receiver both exhibit nonlinear effects. By using a polynomial model to approximate the nonlinearities, we are able to derive closed-form expressions for the SNDR. Special system scenarios are studied to obtain concise closed-form solutions. Finally, several examples are given to illustrate the SNDR analysis.

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2. SYSTEM SETUP

A baseband transceiver model is shown in Fig. 1 where $g(\cdot)$ and $s(\cdot)$ represent memoryless nonlinear functions at the transmitter and at the receiver, respectively. x is the input to the transmitter nonlinearity, and $y = g(x)$ is the transmitter output. Denote by v the additive white Gaussian noise (AWGN), which has zero mean and variance σ_v^2. The signal arriving at the receiver $w = y + v$ experiences another nonlinearity $s(\cdot)$ to yield $z = s(w)$ for further processing.

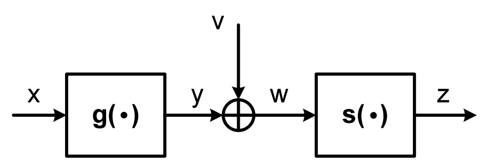

Fig. 1. An AWGN channel with transmitter nonlinearity $g(\cdot)$ and receiver nonlinearity $s(\cdot)$.

In general, $g(\cdot)$ and $s(\cdot)$ can be two unrelated nonlinear functions (including linear functions as special cases). In companding applications [1–3], $s(\cdot)$ is usually chosen as the inverse function of $g(\cdot)$, i.e., $s(\cdot) = g^{-1}(\cdot)$.

3. SNDR ANALYSIS FRAMEWORK

3.1. SNDR Formulation for the Linear Receiver Case

Using the Bussgang Theorem [6], any nonlinear function can be decomposed into a linear part and an uncorrelated distortion part, i.e.,

$$y = g(x) = \alpha x + d, \tag{1}$$

where $\alpha = E[x^* y]/\sigma_x^2$ is chosen so that $E[x^* d] = 0$ ($E[\cdot]$ is the expectation operation, and $(\cdot)^*$ denotes the complex conjugate). The power in d is $E[|d|^2] = E[|y|^2] - |\alpha|^2\sigma_x^2 =$

$E[|y|^2] - |E[x^*y]|^2/\sigma_x^2$. Therefore,

$$\mathrm{SNDR} = \frac{|\alpha|^2\sigma_x^2}{E[|d|^2] + \sigma_v^2} = \frac{|E[x^*y]|^2}{\sigma_x^2 E[|y|^2] - |E[x^*y]|^2 + \sigma_x^2\sigma_v^2}, \quad (2)$$

where

$$E[x^*y] = E[x^*g(x)] = \int_{\mathcal{R}(x)} g(x)x^* f_x(x)dx, \quad (3)$$

$$E[|y|^2] = E[|g(x)|^2] = \int_{\mathcal{R}(x)} |g(x)|^2 f_x(x)dx, \quad (4)$$

where $\mathcal{R}(x)$ is the range of x, and $f_x(x)$ is the probability density function (PDF) of x.

3.2. SNDR Transceiver Framework

Now for the system setup in Fig. 1, we again use the Bussgang Theorem to obtain

$$\mathrm{SNDR} = \frac{|E[x^*z]|^2}{\sigma_x^2 E[|z|^2] - |E[x^*z]|^2}, \quad (5)$$

where $z = s(y+v) = s\left(g(x)+v\right)$. To compute (5), it is necessary to calculate $E[x^*z]$ and $E[|z|^2]$. Since random variables x and v both appear as arguments of the nonlinear mapping $s(\cdot)$, the expectations need to be taken over v and x and the joint PDF $f_{x,v}(x,v)$ must be known. For most communications applications, it is assumed that the signal x and the noise v are mutually independent, implying that $f_{x,v}(x,v) = f_x(x)f_v(v)$. Thus,

$$E[x^*z] = \int_{\mathcal{R}(x,v)} s\left(g(x)+v\right) x^* f_x(x) f_v(v)\, dx\, dv, \quad (6)$$

$$E[|z|^2] = \int_{\mathcal{R}(x,v)} |s\left(g(x)+v\right)|^2 f_x(x) f_v(v)\, dx\, dv, \quad (7)$$

which is straightforward to calculate numerically for any pair of functions $g(\cdot)$ and $s(\cdot)$.

When the receiver is linear, i.e., when $s(w) = w$,

$$\begin{aligned} E[x^*z] &= E[x^*(y+v)] = E[x^*y], && (8)\\ E[|z|^2] &= E[|y+v|^2] = E[|y|^2] + \sigma_v^2. && (9) \end{aligned}$$

Substituting (8) and (9) into (5), the SNDR expression in (2) follows.

3.3. Polynomial Nonlinear Model

From this point on, we assume that Fig. 1 represents a baseband communication system. If $g(\cdot)$ and $s(\cdot)$ can be modeled by polynomials, we follow the convention of [7] and write

$$\begin{aligned} g(x) &= \sum_{k=0}^{K_g} a_{2k+1} x^{k+1} \left(x^*\right)^k, && (10)\\ s(w) &= \sum_{k=0}^{K_s} b_{2k+1} w^{k+1} \left(w^*\right)^k, && (11) \end{aligned}$$

where $a_1, a_3, a_5, ..., a_{2K_g+1}$ are the coefficients that dictate the characteristic of the transmitter nonlinearity $g(\cdot)$ and $b_1, b_3, b_5, ..., b_{2K_s+1}$ describe the receiver nonlinearity $s(\cdot)$. By using curve fitting techniques, these coefficients can be extracted from physical measurements of the device (such as AM-AM and AM-PM measurements of a power amplifier [8]). When v and x are both complex Gaussian distributed; i.e., $v \sim \mathcal{CN}(0, \sigma_v^2)$ and $x \sim \mathcal{CN}(0, \sigma_x^2)$ (such as when x is Nyquist-sampled time-domain OFDM signal), closed-form analysis of the SNDR is feasible since $E[x^*z]$ and $E[|z|^2]$ will be a linear combination of the moments of x and v. The moments of a complex Gaussian random variable can be expressed as [9]

$$E[x^k(x^*)^p] = \begin{cases} k!\, \sigma_x^{2k}, & k = p \\ 0, & k \neq p \end{cases}. \quad (12)$$

Hence, determining the transceiver SNDR when the polynomial coefficients of $g(\cdot)$ and $s(\cdot)$ are known is simply a matter of expressing $E[x^*z]$ and $E[|z|^2]$ in terms of the moments of x and v. The general closed-form SNDR expression is very complicated. However, we can examine two special cases to obtain insight.

3.4. Closed-Form SNDR Expression

Case 1: Linear Transmitter Nonlinear Receiver

When the transmitter is linear, we set $K_g = 0$ in (10), i.e., $g(x) = a_1 x$. Hence, the process z in Fig. 1 can be expressed as

$$\begin{aligned} z &= s(g(x)+v) = \sum_{k=0}^{K_s} b_{2k+1} \left(a_1 x + v\right)^{k+1} \left(a_1^* x^* + v^*\right)^k \\ &= \sum_{k=0}^{K_s} b_{2k+1} \sum_{l=0}^{k+1} \binom{k+1}{l} (a_1 x)^{k+1-l} v^l \\ &\quad \sum_{m=0}^{k} \binom{k}{m} (a_1^* x^*)^{k-m} (v^*)^m. && (13) \end{aligned}$$

In simplifying the $E[x^*z]$ expression, we first realize that because $E[v^l(v^*)^m] \neq 0$ only if $l = m$ (c.f. (12)), we can reduce the double summation $\sum_{l=0}^{k+1}\sum_{m=0}^{k}$ in the $E[x^*z]$ expression into a single summation $\sum_{l=0}^{k}$ and set $m = l$. We then obtain

$$\begin{aligned} E[x^*z] &= \sum_{k=0}^{K_s} b_{2k+1} \sum_{l=0}^{k} \binom{k+1}{l} \binom{k}{l} \\ &\quad E\left[x^*(a_1 x)^{k+1-l}(a_1^* x^*)^{k-l}\right] l!\, \sigma_v^{2l}. && (14) \end{aligned}$$

Next, we recognize that $E[x^{k+1-l}(x^*)^{k+1-l}] = (k+1-l)!\sigma_x^{2(k+1-l)}$ and simplify $E[x^*z]$ to

$$E[x^*z] = a_1 \sigma_x^2 \sum_{k=0}^{K_s} b_{2k+1} (k+1)!\, \gamma^k, \quad (15)$$

where $\gamma = |a_1|^2\sigma_x^2 + \sigma_v^2$.

To find $E[|z|^2]$, we first realize that $u = a_1x+v$ is another complex Gaussian r.v. with $E[|u|^2] = |a_1|^2\sigma_x^2 + \sigma_v^2 = \gamma$. Again utilizing (12), we find

$$E[|z|^2] = \sum_{k_1=0}^{K_s}\sum_{k_2=0}^{K_s} b_{2k_1+1}^* b_{2k_2+1} K!\,\gamma^K, \tag{16}$$

where $K = k_1 + k_2 + 1$. Substituting (15) and (16) into (5), we can obtain the closed-form SNDR expression

$$\text{SNDR} = \frac{|a_1|^2\sigma_x^2 \sum_{k_1=0}^{K_s}\sum_{k_2=0}^{K_s} b_{2k_1+1}^* b_{2k_2+1} S(k_1,k_2)}{\sum_{k_1=0}^{K_s}\sum_{k_2=0}^{K_s} b_{2k_1+1}^* b_{2k_2+1} T(k_1,k_2)}, \tag{17}$$

where

$$S(k_1,k_2) = (k_1+1)!(k_2+1)!\,\gamma^{K-1},$$

and

$$T(k_1,k_2) = \gamma^{K-1}[K!\,\gamma - (k_1+1)!(k_2+1)!\,|a_1|^2\sigma_x^2].$$

For this linear transmitter nonlinear receiver case, we are interested in finding coefficients of $s(\cdot)$ that maximize the SNDR. These optimum b_{2k+1} can be found by setting the partial derivatives of SNDR with respect to b_{2k+1}^* to zero [10]. This allows us to write for example,

$$\frac{\partial |E[x^*z]|^2}{\partial b_{2\tilde{k}+1}^*} E[|z|^2] = \frac{\partial E[|z|^2]}{\partial b_{2\tilde{k}+1}^*} |E[x^*z]|^2, \tag{18}$$

where $\tilde{k}$ is a nonnegative integer.

From (15) and (16), we can calculate

$$\frac{\partial |E[x^*z]|^2}{\partial b_{2\tilde{k}+1}^*} = E[x^*z] a_1^* \sigma_x^2 (\tilde{k}+1)!\,\gamma^{\tilde{k}}, \tag{19}$$

and

$$\frac{\partial E[|z|^2]}{\partial b_{2\tilde{k}+1}^*} = \sum_{k_2=0}^{K_s} b_{2k_2+1} (\tilde{k}+k_2+1)!\,\gamma^{\tilde{k}+k_2+1}. \tag{20}$$

By substituting (19) and (20) into (18) and assuming $E[x^*z] \neq 0$ [1], we simplify (18) to

$$\sum_{k_1=0}^{K_s}\sum_{k_2=0}^{K_s} b_{2k_1+1}^* b_{2k_2+1} (\tilde{k}+k_2+1)!(k_1+1)!\,\gamma^K = \sum_{k_1=0}^{K_s}\sum_{k_2=0}^{K_s} b_{2k_1+1}^* b_{2k_2+1} K!(\tilde{k}+1)!\,\gamma^K. \tag{21}$$

The condition to make (21) valid is $b_{2k+1} = 0, \forall k \neq \tilde{k}$. Therefore, the possible solutions are $(b_1, 0, 0, \cdots, 0)$, $(0, b_3, 0, \cdots, 0)$, $\cdots$, $(0, 0, \cdots, 0, b_{2K_s+1})$. We substitute these solutions into (17) and find that $(b_1, 0, 0, \cdots, 0)$ is the set of coefficients maximizing the SNDR. The maximum SNDR is $|a_1|^2\sigma_x^2/\sigma_v^2$, which is also the signal-to-noise ratio (SNR) at the input to the receiver.

[1] When $E[x^*z] = 0$, SNDR is also zero, which minimizes the SNDR value. Because our objective is to maximize the SNDR, we set $E[x^*z] \neq 0$.

Case 2: Nonlinear Transmitter Linear Receiver

By setting $K_s = 0$ in (11), the receiver is linear, i.e., $s(w) = b_1 w$. So the output process in Fig. 1 is

$$z = s(g(x)+v) = b_1\left(\sum_{k=0}^{K_g} a_{2k+1} x^{k+1}(x^*)^k + v\right). \tag{22}$$

By utilizing (12), we have

$$E[x^*z] = b_1 \sum_{k=0}^{K_g} a_{2k+1}(k+1)!\sigma_x^{2(k+1)}, \tag{23}$$

and

$$E[|z|^2] = |b_1|^2\left(\sum_{k_1=0}^{K_g}\sum_{k_2=0}^{K_g} a_{2k_1+1}^* a_{2k_2+1} K!\sigma_x^{2K} + \sigma_v^2\right). \tag{24}$$

Substituting (23) and (24) into (5), the closed-form SNDR expression for the nonlinear transmitter linear receiver case can be written as

$$\text{SNDR} = \frac{\sum_{k_1=0}^{K_g}\sum_{k_2=0}^{K_g} a_{2k_1+1}^* a_{2k_2+1}(k_1+1)!(k_2+1)!\sigma_x^{2K}}{\sum_{k_1=0}^{K_g}\sum_{k_2=0}^{K_g} a_{2k_1+1}^* a_{2k_2+1}[K! - (k_1+1)!(k_2+1)!]\sigma_x^{2K} + \sigma_v^2}. \tag{25}$$

4. SNDR ANALYSIS EXAMPLES

Case 1: Linear Transmitter

For illustration purposes, let us consider a simple pair of transformations

$$\begin{aligned} g(x) &= a_1 x, & (26)\\ s(w) &= b_1 w + b_3 w^2 w^*. & (27)\end{aligned}$$

Assuming that $v \sim \mathcal{CN}(0,\sigma_v^2)$, $x \sim \mathcal{CN}(0,\sigma_x^2)$, the SNDR expression in (17) can be shown to simplify to

$$\text{SNDR} = \frac{|a_1|^2\sigma_x^2|b_1 + 2b_3\gamma|^2}{|b_1|^2\sigma_v^2 + 2(b_1^*b_3 + b_1b_3^*)\sigma_v^2\gamma + 2|b_3|^2\gamma^2(\gamma+2\sigma_v^2)}. \tag{28}$$

Fig. 2 is a plot of the above SNDR versus b_3, where we assume all the polynomial coefficients are real-valued for simplicity. We can see that the SNDR is maximized at $b_3 = 0$ (corresponding to a linear receiver) for different b_1 values.

Case 2: Linear Receiver

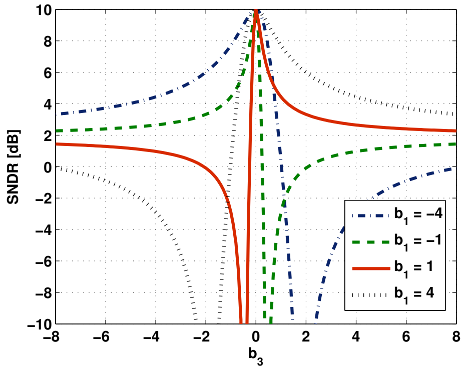

Fig. 2. SNDR vs. b_3 using equations (26) and (27). For all lines, $\sigma_x^2/\sigma_v^2 = 10$dB, $a_1 = 1$, and $\sigma_x^2 = 1$.

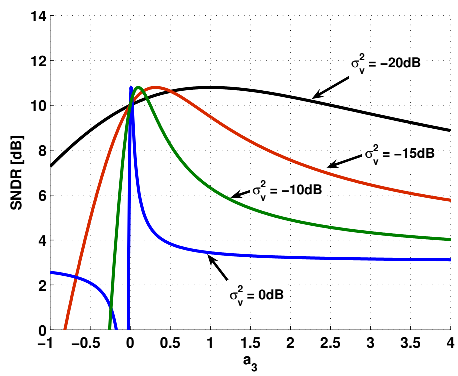

Fig. 3. SNDR vs. a_3 using equations (29) and (30). For all lines, $\sigma_x^2/\sigma_v^2 = 10$dB and $a_1 = 1$.

If, instead,

$$
\begin{aligned}
g(x) &= a_1 x + a_3 x^2 x^*, &(29)\\
s(w) &= b_1 w, &(30)
\end{aligned}
$$

then the SNDR expression in (25) becomes

$$\mathrm{SNDR} = \frac{\sigma_x^2 |a_1 + 2a_3\sigma_x^2|^2}{\sigma_v^2 + 2|a_3|^2\sigma_x^6}. \tag{31}$$

Fig. 3 is a plot of the SNDR versus a_3, which is assumed to be real-valued for simplicity. Interestingly, the SNDR can exceed the SNR by as much as 0.79 dB when some nonlinear distortion is used. The optimizing a_3 is $\sigma_v^2/(a_1\sigma_x^4)$, and the corresponding α in (1) is $a_1 + 2\sigma_v^2/\sigma_x^2$. Such expanding nonlinearity increases the useful signal power by making $|\alpha| > 1$, while keeping the distortion power $E[|d|^2]$ relatively small.

Case 3: Nonlinear Transmitter and Nonlinear Receiver

In reality, nonlinearities can exist at both the transmitter and the receiver. We adopt the cubic nonlinearities for illustration purposes, namely

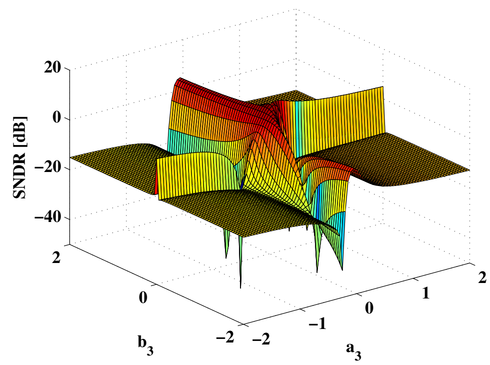

Fig. 4. SNDR vs. a_3 and b_3 using equations (34) and (35). For all lines, $\sigma_x^2/\sigma_v^2 = 10$dB and $a_1 = 1$, $b_1 = 1$.

$$
\begin{aligned}
g(x) &= a_1 x + a_3 x^2 x^*, &(32)\\
s(w) &= b_1 w + b_3 w^2 w^*. &(33)
\end{aligned}
$$

By following similar procedures as in Section 3.4, we can simplify the $E[x^* z]$ and $E[|z|^2]$ expressions as follows:

$$E[x^* z] = \theta b_1 + (2\theta\sigma_v^2 + \beta) b_3, \tag{34}$$

and

$$
\begin{aligned}
E[|z|^2] &= |b_1|^2(\delta + \sigma_v^2) + (b_1 b_3^* + b_3 b_1^*)(\lambda + 4\delta\sigma_v^2 + 2\sigma_v^4)\\
&+ |b_3|^2(\zeta + 9\lambda\sigma_v^2 + 18\delta\sigma_v^4 + 6\sigma_v^6), \qquad (35)
\end{aligned}
$$

where

$$\theta = E[x^* g(x)] = a_1\sigma_x^2 + 2a_3\sigma_x^4,$$

$$
\begin{aligned}
\beta &= E[x^* g(x)^2 g^*(x)] = 2a_1|a_1|^2\sigma_x^4 + 6(a_1^2 a_3^* + 2a_3|a_1|^2)\sigma_x^6\\
&+ 24(a_3^2 a_1^* + 2a_1|a_3|^2)\sigma_x^8 + 120a_3|a_3|^2\sigma_x^{10},
\end{aligned}
$$

$$\delta = E[g^*(x)g(x)] = |a_1|^2\sigma_x^2 + 2(a_1^* a_3 + a_3^* a_1)\sigma_x^4 + 6|a_3|^2\sigma_x^6,$$

$$
\begin{aligned}
\lambda &= E[g^*(x)^2 g(x)^2] = 2|a_1|^4\sigma_x^4 + 12(a_1^* a_3 + a_1 a_3^*)|a_1|^2\sigma_x^6\\
&+ 24(a_1^2(a_3^*)^2 + a_3^2(a_1^*)^2 + 4|a_1|^2|a_3|^2)\sigma_x^8\\
&+ 240(a_1^* a_3 + a_1 a_3^*)|a_3|^2\sigma_x^{10} + 720|a_3|^4\sigma_x^{12},
\end{aligned}
$$

and

$$
\begin{aligned}
\zeta &= E[g^*(x)^3 g(x)^3] = 6|a_1|^6\sigma_x^6 + 72(a_1^* a_3 + a_1 a_3^*)|a_1|^4\sigma_x^8\\
&+ 3\times 5!(a_1^2(a_3^*)^2 + a_3^2(a_1^*)^2 + 3|a_1|^2|a_3|^2)|a_1|^2\sigma_x^{10}\\
&+ 6!(a_1^3(a_3^*)^3 + 9(a_1^* a_3 + a_3^* a_1)|a_1|^2|a_3|^2 + a_3^3(a_1^*)^3)\sigma_x^{12}\\
&+ 3\times 7!(a_1^2(a_3^*)^2 + a_3^2(a_1^*)^2 + 3|a_1|^2|a_3|^2)|a_3|^2\sigma_x^{14}\\
&+ 3\times 8!(a_1^* a_3 + a_1 a_3^*)|a_3|^4\sigma_x^{16} + 9!|a_3|^6\sigma_x^{18}.
\end{aligned}
$$

Substituting (34) and (35) into (5), we can obtain a closed-form SNDR expression. In Fig. 4, SNDR versus a_3 and b_3 is

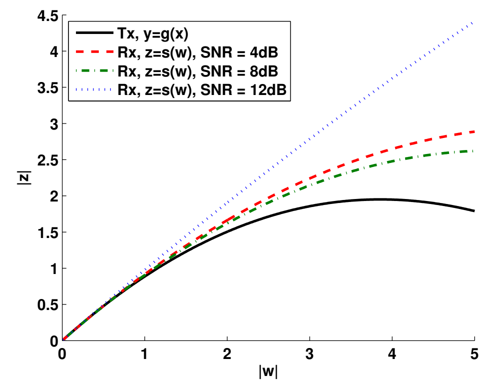

Fig. 5. Plot of the optimal receiver AM-AM characteristic for the given transmitter characteristic.

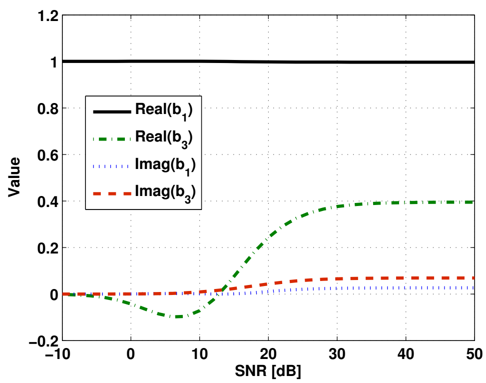

Fig. 6. Plot of the receiver coefficients as the SNR varies.

shown, where the coefficients are assumed to be real-valued for simplicity.

Next, by finding the partial derivatives of the SNDR w.r.t. b_1^* and b_3^* and utilizing (18) it is possible to find the optimizing b_1 and b_3 when $a_1,\ a_3,\ \sigma_x^2$, and σ_v^2 are known by solving

$$\theta^* \frac{E[|z|^2]}{(E[x^*z])^*} = b_1(\delta+\sigma_v^2) + b_3(\lambda + 4\delta\sigma_v^2 + 2\sigma_v^4) \quad (36)$$

and

$$b_1(\lambda + 4\delta\sigma_v^2 + 2\sigma_v^4) + b_3(\zeta + 9\lambda\sigma_v^2 + 18\delta\sigma_v^4 + 6\sigma_v^6) = (2\theta\sigma_v^2 + \beta)^* \frac{E[|z|^2]}{(E[x^*z])^*}, \quad (37)$$

which can be done numerically.

Fig. 5 is a plot of AM-AM characteristic; i.e., $|z| = |s(w)|$ vs. $|w|$ of the optimal receiver when the transmitter has $a_1 = 1 - .2j$ and $a_3 = -0.135 + .01j$. For the receiver nonlinearity in (33), $|s(w)| = |w| \cdot |b_1 + b_3|w|^2|$. For the plot $\sigma_x^2 = 1$ and σ_v^2 is varied to adjust the SNR. The plot shows how the SNDR maximizing cubic curve changes as the noise power changes. Fig. 6 demonstrates the trend for the optimizing values of b_1 and b_3 as the noise level varies. Interestingly, for SNRs below 12dB, the real part of b_3 is negative which means that b_3 is a compressing function. This result shows that for this particular compressing transmitter, to maximize the SNDR, the receiver also has the form of a compressing function. This is an interesting result which implies that the companding technique may not be always desirable from the SNDR perspective.

5. CONCLUSIONS

SNDR is a useful metric to measure the nonlinear distortion in a communication system. In this paper, we perform the SNDR analysis in a transceiver framework, which contains nonlinearities in both the transmitter and receiver. For polynomial nonlinear functions, closed-form SNDR expressions can be obtained. Based on several examples, we show that the SNDR analysis is a powerful tool, which can help us analyze, compare and optimize the transceiver design.

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