MAGNITUDE-SCALED SELECTED MAPPING: A CREST FACTOR REDUCTION SCHEME FOR OFDM WITHOUT SIDE-INFORMATION TRANSMISSION

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ABSTRACT

Selected mapping (SLM) is a distortionless crest factor reduction (CFR) method for orthogonal frequency division multiplexing (OFDM) transmission. With SLM, it is possible to reduce the peak-to-average power ratio (PAR) of an OFDM symbol by several decibels. In this paper, we propose a method for SLM phase sequence detection that does not require side information transmission. We refer to this method as magnitude-scaled SLM, in the sense that it scales the frequency-domain power profile of the OFDM symbol with an envelope function from a set of pre-determined envelope functions. From the envelope of the received symbol, the receiver can detect which envelope and thus which phase sequence was used in the transmission. Also presented in this paper are the theoretical characterizations of the detection error rate (DER) and symbol error rate (SER) in a magnitude-scaled SLM system. Compared with ordinary OFDM without CFR, magnitude-scaled SLM can achieve an order of magnitude SER improvement in a peak-power-limited channel.

Index Terms— Orthogonal frequency division multiplexing, selected mapping, crest factor reduction, peak-to-average power ratio

1. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) is an attractive multi-carrier transmission scheme due to its immunity to inter-symbol interference and robustness to multi-path fading. It has been adopted by several communications standards, such as digital audio broadcasting (DAB), digital video broadcasting (DVB), wireless LAN and wireless MAN. However, one major problem associated with OFDM is its high peak-to-average power ratio (PAR) or crest factor (CF). When a high-PAR signal, such as OFDM, is passed through a high-power amplifier (HPA), the HPA will either operate with a large back-off resulting in very poor power efficiency or operate in its non-linear region, which will generate both in-band distortion and out-of-band spectral regrowth in the transmitted signal. Hence, crest factor reduction (CFR) of the OFDM signal is often necessary.

Selected mapping (SLM) is a distortionless CFR method. It selects an (alternative) representation signal with the minimum PAR from a set of equivalent representations each of which is related to the original OFDM sequence in the frequency domain through a sequence of phase rotations on individual sub-carriers [1]. The receiver must know the index of the selected sequence to retrieve the corresponding phase vector for the phase de-rotation. Explicit side-information transmission will undesirably reduce the data rate; therefore a blind SLM (BSLM) scheme is preferred. Existing BSLM methods include maximum likelihood (ML) detection of the data symbols [2], maximum a posteriori (MAP) detection of the phase sequence [3], phase sequence detection based on pilot subcarriers [4], and phase sequence detection based on constellation shifts [5].

The magnitude-scaled SLM scheme proposed in this paper is an extension of the idea presented in [6], which only worked for constant magnitude (PSK) constellations. The scheme proposed in this paper is applicable regardless of the constellation type. Basically, magnitude-scaled SLM uses a pre-determined set of envelope scaling functions to scale the OFDM symbol, and the amplitude scaling and the phase rotation sequences are linked by a common index. The receiver can then use a specially designed metric to recover the amplitude and phase sequence (index) used in the transmission.

Notations: Upper case and lower case bold face letters represent matrices and column vectors respectively; superscript ^{T} and $^{\mathcal{H}}$ stand for the transpose and the Hermitian transpose, respectively; $E[\cdot]$ is the expectation operator; $\|\mathbf{x}\|_n$ is the ℓ^n-norm of $\mathbf{x}$; $|\mathbf{x}|$ is a vector that is the element-wise magnitude of $\mathbf{x}$; $|\mathcal{A}|$ is the cardinality of set $\mathcal{A}$; $\mathbf{D}_{\mathbf{x}}$ is a diagonal matrix with vector $\mathbf{x}$ on the diagonal; the $N \times N$ discrete Fourier transform (DFT) matrix is denoted by $[\mathbf{Q}]_{n,k} = N^{-1/2} \exp(j2\pi(n-1)(k-1)/N)$.

2. OFDM MODEL

In OFDM, individual subcarriers in the frequency-domain are modulated with constellation points, transformed into the time-domain and transmitted with a cyclic prefix. For PAR anal-

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ysis, the cyclic prefix can be ignored since it has no effect of the symbol PAR. Let the frequency-domain vector of constellation points be $\mathbf{x} = [x_1, x_2, ..., x_{N-1}, x_N]^T$, where x_k is drawn from a R-point constellation and the power in $\mathbf{x}$ is normalized so that $\mathrm{E}\left[\|\mathbf{x}\|_2^2\right] = N$. Using the inverse discrete Fourier transform, the time-domain symbol is $\mathbf{y} = \sqrt{\mathcal{E}_y}\mathbf{Q}^{\mathcal{H}}\mathbf{x}$, where $\mathcal{E}_y$ is the symbol energy of $\mathbf{y}$. The PAR of the transmitted signal is defined by

$$PAR\{\mathbf{y}\} = \frac{\|\mathbf{y}\|_\infty^2}{\mathcal{E}_y}. \tag{1}$$

For transmission through a peak-power-limited channel it is desirable to make the PAR as low as possible. The received baseband frequency-domain signal after synchronization is $\mathbf{z} = \sqrt{\mathcal{E}_y}\mathbf{D_h}\mathbf{x} + \mathbf{n}$, where $\mathbf{D_h}$ is a diagonal matrix with diagonal elements from the channel frequency response vector $\mathbf{h}$ and $\mathbf{n}$ is (complex-valued) white Gaussian noise with zero mean and variance σ_n^2. Finally, assuming perfect channel state information, the estimated transmitted symbol is $\hat{\mathbf{x}} \triangleq \mathbf{D_h}^{-1}\mathbf{z}/\sqrt{\mathcal{E}_y}$.

3. MAGNITUDE-SCALED SLM

In the conventional SLM, M complex-valued vectors $\mathbf{s}^{(m)}$, $1 \leq m \leq M$, are multiplied with $\mathbf{x}$ prior to transmission, to generate $\mathbf{x}^{(m)} \triangleq \mathbf{D}_{\mathbf{s}^{(m)}}\mathbf{x}$. The candidate signal $\mathbf{Q}^{\mathcal{H}}\mathbf{x}^{(m)}$ that produces the lowest PAR among the M possible candidate signals is selected for transmission. Every element in $\mathbf{s}^{(m)}$ has unit magnitude, i.e., $|\mathbf{s}^{(m)}| = \mathbf{1}_{N\times 1}$.

In contrast to conventional SLM, magnitude-scaled SLM does *not* impose the constraint $|\mathbf{s}^{(m)}| = \mathbf{1}_{N\times 1}$. Specifically, the complex-valued scaling vectors are of the form

$$[\mathbf{s}^{(m)}]_k \triangleq p_k^{(m)} e^{j\phi_k^{(m)}}. \tag{2}$$

The distribution of the phase angles $\phi_k^{(m)}$ is chosen so that $E[e^{j\phi_k^{(m)}}] = 0$ which is the condition required for SLM to achieve maximum CFR [7]. This condition is satisfied for example, when $\phi_k^{(m)}$ is i.i.d. uniformly distributed in $\{0, 2\pi\}$. Define $\left[\mathbf{p}^{(m)}\right]_k \triangleq p_k^{(m)}$, where $\mathbf{p}^{(m)}$ is chosen to be a scaled and shifted column of a pseudo-random matrix that has elements of either 1 or -1. Denote column i of the $N \times N$ pseudo-random matrix by $\mathbf{w}^{(i)}$, and define sets of indices $\mathcal{K}_1^{(i)} = \{k \mid [\mathbf{w}^{(i)}]_k = 1\}$, $\mathcal{K}_{-1}^{(i)} = \{k \mid [\mathbf{w}^{(i)}]_k = -1\}$. One example of $\mathbf{w}^{(i)}$ is a column of the Hadamard matrix, which is called a Walsh sequence. For example, given the 8-element Walsh sequence, $\mathbf{w}^{(3)} \triangleq [1, 1, -1, -1, 1, 1, -1, -1]^T$, we have $\mathcal{K}_1^{(3)} = \{1, 2, 5, 6\}$ and $\mathcal{K}_{-1}^{(3)} = \{3, 4, 7, 8\}$. Walsh sequences have the nice property that for $1 < i \leq N$, $|\mathcal{K}_1^{(i)}| = |\mathcal{K}_{-1}^{(i)}|$. The magnitude sequence is chosen so that $p_k^{(m)} = \sqrt{\beta}$ when $k \in \mathcal{K}_1^{(m+1)}$ and $p_k^{(m)} = \sqrt{2-\beta}$ when $k \in \mathcal{K}_{-1}^{(m+1)}$. We require $1 < \beta < 2$, and thus $0 < 2 - \beta < 1$. Therefore,

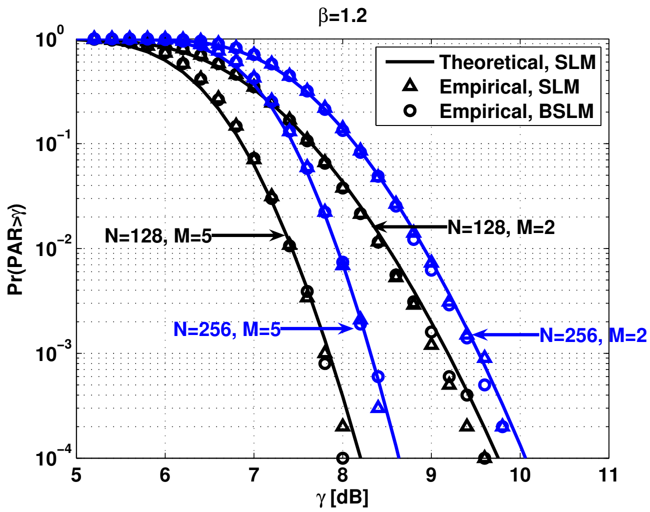

Fig. 1. BSLM vs. SLM CCDF curves for OFDM with 16QAM modulation.

depending on the sub-carrier location k, the power on 50% of the sub-carriers is scaled down to $2 - \beta$, whereas the power on the other 50% of the sub-carriers is scaled up to β; the average power remains the same.

Now the m^{th} time-domain candidate signal is

$$\mathbf{y}^{(m)} = \sqrt{\mathcal{E}_y}\mathbf{Q}^{\mathcal{H}}\mathbf{x}^{(m)} \tag{3}$$

If $\mathbf{p}^{(m)}$ is chosen according to the Walsh sequence, we can guarantee that $\mathrm{E}\left[\|\mathbf{y}^{(m)}\|_2^2\right] = N\mathcal{E}_y$, $\forall m$. The index of the transmitted candidate signal $\mathbf{y}^{(\bar{m})}$ is chosen so that

$$\bar{m} \triangleq \underset{1\leq m\leq M}{\arg\min} \left\|\mathbf{y}^{(m)}\right\|_\infty. \tag{4}$$

The PAR reduction capability of BSLM is evaluated by the complementary cumulative distribution function (CCDF) of the PAR values after amplitude scalings and phase rotations. The results are compared to the theoretical and empirical CCDFs for the conventional SLM (the theoretical CCDF expression can be found in [1]). Four cases are simulated with different combinations of N and M (see Fig. 1). The amplitude-scaling factor was set to $\beta = 1.2$, the constellation used was 16QAM and the phase rotation sequence $e^{j\phi_k^{(m)}}$ was i.i.d. ± 1 with equal probability. From Fig. 1, empirical CCDF of the PAR from our proposed BSLM algorithm and that from the conventional SLM were very close, and they both agreed with the theoretical CCDF very well. Hence, magnitude-scaled SLM is shown to provide the same PAR reduction capability as the conventional SLM.

The received frequency-domain SLM symbol is $\mathbf{z} = \sqrt{\mathcal{E}_y}\mathbf{D_h}\mathbf{x}^{(\bar{m})} + \mathbf{n}$. Assuming perfect channel state information, we can write

$$\hat{\mathbf{x}}^{(\bar{m})} \triangleq \frac{\mathbf{D_h}^{-1}\mathbf{z}}{\sqrt{\mathcal{E}_y}} = \mathbf{x}^{(\bar{m})} + \frac{\mathbf{D_h}^{-1}\mathbf{n}}{\sqrt{\mathcal{E}_y}}. \tag{5}$$

Finally, in order to detect $\bar{m}$ blindly, we can create M receive metrics, one for each possible scaling sequence. The m^{th} metric is

$$G^{(m)} = \sum_{k \in \mathcal{K}_1^{(m+1)}} \left| [\hat{\mathbf{x}}^{(\bar{m})}]_k \right|^2 - \sum_{k \in \mathcal{K}_{-1}^{(m+1)}} \left| [\hat{\mathbf{x}}^{(\bar{m})}]_k \right|^2 \quad (6)$$

It may also be convenient to express the detection metrics as a column vector

$$\begin{bmatrix} G^{(1)} \\ G^{(2)} \\ \vdots \\ G^{(M)} \end{bmatrix} = \left[\mathbf{w}^{(2)} \; \mathbf{w}^{(3)} \; \ldots \; \mathbf{w}^{(M+1)} \right]^T \left| \hat{\mathbf{x}}^{(\bar{m})} \right|^2, \quad (7)$$

where $|\cdot|^2$ is the element-wise magnitude squared value of a vector. Notice that $\mathrm{E}[G^{(\bar{m})}] = N\mathcal{E}_y(\beta - 1)$ and $\mathrm{E}[G^{(m \neq \bar{m})}] = 0$. Thus, we can estimate $\bar{m}$ with

$$\hat{\bar{m}} \triangleq \underset{1 \le m \le M}{\arg\max}\, G^{(m)}. \quad (8)$$

With $\hat{\bar{m}}$, the estimated symbol becomes $\hat{\mathbf{x}} = \mathbf{D}^{-1}_{\mathbf{s}^{(\hat{\bar{m}})}} \hat{\mathbf{x}}^{(\bar{m})}$.

Magnitude-scaled SLM is designed to operate in peak-power-limited channels. Thus, in order to provide a fair comparison of the proposed scheme to traditional OFDM in terms of SER, we must assume a clipping channel. Accordingly, we adopt the linear block scaling OFDM architecture proposed in [8]. In [8], it was demonstrated that by using a linear block scaling architecture the transmitted signal power is actually $1/PAR\{\mathbf{y}\}$, thus the SERs of competing schemes are mostly aptly compared using the peak SNR (PSNR), where $PSNR \triangleq 1/PAR\{\mathbf{y}\}\sigma_n^2$. Assuming perfect detection of $\bar{m}$, a tight upper bound on the SER when QAM is used in an AWGN channel is

$$p_{s|\hat{\bar{m}}=\bar{m}} \le 2\mathrm{Erfc}\left[\sqrt{\frac{3r\beta PSNR}{2(R-1)}}\right] + 2\mathrm{Erfc}\left[\sqrt{\frac{3r(2-\beta)PSNR}{2(R-1)}}\right] \quad (9)$$

where R is the constellation size and $r \triangleq \log_2 R$ [9]. Furthermore, the detection error rate (DER) in an AWGN channel can be approximated by

$$\Pr\left[\hat{\bar{m}} \neq \bar{m}\right] = 1 - \left(1 - \frac{1}{2}\,\mathrm{Erfc}\left[\frac{(\beta-1)\sqrt{N}}{2\sigma PAR\{\mathbf{y}\}}\right]\right)^{M-1} \quad (10)$$

where $\sigma^2 = \sigma_n^4 + 2\sigma_n^2 + \mathcal{E}_y^2\sigma^2_{|x|^2}$ (see the Appendix for details). The validity of this approximation is verified in Fig. 2. The plot shows that the expression in (10) matches very closely with the empirical DER obtained from Monte Carlo simulations.

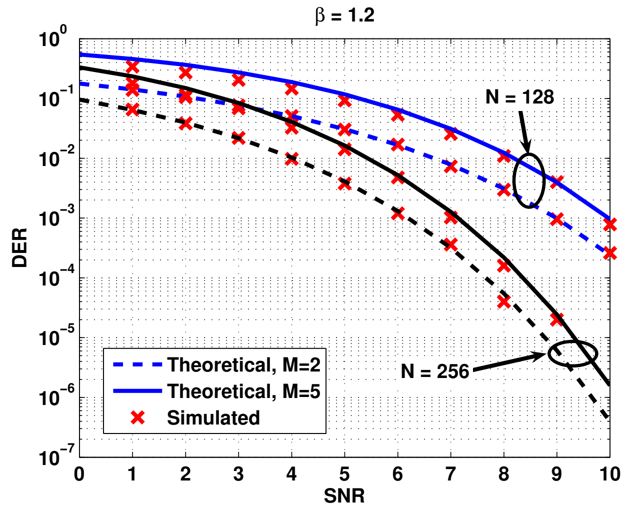

Fig. 2. Simulated and theoretical DER for QPSK, where $SNR = \mathcal{E}_y/\sigma_n^2$.

Finally, the SER of the proposed system can be quantified through $SER = p_{s|\hat{\bar{m}}=\bar{m}} + DER(1 - 1/R - p_{s|\hat{\bar{m}}=\bar{m}})$, where DER is defined in (10). In this case, SER depends on the random variable $PAR\{\mathbf{y}\}$. Obtaining the most precise estimate of the SER requires integrating SER over the probability density function of $PAR\{\mathbf{y}\}$ [10]. However, it is possible to use Jensen's inequality in conjunction with $\mathrm{E}[PAR\{\mathbf{y}\}]$ from [10] to provide a tight closed form lower bound on SER.

In order to realize the full potential of the proposed scheme, it is necessary to optimize β and M, for a given signal power, $\mathcal{E}_y$, channel noise, σ_n^2 and constellation size R to minimize the SER. Because the minimization is difficult to carry out analytically, we instead performed the optimization numerically. Fig. 3 is a plot of the optimal values of β versus $\log_2 N$. As expected, larger values for N lead to decreases in β. Finally, in Fig. 4 the SER for magnitude-scaled OFDM with optimized β is plotted along with conventional OFDM. At 20dB of PSNR, the proposed magnitude-scale SLM scheme (with $M = 20$) outperforms conventional OFDM by a factor of 10 in terms of SER.

4. CONCLUSIONS

In this paper we proposed magnitude-scaled SLM as a CFR method that obviates the need for SLM side-information transmission. The proposed scheme uses envelope functions derived from Walsh sequences to shape the frequency-domain power profile of the OFDM symbol. At the receiver, envelope detection is used in conjunction with a specially designed metric to determine the transmitted SLM phase sequence. To verify the utility of magnitude-scaled SLM, we derived a closed-form bound for the SER and compared it to the SER of conventional OFDM. In the linear block scal-

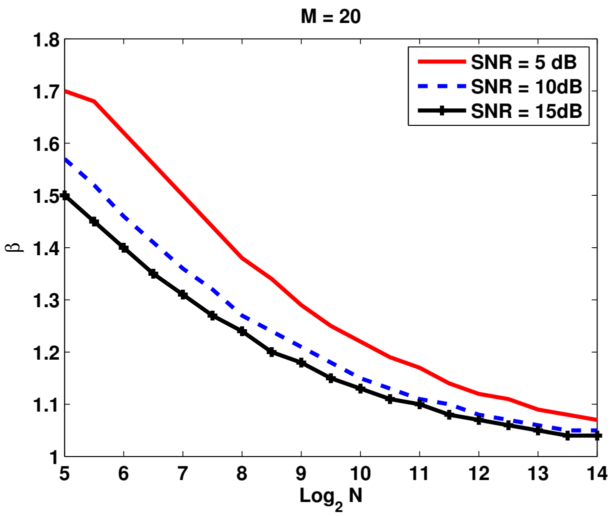

Fig. 3. β versus $\log_2 N$, where $SNR = \mathcal{E}_y/\sigma_n^2$.

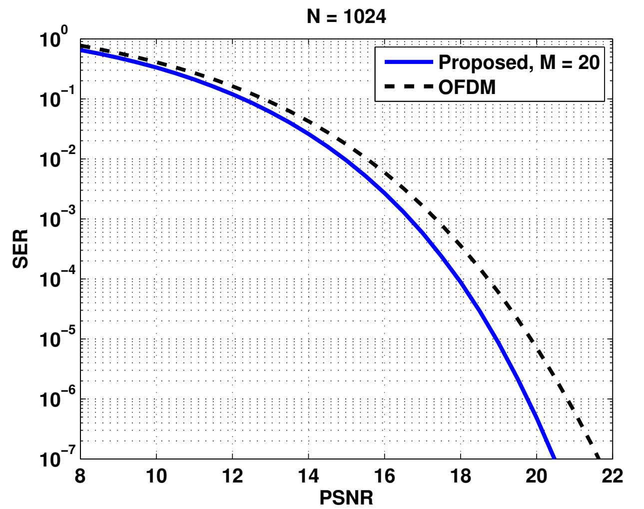

Fig. 4. Total symbol error rate (including DER) versus the PSNR.

Table 1. Power variances for various constellation sizes

R	4	16	64	256	∞
$\sigma^2_{\lvert x\rvert^2}$	0	0.320	0.382	0.396	0.400

ing channel corrupted by AWGN, the magnitude-scaled SLM signals yielded more than 1dB PSNR improvement over the conventional OFDM.

5. APPENDIX

By applying the Central Limit Theorem, we can approximate $G^{(\bar{m})}$ and $G^{(m\neq\bar{m})}$ with Gaussian random variables so that $G^{(\bar{m})} \sim \mathcal{N}\left(N\mathcal{E}_y(\beta-1)/PAR\{\mathbf{y}\}, N\sigma^2\right)$ and $G^{(m\neq\bar{m})} \sim \mathcal{N}\left(0, N\sigma^2\right)$, where $\sigma^2 = \sigma_n^4 + 2\sigma_n^2 + \mathcal{E}_y^2\sigma^2_{|x|^2}$. Values of $\sigma^2_{|x|^2}$ are tabulated in Table 1.

When $M = 2$, we have

$$\Pr\left[\hat{\bar{m}} \neq \bar{m}\right] = \Pr\left[G^{(m\neq\bar{m})} > G^{(\bar{m})}\right] \quad (11)$$

$$= \frac{1}{2}\,\text{Erfc}\left[\frac{\mathcal{E}_y(\beta-1)\sqrt{N}}{2\sigma PAR\{\mathbf{y}\}}\right]. \quad (12)$$

For $M > 2$, assuming that $G^{(m\neq\bar{m})}$ is independent for different m, we can obtain

$$\Pr\left[\hat{\bar{m}} = \bar{m}\right] = \prod_{m\neq\bar{m}} \left\{1 - \Pr\left[G^{(m)} > G^{(\bar{m})}\right]\right\}. \quad (13)$$

Combining (12) and (13) gives rise to (10).

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