COMPARISON OF SELECTED MAPPING AND PARTIAL TRANSMIT SEQUENCE FOR CREST FACTOR REDUCTION IN OFDM

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ABSTRACT

Selected mapping (SLM) and partial transmit sequence (PTS) are two existing distrotionless crest factor reduction (CFR) schemes that have been proposed for orthogonal frequency division multiplexing (OFDM). Previously, it was argued that SLM and PTS have comparable CFR performance but that the latter has lower computational complexity because it uses fewer IFFTs. In this paper, we show that the overall computational complexity of PTS may be higher than that of SLM, but that SLM always has better CFR performance as quantified through a per unit of complexity metric.

1. INTRODUCTION

OFDM is a popular modulation technique with many desirable qualities and has been proposed for the IEEE 802.11a, IEEE 802.16, the European digital audio broadcasting (DAB) and the European digital video broadcasting (DVB) standards. The desirable attributes of OFDM come at the expense of large envelope variations. Such signal envelope or power variations can be difficult for practical power amplifiers (PAs) to accommodate, resulting in either low power efficiency or distortion-inducing signal clips. One popular way to quantify the dynamic range of a signal is through the peak to average power ratio (PAR) or the crest factor (CF) where $PAR = CF^2$.

There has been a significant amount of research devoted to the development of CF reduction (CFR) algorithms for OFDM. An overview of the different approaches can be found in [1]. In this paper, we are interested in comparing two promising distortionless PAR reduction algorithms. The first is selected mapping (SLM), which was first presented in [2]. The second algorithm is partial transmit sequence (PTS), which was proposed in [3] and can be viewed as a generalization of the SLM algorithm. These two methods are popular because they are conceptually simple, they can yield large CFR results, and they offer the possibility of only sacrificing computational complexity for CFR (i.e., techniques exists for SLM and PTS that can avoid side information transmission [4–6]).

Comparisons of SLM and PTS were made in [7]. In [7], is was claimed that the computational complexity of PTS is lower than SLM, but that the two have comparable CFR performance, which we will show is not necessarily true. Unlike [7], we will make quantifiable comparisons based on the CFR performance per unit of complexity of the two schemes.

For the purposes of comparing SLM and PTS, we can use a simple OFDM model where the baseband OFDM symbol $\{x_{n/L}\}_{n=0}^{NL-1}$ is an oversampled IFFT of the data vector $\{X_k\}_{k=0}^{N-1}$, $x_{n/L} = \mathrm{IFFT}\{X_k\}$. That is,

$$x_{n/L} = \sum_{k=0}^{N/2-1} X_k e^{\frac{j2\pi kn}{LN}} + \sum_{k=N/2}^{N-1} X_k e^{\frac{j2\pi(k-N)n}{LN}}$$

where L is the oversampling factor, N is the number of subcarriers, $n \in \{0, 1, ..., LN-1\}$ and $\{X_k\}_{k=0}^{N-1}$ is a sequence of complex numbers drawn from a finite constellation. The baseband PAR is defined as

$$\mathrm{PAR}\left[x_{n/L}\right] = \max_{0 \leq n \leq LN-1} \frac{|x_{n/L}|^2}{\mathrm{E}[|x_{n/L}|^2]}, \tag{1}$$

which is a random variable.

To justify the clipping noise metric, we assume that the transmitted signal has to pass through a predistorter/PA system where the predistorter/PA concatenation can be viewed as a soft clipping operation or the so-called ideal linear PA defined by

$$f_{clip}(x) = \begin{cases} x, & |x| \leq A \\ Ae^{j\angle x}, & |x| > A. \end{cases} \tag{2}$$

With this peak-limited channel defined, any excursion of the envelope of x above A will lead to a signal distortion. We propose to quantify this signal distortion via $\mathrm{E}[(|x_{n/L}|^2 - A^2)^+]$, where $(\cdot)^+$ denotes the positive part of the argument.

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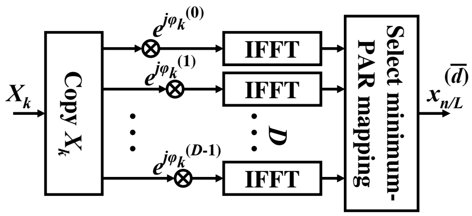

Fig. 1. SLM block diagram.

2. SELECTED MAPPING (SLM)

SLM was first described in [2] as a distortionless CFR method. In SLM, D equivalent data sequences are created each by multiplying the original sequence X_k with one of D phase rotation sequences $e^{j\phi_k^{(d)}}$. So, $X_k^{(d)} = X_k e^{j\phi_k^{(d)}}$, which is used to create $x_{n/L}^{(d)} = \text{IFFT}\{X_k^{(d)}\}$, where $d \in \{0, 1, ..., D-1\}$. A total of D length-NL IFFTs are performed. From these D candidates, the transmitter selects the lowest PAR sequence, $x_{n/L}^{(\bar{d})}$, for transmission where

$$\bar{d} = \underset{0 \leq d \leq D-1}{\arg\min} \ \text{PAR}\left[x_{n/L}^{(d)}\right]. \tag{3}$$

Fig. 1 is a block diagram of the SLM technique.

It is assumed that the transmitter and the receiver have the table of D length-N phase sequences $e^{j\phi_k^{(d)}}$. However, in order to recover the original data sequence X_k the receiver must determine $\bar{d}$. To distinguish $\bar{d}$ from the D possibilities, $\log_2(D)$ bits are needed. Since the receiver will not be able to determine the original data without $\bar{d}$, the side-information bits are extremely important and it may be necessary to allocate additional redundancy bits to ensure accurate recovery of $\bar{d}$.

Because side-information transmission may not be practical for SLM systems with large D, we need to make a comparison between SLM and PTS assuming a distribution for $\phi_k^{(d)}$ that will enable blind ML detection in the receiver. One distribution that has been demonstrated to have excellent ML-detection performance is i.i.d. Φ uniformly distributed over $[0, 2\pi)$ [6]. Obviously, with this choice of distribution, the complex multiplication to create $X_k^{(d)}$ can not be implemented by only sign changes as alluded to in [2] and must instead be executed with full-complexity complex multiplications.

3. PARTIAL TRANSMIT SEQUENCE (PTS)

PTS is a CFR technique that takes advantage of the linearity of the IFFT to reduce the number of IFFTs necessary to create multiple signal representations. In a PTS system, $\{X_k\}_{k=0}^{N-1}$ is partitioned into V non-overlapping sub-blocks $\{X_{\mathcal{V}_v}\}_{v=0}^{V-1}$ with indices in the sets $\{\mathcal{V}_v\}_{v=0}^{V-1}$. That

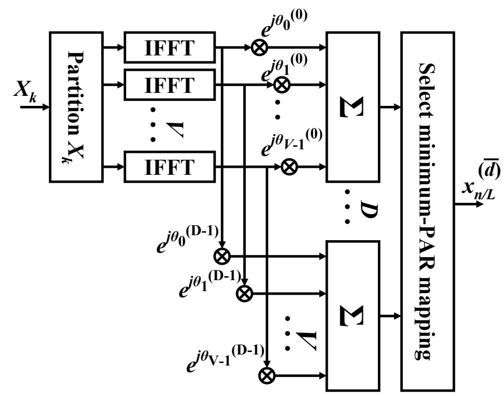

Fig. 2. PTS block diagram.

is $\bigcup_{v=0}^{V-1} \mathcal{V}_v = \{0, 1, ..., N-1\}$ and $\sum_{v=0}^{V-1} X_{\mathcal{V}_v} = X_k \; \forall \, k \in \{0, 1, ..., N-1\}$. To generate D PTS signal representations, each of the sub-blocks is scaled by a complex constant $e^{j\theta_v^{(d)}}$ and added together so that

$$x_{n/L}^{(d)} = \sum_{v=0}^{V-1} e^{j\theta_v^{(d)}} \underbrace{\text{IFFT}\{X_{\mathcal{V}_v}\}}_{x_{n/L,v}}, \tag{4}$$

where $d \in \{0, 1, ..., D-1\}$. Finally, the transmitted signal $x_{n/L}^{(\bar{d})}$ is chosen according to (3) similar to the SLM method. In (4), a total of V length-NL IFFTs are performed to create any number D of alternative signal mappings $x_{n/L}^{(d)}$. The small number of IFFTs (V as opposed to D) relative to SLM was one main justification for proposing PTS [3]. However, as we will show in Section IV, the number of IFFTs should not be the only complexity consideration as the length-NL multiplications and additions necessary to combine $x_{n/L,v}$ in order to create $x_{n/L}^{(d)}$ in (4) can contribute significantly to the overall complexity of PTS. Fig. 2 is a block diagram of the PTS technique.

From (4) it is apparent that SLM can be regarded as a special case of PTS where each sub-block contains only one subcarrier each (e.g. $\mathcal{V}_v = v|_{v \in [0, N-1]}$). A general question is whether PTS is less complex than SLM for any V and D and, if so, are the CFR capabilities of PTS for that V and D comparable to the CFR capabilities of SLM for the same D? Before we can answer this question we need to discuss some of the options for choosing PTS parameters.

The fact that PTS can be viewed as a special case of SLM, can be used to show that a PTS system can use a SLM ML decoder to recover $\bar{d}$ [4]. Again, as with SLM, the performance of the ML decoder is dependent on the distribution of $\theta_v^{(d)}$. Accordingly, if $\theta_v^{(d)}$ is distributed uniformly over $[0, 2\pi)$, as opposed to for example, taking on discrete values such as $\{0, \pi, \pm\pi/2\}$, excellent ML detection performance will be possible. The implication here is that length-NL complex multiplications will be necessary, as opposed to simple

Table 1. Number of real operations for SLM and PTS

$A_{SLM} = DNL(3\log N + 2) + 2N(D-1)$
$M_{SLM} = 2DNL(\log N + 2) + 4N(D-1)$
$A_{PTS} = 4(D-1/2)NLV - DNL + VA_{IFFT}$
$M_{PTS} = 4(D-1)NLV + 2DNL + VM_{IFFT}$

sign changes to calculate the product in (4).

The final consideration in a PTS system is the choice of the partitions $\mathcal{V}_v$. Some authors have mentioned that it is possible to choose the $\mathcal{V}_v$ in order to exploit the IFFT structure and reduce the number of computations required to compute IFFT$\{X_{\mathcal{V}_v}\}$. On the other hand, it has been shown that the PAR minimizing choice for $\mathcal{V}_v$ is random equally-sized subblocks [8]. If the IFFT is designed especially for the specific set of PTS partitions, then it is possible to exploit the sparseness of $X_{\mathcal{V}_v}$ to use reduced complexity IFFTs [9].

Although several methods have been proposed to find $\theta_v^{(\bar{d})}$ without having to search all D possibilities [10–12], in this paper we are interested in determining the CFR capability per unit of computational complexity of PTS and SLM under a side information constraint (which can also be thought of as a receiver complexity constraint). Thus, we opt to analyze PTS using a set of phase constants that are i.i.d. random, where the PAR-minimizing set of phase constants is determined by exhaustive search. Performing an exhaustive search guarantees that $\theta_v^{(\bar{d})}$ will be the PAR-minimizing sequence among the set $\theta_v^{(d)}$, $d \in \{0, 1, ..., D-1\}$.

4. COMPUTATIONAL COMPLEXITY

The complexity comparison between SLM and PTS will take into account the number of multiplications and additions necessary in terms of the algorithm parameters. Also, we will assume, without any CFR sacrifice [13], that $\theta_v^{(0)} = 0$ and that $\phi_k^{(0)} = 0$, i.e. the first of D signal mappings is just the original OFDM symbol.

In general, a complex multiplication takes four real multiplications and two real additions. On the other hand a complex addition requires two real additions. In Table 1 we have summarized the computational requirement of each scheme, where A_{IFFT} and M_{IFFT} are the number of real additions and real multiplication required for each sparse PTS IFFT.

The complexity of PTS and SLM is quantified through a parameter f that is the number of addition instructions required for each multiplication operation. So the overall complexity is $C_{SLM} = A_{SLM} + fM_{SLM}$ for SLM and $C_{PTS} = A_{PTS} + fM_{PTS}$ for PTS, where A_{SLM}, M_{SLM}, A_{PTS} and M_{PTS} are defined in Table 1.

Fig. 3 is a plot of the ratio C_{PTS}/C_{SLM} versus the number of PTS partitions V. The plot contains lines for $N = \{64, 256, 1024\}$ and $D = \{2, 4, 16\}$ using $L = 1$ and $f = 4$. Fig. 3 shows that the ratio is monotonically increasing in V. The point of utmost interest is where the lines cross $C_{PTS}/C_{SLM} = 1$. For $C_{PTS}/C_{SLM} > 1$, PTS is more computationally complex, whereas, when $C_{PTS}/C_{SLM} < 1$ SLM is more complex. The plot shows that when $D = 2$, PTS is always more complex. Even for a large value $D = 16$, SLM is less complex than PTS when $V > 6$. These are surprising results considering that PTS was designed to have a *lower* complexity than SLM.

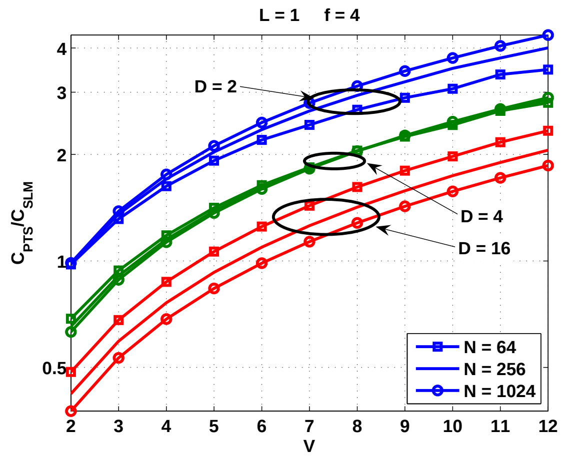

Fig. 3. Plot of C_{PTS}/C_{SLM} versus the number of PTS partitions V.

5. SLM/PTS COMPARISON

From the PAR CCDF-minimizing criteria outlined in [13], we infer that SLM will result in a lower PAR than PTS for a given number of mappings D. With Fig. 3, we can see that large values of V lead to increased computational complexity for PTS. Together, these two observations imply that PTS will be inferior to SLM in complexity and in CFR capability for all values of V that cause $C_{PTS}/C_{SLM} > 1$.

The next question is whether SLM should be used over PTS when V is so small that $C_{PTS}/C_{SLM} \geq 1$. To draw conclusions we will analyze the performance of each scheme using the clipping noise metric, $\mathrm{E}[(|x_{n/L}|^2 - A^2)^+]$. When using this clipping noise metric, it is desirable to slightly reformulate SLM and PTS. The reformulation would mean that the signal that produces the lowest power distortion among the D alternatives $\{x_{n/L}^{(d)}\}_{d=0}^{D-1}$ is transmitted [14]. This differs from traditional SLM and PTS where the signal with the lowest PAR is selected for transmission. Stated precisely, the transmitted signal $x_{n/L}^{(\tilde{d})}$ is selected according to

$$\bar{d} = \arg\min_{0 \leq d \leq D-1} \sum_{n=0}^{NL-1} \left(\left| x_{n/L}^{(d)} \right|^2 - A^2 \right)^+ . \tag{5}$$

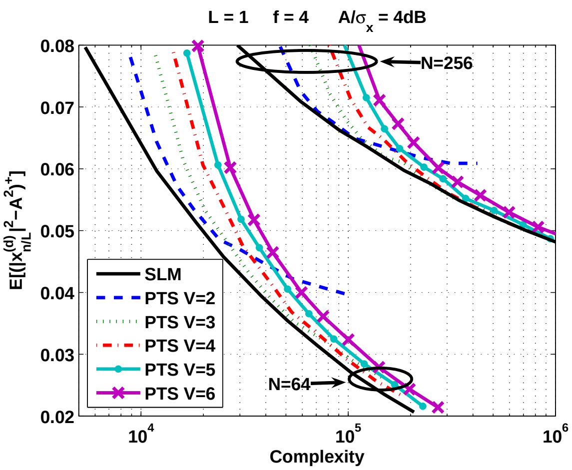

Fig. 4. Plot of $\mathrm{E}[(|x_{n/L}^{(\widetilde{d})}|^2 - A^2)^+]$ versus the complexity (C_{PTS} or C_{SLM}).

Fig. 4 is a plot of $\mathrm{E}[(|x_{n/L}^{(\widetilde{d})}|^2 - A^2)^+]$ versus the complexity for PTS with $V = \{2, 3, 4, 5, 6\}$ and for SLM, where $N = \{64, 256\}$. To create the plot the clipping level was set so that $A/\sigma_x = 4$dB, where σ_x^2 is the variance of X_k. The plot shows that SLM slightly outperforms PTS for all of the cases plotted.

6. CONCLUSIONS

In this paper we focused on two popular distortionless CFR techniques, namely PTS and SLM and analyzed their computational complexity and resulting CFR performance. It was already known that SLM can produce multiple time-domain signals that are asymptotically-independent, while the alternative signals generated by PTS are interdependent. This interdependency necessarily implies that PTS will have some CFR capability degradation compared to SLM for a given number of mappings. However, it has been assumed in [7] that the computational complexity of PTS would be much less than that of SLM so that the computational savings of PTS outweighed the CFR advantages of SLM.

Surprisingly, we found that SLM is actually less computational complex than PTS when more than a couple of PTS partitions are used. In order to compare PTS and SLM $\mathrm{E}[(|x_{n/L}^{(\widetilde{d})}|^2 - A^2)^+]$, which reflects clipping distortion, as a metric. The results showed, for a given amount of computational complexity, SLM performs better than PTS. Even PTS with an optimized V does not beat the performance of SLM in the suggested metric across all complexities. We conclude that, SLM is the preferred method because it is conceptually simpler and because it does not require any off-line complexity optimization with respect to V as would be recommended for PTS.

7. ACKNOWLEDGEMENTS

The authors would like to thank Chunming Zhao for his valuable suggestions regarding the computational complexity analysis. [1]

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