PILOT DESIGN FOR IEEE 802.16 OFDM AND OFDMA

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ABSTRACT

In this paper we apply a new pilot design optimization technique to the IEEE 802.16a WirelessMAN standard. Using the specifications from 802.16a, we demonstrate that significant SER improvements are possible for 802.16a in a Rayleigh fading channel by judiciously choosing where the pilots are placed and the power contained in each pilot. Specifically, for the OFDM mode, we show that up to 13dB SNR improvement is possible by simply modifying the pilots. For the OFDMA mode we demonstrate that a more modest but still significant improvement of 1.8dB is possible with proper pilot design.

Index Terms— Orthogonal frequency division multiplexing (OFDM), pilot symbol assisted modulation (PSAM), pilot design.

1. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) is a popular method in wireless high-speed communications schemes. Pilot symbol assisted modulation (PSAM) was proposed as a low complexity way to estimate multipath channels and remove their effects from a received OFDM symbol [1]. Since its proposal, PSAM has been widely adopted in OFDM-based commercial wireless communications standards. In this paper, we are interested in analyzing and optimizing the pilot designs used in the Institute of Electrical and Electronics Engineers (IEEE) 802.16a WirelessMAN standard [2] (a.k.a WiMax). Specifically, we will apply the pilot design optimization techniques presented in [3] and compare the resulting near-optimal pilot design with the pilot designs currently specified by the standards.

In [4] it was proven that the optimal pilot design for OFDM systems is one that contains evenly-spaced constant-power pilots. However, this result did not extend to OFDM systems with null edge subcarriers. When a so-called "guard band" is implemented and is large enough to make the evenly space pilot design impossible, an alternative design is necessary [5]. In response to this problem, a systematic procedure for designing pilots in null-subcarrier OFDM system was outlined in [3]. In that paper, the highly non-linear discontinuous pilot design problem was broken into several independent and solvable optimization problems. With this technique, it was demonstrated in [3] that large improvements in channel estimation performance and symbol error rate (SER) performance are possible compared to evenly-spaced constant-power pilot designs when guard bands are present.

Notation: Upper case and lower case bold faced letters represent matrices and column vectors respectively; $\mathbf{X}^T$ and $\mathbf{X}^{\mathcal{H}}$ stand for the transpose and the Hermitian transpose of $\mathbf{X}$, respectively; E[$\cdot$] is the expectation operator; $\|\mathbf{x}\|_n$ is the ℓ^n norm of $\mathbf{x}$; $|\mathbf{x}|$ is a vector that is the element-wise magnitude of $\mathbf{x}$; $\mathbf{A}^+ = (\mathbf{A}^{\mathcal{H}}\mathbf{A})^{-1}\mathbf{A}^{\mathcal{H}}$ is the pseudoinverse of matrix $\mathbf{A}$; $|\mathcal{A}|$ is the cardinality of set $\mathcal{A}$; $((\cdot))_N$ is the modulo N operation; int($\cdot$) rounds the argument to the nearest integer; $\mathbf{D}_{\mathbf{x}}$ is a diagonal matrix with vector $\mathbf{x}$ on the diagonal; the $N \times N$ discrete Fourier transform (DFT) matrix is denoted by $[\mathbf{Q}]_{k,n} = N^{-1/2}\exp(-j2\pi(n-1)(k-1)/N)$.

To outline the procedure put forward in [3], start by defining frequency domain symbol $\mathbf{x} = [x_1, x_2, ..., 0, ..., 0, ..., x_{N-1}, x_N]^T$. The received baseband frequency-domain signal after synchronization and cyclic prefix (CP) removal[1] is $\mathbf{y} = \sqrt{\mathcal{E}_s}\mathbf{D}_{\mathbf{h}}\mathbf{x} + \mathbf{w}$, where $\mathbf{w}$ is additive white complex Gaussian noise, (i.e. $\mathbf{w} \sim \mathcal{CN}(0, \sigma_n^2\mathbf{I}_N)$) and $\mathbf{h}$ is the frequency response of the channel. Note that $\mathbf{h} = \mathbf{Q}_L\mathbf{h}^{(t)}$, where $\mathbf{h}^{(t)}$ is a length-L vector of the channel impulse response and $\mathbf{Q}_L$ is the first L columns of the DFT matrix $\mathbf{Q}$.

Using the set of indices in $\mathbf{x}$ corresponding to the data carriers $\mathcal{K}_d$ and the pilot carriers $\mathcal{K}_p$, define the two matrices, $\mathbf{Q}_d$ and $\mathbf{Q}_p$, which transforms the impulse response of the channel to the data and pilot subcarriers, respectively. Here, $[\mathbf{Q}_p]_{k,n} = N^{-1/2}\exp(j2\pi(n-1)(k-1)/N)$, where $k \in \mathcal{K}_p$ and $n \in \{1, 2, ..., L\}$ and $[\mathbf{Q}_d]_{k,n} = N^{-1/2}\exp(j2\pi(n-1)(k-1)/N)$, where $k \in \mathcal{K}_d$ and $n \in \{1, 2, ..., L\}$.

Using the least-squares channel estimator, the MSE of the channel estimate in the data subcarriers is approximated by $\mathbf{z} \approx \operatorname{diag}\left\{\frac{\sigma_n^2}{\mathcal{E}_p}\mathbf{Q}_d\mathbf{Q}_p^+\mathbf{D}_{|\mathbf{x}_p|^{-2}}\mathbf{Q}_p^{\mathcal{H}+}\mathbf{Q}_d^{\mathcal{H}}\right\}$, where $\mathbf{x}_p \triangleq$

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[1]The CP is assumed to be large enough so that inter-symbol interference is avoided (i.e. $CP \geq L-1$).

$[\mathbf{x}]_k,\ k \in \mathcal{K}_p$, i.e. the modulated values in the pilot subcarriers. Furthermore, from [3], the maximum likelihood estimator of $\mathbf{x}_d$ has MSE

$$\mathbf{e} = \operatorname{diag}\left\{\mathbf{D}_{\mathbf{z}} + \frac{\sigma_n^2}{\mathcal{E}_d}\mathbf{I}_{|\mathcal{K}_d|}\right\}. \tag{1}$$

It can be shown that it is desirable to minimize the $\|\mathbf{e}\|_\infty$ in order to reduce the symbol error rate [3], which leads to the optimization problem

$$\begin{aligned} \arg \min_{\mathrm{E}[|\mathbf{x}|],\ \mathcal{E}_p,\ \mathcal{K}_p} \quad & \|\mathbf{e}\|_\infty \\ \text{subject to} \quad & \mathcal{E}_p + \mathcal{E}_d = \mathcal{E}_s, \\ & \mathbf{x}_n = \mathbf{0}_{|\mathcal{K}_n|\times 1}, \end{aligned} \tag{2}$$

where $\mathcal{E}_p$ is the energy in the pilot subcarriers, $\mathcal{E}_d$ is the energy in the data subcarriers and $\mathbf{x}_n$ are the null subcarriers. However, solving (2) to find the true minimum is a difficult, nonlinear, discontinuous, problem. In [3] a sub-optimal method of minimization was proposed that separated the full optimization problem into several tractable optimization steps.

By parameterizing the pilot positions $\mathcal{K}_p$ with a cubic polynomial, (2) can be simplified to a problem with continuous inputs. Using further assumptions about the pilot positions it is possible to specify the pilot positions in two continuous variables δ and a_3, whose domain is bounded[2] (see [3] for details). For an arbitrary set of pilot indices $\{k_1, k_2, ..., k_{|\mathcal{K}_p|}\}$, it is possible to show that the optimization problem

$$\begin{aligned} \arg \min_{|\mathbf{x}_p|^{-2}} \quad & \|\mathbf{z}\|_\infty \\ \text{subject to} \quad & \|\mathbf{x}_p\|_2^2 = \mathcal{E}_p|\mathcal{K}_p|, \\ & \mathcal{K}_p = \{k_1, k_2, ..., k_{|\mathcal{K}_p|}\}. \end{aligned} \tag{3}$$

is convex and easily solved using conventional convex optimization methods. Finally, differentiating (1) gives

$$\mathcal{E}_p = \mathcal{E}_s \frac{\|\mathbf{z}\|_\infty - \sqrt{\|\mathbf{z}\|_\infty}}{\|\mathbf{z}\|_\infty - 1} \tag{4}$$

and $\mathcal{E}_d = \mathcal{E}_s - \mathcal{E}_p$. Using these results the procedure for selecting the OFDM pilot parameters can be reduced to a simple grid search over the domain of $(\delta,\ a_3)$. The psuedo-code algorithm for this procedure is

1. Initialize $i = 1$.
2. Select a $\delta^{(i)}$ and $a_3^{(i)}$ and find $\mathcal{K}_p^{(i)}$.
3. Use $\mathcal{K}_p^{(i)}$ to solve (3) for $|\mathbf{x}_p^{(i)}|$.
4. Use $\mathcal{K}_p^{(i)}$ and $|\mathbf{x}_p^{(i)}|$ to find $\mathcal{E}_p^{(i)}$ and $\mathcal{E}_d^{(i)}$ via (4).
5. Calculate $\|\mathbf{e}^{(i)}\|_\infty$ from $\mathbf{e}^{(i)}$ in (1).
6. If $\overline{MSE} > \|\mathbf{e}^{(i)}\|_\infty$ or $i = 1$, set $\overline{MSE} = \|\mathbf{e}^{(i)}\|_\infty$ and $\bar{i} = i$.
7. If $i = i_{max}$, quit, else, set $i = i + 1$ and go to Step 2.

[2] δ determines the positions of the pilots on the two edges of the passband and a_3 is related how the pilot spacing changes across the band.

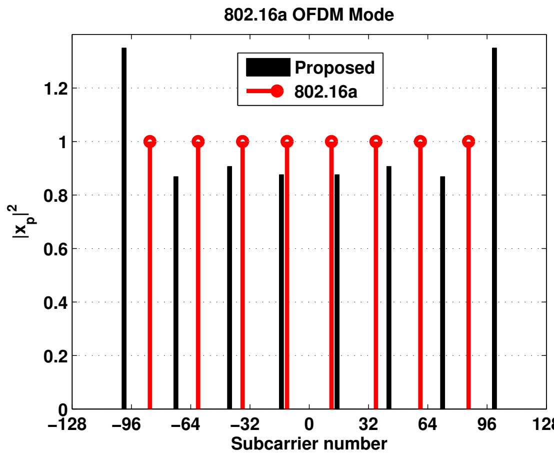

Fig. 1. Default and proposed pilot design for 802.16a OFDM mode, where $L = 8$.

When the algorithm quits, the minimum MSE is stored in $\overline{MSE}$ and the minimizing parameters are $\delta^{(\bar{i})}$, $\mathcal{K}_p^{(\bar{i})}$, $|\mathbf{x}_p^{(\bar{i})}|$, $\mathcal{E}_p^{(\bar{i})}$ and $\mathcal{E}_s^{(\bar{i})}$.

In the following sections we will examine the MSE of OFDM pilot configurations found in the IEEE 802.16a. For each configuration we will also use the procedure outlined in this section to find the near-optimal pilot design. Our goal is to illustrate how simple changes to the existing standardized pilot designs can lead to large channel estimate MSE improvements.

2. IEEE 802.16 OFDM

The IEEE 802.16a standard contains three possible physical layer modes: Single carrier, OFDM, and orthogonal frequency division multiple access (OFDMA) [2]. In this section we will discuss the OFDM mode and demonstrate the near optimal pilot design.

For IEEE 802.16a OFDM mode, the transmission frame is segmented into several parts. Of relevance here are the preamble and the data-carrying parts of the frame. The preamble is used for synchronization purposes including channel estimation. Additionally, each data-carrying symbol contains several pilots, which can be used for fine synchronization and also for channel estimation. In a data-carrying symbol 200 subcarriers of the 256 subcarrier window are used for data and pilots. Of the other 56 subcarriers, 28 are null in the lower-frequency guard band, 27 are nulled in the upper-frequency guard band and one is the DC subcarrier which is nulled. Of the 200 used subcarriers, 8 are allocated as pilots, while the remaining 192 are used for data transmission. The pilot positions specified by the standard are $\mathcal{K}_{p,OFDM} = \{-84, -60, -36, -12, 12, 36, 60, 84\}$, which all contain the same amount of power. Additionally, the pilot to data power ratio, $\beta \triangleq \frac{\mathcal{E}_p}{\mathcal{E}_d}$, is $\beta_{OFDM} = 1/24$. After following the pilot

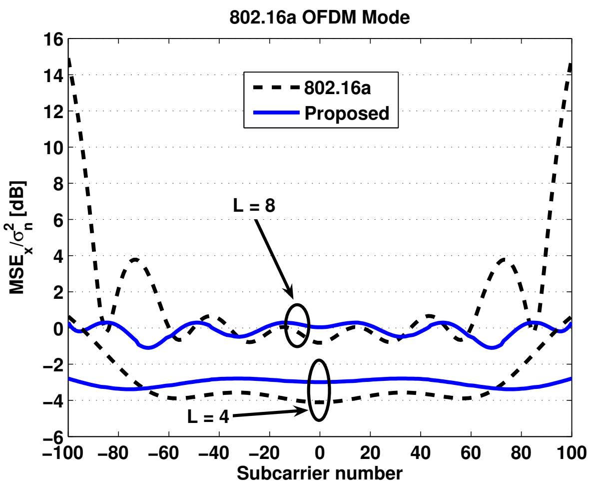

Fig. 2. Channel estimate MSE of the 802.16a pilot design and of the proposed pilot design for two different channel lengths, L.

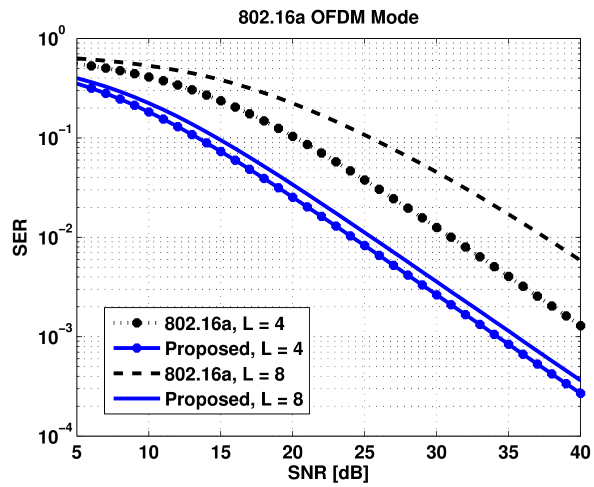

Fig. 3. SER of the 802.16a pilot design and of the proposed pilot design for two different channel lengths, L.

design procedure in [3], we find that $\mathcal{K}_p^* = \{-100, -72, -43, -15, 15, 43, 72, 100\}$ for $L = 4$ and $L = 8$. Fig. 1 is a plot of the two pilot designs when $L = 8$. The most notable difference between the two designs is that the near-optimal design places pilots at the edges of the guard band. Also, close the edges of the pilot/data band the near-optimal pilot design has a smaller pilot spacing than near the center of the band. These two pilot design features work to create an improved channel estimate near the guard band.

Using the (4), we have $\beta^* = 1.03$ for $L = 8$ and $\beta^* = 0.73$ for $L = 4$. Fig. 2 is a plot of the normalized channel estimate MSE, MSE/σ_n^2, for the proposed pilot design and for the pilot design specified in the 802.16a standard for OFDM mode operation. The plot shows that the standard pilot design does a poor job of estimating the channel in the subcarriers near the guard band. Conversely, the proposed pilot design is capable of a relatively flat channel estimate MSE across all data subcarriers.

Fig. 3, is a plot of the SER for the two pilot designs for different channel lengths. In this case, the L channel taps are $\mathcal{CN}(0, \frac{1}{L}\mathbf{I}_L)$. The plot shows that, in an L-tap Rayleigh fading channel, the proposed pilot design leads to 13dB SNR improvement when $L = 8$ and a 7dB SNR improvement when $L = 4$.

3. IEEE 802.16 OFDMA

The OFDMA mode in 806.16a is different from the OFDM mode discussed in the last section. In the OFDMA mode, the transmission band is made up of 2048 subcarriers that are partitioned into two guard bands (consisting of null subcarriers) and 32 subchannels of 53 subcarriers each. Also, there are two main transmission options: uplink (UL), downlink (DL) and a third option known as adaptive antenna system (AAS) that is employed when multiple transmit antennas are available. For the AAS option the UL and DL carrier allocations are identical.

Downlink: The DL channel goes from the base station (BS) to all of the individual subscriber stations (SSs). In the DL direction an all-pilot premable is not sent prior to the information symbols. Thus, it is reasonable to assume that the pilots contained in the information symbols, are needed to estimate the channel.

For the DL channel, the pilots are partitioned into a set of 32 constant pilots and a set of 142 variable pilots, whose positions vary based on the symbol number. Also, the fixed pilots are designed so that exactly 8 pilots from each set are the same for every symbol. Thus, in total there are $\mathcal{K}_p = 166$ pilots. In the left guard band there are 173 subcarriers and 172 subcarriers in the right guard band. Also, the DC subcarrier is nulled. So, in the data/pilot band, each of the 32 subchannels is made up of 48 contiguous non-pilot, non-DC, subcarriers.

Uplink: In the UL direction the subcarrier allocation is defined per subchannel and is variable depending on the symbol number, which can take on values $S \in \{0, 1, ..., 12\}$. In each subchannel the 53 subcarriers are numbered from 1 to 53. The pilot allocation for each subchannel is made up of 4 variable pilots at base positions $\mathcal{K}_{p,sub}^{(base)} = \{1, 14, 28, 41\}$ and one constant pilot at position 27. For symbol number S variable pilots are at positions $\mathcal{K}_{p,sub} = \mathcal{K}_{p,sub}^{(base)} + S$ and the constant pilot is at position 27. The pilots are designed such that the variable pilots never occupy position 27, which means that every subchannel contains exactly five pilots regardless of the symbol number.

For the UL direction an all-pilot preamble is sent prior to the transmission of a burst of information symbols. Such a preamble symbol can be used for synchronization and channel estimation purposes. However, because there are also a

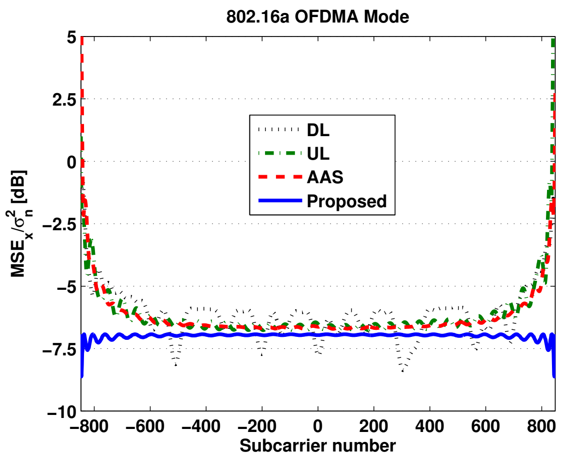

Fig. 4. Channel estimate MSE of the 802.16a pilot design and of the proposed pilot design for OFDMA.

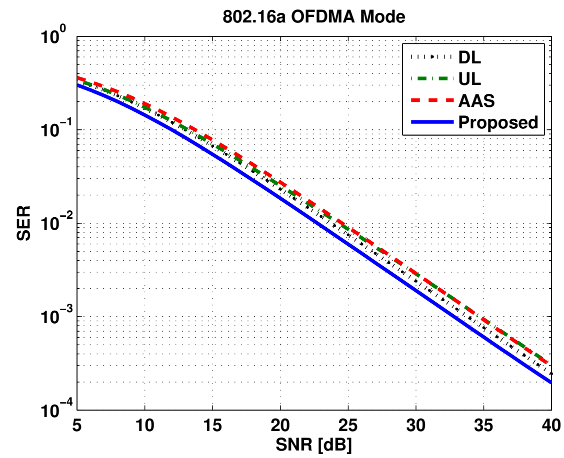

Fig. 5. SER of the 802.16a pilot design and of the proposed pilot design for OFDMA.

large number of pilots contained in the information symbols, it is reasonable to assume that these pilot might be used for channel estimation as well.

Adaptive Antenna System: Unlike the standard UL and DL options, the pilot positions for the AAS option are all constant regardless of the symbol number. Specifically, in each subchannel the pilot positions are $\{6, 17, 29, 39, 50\}$. Similar to the standard UL and DL options, the DL direction in AAS does not use an all-pilot preamble, while the UL direction does.

OFDMA Pilot Designs: Notice that the pilot design for the UL and DL options are dependent on the symbol number. For comparison purposes we choose to use the pilot design for symbol number $S = 0$ in both cases. In addition to pilot position, the pilot amplitude also plays an important part in the system performance. For each of the OFDMA pilot design, the pilots are all modulated with values $\pm 4/3$, where the sign is chosen pseudo randomly. Thus, for the DL option, $\beta_{DL} \approx 0.185$ and for the UL and AAS option $\beta_{AAS} = \beta_{UL} \approx 0.192$.

For the proposed near-optimal pilot design, the specifications from the AAS and UL options were used (i.e. $|\mathcal{K}_p| = 160$, $|\mathcal{K}_n| = 352$ and $|\mathcal{K}_d| = 1536$). The pilot parameters for the near-optimal design are $\beta^* = 0.452$ and $\mathcal{K}_p^* = \big\{((\text{int}[-.0006666x^3 + 0.159x^2 + 2.34x - 1200.15)]))_N | x \in \{0, 1, ..., 159\}\big\}$. Fig. 4 is a plot of the channel estimate MSE for the three 802.16a OFDMA options and for the proposed pilot design when the channel length is $L = 48$. In Fig. 5 there is a plot of the SER for the three options and for the proposed design. Among the OFDMA options, the DL pilot design performs better than the other two options by about 0.8dB of SNR. However, this comparison is not totally fair because the DL design utilizes 6 more pilots than any of the other designs plotted. Despite this handicap, the proposed pilot design still outperforms the DL design by about 1dB and the other two designs by about 1.8dB.

4. CONCLUSIONS

In this paper we presented several design examples using the pilot design procedure presented in [3]. The design examples were generated using the system specifications from the IEEE 802.16a standard. As expected the near-optimal pilot designs outperformed the 802.16a design in every situation. For the OFDM mode, the proposed design saved 13dB of SNR for Rayleigh channels with 8 tap impulse responses. In the OFDMA mode, even for a relatively short channel impulse response, the proposed design realized from 1 to 1.8dB SNR improvement depending on the OFDMA option implemented. The results show that by simply altering the pilot design, it is possible to realize significant SER reductions in IEEE 802.16a communications systems[3].

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