Comparing Selected Mapping and Partial Transmit Sequence for PAR Reduction

Robert J. Baxley and G. Tong Zhou

***Abstract*—Selected mapping (SLM) and partial transmit sequence (PTS) are two existing distortionless peak-to-average power ratio (PAR) reduction schemes that have been proposed for orthogonal frequency division multiplexing (OFDM). Previously, it was argued that SLM and PTS have comparable PAR reduction performance but that the latter has lower computational complexity because it uses fewer IFFTs. In this paper, we show that the overall computational complexity of PTS is only lower than that of SLM in certain cases, and that SLM always has better PAR reduction performance. We compare the two schemes using three different performance metrics by assuming a given amount of computational complexity that can be afforded. Using the metrics, we show that SLM outperforms PTS for a given amount of complexity.**

***Index Terms*—Orthogonal frequency division multiplexing (OFDM), partial transmit sequence (PTS), selected mapping (SLM).**

I. Introduction

OFDM is a popular modulation technique with many desirable qualities and has been proposed for the IEEE 802.11a, IEEE 802.11g, IEEE 802.16, the European digital audio broadcasting (DAB) and the European digital video broadcasting (DVB) standards. The desirable attributes of OFDM come at the expense of large envelope variations. Such signal envelope or power variations can be difficult for practical power amplifiers (PAs) to accommodate, resulting in either low power efficiency or distortion-inducing signal clips. The dynamic range of a signal is usually quantified through the peak to average power ratio (PAR) or the crest factor (CF) where $PAR = CF^2$.

There has been a significant amount of research devoted to the development of PAR reduction algorithms for OFDM. An overview of the different approaches can be found in [1]–[3]. In this paper, we are interested in comparing two distortionless PAR reduction algorithms. The first is selected mapping (SLM), which was first presented in [4]. The second algorithm is partial transmit sequence (PTS), which was proposed in [5] and can be viewed as a generalization of the SLM algorithm. Since the initial publication of the algorithms, many proposals have been made to refine the algorithms, including complexity reductions [6]–[11], PTS phase optimization [12]–[16], techniques to obviate the transmission of side information [17], [18], SLM/PTS combination approaches [19], and extensions to MIMO-OFDM [20].

The objective of this paper is to come up with some common ground metrics for comparing SLM and PTS. Comparisons of SLM and PTS were made in [21]. In [21], it was claimed that the computational complexity of PTS is lower than that of SLM, but that the two have comparable PAR reduction performance, which we will show is not necessarily true. Unlike [21], we will make quantifiable comparisons based on the PAR reduction per unit of complexity of the two schemes.

For the purposes of comparing SLM and PTS, we can use a simple OFDM model where the baseband OFDM symbol $\{x_{n/L}\}_{n=0}^{NL-1}$ is an oversampled IFFT output of the data vector $\{X_k\}_{k=0}^{N-1}$, a sequence of (complex) numbers drawn from a finite constellation. That is,

$$x_{n/L} = \frac{1}{\sqrt{LN}}\left(\sum_{k=0}^{N/2-1} X_k e^{\frac{j2\pi kn}{LN}} + \sum_{k=N/2}^{N-1} X_k e^{\frac{j2\pi(k-N)n}{LN}}\right) = \mathrm{IFFT}\{X_k\}, \quad (1)$$

where L is the oversampling factor, N is the number of subcarriers, and $\mathrm{IFFT}\{\cdot\}$ is NL-point oversampled IFFT indexed by n/L, where $n \in \{0, 1, \ldots, LN-1\}$. The baseband PAR is defined as

$$\mathrm{PAR}\{x_{n/L}\} = \frac{\max_{0\le n\le LN-1} |x_{n/L}|^2}{\mathrm{E}\left[|x_{n/L}|^2\right]}, \quad (2)$$

which is a random variable.

It is sufficient to examine only the baseband PAR as it is approximately one half of the passband PAR [1]. Also, the cyclic prefix attached to OFDM symbols to combat inter-symbol interference can be ignored for the purposes of PAR analysis as the prefix will not produce a peak that is not already present in $x_{n/L}$. Finally, it has been shown in [22] that when $L \ge 4$ the envelope of $x_{n/L}$ approximates the continuous-time envelope.

In the next section, we will review SLM and PTS and investigate the relationship between the two schemes. Section III provides computational complexity analysis for both schemes. Section IV introduces three PAR reduction metrics and uses them to compare SLM and PTS. Finally our concluding remarks are provided in Section V.

II. Selected Mapping and Partial Transmit Sequence

Selected mapping (SLM): SLM was first described in [4] as a distortionless PAR reduction method. In SLM, D equivalent data sequences are created each by rotating the phases of the original sequence X_k by a distinct sequence $\phi_k^{(d)}$; i.e.,

$$X_k^{(d)} = X_k e^{j\phi_k^{(d)}}, \quad (3)$$

Manuscript received January 13, 2007; revised April 30, 2007. This work was supported in part by the US Army Research Laboratory Communication and Networks Collaborative Technology Alliance program and by the US National Science Foundation under the Graduate Research Fellowship Program. Some results of this paper were presented at the IEEE Military Communications ConferenceWashington, DC, USAOctober2006.

The authors are with the School of Electrical and Computer Engineering, Georgia Institute of Technology, Atlanta, GA 30332-0250, USA (e-mail: baxley@ece.gatech.edu; gtz@ece.gatech.edu).

Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/TBC.2007.908335

which is used to create

$$x_{n/L}^{(d)} = \mathrm{IFFT}\left\{X_k^{(d)}\right\} \tag{4}$$

where $d \in \{0, 1, \ldots, D-1\}$. A total of D length-NL IFFTs are performed. From these D candidates, the transmitter selects the lowest PAR sequence, $x_{n/L}^{(\bar{d})}$, for transmission where

$$\bar{d} = \underset{0 \leq d \leq D-1}{\arg\min}\ \mathrm{PAR}\left\{x_{n/L}^{(d)}\right\}. \tag{5}$$

It is assumed that the transmitter and the receiver have the table of D length-N phase sequences $\phi_k^{(d)}$. However, in order to recover the original data sequence X_k, the receiver must determine $\bar{d}$. To distinguish $\bar{d}$ from the D possibilities, $\log_2(D)$ bits are needed. Because side information transmission decreases the information throughput, several authors have proposed blind techniques for recovering $\bar{d}$ based only on the received data and the known phase table [17], [18], [23]–[25]. In the computational analyses in this paper, we will assume that the blind maximum-likelihood side information recovery technique is employed, which implies that full complexity complex multiplications will be required to compute (3).

Partial transmit sequence (PTS): In a PTS system, $\{X_k\}_{k=0}^{N-1}$ is partitioned into V non-overlapping sub-blocks $\{X_{\mathcal{V}_v}\}_{v=0}^{V-1}$ with indices in the sets $\{\mathcal{V}_v\}_{v=0}^{V-1}$. That is

$$\bigcup_{v=0}^{V-1} \mathcal{V}_v = \{0, 1, \ldots, N-1\} \tag{6}$$

and

$$\sum_{v=0}^{V-1} X_{\mathcal{V}_v} = X_k, \quad \forall\, k \in \{0, 1, \ldots, N-1\}. \tag{7}$$

To generate D PTS signal representations, each of the sub-blocks is scaled by a complex constant $e^{j\theta_v^{(d)}}$ and added together so that

$$x_{n/L}^{(d)} = \mathrm{IFFT}\left\{\sum_{v=0}^{V-1} e^{j\theta_v^{(d)}} X_{\mathcal{V}_v}\right\} \sum_{v=0}^{V-1} e^{j\theta_v^{(d)}} \underbrace{\mathrm{IFFT}\{X_{\mathcal{V}_v}\}}_{x_{n/L,v}}, \tag{8}$$

where $d \in \{0, 1, \ldots, D-1\}$. Finally, the transmitted signal $x_{n/L}^{(\bar{d})}$ is chosen according to (5) similar to the SLM method.

The PTS method originally described in [5] is a special case of the technique we have described here. In [5], it is assumed that the phase parameters take on values from a finite set, $\mathcal{P}$, so that $\theta_v^{(d)} \in \mathcal{P}$. Next, every possible set of phase combinations is tested to find the combination that produces that lowest PAR sequence. Thus, for traditional PTS, $D = V^{|\mathcal{P}|}$. The only modification we have made is to allow non-exhaustive searches over the phase sequence space so that $D \leq V^{|\mathcal{P}|}$. A similar development was used in [21] by the authors of [5].

In (8), a total of V length-NL IFFTs are performed, regardless of the size of D. The small number of IFFTs (V as opposed to D, assuming $V < D$) relative to SLM was one main justification for proposing PTS [5]. However, as we will show in Section IV, the number of IFFTs should not be the only complexity consideration since the length-NL multiplications and additions necessary to create $x_{n/L}^{d}$ in (8) can contribute significantly to the overall complexity of PTS.

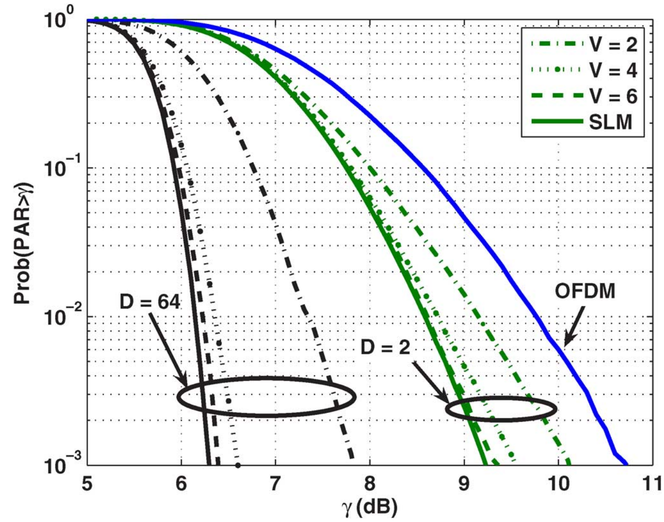

Fig. 1. Plot of the CCDF of the PAR for SLM and PTS with $L = 4$.

From (8) it is apparent that SLM can be regarded as a special case of PTS where each sub-block contains only one subcarrier (e.g. $\mathcal{V}_v = v|_{v \in [0, N-1]}$). In analyzing the PAR-minimizing values of $\theta_v^{(d)}$, it is convenient to view PTS as a special case of SLM where the SLM phases are constant over each sub-block, i.e. $\phi_{\mathcal{V}_v}^{(d)} = \theta_v^{(d)}$. Recall from [26] that if $\phi_k^{(d)}$ are realizations of an i.i.d. random variable Φ with $\mathrm{E}[e^{j\Phi}] = 0$, then the corresponding PAR CCDF curve will be minimized. In PTS, the i.i.d. condition is violated for $V < N$. So PTS with $V \neq N$ will not be able to achieve as much PAR reduction as SLM for a given D. In order words, the result from [26] can be applied to PTS to prove that PTS will have worse PAR reduction performance than SLM for a given amount of side information. A similar conclusion, based on simulation results, was reached in [21] by the original authors of PTS.

Fig. 1 is a plot of the PAR CCDF for PTS and SLM when the oversampling factor is $L = 4$. The plot illustrates the PAR reduction degradation that PTS suffers when V is small. However, with $\mathrm{V} = 6$, the PTS CCDF is quite close to its lower bound which is the SLM CCDF, achievable when $V = N$.

In addition to reducing the number of IFFTs, the other main justification for PTS is that it may be possible to find $\theta_v^{(\bar{d})}$ without having to search all D possibilities, thus reducing the amount of computation [5]. Many optimization methods have been proposed [14]–[16]. However, some of them are misguided because they do not take into account the large amount of side information required to convey the optimized parameters $\theta_v^{(\bar{d})}$ to the receiver. That is, they perform a sub-optimal discrete optimization over a very large set $\theta_v^{(d)}$, from which they have to convey $\theta_v^{(\bar{d})}$ to the receiver.

In this paper, we are interested in determining the PAR reduction capability per unit of computational complexity of PTS and SLM under a side information constraint. In order for the conclusions to not depend on a particular optimization

TABLE I
NUMBER OF REAL OPERATIONS FOR SLM AND PTS

SLM additions (A_{SLM})	$DNL(3\log N + 2) + 2N(D - 1)$
SLM multiplications (M_{SLM})	$2DNL(\log N + 2) + 4N(D - 1)$
PTS additions (A_{PTS})	$4(D - 1/2)NLV - DNL + VA_{IFFT}$
PTS multiplications (M_{PTS})	$4(D - 1)NLV + 2DNL + VM_{IFFT}$

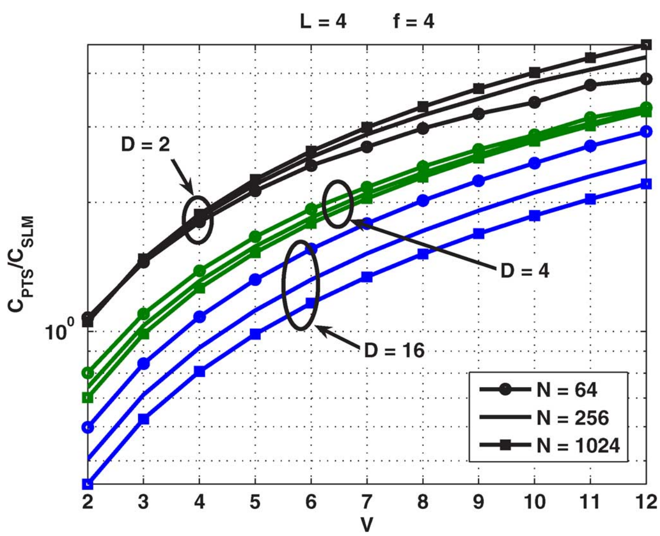

Fig. 2. Plot of C_{PTS}/C_{SLM} versus number of PTS partitions V for $L = 4$.

method, we opt to analyse PTS using a set of phase constants that are i.i.d. random, where the PAR-minimizing set of phase constants is determined by exhaustive search. Performing an exhaustive search guarantees that $\theta_v^{(\tilde{d})}$ will be the PAR-minimizing sequence among the set $\theta_v^{(d)}$, $d \in \{0, 1, \ldots, D-1\}$.

Two issues in PTS need to be discussed before analyzing its complexity. The first is whether it is possible to choose the values of $\theta_v^{(d)}$ such that the multiplication in (8) can be implemented with simple sign changes. It was shown in [17] that in order to use a blind maximum-likelihood receiver to detect $\tilde{d}$, full-complexity complex multiplications are required. The other consideration is whether it is possible to choose the sets $\mathcal{V}_v$ so that the IFFT in (8) can be performed at a reduced complexity. It has been shown that the PAR minimizing choice for $\mathcal{V}_v$ is random equally-sized sub-blocks, which do not generally allow for a complexity reducing structure [13]. On the other hand, If the IFFT is designed especially for the specific set of PTS partitions, then it is possible to exploit the sparseness of $X_{\mathcal{V}_v}$ to reduce the complexity of IFFTs [27].

III. Computational Complexity

This section will outline the computational complexity involved in SLM and PTS. We will assume that $\theta_v^{(0)} = 0$ and that $\phi_k^{(0)} = 0$, i.e. the first of D signal mappings is just the original OFDM symbol.

For SLM, as indicated in (3), $N(D-1)$ complex multiplications are required to create $X_k^{(d)}$, $d \in \{0, 1, \ldots, D-1\}$. Next, D length-NL IFFTs are needed to generate $x_{n/L}^{(d)}$, $d \in \{0, 1, \ldots, D-1\}$. Each oversampled IFFT requires $NL/2 \log N + NL/2$ complex multiplications and $NL \log N$ complex additions (all logarithms are base 2, i.e. $\log N \triangleq \log_2 N$) [11]. Finally, $|x_{n/L}^{(d)}|^2$ must be calculated at each n to determine the PAR which comes at the expense of $2DLN$ real multiplications and DLN real additions.

In (8), the generation of a PTS symbol requires V length-NL oversampled IFFTs to create $x_{v/L,v}$. To make the comparison fair, we assume that each IFFT is especially designed to exploit the sparseness of $X_{\mathcal{V}_v}$. With this assumption we can calculate the mean[1] number of multiplications and additions required for each IFFT in terms of the sparseness of $X_{\mathcal{V}_v}$ [27]. In the context of PTS, the sparseness of $X_{\mathcal{V}_v}$ is the proportion of its entries that are non-zero, which is $1/V$. According to (8), $VNL(D-1)$ complex multiplications are needed to create $e^{j\theta_v^{(d)}} x_{v/L,v}$, $d \in \{0, 1, \ldots, D-1\}$, which are combined through $(V-1)NLD$ complex additions to generate $x_{n/L}^{(d)}$. Finally, just as in SLM, the cost of calculating the $\mathrm{PAR}\{x_{n/L}^{(d)}\}$, $d \in \{0, 1, \ldots, D-1\}$, is $2DLN$ real multiplications and DLN real additions.

In general, a complex multiplication takes four real multiplications and two real additions. On the other hand, a complex addition requires two real additions. In Table I, we have summarized the computational requirement of each scheme, where A_{IFFT} and M_{IFFT} are the number of real additions and real multiplication required for each sparse PTS IFFT.

The complexity of PTS and SLM is quantified through a parameter f that is the number of addition instructions required for each multiplication operation. So the overall complexity is

$$C_{SLM} = A_{SLM} + fM_{SLM}$$
$$C_{PTS} = A_{PTS} + fM_{PTS},$$

where A_{SLM}, M_{SLM}, A_{PTS} and M_{PTS} are defined in Table I.

Fig. 2 is a plot of the ratio C_{PTS}/C_{SLM} versus the number of PTS partitions V. The plot contains lines for $N = \{64256, 1024\}$ and $D = \{2, 4, 16\}$ with $L = 4$ and $f = 4$. Note that Fig. 2 shows that the ratio is monotonically increasing in V. The point of utmost interest is where the lines cross $C_{PTS}/C_{SLM} = 1$. For $C_{PTS}/C_{SLM} > 1$, PTS is more computationally complex, whereas, when $C_{PTS}/C_{SLM} < 1$ SLM is more complex. The plot shows that when $D = 2$, PTS is always more complex. Even for a large value $D = 16$, SLM is less complex than PTS when $V > 5$. These are surprising results considering that PTS was designed to have a *lower* complexity than SLM.

IV. SLM/PTS Comparison

From the PAR CCDF-minimizing criteria outlined in [26], we infer that SLM will result in a lower PAR than PTS for a given number of mappings D; this is empirically demonstrated in Fig. 1. With Fig. 2, we can see that large values of V lead to increased computational complexity for PTS. Together, these two observations imply that PTS will be inferior to SLM in complexity and in PAR reduction capability for all values of V that cause $C_{PTS}/C_{SLM} > 1$.

[1]The mean complexity in [27] is averaged over all possible input vectors with the specified sparseness. As a point of reference, a 256-point IFFT of a 25% non-zero vector requires 78% as many complex multiplications and 70% as many complex additions as a full 256-point IFFT.

The next question is: should SLM be used over PTS when V is so small that $C_{PTS}/C_{SLM} \leq 1$? To make this comparison, we must first find some metric that quantifies the performance of each PAR reduction scheme. The most obvious candidate is the PAR at a predetermined CCDF level. But such a metric is sensitive to the probability level chosen. Also, the PAR at a certain probability level does not translate in any obvious way to a meaningful system metric like bit error rate or PA power efficiency. Instead, we advocate three separate metrics all of which have a more tangible meaning than the PAR at a certain probability level.

To simplify the analysis we assume a soft clipping operation or the so-called ideal linear (or linearized) PA defined by

$$g_{clip}(x) = \begin{cases} x, & |x| \leq A \\ Ae^{j\angle x}, & |x| > A. \end{cases} \tag{9}$$

A popular linearization technique for PAs is called predistortion [28]. With predistortion, it is possible to realize an overall PA response that resembles (9).

A. Power Efficiency-Based Metric

The first metric we are going to introduce assumes a system where each OFDM symbol is scaled digitally by a factor $\sqrt{\alpha}$ before being sent to the PA [29]. Such a scaling operation appears as flat fading to the receiver and can be thought of as part of the multipath channel. The scaling factor is chosen so that the peak power of the transmitted symbol $x_{n/L}^{(\tilde{d})}$ is exactly equal to A^2. With this definition,

$$\alpha = \frac{A^2}{\max\limits_{0 \leq n \leq NL-1} \left| x_{n/L}^{(\tilde{d})} \right|^2}. \tag{10}$$

The average symbol power can be shown to be proportional to $\mathrm{E}[1/\mathrm{PAR}\{x_{n/L}^{(\tilde{d})}\}]$. Also, from [30], we can express the mean power efficiency, η of a class A power amplifier using

$$\eta = \mathrm{E}\left[\frac{1}{2PAR}\right]. \tag{11}$$

Accordingly, we will use this metric to quantify the performance of PTS and SLM when the minimum PAR criterion is implemented.

Fig. 3 is a plot of the class A PA power efficiency, η, versus the complexity for PTS with $V = \{2, 3, 4\}$ and for SLM, where $N = \{64, 256\}$. As a point of reference, assuming $L = 4$ and $f = 4$, conventional OFDM with $N = 64$ subcarriers or $N = 256$ subcarriers has complexity $C \approx 2 \cdot 10^4$ and $C \approx 10^5$, respectively. When $D = 32$, the SLM complexity numbers are $C_{SLM} \approx 7 \cdot 10^5$ and $C_{SLM} \approx 3 \cdot 10^6$ respectively for 64-subcarrier and 256-subcarrier SLM OFDM.

The plot shows that the performance of PTS is sensitive to the number of partitions V. There is not a single choice of V that is optimal over the entire complexity range so V should be chosen so that η is maximized for the complexity value of interest.

For both $N = 64$ and $N = 256$, there is no single value of V that is better than the other values of V across all of complexity values shown in Fig. 3. However, since we have the freedom of choosing the number of partitions V in PTS, we can select the best V for each allowed complexity value. For $N = 64$, if no more than $5 \cdot 10^4$ complexity is allowed, then the best choice is $V = 2$ because it maximizes efficiency. Similarly, if no more than $5 \cdot 10^5$ complexity is allowed when $N = 256$, then $V = 2$ is also the best choice. In any case, even when V is optimized for maximum efficiency, SLM will outperform PTS. However, the trends in Fig. 3 show that the higher the allowed complexity, the less the difference is between η for PTS and SLM. For practical levels of complexity, SLM tends to be a significantly better choice than PTS.

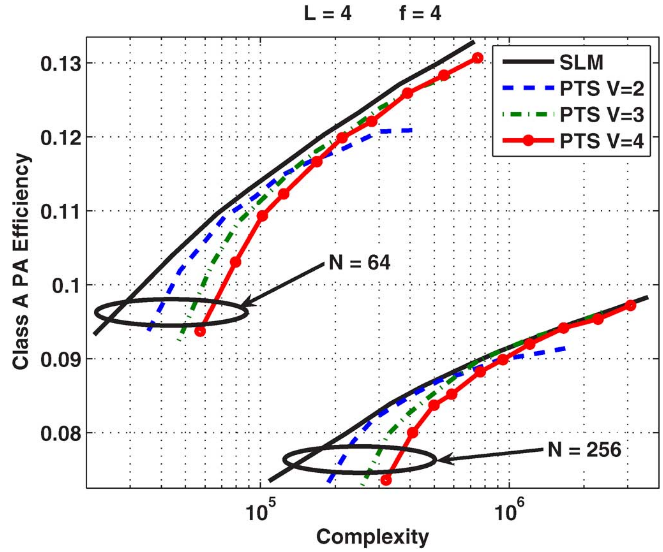

Fig. 3. Plot of η versus the complexity (C_{PTS} or C_{SLM}). Each line in the plot contains 32 points corresponding to the set of $D \in \{1, \ldots, 32\}$.

B. Clipping Power-Based Metric

In using η as the performance metric, we assumed that each symbol is scaled to maximally utilize the linear range of the PA. For high PAR signals, this assumption may lead to an unacceptably low SNR. One solution is to assume that some part of the signal will be clipped by the predistorter/PA soft limiter. By anticipating the irreversible clipping distortion that will be introduced, it is possible to reformulate the SLM and PTS selection criterion to mitigate the effect of the distortion. The reformulation would mean that the signal that produces the lowest power distortion among the D alternatives $\{x_{n/L}^{(d)}\}_{d=0}^{D-1}$ is transmitted. This differs from conventional SLM and PTS where the signal with the lowest PAR is selected for transmission. Stated precisely, the index of the transmitted signal $x_{n/L}^{(\tilde{d})}$ is selected according to

$$\tilde{d} = \underset{0 \leq d \leq D-1}{\arg\min} \sum_{n=0}^{NL-1} \left| x_{n/L}^{(d)} - g_{clip}\left(x_{n/L}^{(d)}\right) \right|^2. \tag{12}$$

Similar SLM metrics have been presented in [31], [32]. Note that the blind detection techniques used to recover $\tilde{d}$ described in [17], [18] will still be applicable for $\tilde{d}$ recovery regardless of the selection criterion. Therefore, no blind receiver modification would be necessary to accommodate the selection criterion in (12).

There are two main reasons to select the transmitted signal based on minimum distortion noise, the first is to limit the amount of inband distortion, which leads to bit errors. The second is to limit the amount of spectral regrowth. We will show in the next section that a better selection criteria exists for limiting the number of bit errors; however the minimum distortion metric is excellent at reducing the amount of spectral regrowth.

The spectral regrowth can be quantified through a popular metric known as the adjacent channel leakage ratio (ACLR). Define the frequency domain of the clipped signal to be

$$\bar{X}_k^{(d)} = \frac{1}{\sqrt{LN}} \sum_{n=0}^{NL-1} g_{clip}\left(x_{n/L}^{(d)}\right) e^{\frac{-j2\pi kn}{LN}}, \tag{13}$$

where $k \in \{0, 1, \ldots, LN-1\}$. Define the set of adjacent channel subcarriers $\mathcal{A} = \{N/2, N/2+1, \ldots, N-1, LN-N, LN-N, \ldots, LN-1-N/2\}$ and the set of in-band subcarriers $\mathcal{I} = \{0, 1, \ldots, N/2-1, LN-N/2, LN-N/2+1, \ldots LN-1\}$. Thus,

$$ACLR = \frac{\mathrm{E}\left[\sum_{k\in\mathcal{A}} \left|\bar{X}_k^{(\tilde{d})}\right|^2\right]}{\mathrm{E}\left[\sum_{k\in\mathcal{I}} \left|\bar{X}_k^{(\tilde{d})}\right|^2\right]}, \tag{14}$$

which is a measure of how much distortion power from the in-band signal "leaks" in to the adjacent frequency bands.

Fig. 4 is a plot of the ACLR versus the complexity for PTS with $V = \{2,3,4\}$ and for SLM, where $N = \{64, 256\}$, and with $1 \leq D \leq 32$. The complexity for PTS and SLM using the section criterion in (12) are slightly different from the complexity calculations formulated in Section IV. Namely, $(2N-1)LD$ more real additions are necessary. To create the plot, the clipping level was set so that $A/\sigma_x = 4$ dB, where σ_x^2 is the variance of X_k. As with the power efficiency metric in Section IV-A, the performance of PTS depends on the choice of V. For the best performance, V should be selected so that the ACLR is minimized at the complexity level of interest. The plot shows that SLM slightly outperforms PTS even when the minimizing V is selected.

C. Intermodulation Distortion-Based Metric

In [33]–[35], a case was made for selecting the transmitted symbol based on the worst case intermodulation distortion (IMD) power. For justification, those papers showed that the BER induced by clipping is dominated by the subcarrier with the largest IMD. Here we define the IMD E_k in the kth subcarrier as

$$E_k^{(d)} = X_k^{(d)} - \bar{X}_{k\in\mathcal{I}}^{(d)}, \tag{15}$$

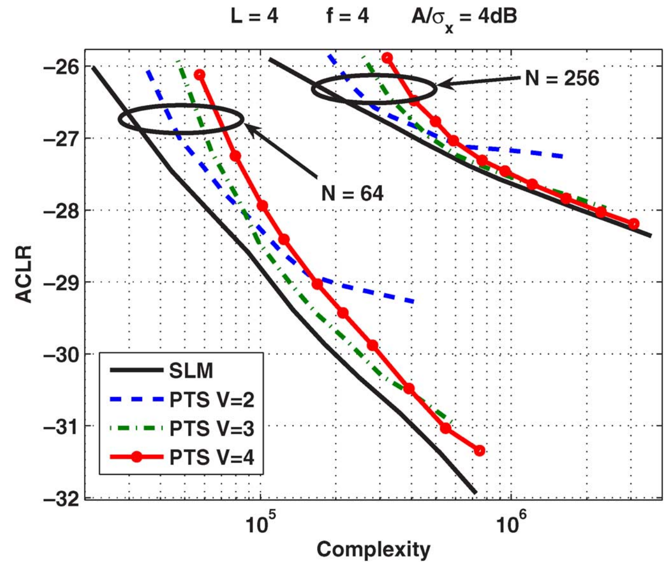

Fig. 4. Plot of the ACLR in decibel scale versus the complexity (C_{PTS} or C_{SLM}). Each line in the plot contains 32 points corresponding to the set of $D \in \{1, 2, \ldots, 32\}$.

where $\bar{X}_k^{(d)}$ is defined in (13). With this, the index of the transmitted signal $x_{n/L}^{(\breve{d})}$ is selected according to

$$\breve{d} = \underset{0\leq d\leq D-1}{\arg\min} \left(\max_{0\leq k\leq N-1} \left|E_k^{(d)}\right| \right), \tag{16}$$

which can be recovered blindly without any modification to the blind receivers described in [17], [18]. As noted in [35], this selection criterion requires additional computational complexity over the conventional minimum PAR selection criterion in (5). Specifically, the additional computational complexity is D FFT operations and NLD complex additions. To compare the performance of SLM and PTS when both schemes use an IMD-based selection criteria, we suggest that BER be used.

Fig. 5 and Fig. 6 are plots of the BER versus the complexity for PTS with $V = \{2,3,4\}$ and for SLM, where $N = \{64, 256\}$, and with $1 \leq D \leq 32$. The BER is calculated assuming that X_k is drawn from a QPSK constellation. Also, a noiseless Rayleigh fading channel, where the receiver has perfect channel state information, is assumed so that the received signal is $H_k(X_k^{(\breve{d})} + E_k^{(\breve{d})})$. Here, H_k has a complex Gaussian distribution with zero mean and variance one (i.e. $H_k \sim \mathcal{CN}(0,1)$). Finally, it is assumed that $\breve{d}$ is detected without error.

To create the plots two different clipping levels were used $A/\sigma_x = 4$ dB and $A/\sigma_x = 6$ dB, where σ_x^2 is the variance of X_k. For the 4 dB clipping level, the BER may be too low for practical applications. However, the 6dB clipping does achieve an adequate BER for practical systems.

The plot shows that it is beneficial to select V so that, at the complexity level of interest, the BER is minimized. The figure also illustrates that, for all of the values plotted, SLM has better BER performance than PTS regardless of the number of PTS partitions employed. The difference is particularly pronounced for the $N = 64$, $A/\sigma_x = 6$ dB case where SLM significantly outperforms PTS.

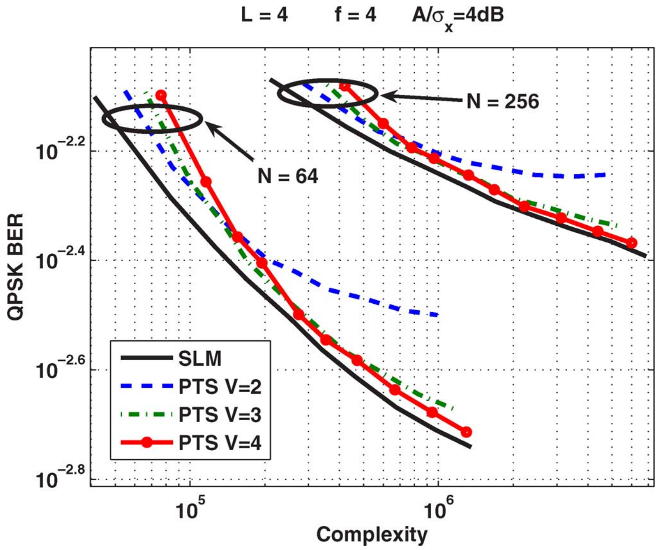

Fig. 5. Plot of QPSK BER versus the complexity (C_{PTS} or C_{SLM}), with a clipping level of 4 dB. Each line in the plot contains 32 points corresponding to the set of $D \in \{1, 2, \ldots, 32\}$.

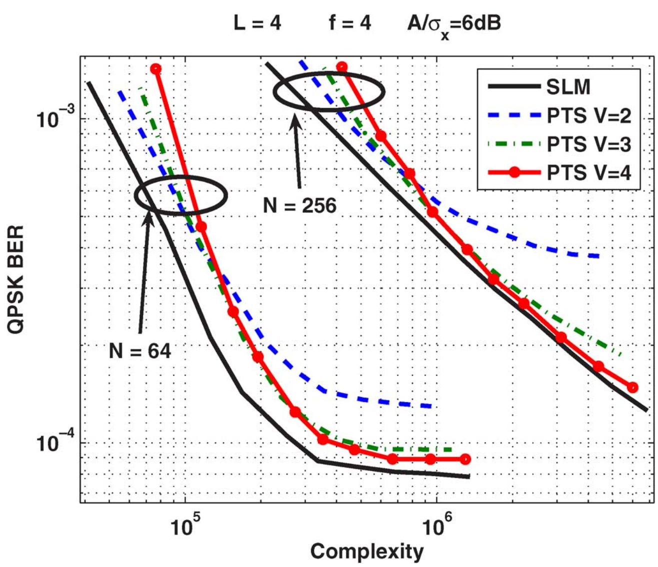

Fig. 6. Plot of QPSK BER versus the complexity (C_{PTS} or C_{SLM}), with a clipping level of 6 dB. Each line in the plot contains 32 points corresponding to the set of $D \in \{1, 2, \ldots, 32\}$.

V. Conclusions

In this paper we focused on two popular distortionless PAR reduction techniques, namely PTS and SLM and analysed their computational complexity and resulting PAR reduction performance. It was already known that SLM can produce multiple time-domain signals that are asymptotically independent, while the alternative signals generated by PTS are interdependent. This interdependency necessarily implies that PTS will have some PAR reduction capability degradation compared to SLM for a given number of mappings. However, it has been assumed in [21] that the computational complexity of PTS would be much less than that of SLM so that the computational savings of PTS would outweigh the PAR reduction advantages of SLM.

Surprisingly, we found that SLM is actually less computational complex than PTS when more than a couple of PTS partitions are used. In order to compare PTS and SLM we used three different selection criteria and, correspondingly, three different metrics. The results showed, for a given amount of computational complexity, SLM performs better than PTS in all three metrics. Even PTS with an optimized V does not beat the performance of SLM in any of the metrics across all complexities. In summary, SLM is preferred to PTS because (i) SLM is conceptually simpler; (ii) SLM does not require any off-line complexity optimization with respect to V as would be recommended for PTS; (iii) SLM performs better than *optimized* PTS.

Acknowledgment

The authors would like to thank the reviewers for their detailed comments and suggestions that have helped the presentation of this paper.

References

[1] J. Tellado, *Multicarrier Modulation with Low PAR: Applications to DSL and Wireless*. Norwell, MA: Kluwer Academic Publishers, 2000.

[2] R. J. Baxley, "Analyzing Selected Mapping for Peak-to-Average Power Reduction in OFDM," Master of Science Thesis, Georgia Institute of Technology, Atlanta, GA, May 2005.

[3] S. H. Han and J. H. Lee, "An overview of peak-to-average power ratio reduction techniques for multicarrier transmission," *IEEE Wireless Communications*, vol. 12, no. 2, pp. 56–65, Apr. 2005.

[4] R. W. Bauml, R. F. H. Fischer, and J. B. Huber, "Reducing the peak-to-average power ratio of multicarrier modulation by selected mapping," *IEE Electronics Letters*, vol. 32, no. 22, pp. 2056–2057, Oct. 1996.

[5] S. H. Muller and J. B. Huber, "OFDM with reduced peak-to-average power ratio by optimum combination of partial transmit sequences," *IEE Electronics Letters*, vol. 33, no. 5, pp. 368–369, Feb. 1997.

[6] N. T. Hieu, S. W. Kiom, and H. G. Ryu, "PAPR reduction of the low complexity phase weighting method in OFDM communication system," *IEEE Trans. on Consumer Electronics*, vol. 51, no. 3, pp. 776–782, Aug. 2005.

[7] D. W. Lim, S. J. Heo, J. S. No, and H. Chung, "A new PTS OFDM scheme with low complexity for PAPR reduction," *IEEE Trans. on Broadcasting*, vol. 52, no. 1, pp. 77–82, Mar. 2006.

[8] O. J. Kwon and Y. H. Ha, "Multi-carrier PAP reduction method using sub-optimal PTS with threshold," *IEEE Trans. on Broadcasting*, vol. 49, no. 2, pp. 232–236, Jun. 2003.

[9] L. Yang, R. S. Chen, Y. M. Siu, and K. K. Soo, "PAPR reduction of an OFDM signal by use of PTS with low computational complexity," *IEEE Trans. on Broadcasting*, vol. 52, no. 1, pp. 83–86, Mar. 2006.

[10] S. H. Han and J. H. Lee, "PAPR reduction of OFDM signals using a reduced complexity PTS technique," *IEEE Signal Processing Letters*, vol. 11, no. 11, pp. 887–890, Nov. 2004.

[11] C. L. Wang and Y. Ouyang, "Low-complexity selected mapping schemes for peak-to-average power ratio reduction in OFDM systems," *IEEE Trans. on Signal Processing*, vol. 53, no. 12, pp. 4652–4660, December 2005.

[12] A. Alavi, C. Tellambura, and I. Fair, "PAPR reduction of OFDM signals using partial transmit sequence: An optimal approach using sphere decoding," *IEEE Communications Letters*, vol. 9, no. 11, pp. 982–984, Nov. 2005.

[13] S. G. Kang, J. G. Kim, and E. K. Joo, "A novel subblock partition scheme for partial transmit sequence OFDM," *IEEE Trans. on Broadcasting*, vol. 45, no. 3, pp. 333–338, Sept. 1999.

[14] L. J. Cimini and N. R. Sollenberger, "Peak-to-average power ratio reduction of an OFDM signal using partial transmit sequences," in *Proc. of the IEEE International Conference on Communications*, June 1999, vol. 1, pp. 511–515.

[15] H. Ryu and K. Youn, "A new PAPR reduction scheme: SPW (subblock phase weighting)," *IEEE Trans. on Consumer Electronics*, vol. 48, pp. 81–89, Feb. 2002.

[16] W. S. Ho, A. S. Madhukumar, and F. Chin, "Peak-to-average power reduction using partial transmit sequences: A suboptimal approach based on dual layered phase sequencing," *IEEE Trans. on Broadcasting*, vol. 49, no. 2, pp. 225–231, June 2003.

[17] A. D. S. Jayalath and C. Tellambura, "SLM and PTS peak-power reduction of OFDM signals without side information," *IEEE Trans. on Wireless Communications*, vol. 4, no. 5, pp. 2006–2013, Sept. 2005.

[18] R. J. Baxley and G. T. Zhou, "MAP metric for blind phase sequence detection in selected mapping," *IEEE Trans. on Broadcasting*, vol. 51, no. 4, pp. 565–570, December 2005.

[19] Y. H. You, W. G. Jeon, J. H. Paik, and H. K. Song, "A simple construction of OFDM-CDMA signals with low peak-to-average power ratio," *IEEE Trans. on Broadcasting*, vol. 49, no. 4, pp. 403–407, Dec. 2003.

[20] M. S. Baek, M. J. Kim, Y. H. You, and H. K. Song, "Semi-blind channel estimation and PAR reduction for MIMO-OFDM system with multiple antennas," *IEEE Trans. on Broadcasting*, vol. 50, no. 4, pp. 414–424, Dec. 2004.

[21] S. H. Muller and J. B. Huber, "A comparison of peak power reduction schemes for OFDM," in *Proc. IEEE Global Telecommunications Conference*, Nov. 1997, vol. 1, pp. 1–5.

[22] H. Ochiai and H. Imai, "On the distribution of the peak-to-average power ratio in OFDM signals," *IEEE Trans. Communications*, vol. 49, pp. 282–289, Feb. 2001.

[23] N. Chen and G. T. Zhou, "Peak-to-average power ratio reduction in OFDM with blind selected pilot tone modulation," *IEEE Trans. on Wireless Communications*, vol. 5, no. 6, August 2006.

[24] G. T. Zhou, "Peak-to-average power ratio reduction in OFDM: blind selected mapping for PSK inputs," in *Proc. IEEE Intl. Conference on Acoustics, Speech, and Signal Processing*, Toulouse, France, May 2006, pp. 293–296.

[25] S. OHara, B. Chen, and J. Periard, "A bandwidth efficient peak power reduction scheme for multicarrier modulation using selected mapping," in *Proc. Conference on Information Sciences and Systems*, March 2003.

[26] G. T. Zhou and L. Peng, "Optimality condition for selected mapping in OFDM," *IEEE Trans. on Signal Processing*, vol. 54, no. 8, pp. 3159–3165, August 2006.

[27] R. J. Baxley and G. T. Zhou, "Computational complexity analysis of FFT pruning—A Markov modeling approach," in *Proc. 12th IEEE Digital Signal Processing Workshop*, Grand Teton National Park, Wyoming, September 2006.

[28] R. Raich, H. Qian, and G. T. Zhou, "Orthogonal polynomials for power amplifier modeling and predistorter design," *IEEE Trans. on Vehicular Technology*, vol. 53, no. 5, pp. 1468–1479, September 2004.

[29] H. Ochiai, "Performance analysis of peak power and band-limited OFDM system with linear scaling," *IEEE Trans. on Wireless Communications*, vol. 2, no. 5, pp. 1055–1065, Sept. 2003.

[30] S. C. Cripps*, RF Power Amplifiers for Wireless Communications*. Norwood, MA: Artech House, 1999.

[31] Q. Lu, L. Gui, and X. Z. Fang, "A new scheme to mitigate the OFDM high PAR problem by minimizing the signal's nonlinear distortion caused by HPA," *IEEE Trans. on Broadcasting*, vol. 52, no. 4, pp. 576–578, Dec. 2004.

[32] X. Lei, Y. Tang, S. Li, and Y. Li, "A minimum clipping power loss scheme for mitigating the clipping noise in OFDM," in *Proc. IEEE Global Telecommunications Conference*, Dec. 2003, vol. 1, pp. 6–9.

[33] M. R. D. Rodrigues and I. J. Wassell, "A novel coding strategy to improve the error probability performance of non-linearly distorted OFDM signals," in *Proc. IEEE Vehicular Technology Conf.*, Oct. 2003, vol. 1, pp. 294–298.

[34] M. R. D. Rodrigues and I. J. Wassell, "SLM and PTS based on an IMD reduction strategy to improve the error probability performance of non-linearly distorted OFDM signals," in *Proc. IEEE Int. Conf. Communications*, Jun. 2004, vol. 2, pp. 857–861.

[35] M. R. D. Rodrigues and I. J. Wassell, "IMD reduction with SLM and PTS to improve the error-probability performance of nonlinearly distorted OFDM signals," *IEEE Trans. on Vehicular Technology*, vol. 55, no. 2, pp. 537–548, Mar. 2006.