Nonlinear Spreading for Communications in Co-Band Interference Channels

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***Abstract*—Significant recent research effort has been devoted to mitigating the detrimental nonlinear effects that physical components play on communications signals. In this paper, we take a different view of system nonlinearity and show how it can be leveraged to increase system performance. Specifically, we propose a spread spectrum modulation called nonlinear spreading to increase link robustness in co-band interference channels and show that large performance benefits are possible. We also show how the power spectral density (PSD) of a nonlinearity spread function can be controlled by shaping the input PSD.**

I. Introduction

Physical component nonlinearity is a fact of life in real world communications systems. Traditionally, such nonlinearity is treated as a nuisance that needs to be mitigated so that the overall channel can be viewed as linear. In this paper, we analyze a situation where existing component nonlinearity can be leveraged with intentionally induced nonlinearity to improve system performance. In particular, we are interested in systems that are bandwidth constrained by the digital-to-analog converter (DAC). For such a system, we demonstrate how the bandwidth can be spread in the analog domain with careful choice of nonlinearity to achieve robustness against narrow-band interferers.

Notationally, assume their is an information symbol in the digital domain that is expressed as a vector $\mathbf{x}$ drawn from a finite set $\mathcal{A}$, i.e. $\mathbf{x} \in \mathcal{A}$. After $\mathbf{x}$ is created digitally, it passes through a DAC with maximum bandwidth B_x to create an analog signal

$$x(t) = \sum_n x_n q\left(t - \frac{n}{B_x}\right), \tag{1}$$

where $q(t)$ is the DAC shaping function with bandwidth B_x. For the purposes of this work, $q(t)$ is considered to be a sync function with a ideal unit low-pass response on $f \in \left(-\frac{B_x}{2}, \frac{B_x}{2}\right]$ and a zero response at all other frequencies. Practical DACs achieve a close approximation of this characteristic using a sample and hold architecture followed by an anti-aliasing filter. However, in future work, it may of interest to study a DAC that does not employ filtering.

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The objective in this paper is two-fold: i) establish that nonlinear spreading can be used to decrease the symbol error rate (SER) in a co-band interference channel, and ii) formulate a method for controlling the nonlinear output PSD shape for a given nonlinearity. This formulation can be used to optimize the PSD under various objective functions like BER, eavesdropper detection, etc.

II. Co-Band Interference

For the channel we assume static multipath fading $h(t)$, additive white Gaussian noise (AWGN) $w(t)$, and a band-limited Gaussian interferer $z(t)$ with power spectral density (PSD) $S_z(f)$ and maximum bandwidth B_z.

Naturally, if the interferer overlaps in frequency with the signal of interest and has high enough power, recovering the original signal will be difficult. Classically, spread spectrum (SS) waveforms like direct sequence SS (DSSS) and frequency hopping SS (FHSS) have been employed to mitigate the detrimental effects of co-band intereferes [1]. These methods can be extremely useful to this end [2], [3]. However, here we explore the idea of nonlinear analog-based spreading to combat co-band interference. This problem is of interest because DACs only have a finite bandwidth, so digital domain spreading will ultimately be limited by the DAC bandwidth.

Instead of digital spreading, we assume the analog waveform is spread via some nonlinear analog transform $g(x(t))$, where it is assumed that $g(\cdot)$ is memoryless and includes any nonlinear effects induced by the power amplifier or other physical componentry. Therefore, the received signal is

$$y(t) = h(t) \otimes g(x(t)) + z(t) + w(t), \tag{2}$$

where $\otimes$ is the convolution operator $x(t) \otimes y(t) = \int x(t-\tau)y(t)d\tau$.

As we will show in the following section, for Gaussian input signals, a nonlinear spreading function will necessarily lead the power spectral density (PSD) of $g(x)$, $S_{g(x)}(f)$, to have a larger bandwidth than the PSD $S_x(f)$.

In the following it is shown that when the nonlinear function is known, the larger bandwidth can result in robustness again co-band interferers. To verify that nonlinear spreading can result in superior co-band inference SER performance we utilize orthogonal frequency division multiplexing (OFDM) modulation as an example where the digital time-domain samples are created with an inverse discrete Fourier Transform (IDFT) according to

$$x_n = \sum_{k=0}^{N-1} X_k e^{j2\pi nk/N}. \tag{3}$$

The data symbols $\{X_k\}_{k=0}^{N-1}$ are drawn from a N-dimensional constellation $\hat{\mathcal{A}}$ with cardinality $|\hat{\mathcal{A}}|$. At the receiver, we assume perfect knowledge of the transmitter nonlinearity $g(\cdot)$ and of the channel $h(t)$. Because of the cyclic prefix prepended to the OFDM symbol (i.e., the actual transmit signal is $[x_{N-1-M:N-1},\ x_{0:N-1}]$, any nonlinear distortion, as long as it does not have memory, will still allow a discrete Fourier Transform operation at the receiver to diagonalize the channel up to channel order $M+1$ (see [4] for details on the cyclic prefix). Thus, as with standard OFDM, we consider the frequency domain symbol without inter-symbol interference (ISI).

The received signal $y(t)$ is sampled to create y_n. The sampling will have a rate of $1/B_r$ so that the digital bandwidth is $B_x < B_r < B_y$, which means that some aliasing will occur. Next, the cyclic prefix is discarded and the symbol is transformed to the frequency domain where

$$Y_l = H_l \left(\sum_{k=0}^{N-1} \alpha_k^{(l)} X_l \right) + Z_l + W_l, \tag{4}$$

where $\alpha_k^{(l)}$ are the mixing mixing coefficients that result from the nonlinear function $g(\cdot)$ and the capital case variables $Y_l,\ Z_l,\ W_l$, and X_l are the DFT coefficients of their respective lower-case counterparts. Also, in (4), $l = \{0, 1, ..., L-1\}$ where $L = NB_r/B_x$.

Now, instead of calculating these coefficients $\alpha_k^{(l)}$ explicitly, to decode the received symbol, we instead compute the new constellation $\tilde{\mathcal{A}}$ that results when from passing the OFDM symbol through the nonlinearity $g(x)$. Then we detect over this set with maximum likelihood sequence detection [5]. That is,

$$\hat{\mathbf{x}} = \arg\min_{\mathbf{x}} \sum_l |Y_l - \hat{H}_l \text{IFFT}_l g(\mathbf{x}))|^2. \tag{5}$$

The results are shown in Fig. 1. In the plot, $B_r/B_x = 2$, $SNR = \sigma_x^2/\sigma_w^2$, $N = 16$, and $S_z(f) = \sigma_z^2$ for $f \in \left(-\frac{B_x}{2}, \frac{B_x}{2}\right]$, with $\sigma_z^2 = \sigma_x^2$ (i.e., a 0dB signal to interference ratio (SIR)). For $f \in \left(-\frac{B_x}{2}, \frac{B_x}{2}\right]$, $S_z(f) = 0$.

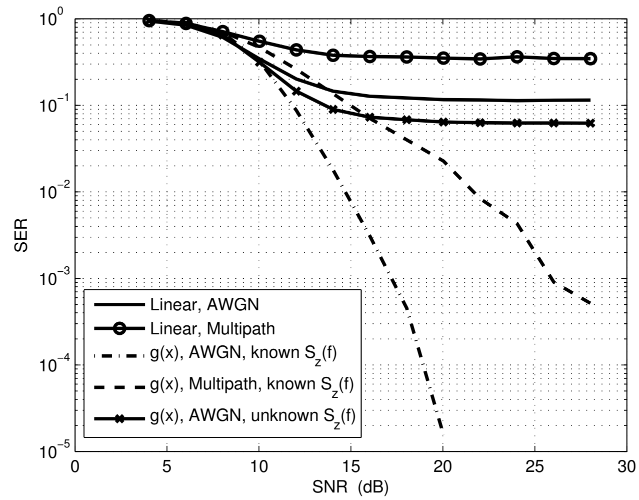

Fig. 1. Symbol error rate for the linear and nonlinear spreading transmitter.

The constellation points are chosen randomly from all 16-OFDM QPSK possibilities so that $|\hat{\mathcal{A}}| = 64$. For a linear transmitter, their is no spectral spreading so, as the plot shows, the SER floor is dictated by the SIR. In contrast, for the nonlinearly spread signal, when $S_z(f)$ is known, the SER performance continues to improve with decreasing σ_w^2 (increasing SNR). When $S_z(f)$ is not known, there is a performance limit even for the spread signal.

III. PSD Shaping

As discussed in the last section, a nonlinear transform $g(\cdot)$ of a signal x effects the shape of the transformed signal PSD, $S_{g(x)}(f)$. Here, we formulate a method for systematically controlling the output PSD, $S_{g(x)}(f)$ by adjusting the input PSD $S_x(f)$. The ultimate objective is to optimize the waveform shape to maximize performance (e.g., SER performance in an interference channel, probability of detection performance in an eavesdropper channel, etc.). For this analysis, we assume the input signal is colored Gaussian where the user has control over the PSD as is the case with OFDM. Based on the work in [6], the output spectrum can be expressed as

$$S_{g(x)}(f) = \sum_{p=0}^{\infty} \frac{c_p}{(\sigma_x^2)^{2p+1}} \bigotimes_{k=1}^{2p+1} S_x(f), \tag{6}$$

where $\bigotimes$ is the multiple convolution operator, e.g. $\bigotimes_{k=1}^{2} S_x(f) = S_x(f) \otimes S_x(f)$, and where c_p is a coefficient that describes the nonlinearity $g(x)$ and is defined by

$$c_p = \frac{1}{\sigma_x^4(p+1)} \left| \int g(r) r^2 e^{-r^2/2\sigma_x^2} L_p^{(1)}\left(\frac{r^2}{\sigma_x^2}\right) dr \right|^2, \tag{7}$$

where the Laguerre function is defined as

$$L_p^{(k)}(x) = \frac{x^{-k} e^x}{p!} \left(\frac{d}{dx}\right)^p (x^{p+k} e^{-x}). \tag{8}$$

Now, as the goal is to shape $S_x(f)$, over which we have full control so that the output PSD $S_{g(x)}(f)$ is as close as possible (defined by the $\mathcal{L}^2$ norm in this case; other norms can be considered) to a certain desired PSD $\tilde{S}(f)$ for a given nonlinear function $g(\cdot)$. Equivalently, we are trying to solve the optimization problem

$$\begin{aligned} \underset{S_x(f)}{\text{minimize}} \quad & \int |S_{g(x)}(f) - \tilde{S}(f)|^2 df \\ \text{subject to} \quad & S_x(f) = 0, \ f \notin \left(-\frac{B_x}{2}, \frac{B_x}{2}\right]. \end{aligned} \tag{9}$$

For instance, consider an example where the objective is to create a flat output PSD with three times the bandwidth of the input waveform, i.e. $\tilde{S}(f) = 1, \ f \in (-3B_x/2, 3B_x/2]$. Because (9) is not convex, we utilize a genetic search algorithm to find a near optimal input PSD function [7].

As an example, the coefficients, c_p, from (6) and the optimized output PSDs are presented for two nonlinearities. First we examine the the hard clipper nonlinearity, $g(x) = e^{j\angle x}$ where we calculated the coefficients to be

$$c_p = \frac{1}{\sigma_x^2(p+1)} \left(\frac{\Gamma\left(\frac{1}{2}+p\right)}{2p!}\right)^2. \tag{10}$$

We also examined the log-magnitude nonlinearity with response $g(x) = \log(|x|)e^{j\angle x}$, which has coefficients

$$c_p = \frac{(p+1)!}{\sigma_x^2(p+1)} \left(\sum_{m=0}^{p} (-1)^m \frac{\Gamma\left(\frac{3}{2}+m\right)\left(\log(\sigma_x^2) + \Psi\left(\frac{3}{2}+m\right)\right)}{(p-m)!(m+1)!m!}\right)^2, \tag{11}$$

where $\Psi(x) = \frac{\partial}{\partial x}\log(\Gamma(x))$. Fig. 2 is a plot of the input, objective, and output PSD for the two nonlinear spreading functions above where the input PSD is a near solution to (9) found via genetic algorithm search. The plot shows that the choice of nonlinearity is very important in achieving the objective of broadening the spectral bandwidth. For the hard clipper, even the optimized output PSD has relatively low power at $f = 3B_x/2$. This is a result of the coefficients c_p diminishing very quickly in p. On the other hand, when the log-magnitude spreading function is used, optimization can can ensure that the objective output is almost met.

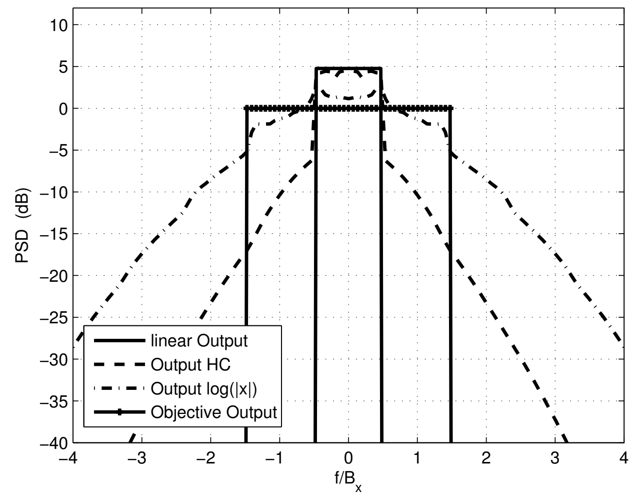

Fig. 2. PSD of input, objective and nonlinear spreading functions (hard clipper: $g(x) = e^{j\angle x}$; and the log-magnitude function: $g(x) = \log(|x|)e^{j\angle x}$).

IV. Conclusions

This paper provides the justification for leveraging system nonlinearities in order to increase system robustness in the presence of narrow-band interference. Furthermore, it was demonstrated how the PSD of the transmitted signal can be shaped using both nonlinearity selection and input PSD selection. The resulting NLSS scheme is practical for tactical communications settings where the transmitter is band-limited by the transmit DAC.

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