ON THE EVM CALCULATION OF CLIPPED OPTICAL OFDM SIGNALS

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ABSTRACT

Error vector magnitude (EVM) is a commonly used metric to characterize distortions. We numerically calculate the EVM of clipped optical orthogonal frequency division multiplexing (OFDM) signals and compare with lower bounds.

Keywords— DC biased optical OFDM (DCO-OFDM), Asymmetrically clipped optical OFDM (ACO-OFDM), Error vector magnitude (EVM), Clipping

1. INTRODUCTION

Recently, orthogonal frequency division multiplexing (OFDM) has been considered for optical wireless communication (OWC) systems thanks to its ability to boost data rates. DC biased optical OFDM (DCO-OFDM) [1] and asymmetrically clipped optical OFDM (ACO-OFDM) [2] are two popular optical OFDM techniques. In OWC systems, baseband signals outside the dynamic range of a light emitting diode (LED) are clipped, resulting in nonlinear distortions. Error vector magnitude (EVM) is a frequently used performance metric in modern communication standards. In references [3, 4], the EVM is measured by simulations for varying power back-off and biasing levels. In this paper, we describe an approach to numerically calculate the EVM. We formulate the EVM minimization problem as a convex linear optimization problem and obtain an EVM lower bound.

2. SYSTEM MODEL

In an OFDM system, a discrete time-domain signal $\mathbf{x} = [x[0], x[1], \ldots, x[N-1]]$ is generated by applying inverse FFT (IFFT) operation to a frequency-domain signal $\mathbf{X} = [X_0, X_1, \ldots, X_{N-1}]$, where N is the size of IFFT assumed to be an even number in this paper. In OWC systems, baseband signals must be real valued and unipolar (positive valued). To ensure that $\mathbf{x}$ is real valued, $\mathbf{X}$ must be Hermitian symmetric. A positive valued signal $y[n]$ can be generated by adding DC-bias B to $x[n]$. In this paper, we assume that a pre-distorter [4] has perfectly linearized the LED between the interval $[V_L, V_H]$. The input signal $y[n]$ outside the range will be clipped. According to the Central Limit Theorem, $x[n]$ is approximately Gaussian distributed with zero mean and variance σ^2. As a result, $x[n]$ tends to occupy a large dynamic range. Clipping $y[n]$ is equivalent to clipping $x[n]$ as

$$\bar{x}[n] = \begin{cases} c_u, & x[n] > c_u \\ x[n], & c_l \le x[n] \le c_u, \\ c_l, & x[n] < c_l \end{cases} \tag{1}$$

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where $c_u = V_H - B$ denotes the upper clipping level for $x[n]$, $c_l = V_L - B$ denotes the lower clipping level for $x[n]$. To facilitate the analysis, we define the clipping ratio γ and the biasing ratio ς as

$$\gamma \triangleq \frac{V_H - V_L}{2\sigma} = \frac{c_u - c_l}{2\sigma}, \qquad \varsigma \triangleq \frac{B - V_L}{V_H - V_L} = \frac{-c_l}{c_u - c_l}. \tag{2}$$

Thus, the upper clipping level can be written as $c_u = 2\sigma\gamma(1-\varsigma)$ and the lower clipping level can be written as $c_l = -2\sigma\gamma\varsigma$. In the following, we shall use superscripts $^{(D)}$ and $^{(A)}$ to indicate DCO-OFDM and ACO-OFDM, respectively. In this paper, we will focus on the analysis of two widely studied optical OFDM techniques, namely, DCO-OFDM and ACO-OFDM. Let $\mathcal{K}_d$ denote the set of information-bearing subcarriers with cardinality $|\mathcal{K}_d| = d$. In DCO-OFDM, $\mathcal{K}_d^{(D)} = \{1, 2, \ldots, N/2-1, N/2+1, \ldots, N-2, N-1\}$ and $|\mathcal{K}_d^{(D)}| = N-2$. In ACO-OFDM, only odd subcarriers carry data. Thus, $\mathcal{K}_d^{(A)} = \{1, 3, \ldots, N-1\}$ and $|\mathcal{K}_d^{(A)}| = N/2$. It was shown in reference [2] that $x^{(A)}[n]$ is half negative symmetric. Clipping the negative parts of $x^{(A)}[n]$ will not distort the odd subcarriers of frequency-domain signals but only reduce their magnitude by a factor of 2. Hence, the lower clipping level can be chosen as $c_l = 0$, and correspondingly, $\varsigma = 0, c_u = 2\gamma\sigma$.

3. EVM CALCULATION

EVM is a figure of merit for distortions. Let $\mathbf{X}^\dagger = [X_0^\dagger, X_1^\dagger, \ldots, X_{N-1}^\dagger]$ denote the N-length FFT of the modified time-domain signal $\mathbf{x}^\dagger$. The corresponding EVM is

$$\xi \triangleq \sqrt{\frac{\mathcal{E}\left[\sum_{k\in\mathcal{K}_d} |X_k - X_k^\dagger|^2\right]}{\mathcal{E}\left[\sum_{k\in\mathcal{K}_d} |X_k|^2\right]}}, \tag{3}$$

where $\mathcal{E}[\cdot]$ denotes statistical expectation. We denote the clipping error power by $\bar{P}_{\gamma,\varsigma} \triangleq \sum_{k\in\mathcal{K}_d} \mathcal{E}[|X_k - \bar{X}_k|^2]$.

In DCO-OFDM, the sum distortion power on the 0th and $N/2$th subcarriers is small relative to the total distortion power of N subcarriers, thus we approximate $\bar{P}_{\gamma,\varsigma}^{(D)}$ as

$$\begin{aligned} \bar{P}_{\gamma,\varsigma}^{(D)} &\approx \sum_{n=0}^{N-1} \mathcal{E}[|x^{(D)}[n] - \bar{x}^{(D)}[n]|^2] \qquad (4) \\ &= N\sigma^2 \Big(1 + 4\gamma^2(1-\varsigma)^2 - 2\gamma\varsigma\phi(2\gamma\varsigma) \\ &\quad -2\gamma(1-\varsigma)\phi(2\gamma(1-\varsigma)) - \Phi(2\gamma(1-\varsigma)) + \Phi(-2\gamma\varsigma) \\ &\quad -4\gamma^2(1-\varsigma)^2\Phi(2\gamma(1-\varsigma)) + 4\gamma^2\varsigma^2\Phi(-2\gamma\varsigma) \Big), \end{aligned}$$

where $\phi(x)$ and $\Phi(x)$ denote the Gaussian probability density function and the Gaussian cumulative distribution function, respectively. Then we obtain the EVM for clipped DCO-OFDM as $\xi^{(D)}_{\gamma,\varsigma} = \sqrt{\bar{P}^{(D)}_{\gamma,\varsigma}/N\sigma^2}$. After examining the first order partial derivative and the second order partial derivative of $\bar{P}^{(D)}_{\gamma,\varsigma}$ with respect to ς, we observe that $\partial\bar{P}^{(D)}_{\gamma,\varsigma}/\partial\varsigma = 0$ when $\varsigma = 0.5$, and $\partial^2\bar{P}^{(D)}_{\gamma,\varsigma}/\partial\varsigma^2 > 0$ for all ς. Therefore, $\varsigma^\star = 0.5$ is the optimum biasing ratio which minimizes $\bar{P}^{(D)}_{\gamma,\varsigma}$. We can show that EVM for clipped DCO-OFDM at the optimum biasing ratio is [5]

$$\xi^{(D)}_{\gamma,\varsigma^\star} = \sqrt{2(1+\gamma^2)\Phi(-\gamma) - 2\gamma\phi(\gamma)} \tag{5}$$

In ACO-OFDM, clipping at 0 and c_u is equivalent to clipping symmetrically at $-c_u$ and c_u in terms of EVM [5]. Clipping symmetrically can keep the half negative symmetric property. Moreover, no distortions will fall on the even subcarriers. According to the Parsevals Theorem, we can compute the error power for clipping at $-c_u$ and c_u as

$$\bar{P}^{(A)}_{\gamma,0} = \sum_{n=0}^{N-1} \mathcal{E}[|x^{(A)}[n] - \bar{x}^{(A)}[n]|^2] \tag{6}$$

$$= 2N\sigma^2\Big(-2\gamma\phi\left(2\gamma\right) + \Phi\left(-2\gamma\right) + 4\gamma^2\Phi\left(-2\gamma\right)\Big)$$

Then we obtain the EVM for the ACO-OFDM scheme as [5]

$$\xi^{(A)}_{\gamma,0} = \sqrt{-4\gamma\phi\left(2\gamma\right) + 2\Phi\left(-2\gamma\right) + 8\gamma^2\Phi\left(-2\gamma\right)} \tag{7}$$

4. EVM LOWER BOUND

In this section, we formulate the EVM minimization problem to find the EVM lower bound given the dynamic range constraint $2\gamma\sigma$. Considering the Hermitian symmetry condition, there are $N/2+1$ subcarriers per symbol which can be adjusted. Let $\mathbf{C} = [C_0, C_1, \ldots, C_{N-1}]$ denote the Hermitian symmetric distortion introduced in the frequency-domain. The modified time-domain signal becomes $\hat{x}[n] = x^{(r)}[n] + \text{IFFT}\{C_k\}[n]$, where $x^{(r)}[n]$ represents the reference signal. For DCO-OFDM, $x^{(r)}[n] = x^D[n]$. For ACO-OFDM, $x^{(r)}[n]$ is obtained by negatively clipping $x^A[n]$. The symbol-wise distortion minimization can be formulated as a linear optimization problem; i.e.,

$$\begin{aligned} &\text{minimize} \quad \sum_{k\in\mathcal{K}_d} |C_k|^2 \\ &\text{subject to} \quad \max\big(\hat{x}[n]\big) - \min\big(\hat{x}[n]\big) \le 2\gamma\sigma \\ &\qquad\qquad \hat{x}[n] = x^{(r)}[n] + \text{IFFT}\{C_k\}[n], \quad C_0, C_{N/2} \in \mathcal{R} \end{aligned} \tag{8}$$

Since the distortion of each symbol is minimized by the above convex optimization problem, the EVM of $\hat{x}[n]$ (which is proportional to $\sqrt{\mathcal{E}\left[\sum_{k\in\mathcal{K}_d}|C_k|^2\right]}$) serves as the lower bound for the given dynamic range.

5. NUMERICAL RESULTS

The EVM analyses for clipped DCO-OFDM and ACO-OFDM are validated through computer simulations. In the simulations, we chose $N = 512$, and QPSK modulation. 1000 OFDM symbols were generated based on which we calculated the EVM. We compared the EVM for clipped DCO-OFDM with biasing ratio 0.5, EVM for clipped ACO-OFDM with biasing ratio 0, and with their respective lower bounds. To obtain the lower bounds we used `CVX`, a package for specifying and solving convex programs [6]. The resulting EVM curves are plotted in Fig. 1. The approximate 6 dB gap between DCO-OFDM and ACO-OFDM is due to the half negative symmetry property of ACO-OFDM. The EVM for ACO-OFDM achieves its lower bound. For DCO-OFDM, the gap with lower bound increases with the clipping ratio.

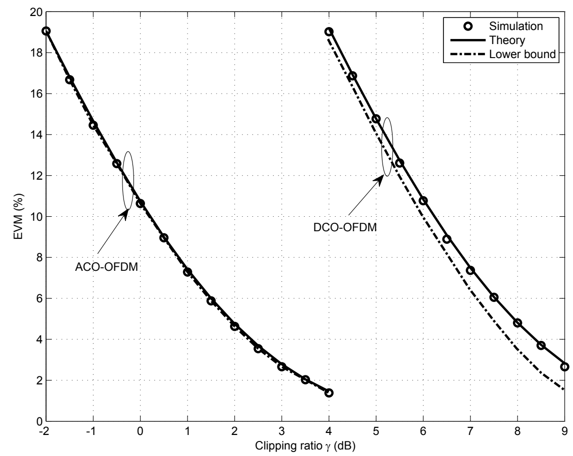

Fig. 1. EVM as a function of clipping ratio for DCO-OFDM, ACO-OFDM and lower bounds

6. CONCLUSIONS

In this paper, we numerically calculated the EVM and compared with the corresponding lower bound. Clipped ACO-OFDM can achieve the lower bound while there is some potential for DCO-OFDM to achieve a lower EVM. Many technical details and derivations are omitted here due to space limitations. The full analyses can be found in [5].

7. REFERENCES

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