ON THE PTS METHOD AND BER-MINIMIZING POWER ALLOCATION OF THE MULTI-CHANNEL OFDM SYSTEM

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ABSTRACT

In multicarrier multiple-access systems, it is possible to serve multiple users at high data rates from a single base station. In this paper, we propose the multi-channel partial transmit sequences (MCPTS) method for multiple-access OFDM peak-to-average power ratio (PAR) reduction. By applying the PTS phase rotations to each access channel separately and assuming that the rotations can be detected by each user as a channel fading, MCPTS avoids two major disadvantages of standard PTS: i) the phase rotations can be detected without side information, and ii) the set of possible phase rotation values does not need to be finite. In addition to MCPTS, we show an iterative optimization method to minimize the average BER by allocating the power in each channel subject to a peak-power constraint. The results show a significant BER improvement.

1. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) has been adopted by various modern communication standards because of its high spectral efficiency and robustness against frequency-selective fading channels [1–3]. However, a potentially large dynamic range, which is usually measured by peak-to-average power ratio (PAR), exists in the time-domain OFDM waveform and rendering the power amplifier (PA) inefficient. OFDM-based frequency-division multiple access systems were also proposed in these communication standards. The frequency spectrum is channelized into adjacent fragments on each of which a (or multiple) separate OFDM user(s) can communicate. For instance, 5-20MHz and 1.25-20MHz channelization schemes were used in IEEE 802.11a and 802.16 standards, respectively [1, 2]. In the downlink scenario, the base station transmits multi-channel signals through a single high-power PA. The waveforms overlap in the time domain thus the multi-channel system has an even more challenging PAR problem.

Many PAR reduction methods have been proposed for OFDM systems with regard to diversified considerations, e.g. clipping methods [4], partial transmit sequence (PTS) [5] and waveform optimization methods [6, 7]. However, the PAR reduction problem for multi-channel OFDM systems has not been well addressed. In this paper, a PTS-based PAR reduction algorithm, named multi-channel PTS (MCPTS), is proposed for the multi-channel OFDM system. It has several advantages over the traditional PTS method. Additionally, a joint MCPTS and power allocation method is presented to minimize the average bit error rate (BER) for the peak-power-limited PA.

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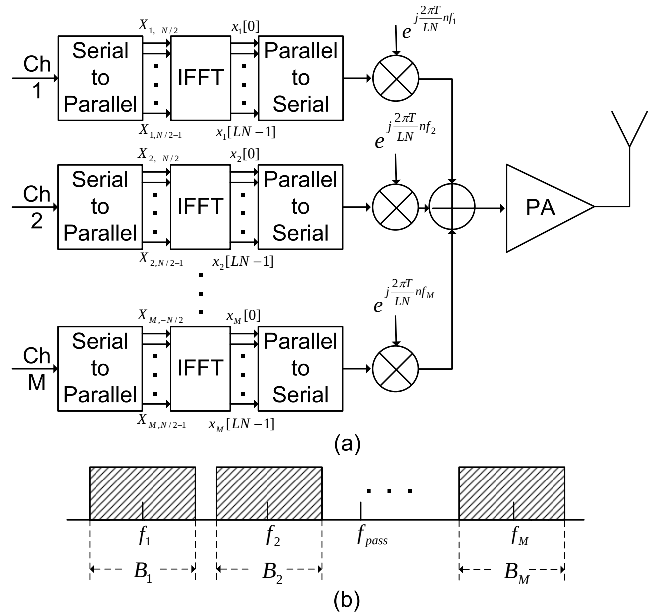

Fig. 1. The (a) structure and (b) frequency spectrum diagram of the multi-channel OFDM base station.

2. SYSTEM MODEL

The multi-channel OFDM base station considered in this paper is shown in Fig. 1. Independent OFDM signals are transmitted on M different (but usually adjacent) frequency bands, whose center frequencies are f_m $(m = 1, \cdots, M)$ and satisfy $f_{m+1} - f_m \geq \frac{1}{2}(B_{m+1} + B_m)$ $(m = 1, \cdots, M-1)$ where B_m is the frequency bandwidth of the mth channel.

For each of the OFDM signals, data are transmitted on N_m orthogonal subcarriers which make up the OFDM symbol, denoted as $\mathbf{X}_m = [X_{m,-N_m/2}, \cdots, X_{m,N_m/2-1}]^T$. For notational simplicity, $B_m = B$ and $N_m = N$ are assumed in this paper. An L-times oversampling inverse FFT (LN-point IFFT) operation is performed to generate the mth channel's baseband time-domain samples

$$x_m[n] = \frac{1}{\sqrt{LN}} \sum_{k=0}^{LN-1} X_{m,k}^{\mathrm{ZP}} e^{\frac{j2\pi kn}{LN}},\ n = 0, \cdots, LN-1, \quad (1)$$

where $\mathbf{X}_m^{\mathrm{ZP}} = [X_{m,0}, \cdots, X_{m,\frac{N}{2}-1}, 0, \cdots, 0, X_{m,-\frac{N}{2}}, \cdots, X_{m,-1}]^T$ is generated by zero-padding $\mathbf{X}_m$ with $(L-1)N$ zeros. Then, the baseband signals will be up-converted to passband and combined as the input to the PA, e.g.

$$x_p[n] = \sum_{m=1}^{M} x_m[n] e^{\frac{j2\pi nTf_m}{LN}}, \quad (2)$$

where T is the symbol duration. The center frequency of the above passband signal can be found as $f_{\mathrm{pass}} = \frac{1}{2}(f_1 + f_M) + \frac{1}{4}(B_M - B_1)$ and the equivalent baseband signal is $\mathbf{x} = [x[0], \cdots, x[LN-1]]^T$ with $x[n] = x_p[n] e^{-\frac{j2\pi nTf_{\mathrm{pass}}}{LN}}$, which can be calculated by using IFFT. Since the baseband PAR can be used to infer the passband signal's dynamic range [8], in this paper we consider the symbol-wise PAR defined as

$$\mathrm{PAR}(\mathbf{x}) = \frac{\|\mathbf{x}\|_\infty^2}{\frac{1}{LN}\|\mathbf{x}\|_2^2}. \quad (3)$$

PAs are generally peak-power limited. For a PA with predistortion, the soft limiter characteristic can be adopted [9]. Assuming an output peak-power limit of P_{peak}, nonlinear distortion occurs when $|x[n]|^2 > P_{\mathrm{peak}}$ (i.e. clipping). Symbol-wise linear scaling (SWLS) can be used so that no clipping occurs, in which the OFDM symbol is modified such that $\mathbf{x}_{\mathrm{SWLS}} = P_{\mathrm{peak}}^{1/2}\mathbf{x}/\|\mathbf{x}\|_\infty$. Thus, reduction in the PAR leads to increase in the average transmit power [4].

3. MULTI-CHANNEL PARTIAL TRANSMIT SEQUENCES (MCPTS)

Partial transmit sequence (PTS) approach has been proposed to reduce the PAR of OFDM signals [5]. The main idea is to produce U representations of the same OFDM symbol and transmit the one with the smallest PAR. At first, the OFDM subcarriers are partitioned into S disjoint sets $\mathbf{X} = \bigcup_{s=1,\cdots,S} \hat{\mathbf{S}}_s$, $\hat{\mathbf{S}}_m \bigcap \hat{\mathbf{S}}_n = \emptyset$ $(m \neq n,\ 1 \leq m, n \leq S)$. Then, these subcarrier sets are combined after independent phase rotations as

$$\mathbf{X}^{(u)} = \sum_{s=1}^{S} e^{j\theta_s^{(u)}} \mathbf{S}_s,\ u = 1, \cdots, U, \quad (4)$$

where $\Theta^{(u)} = [e^{j\theta_1^{(u)}}, \cdots, e^{j\theta_S^{(u)}}]^T$ $(u = 1, \cdots, U)$ are called phase sequences. The superscript is used for the index of representations and the subscript is the index of partitions. The time-domain waveform $\mathbf{x}^{(u)}$ and the corresponding PAR value can be calculated for each $\mathbf{X}^{(u)}$.

In this paper, the PTS method is extended to the multi-channel OFDM system, referred to as the multi-channel PTS (MCPTS) method. In this case, each channel can be regarded as a disjoint subcarrier set. The partition is naturally determined by channelization. The number of disjoint sets is $S = M$ and $\mathbf{S}_s = \mathbf{X}_m$ $(s = m = 1, \cdots, M)$ in Eq. (4). The M-by-1 phase sequences will then be used to generate U different combinations of phase-rotated signals, and the PTS approach becomes

$$\begin{aligned} \underset{u \in \{1,\cdots,U\}}{\text{minimize}} \quad & \mathrm{PAR}(\mathbf{x}^{(u)}) && (5) \\ \text{subject to} \quad & \mathbf{X}_m^{(u)} = e^{j\theta_m^{(u)}} \mathbf{X}_m,\ m = 1, \cdots, M && (6) \\ & \mathbf{x}_m^{(u)} = \mathrm{IFFT}\{\mathbf{X}_m^{(u)}\} \text{ as in Eq. (1)} && (7) \\ & x^{(u)}[n] = \sum_{m=1}^{M} x_m^{(u)}[n] e^{\frac{j2\pi nT}{LN}(f_m - f_{\mathrm{pass}})}. && (8) \end{aligned}$$

MCPTS has the following advantages over PTS:

1. Neither side information nor receiver-side modification is needed. The phase rotation is equivalently part of the channel response and can be recovered by the channel estimation capability of each OFDM channel;
2. Since no side information should be transmitted, phase rotation sequences can take on any value. Additionally, optimization techniques, such as the particle swarm optimization [10], can be used for finding the optimal phase sequence.

4. JOINT MCPTS AND BER-MINIMIZING POWER ALLOCATION

For peak-power-limited PA, reducing PAR can increase the effective average output power. However, for multiple fading channels, power allocation should also be designed so that the potential average power increase can be effectively utilized. Accordingly, we propose that each subcarrier in each channel be scaled so that the average power of the kth subcarrier in the mth channel is $P_{m,k}$, i.e. $\bar{X}_{m,k} = X_{m,k}(P_{m,k}/E[|X_{m,k}|^2])^{\frac{1}{2}}$. We assume the transmitter knows the CSI, including the frequency response $h_{m,k}$ and the power of the Gaussian channel noise $\sigma_{m,k}^2$. The signal-to-noise ratio (SNR) of a given subcarrier is

$$\mathrm{SNR}_{m,k} = P_{m,k}\frac{|h_{m,k}|^2}{\sigma_{m,k}^2} = \frac{P_{m,k}}{\hat{\sigma}_{m,k}^2}, \quad (9)$$

where $\hat{\sigma}_{m,k}^2 = \sigma_{m,k}^2/|h_{m,k}|^2$ is the equivalent channel noise power. The values for $P_{m,k}$ can be determined to minimize BER.

In this paper, we assume quadrature phase-shift keying (QPSK) is used on every subcarrier, and the average power of all subcarriers in one channel is the same, e.g. $P_{m,k} = P_m$. The optimal BER-minimizing power allocation problem can be formulated as:

$$\underset{P_m, m\in\{1,\cdots,M\}}{\text{minimize}} \quad \frac{1}{MN}\sum_{m=1}^{M}\sum_{k=-N/2}^{N/2-1} Q\left(\sqrt{\frac{P_m}{\hat{\sigma}^2_{m,k}}}\right) \tag{10}$$

$$\text{subject to} \quad \sum_{m=1}^{M} NP_m = \|\bar{\mathbf{x}}\|_2^2 \le LN\frac{P_{\text{peak}}}{\text{PAR}(\bar{\mathbf{x}})} \tag{11}$$

$$\bar{x}[n] = \sum_{m=1}^{M} \bar{x}_m[n] e^{\frac{j2\pi nT}{LN}(f_m - f_{\text{pass}})} \tag{12}$$

$$\bar{\mathbf{x}}_m = \text{IFFT}\{\bar{\mathbf{X}}_m\},\ \forall m \in \{1,\cdots,M\},$$

where $Q(c) = \text{erfc}(c/\sqrt{2})/2$ is the bit error probability of the QPSK signal with Gray mapping over an AWGN channel [11, P. 271]. Because the convexity of this problem is not clear, it is hard to solve.

Next, we propose an iterative algorithm to find a suboptimal solution to this problem. First, for an initial objective average power $P^0_{\text{av}} \triangleq \frac{L\cdot P_{\text{peak}}}{\text{PAR}^0}$, the BER-minimizing power allocation solves the Lagrangian problem

$$\frac{\partial}{\partial P_m}\left[-\sum_{m=1}^{M}\sum_{k=-N/2}^{N/2-1} Q\left(\sqrt{\frac{P_m}{\hat{\sigma}^2_{m,k}}}\right) - \lambda\sum_{m=1}^{M} P_m\right] = 0 \tag{13}$$

$$\sum_{m=1}^{M} P_m = P^0_{\text{av}}. \tag{14}$$

After some manipulations, Eq. (13) becomes

$$\sum_{k=-N/2}^{N/2-1} \frac{1}{\sqrt{\hat{\sigma}^2_{m,k} P_m}} e^{-\frac{P_m}{2\hat{\sigma}^2_{m,k}}} = 2\sqrt{2\pi}\lambda, \tag{15}$$

for $m = 1,\cdots,M$. P_m is a function of the Lagrangian parameter λ. By Eq. (14), λ can be determined and results in P_m's. Unfortunately, although P_m is one-to-one with λ, the functional relationship cannot be described in closed form. But by combining Eq. (14) and Eq. (15), P_m's can be determined numerically.

Because changing the power in each channel with a factor of $\sqrt{P_m}$ will change the PAR, to minimize the BER, PAR minimization in MCPTS needs to include this scaling as part of the optimization. By replacing Eq. (6) with

$$\bar{\mathbf{X}}^{(u)}_m = \sqrt{P_m} e^{j\theta^{(u)}_m} \bar{\mathbf{X}}_m,\ m = 1,\cdots,M, \tag{16}$$

the minimum PAR, i.e. $\text{PAR}_{\min} = \min_{u\in\{1,\cdots,U\}} \text{PAR}(\bar{\mathbf{x}}^{(u)})$, can be found as shown in the MCPTS method in (5)-(8).

Denote $0 < \beta < 1$ as the convergence parameter. If $\beta\text{PAR}^0 \le \text{PAR}_{\min} \le \text{PAR}^0$, the objective average power can be achieved by the MCPTS approach and we have a near-optimal solution in terms of both the power allocation and the phase sequence. Otherwise, either the objective average power cannot be reached or the MCPTS yields a much greater average power. In both cases, a new iteration begins with $P^0_{\text{av}} = \frac{L\cdot P_{\text{peak}}}{\text{PAR}_{\min}}$.

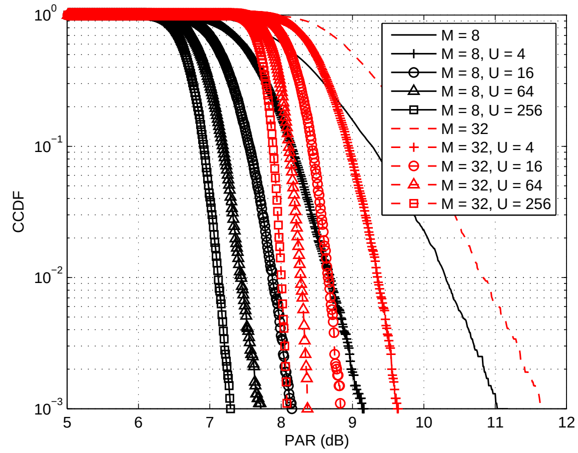

Fig. 2. CCDF curves of the PAR of the original and MCPTS multi-channel OFDM signals for different numbers of channels ($M = 8, 32$) and numbers of phase sequences ($U = 4, 16, 64, 256$).

5. RESULTS

We present some examples to illustrate the performance improvement. In the simulations, adjacent channels were assumed without using guard bands, i.e. $f_{m+1} - f_m = B$. The number of subcarriers was $N = 64$ for every channel. Phase sequences with $\theta^{(u)}_m \in \{0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}\}$ were used in MCPTS with each phase having an equal probability of occurring.

5.1 MCPTS

The complementary cumulative distribution function (CCDF) curves of the PAR are plotted in Fig. 2. The constellation was QPSK although our experiences indicated that the PAR reduction performance is insensitive to the constellation choice. The results show that the PAR reduction performance of MCPTS depends on the system parameters and the number of phase sequences being used. A tradeoff between the extra MCPTS complexity and the PAR reduction performance should be considered given implementation constraints.

5.2 Joint MCPTS and power allocation

In this section, both the number of channels (M) and the oversampling rate (L) were set to 4. $\beta = 0.9$ was used as the convergence parameter. In addition, the equivalent channel noise power was assumed the same for all subcarriers in one channel, i.e $\hat{\sigma}^2_{m,k} = \hat{\sigma}^2_m$, but were independent and identically-distributed Rayleigh random variables.

Fig. 3 shows the average BER curves as the function of the peak signal-to-noise power ratio (PSNR), which is defined as

$$\text{PSNR} \triangleq \frac{P_{\text{peak}}}{\frac{1}{L}\sum_{m=1}^{M} E\left[\hat{\sigma}^2_m\right]}, \tag{17}$$

where the denominator is the equivalent average noise power of each time-domain sample. Several methods were used for comparison. In the first method, the average power is equally allocated among channels. It is shown to have the

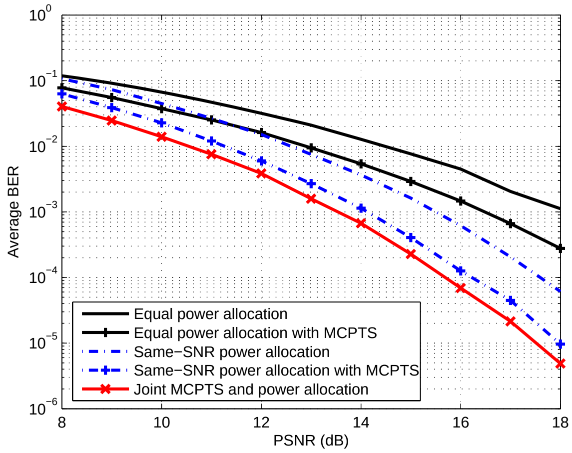

Fig. 3. Average BER versus PSNR curves for $M = 4$ multi-channel OFDM systems. $U = 128$ phase sequences are used in MCPTS.

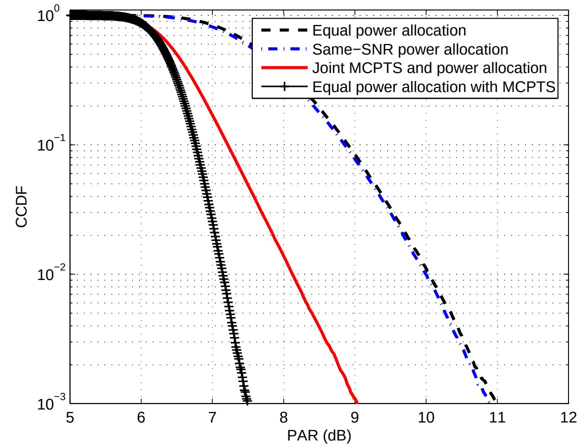

Fig. 4. CCDF curves of the PAR for $M = 4$ multi-channel OFDM systems. $U = 128$ phase sequences are used in MCPTS.

worst average BER on this multiple Rayleigh fading channel. The second method allocates the power so that every channel has the same SNR and average BER. Both of these methods can be easily combined with the proposed MCPTS so that the BER performance is further improved. The last one is the joint MCPTS and BER-minimizing power allocation approach. This method achieves the best average BER performance because of its BER-minimizing property and using MCPTS which reduces PAR to increase the effective average power for the peak-power-limited PA.

However, the resulting PAR of this iterative joint MCPTS and power allocation method might be greater than using MCPTS for a constant power allocation scheme, as shown in Fig. 4. Intuitively, this joint method optimizes both the magnitude and the phase of each OFDM channel, as shown in Eq. (16). Although the potential average power increase is less than MCPTS, the power is more efficiently utilized among channels.

6. CONCLUSION

In this paper, the multi-channel partial transmit sequence (MCPTS) PAR reduction method is proposed for multi-channel OFDM systems. We illustrated how MCPTS can be used in multi-channel systems to reduce the PAR and the BER without side information. In addition, using the assumption of perfect CSI at transmitters, we demonstrated how the BER can be further decreased with a joint MCPTS and BER-minimizing power allocation approach based on multi-channel power allocation optimization. For future research, we would like to show how this scheme can be extended from optimal power allocation to optimal bit allocation for increasing throughput. We also hope to describe a general MCPTS framework for other types of multiple access systems.

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