IMPULSES INJECTION FOR PAPR REDUCTION IN VISIBLE LIGHT OFDM COMMUNICATIONS

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ABSTRACT

Orthogonal frequency division multiplexing (OFDM) in visible light communications (VLC) inherits the disadvantage of high peak-to-average power ratio (PAPR) from OFDM in radio frequency (RF) communications. VLC-OFDM differs from RF-OFDM in that there are two PAPRs to be reduced, namely upper PAPR and lower PAPR. In this paper, we propose a distortion-based impulse injection method to achieve PAPR reduction in bit-loaded VLC-OFDM systems. To minimize the in-band constellation-wise error vector magnitude (EVM), a quadratically constrained linear optimization problem is formulated. This scheme is advantageous over simple clipping and prior works in that it can wisely allocate less distortions to higher order constellations with a few more variables.

Index Terms— Visible light communications (VLC), orthogonal frequency division multiplexing (OFDM), peak-to-average power ratio (PAPR), error vector magnitude (EVM)

1. INTRODUCTION

Visible light communications (VLC) leverage white light emitting diodes (LEDs) to provide illumination and "green communication" simultaneously. Motivated by more and more crowded radio frequency (RF) spectrum and fast growing solid-state lighting (SSL) technology, VLC has attracted extensive attentions recently [1, 2, 3]. Simple and low-cost intensity modulation and direct detection (IM/DD) techniques are employed in VLC, thus only signal intensity information, not phase information, is modulated. White LEDs exhibit low pass filter characteristic [4]. To best make use of available bandwidth of LEDs and boost the achievable data rates, orthogonal frequency division multiplexing (OFDM) has been considered for VLC [5, 6, 7]. OFDM can support bit-loading technology, which allows allocating different numbers of bits to different subcarriers based on signal-to-noise power ratio (SNR) [8, 9]. IM/DD requires the OFDM baseband signal to be real-valued and unipolar (positive-valued). Therefore, Hermitian symmetry must be satisfied in the frequency-domain and bipolar-to-unipolar module is required.

VLC-OFDM inherits the disadvantage of high peak-to-average power ratio (PAPR) from RF-OFDM [10]. High PAPR makes VLC-OFDM very sensitive to nonlinearity of LEDs [11]. Moreover, high PAPR requires large biasing to convert the bipolar OFDM signal into unipolar version, which makes the system optical power inefficient [12]. Therefore, PAPR reduction is necessary in VLC-OFDM system. Among PAPR reduction schemes proposed for RF-OFDM communications [10], the distortion based methods [13, 14, 15, 16, 17, 18] are in particular favored for practical scenario because the modification of receiver is avoided. However, there are some differences between RF-OFDM and VLC-OFDM to prevent directly applying conventional PAPR reduction methods to VLC-OFDM systems. First, RF-OFDM baseband signal is complex-valued and VLC-OFDM baseband signal is real-valued. Real-valued OFDM signal has two PAPRs to be reduced, namely upper PAPR and lower PAPR [19, 20]. Second, in RF communicators, both in-band distortions and out-of-band power leakage should be taken into account in PAPR reduction schemes. However, in VLC-OFDM systems, because LED acts as a low-pass filter, the out-of-band subcarriers cannot be leveraged to transmit information to other users. Thus, we do not need to concern about the out-of-band interferences. This actually gives us more headroom to develop PAPR reduction schemes for VLC-OFDM systems.

Motivated by [18], in this paper, we propose a distortion-based impulse injection method to reduce the UPAPR and LPAPR in bit-loaded VLC-OFDM systems. Given deterministic dynamic range constraints, instead of minimizing distortions equally for the in-band subcarriers as in [16, 18], we aim to minimize the in-band constellation-wise error vector magnitude (EVM). The proposed scheme can wisely allocate less distortions to subcarriers modulated by higher order constellations.

2. PAPR OF VLC-OFDM

In VLC systems, intensity modulation (IM) is employed at the transmitter. The forward signal drives the LED which in turn converts the magnitude of the input electric signal into optical intensity. The human eye cannot perceive fast-changing variations of the light intensity, and only responds to the average light intensity. Direct detection (DD) is employed at the receiver. A photodiode (PD) transforms the received optical power into the amplitude of an electrical signal.

In an OFDM system, a discrete time-domain symbol sequence $\{x_n\}_{n=0}^{N-1}$ is generated by applying the inverse DFT (IDFT) operation to a frequency-domain sequence $\{X_k\}_{k=-N/2}^{N/2-1}$ as

$$\begin{aligned} x_n &= \mathrm{IDFT}(X_k) & (1) \\ &= \frac{1}{\sqrt{N}} \sum_{k=-N/2}^{N/2-1} X_k \exp\left(\frac{j2\pi kn}{N}\right), 0 \le n \le N-1, \end{aligned}$$

where $j = \sqrt{-1}$ and N is the size of IDFT. The resulting time-domain signals $\{x_n\}_{n=0}^{N-1}$ are complex-valued, including in-phase and quadrature components. However, in a VLC system using LED, IM/DD schemes require the input signal of LEDs to be real-valued.

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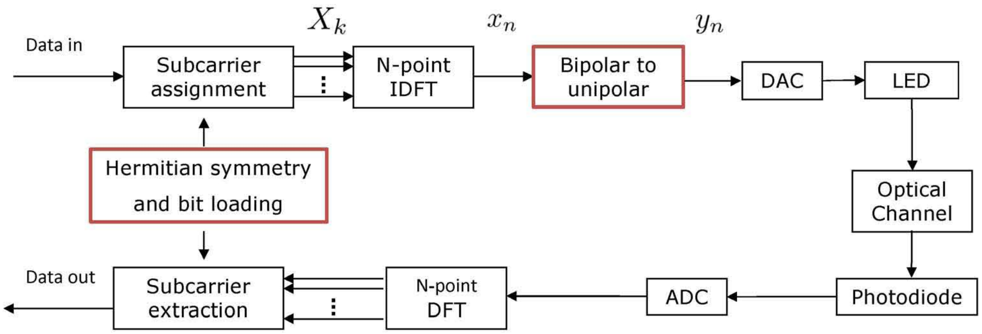

Fig. 1. OFDM system model in visible light communications

Therefore, complex-valued time-domain OFDM signals in (1) cannot be used in VLC directly.

Fig. 1 shows the OFDM system model in visible light communications. In general, VLC-OFDM differs from conventional OFDM systems in two ways: a Hermitian symmetry block ensuring the time-domain signal is real valued, and a bipolar-to-unipolar block converting the bipolar signal into positive signal. According to the property of the inverse Fourier transform, a real-valued time-domain signal x_n corresponds to a frequency-domain signal X_k that is Hermitian symmetric; i.e., $X_k = X^*_{-k}, 1 \leq k \leq N/2-1$, where $*$ denotes complex conjugate. The 0th and $-N/2$th subcarrier are null; i.e., $X_0 = 0$, $X_{-N/2} = 0$. Since the DC component is zero ($X_0 = 0$), x_n has zero mean and therefore is bipolar. Normally, bit loading are employed in VLC-OFDM systems to work with the frequency-selectivity of LEDs. For VLC, the the LED has to emit white light, which includes the whole visible light spectrum from 375 to 780 nm (400 to 800 THz), to provide lighting. Phosphorescent LED uses the blue LED chip coated with a yellow phosphor, which is the most popular white LED in the market due to its low cost. However, the slow response of phosphor limits the 3-dB modulation bandwidth of the phosphorescent white LEDS to only few MHz. A blue filtering can be operated at the receiver to increase the modulation bandwidth to 20 MHz. Thus, white LED acts as a low-pass filter [4]. OFDM enable the bit loading for each subcarrier to make the best use of the available modulation bandwidth. Generally, more bits are allocated to low-frequency subcarriers and fewer bits are allocated to high-frequency subcarriers. Let us define the in-band indices to be set $\mathcal{I} : [-N/2, N/2-1]$. Based on the number of bits loaded on each subcarrier, we divide the in-band subcarriers into a number of non-overlapped subsets; i.e., $\mathcal{I} = \mathcal{I}_1 \cup \mathcal{I}_2 \cup \cdots \cup \mathcal{I}_V$, where V is the number of subsets. Each subset of subcarriers are modulated by the same constellation type.

LEDs place dynamic range constraints $[I_L, I_H]$ on the input driving signals, where I_L denotes the minimum input current to turn on the LED, and I_H denotes the maximum input current to prevent overheating the LED. Since I_L is always a positive value, the original bipolar OFDM signal x_n has to be converted into unipolar (positive) signal. In this paper, we focus on the DC biased bipolar-to-unipolar conversion scheme, where the input signal of LEDs y_n is obtained via adding a biasing B to the original OFDM signal x_n; i.e.,

$$y_n = x_n + B. \tag{2}$$

Adding a biasing will only affect the DC component X_0, which is not an issue since we do not put data on the DC subcarrier.

It is well-known that the OFDM time-domain signal has high peak-to-average power ratio (PAPR) , which is defined as

$$\text{PAPR} \triangleq \frac{\max\limits_{0\leq n\leq N-1} |x[n]|^2}{\sigma_x^2}, \tag{3}$$

where σ_x^2 is the variance of x_n. PAPR is a random variable and varies sequence by sequence. For the real-valued bipolar signal $\{x_n\}_{n=0}^{N-1}$, the square of the maximum value $\left(\max\limits_{0\leq n\leq N-1} x_n\right)^2$ can be seen as the upper peak power, and the square of the minimum value $\left(\min\limits_{0\leq n\leq N-1} x_n\right)^2$ can be seen as the lower peak power. In VLC, we previously demonstrated that the performance of OFDM is directly related with upper PAPR (UPAPR) and lower PAPR (LPAPR) [12, 19, 21], where the UPAPR of x_n is defined as $\mathcal{U}(x_n) \triangleq \left(\max\limits_{0\leq n\leq N-1} x_n\right)^2 / \sigma_x^2$, and the LPAPR of x_n is defined as $\mathcal{L}(x_n) \triangleq \left(\min\limits_{0\leq n\leq N-1} x_n\right)^2 / \sigma_x^2$.

Oversampled OFDM signals are often generated to approximate the PAPR of continuous-time OFDM signals. It is shown that an oversampling ratio $L = 4$ is enough so that the PAPR before the digital to analog converter (DAC) can accurately describe the PAPR after the DAC [22]. Typically, we generate oversampled time-domain OFDM samples by zero-padding in the frequency domain [14]. Let us define the out-of-band indices to be the set $\mathcal{O} = [-NL/2, -N/2-1] \cup [N/2, LN/2-1]$, the zero padded version of X_k can be expressed as

$$X_k^{(L)} = \begin{cases} X_k, & k \in \mathcal{I} \\ 0, & k \in \mathcal{O} \end{cases}. \tag{4}$$

An LN length IDFT is then applied to convert the frequency-domain sequence $\{X_k^{(L)}\}_{k=-LN/2}^{LN/2-1}$ into an L times oversampled time-domain sequence $\{x_n^{(L)}\}_{n=0}^{LN-1}$.

Large UPAPR and LPAPR make VLC-OFDM signals very sensitive to nonlinearity of LEDs [11]. Moreover, according to (2), high LPAPR requires large biasing B to convert the bipolar OFDM signal into unipolar version [12], which makes the system optical power inefficient. Based on the above motivations, we aim to reduce both UPAPR and LPAPR in VLC-OFDM system.

3. IMPULSES INJECTION METHOD

The most straightforward way to reduce the UPAPR and LPAPR is clipping the upper peak and lower peak. Let c_u denotes the upper clipping level and c_l denote the lower clipping level, the clipped signal is given by

$$\bar{x}_n^{(L)} = \begin{cases} c_u, & n \in \mathcal{P}_u, \text{ i.e., } x_n^{(L)} > c_u \\ x_n^{(L)}, & n \notin \mathcal{P}_u \cup \mathcal{P}_l, \text{ i.e., } c_l \le x_n^{(L)} \le c_u \\ c_l, & n \in \mathcal{P}_l, \text{ i.e., } x_n^{(L)} < c_l \end{cases} \tag{5}$$

Normally, c_u is positive and c_l is negative. In (5), $\mathcal{P}_u$ is a set of "upper peak points" where $x_n^{(L)} > c_u$ and $\mathcal{P}_l$ is a set of "lower peak points" where $x_n^{(L)} < c_l$. If we write

$$\bar{x}_n^{(L)} = x_n^{(L)} + e_n^{(L)}, \tag{6}$$

we can also say that we have injected time-domain impulse sequence $e_n^{(L)} = \sum_{m\in\mathcal{P}_u\cup\mathcal{P}_l} \rho_m \delta_{n-m}$ into $x_n^{(L)}$ to produce a distorted signal $\bar{x}_n^{(L)}$, where $\rho_m = \bar{x}_m^{(L)} - x_m^{(L)}$ and δ_{n-m} is the Kronecker delta function. By taking LN-length DFT on both sides of (6), we can obtain in the frequency-domain, $\bar{X}_k^{(L)} = X_k^{(L)} + E_k^{(L)}$, where $E_k^{(L)}$ can be expressed in term of ρ_m as

$$E_k^{(L)} = \frac{1}{\sqrt{LN}} \sum_{m\in\mathcal{P}_u\cup\mathcal{P}_l} \rho_m e^{-j2\pi km/LN}, \tag{7}$$

In the simple clipping framework, both the impulse locations $m \in \mathcal{P}_u \cup \mathcal{P}_l$ and the impulse values ρ_m of the distortion signal $e_n^{(L)}$ are fixed per given signal $x_n^{(L)}$, clipping levels c_u and c_l. Instead of that, we design a distortion waveform

$$d_n^{(L)} = \sum_{m\in\mathcal{P}_u\cup\mathcal{P}_l\cup\mathcal{S}} \beta_m \delta_{n-m}, \tag{8}$$

and the corresponding modified signal

$$\tilde{x}_n^{(L)} = x_n^{(L)} + d_n^{(L)}. \tag{9}$$

In (8), the impulse values β_m are real-valued parameters to be optimized, and we allow additional degrees of freedom by including more impulse locations $m \in \mathcal{S}$. Denote $S = |\mathcal{S}|$, which is the size of the set $\mathcal{S}$. We refer to the smallest S values of the $|x_n^{(L)}|$ as "small points". The rationale for including the small points $\mathcal{S}$ is that they offer the most headroom or space to maneuver for optimization, so $\tilde{x}_n^{(L)}, n \in \mathcal{S}$ will not exceed $[c_l, c_u]$ easily. The number of small points S is designed parameter. In general, the larger the S, the more degrees of freedom and thus the better performance but the higher the computational load.

The objective of the impulse injection method is to find values for β_m to ensure that $c_l \le \tilde{x}_n^{(L)} \le c_u, \forall n$, while minimizing in-band distortions. Let $D_k^{(L)}$ denote the LN-length DFT of $d_n^{(L)}$, i.e.,

$$D_k^{(L)} = \frac{1}{\sqrt{LN}} \sum_{m\in\mathcal{P}_u\cup\mathcal{P}_l\cup\mathcal{S}} \beta_m e^{-j2\pi km/LN}. \tag{10}$$

It can be seen that distortions spread over both in-band and out-of-band subcarriers. Because LEDs act as a low pass filter [4], we assume that the out-of-band subcarriers are not leveraged by other users. In other words, the out-of-band power leakage is not an issue in VLC. Thus, we only concern about in-band distortions $D_k^{(L)}, k \in \mathcal{I}$. Error vector magnitude (EVM) is a widely used figure of merit in literatures [7, 23] and standards to quantify the in-band distortions, which is defined as

$$\text{EVM}_{\mathcal{I}} \triangleq \sqrt{\frac{\sum_{k\in\mathcal{I}} |D_k^{(L)}|^2}{\sum_{k\in\mathcal{I}} |X_k^{(L)}|^2}}. \tag{11}$$

To ensure reliable communication, the in-band EVM of transmitted signals should be kept as minimum as possible. In previous work [16, 18], all in-band subcarriers are treated equally in term of EVM minimization. However, in OFDM systems with bit loading, high order constellations are more sensitive to noises than lower order constellations. For example, in LTE [24], the EVM thresholds are 17.5% (4-QAM), 12.5% (16-QAM), and 8.0% (64-QAM), respectively. Thus, it is necessary for us to examine the constellation-wise EVM for a specific subcarriers subset $\mathcal{I}_v$, which is defined as

$$\text{EVM}_{\mathcal{I}_v} \triangleq \sqrt{\frac{\sum_{k\in\mathcal{I}_v} |D_k^{(L)}|^2}{\sum_{k\in\mathcal{I}_v} |X_k^{(L)}|^2}}. \tag{12}$$

In this paper, rather than simply minimizing in-band EVM, we aims to minimize the constellation-wise EVMs with different weights as

$$\begin{aligned} &\text{minimize} && g && (13) \\ &\text{subject to} && \text{EVM}_{\mathcal{I}_v} \le \omega_v g, \quad \forall v, && (14) \end{aligned}$$

where ω_v denotes the weight for the subset $\mathcal{I}_v$. Note that g is an intermediate variable and has no physical meaning. Generally, the higher order constellations the subset $\mathcal{I}_v$ employs, the smaller the weight ω_v applies. The inequality (14) is equivalent to

$$\sum_{k\in\mathcal{I}_v} |D_k^{(L)}|^2 \le \omega_v^2 g^2 \sum_{k\in\mathcal{I}_v} |X_k^{(L)}|^2, \forall v \tag{15}$$

As a summary, we formulate the constellation-wise EVMs minimization problem as follows:

$$\begin{aligned} \underset{\beta_m}{\text{minimize}} \quad & g^2 \\ \text{subject to} \quad & \text{Amplitude constraint} \\ & c_l \le x_m^{(L)} + \beta_m \le c_u, \quad m \in \mathcal{P}_u \cup \mathcal{P}_l \cup \mathcal{S} \\ & \text{Constellation-wise EVM constraint} \\ & \sum_{k\in\mathcal{I}_v} |D_k^{(L)}|^2 \le \omega_v^2 g^2 \sum_{k\in\mathcal{I}_v} |X_k^{(L)}|^2, \forall v \\ & D_k^{(L)} = \frac{1}{\sqrt{LN}} \sum_{m\in\mathcal{P}_u\cup\mathcal{P}_l\cup\mathcal{S}} \beta_m e^{-j2\pi km/LN} \end{aligned} \tag{16}$$

To solve the quadratically constrained linear optimization problem (16), we used CVX, a package for specifying and solving convex programs [25]. A customized interior point method can be developed for real-time implementation [17, 26].

4. SIMULATION

In this section, we illustrate the proposed impulses injection method with simulations. We generated 10000 VLC-OFDM blocks with $N = 128$, $L = 4$, and $V = 3$. The bit loading scheme is listed in Table 1. The constellation-wise weights for 64-QAM, 16-QAM, and 4-QAM are $\omega_1 = 1$, $\omega_2 = 1.56$, and $\omega_3 = 2.18$, respectively. The number of small points S is chose to be the equal to 5 %, 10

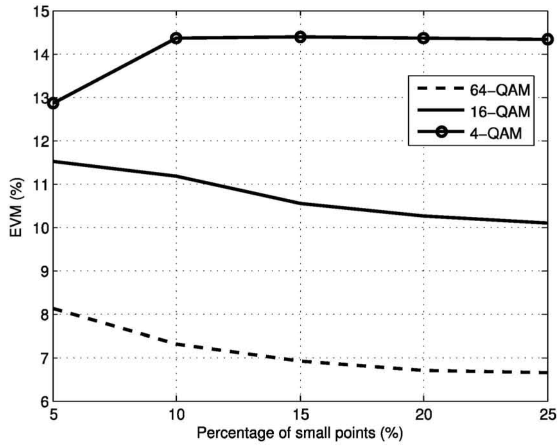

Fig. 2. Constellation-wise EVMs as a function of the percentage of small points with $\gamma_l = \gamma_u = 6$ dB.

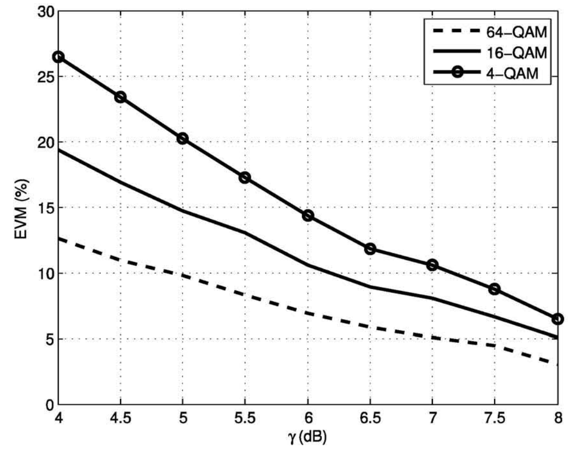

Fig. 3. Constellation-wise EVMs as a function of γ with 15 % small points.

Table 1. Bit loading for different subcarriers subsets ($N = 128$)

Subcarriers subset	Constellation type
$\mathcal{I}_1 = \{-21, \ldots, -2, -1\} \cup \{1, 2, \ldots, 21\}$	64-QAM
$\mathcal{I}_2 = \{-42, \ldots, -23, -22\} \cup \{22, 23, \ldots, 42\}$	16-QAM
$\mathcal{I}_3 = \{-63, \ldots, -44, -43\} \cup \{43, 44, \ldots, 63\}$	4-QAM

%, 15 %, 20 %, and 25 % of the block length ($LN = 512$) for each OFDM block. Fig. 2 shows the constellation-wise EVMs as a function of the percentage of small points with $\gamma_u = \gamma_l = 6$ dB, where $\gamma_u \triangleq c_u^2/\sigma_x^2$ and $\gamma_l \triangleq c_l^2/\sigma_x^2$. It is interesting that the EVM for 4-QAM become worse with increasing number of small points. This can be explained that when S is small, there are not enough variables to be optimized to "allocate" the distortions as we intended. Lower EVM for 4-QAM means higher EVM for 16-QAM and 64-QAM, which is not good for the general system because higher order constellations are less robust to distortions. The selection of the number of small points should be based on the EVM performance and computational load. Fig. 3 shows the constellation-wise EVMs as a function of γ with 15 % small points, where $\gamma = \gamma_l = \gamma_u$. Note that γ_u is not necessarily equal to γ_l. We can choose a greater γ_u if upper peak is less constrained than lower peak, and vise versa. Fig. 4 compares the constellations and EVMs between proposed scheme and simple clipping with $\gamma_l = \gamma_u = 5.5$ dB and 15 % small points. It can be seen that given the same dynamic range $[c_l, c_u]$, the proposed scheme can wisely allocate more distortions to lower order constellations, and less distortions to higher order constellations, which is desired for bit-loaded OFDM systems.

5. CONCLUSIONS

In this paper, we proposed a time-domain impulse injection method to reduce the UPAPR and LPAPR in bit-loaded VLC-OFDM systems. A quadratically constrained linear optimization problem was formulated to solve values for the injected impulses. Constellation-wise EVMs with different weights were minimized in the formulation. This is an effective PAPR reduction method since the UPAPR and LPAPR can be guaranteed to not exceed prescribed thresholds. It is advantageous over simple clipping since it can wisely allocate less distortions to higher order constellations with a few more variables.

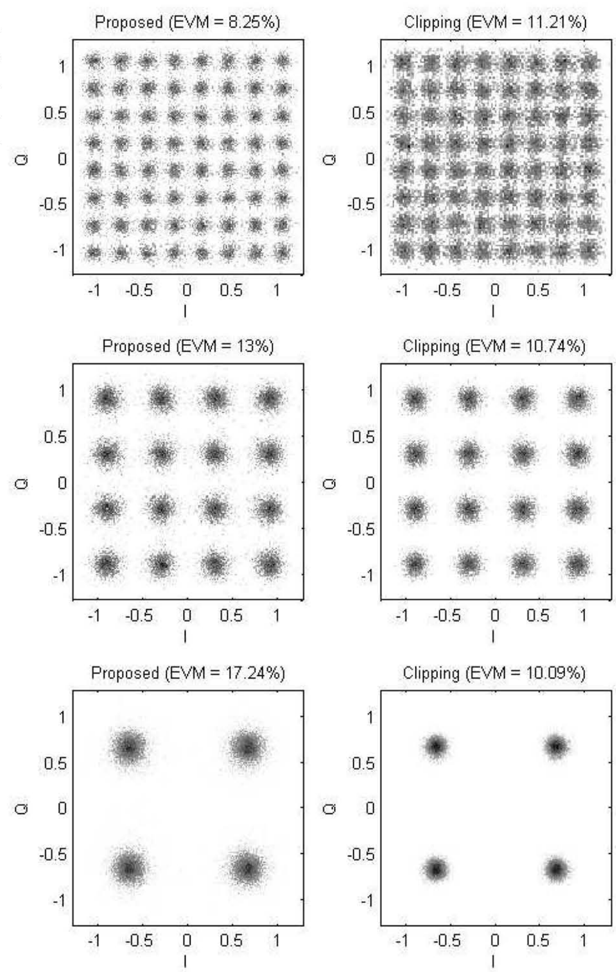

Fig. 4. Comparison of constellations and EVMs between proposed scheme and simple clipping.

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