# MULTI-USER MISO BROADCASTING FOR INDOOR VISIBLE LIGHT COMMUNICATION

*Zhenhua Yu*[⋆] *Robert J. Baxley*[†] *G. Tong Zhou*[⋆]

[⋆] School of Electrical and Computer Engineering, Georgia Institute of Technology, Atlanta, GA, 30332 USA
[†] Georgia Tech Research Institute, Atlanta, GA, 30332 USA

**ABSTRACT**

In this paper, we study multi-user multi-input single-output (MU-MISO) broadcasting for visible light communication (VLC). VLC differs from radio frequency communication in both baseband signal format and optical power constraints. We propose a precoding and biasing model for MU-MISO transmitter design in indoor VLC. We formulate and solve optimization problems for maximizing max-min fairness or throughput subject to per transmitting unit optical power constraint. We apply linear zero forcing and zero forcing dirty paper coding techniques and compare them in simulations.

***Index Terms*—** Visible light communication, multi-input single-output (MISO), multi-user, zero forcing, dirty paper coding

## 1. INTRODUCTION

With a rapidly growing demands for wireless data and the saturation of radio frequency (RF) spectrum, visible light communication (VLC) [1, 2, 3, 4] has become a promising candidate to complement conventional RF communication, especially for indoor short-range applications. VLC uses white light emitting diodes (LEDs), which already provide illumination and are quickly becoming the dominant lighting source, to transmit data. At the receiving end, a photo diode (PD) or an image sensor is used as the light detector. VLC has many advantages, including low-cost front ends, energy efficient transmission, no electromagnetic interference, no eye safety constraints like infrared, high security, and so on. In VLC, simple and low-cost intensity modulation and direct detection (IM/DD) techniques are employed, which means that only signal intensity, not phase information, is modulated. At the transmitter, white LED converts the amplitude of the electric signal into the intensity of the optical signal, while at the receiver, a PD or an image sensor generates an electric signal proportional to the intensity of the received optical signal. The IM/DD requires that the electric signal must be real-valued and unipolar (positive-valued).

Recently, to boost the achievable data rates, multi-input multi-output (MIMO) techniques [5, 6, 7] have been considered for VLC . In VLC with MIMO, multiple light sources transmit information to receiver arrays. However, optical MIMO channels are not as decorrelated as RF MIMO channels. The size of receiver arrays has to be large enough to ensure that the channel matrix is full-rank [6]. Therefore, multi-input single-output (MISO) techniques are more suitable for practical indoor VLC scenarios. Multi-user multi-input single-output (MU-MISO) broadcasting is widely studied in RF communication [8, 9, 10]. However, there has not been much research done on MU-MISO for VLC, which differs from RF

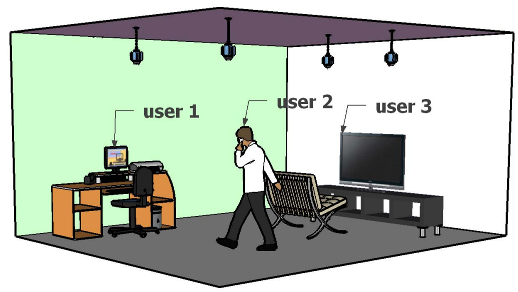


**Fig. 1**. MU-MISO broadcasting scenario in indoor visible light communication (four transmitting units and three users).

in two important ways: i) The RF baseband signals are complex-valued whereas time-domain signals in the VLC system are real-valued and nonnegative; and ii) the main power limitation for VLC is the average optical power, rather than average electrical power in RF communication. Therefore, most theories and methods developed for RF MU-MISO are not directly applicable to optical MU-MISO.

In this paper, we will investigate transmitter design for MU-MISO broadcasting for indoor VLC. We will apply two precoding techniques, namely, linear zero forcing and zero forcing dirty paper coding (ZF-DPC), to eliminate interferences at each user. In addition, the illumination functionality in VLC implies that each transmitting unit has its own optical power constraint. We will formulate optimization problems for maximizing max-min fairness or throughput subject to per transmitting unit optical power constraint. We will compare linear zero forcing to ZF-DPC for different distances between users in the simulations section.

## 2. SYSTEM DESCRIPTION

In an MU-MISO broadcasting system, multiple transmitting units cooperate to broadcast information to multiple users. Only one receiving unit is installed for each user. Let us consider $N_T$ transmitting units and $K$ users, where $N_T \geq K$. In VLC, intensity modulation is employed at the transmitter and direct detection is utilized at the receiver. At the $n$th transmitting unit, where $n = 1, 2, \dots, N_T$, the information-bearing electric signal $y_n$ drives the LED. The LED gives out light intensity $O_n$, which is proportional to the magnitude of the input electric signal $y_n$. The human eye can not perceive the instantaneous variations of the light intensity but only the average light intensity $E(O_n)$, where $E(\cdot)$ stands for statistical expectation. At the $k$th user, where $k = 1, 2, \dots, K$, the PD converts the received optical intensity into an electrical power. Fig. 1 shows the typical scenario of MU-MISO broadcasting in indoor VLC with four trans-

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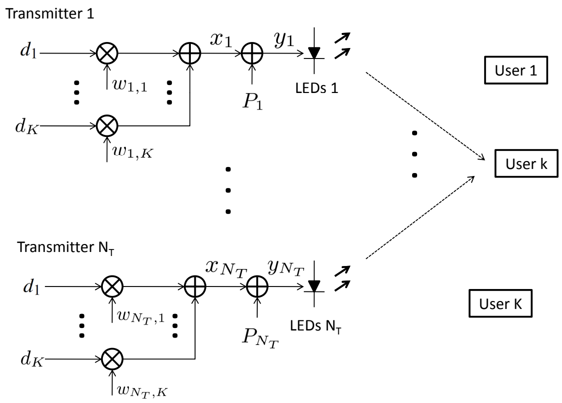


**Fig. 2**. The MU-MISO VLC transmission framework.

mitting units and three users. Because of the presence of intensity modulation and the requirement of illumination, three major constraints are placed on the input electric signal $y_n$ at the transmitter:

- The input electric signal must be real-valued; i.e. $y_n \in \mathbb{R}, \forall n$;
- The input electric signal must be nonnegative; i.e. $y_n \geq 0, \forall n$;
- The expectation of the input electric signal must be equal to some value that is determined by the illumination level; i.e. $E(y_n) = P_n, \forall n$.

Now we describe the design of the transmitters. Fig. 2 shows the diagram of the transmitters. Let $d_k$ denote the real-valued symbol to be transmitted to the $k$th user. We assume $d_k$ is zero mean normalized to the range $[-1, 1]$. At the $n$th transmitting unit, $d_k$ is multiplied by a well-designed precoding weight $w_{n,k}$. To generate a real-valued input electric signal, $w_{n,k}$ must be real-valued as well. By adding up all the weighted symbols for $K$ users at the $n$th transmit chain, we obtain

$$x_n = \sum_{k=1}^{K} d_k w_{n,k}. \tag{1}$$

However, $x_n$ can not serve as the input signal to the LED directly because it may take on negative values. To produce an input signal with mean value $P_n$, a bias $P_n$ should be added to $x_n$; i.e.,

$$y_n = x_n + P_n. \tag{2}$$

To ensure nonnegativity of $y_n$, we need

$$x_n \geq -P_n. \tag{3}$$

Since $d_k \in [-1, 1]$, we infer that the dynamic range of $x_n$ is

$$-\sum_{k=1}^{K} |w_{n,k}| \leq x_n \leq \sum_{k=1}^{K} |w_{n,k}|. \tag{4}$$

The following constraint on the weights can ensure Eq. (3):

$$\sum_{k=1}^{K} |w_{n,k}| \leq P_n. \tag{5}$$

Light propagates from the LED to the receiver. For VLC, we only consider the line-of-sight (LOS) propagation path. The channel gain from the $n$th transmitting unit to the $k$th user is given by [6, 11]

$$h_{n,k} = \begin{cases} \frac{\rho_k A_k}{\iota_{n,k}^2} R(\phi_{n,k}) \cos(\theta_{n,k}), & \theta_{n,k} \leq \theta_{c,k} \\ 0, & \theta_{n,k} \leq \theta_{c,k} \end{cases} \tag{6}$$

where $\phi_{n,k}$ denotes the angle of emission with respect to the $k$th transmitting unit, $\theta_{n,k}$ denotes the angle with respect to the $k$th user, $\iota_{n,k}$ represents the distance between the $n$th transmitting unit and the $k$th user, $\theta_{c,k}$ is the receiver filed of view (FOV) for the $k$th user, $\rho_k$ is the photodetector responsivity, $A_k$ is the collection area in the $k$th user:

$$A_k = \frac{q_k^2}{\sin^2(\theta_{c,k})} A_{PD,k}, \tag{7}$$

where $A_{PD,k}$ is the photodetector area, and $q_k$ is the refractive index of optical concentrator. In Eq. (6), $R(\phi_{n,k})$ denotes the Lambertian radiant intensity:

$$R(\phi_{n,k}) = \frac{(m+1)\cos^m(\phi_{n,k})}{2\pi}, \tag{8}$$

where $m$ is the order of Lambertian emission mode number [6, 11].

Let $\mathbf{w}_k = [w_{1,k}, w_{2,k}, \ldots, w_{N_T,k}]^T$ denote the real-valued precoding vector for the $k$th user. Let $\mathbf{h}_k = [h_{1,k}, h_{2,k}, \ldots, h_{N_T,k}]^T$ denote the channel gain vector seen from the $k$th user. After removing the DC component, the received signal at the $k$th user can be written as

$$\begin{aligned} r_k &= \sum_{n=1}^{N_T} x_n h_{n,k} + z_k \\ &= \mathbf{h}_k^T \mathbf{w}_k d_k + \sum_{j \neq k} \mathbf{h}_k^T \mathbf{w}_j d_j + z_k, \end{aligned} \tag{9}$$

where the first term $\mathbf{h}_k^T \mathbf{w}_k d_k$ is the desired signal, the second term $\sum_{j\neq k} \mathbf{h}_k^T \mathbf{w}_j d_j$ is interference from other users, and $z_k$ denotes noise at the user $k$. In VLC, $z_k$ is assumed to be real-valued Gaussian distributed with zero-mean and variance $\sigma_k^2$ [6, 11]:

$$\sigma_k^2 = 2eP_{s,k}B + 2e\rho_k \chi_{amb} A_k 2\pi(1-\cos(\theta_c))B + i_{amp}^2 B, \tag{10}$$

where $e$ is the electronic charge, $P_{s,k}$ is the average received optical power at the $k$th user,

$$P_{s,k} = \sum_{n=1}^{N_T} P_n h_{n,k}, \tag{11}$$

$\chi_{amb}$ is the ambient photocurrent, $B$ is the bandwidth, and $i_{amp}$ is the preamplifier noise current density.

## 3. PRECODING DESIGN

In this section, we design the precoding weights to eliminate the inference in Eq. (9) while satisfying the per transmitting unit optical power constraints in Eq. (5). We can rewrite Eq. (9) in matrix form as

$$\mathbf{r} = \mathbf{HWd} + \mathbf{z} \tag{12}$$

where $\mathbf{H} = [\mathbf{h}_1, \mathbf{h}_2, \ldots \mathbf{h}_K]^T \in \mathbb{R}^{K \times N_T}$ represents the channel matrix, $\mathbf{W} = [\mathbf{w}_1 \quad \mathbf{w}_2 \quad \ldots \quad \mathbf{w}_K] \in \mathbb{R}^{N_T \times K}$ represents the precoding weights in matrix form, and $\mathbf{d} = [d_1, d_2, \ldots, d_K]^T$ denotes the symbol vector.

We consider two well-known precoding techniques for eliminating the interference:

- Linear zero-forcing [12];
- Zero-forcing dirty paper coding (ZF-DPC) [5, 10].

Linear zero-forcing eliminates the interference $\sum_{j\neq k} \mathbf{h}_k^T \mathbf{w}_j d_j$ from the other users by requiring the matrix multiplication $\mathbf{HW}$ to be a diagonal matrix

$$\mathbf{HW} = \mathrm{diag}(\boldsymbol{\gamma}), \tag{13}$$

where $\gamma_k > 0$ represents the symbol gain for $d_k$. The precoding matrix is given by

$$\mathbf{W} = \mathbf{H}^T(\mathbf{HH}^T)^{-1}\mathrm{diag}(\boldsymbol{\gamma}). \tag{14}$$

ZF-DPC treats the interference $\sum_{j<k} \mathbf{h}_k^T \mathbf{w}_j d_j$ caused by users $j < k$ as known noncausally. With dirty paper coding, noncausal interferences can be eliminated without loss of information. Thus, ZF-DPC only requires the matrix product $\mathbf{HW}$ to be a lower triangular matrix

$$\mathbf{HW} = L(\boldsymbol{\gamma}), \tag{15}$$

where $L(\boldsymbol{\gamma})$ represents a lower triangular matrix with a diagonal $\boldsymbol{\gamma}$. We can obtain QR decomposition of the channel matrix $\mathbf{H} = \mathbf{GQ}$, where $\mathbf{G} \in \mathbb{R}^{K\times K}$ is a lower triangular matrix and $\mathbf{Q} \in \mathbb{R}^{K\times N}$ is an orthogonal matrix ($\mathbf{QQ}^T = \mathbf{I}$). Then the precoding matrix $\mathbf{W}$ can be designed as

$$\mathbf{W} = \mathbf{Q}^T\mathrm{diag}(1/\mathbf{g})\mathrm{diag}(\boldsymbol{\gamma}). \tag{16}$$

where $g_i = \mathbf{G}_{ii}$.

In summary, the precoding matrix takes the general form

$$\mathbf{W} = \mathbf{C}\mathrm{diag}(\boldsymbol{\gamma}), \tag{17}$$

where

$$\mathbf{C} = \begin{cases} \mathbf{H}^T(\mathbf{HH}^T)^{-1}, & \text{Linear zero forcing} \\ \mathbf{Q}^T\mathrm{diag}(1/\mathbf{g}), & \text{ZF-DPC} \end{cases} \tag{18}$$

We can express the constraint (5) in matrix form

$$\mathrm{abs}\,(\mathbf{C})\,\boldsymbol{\gamma} \leq \mathbf{p}, \tag{19}$$

where $\boldsymbol{p} = [P_1,\, P_2, \ldots, P_{N_T}]^T$. In the above, $\mathrm{abs}(\cdot)$ represents an element-wise absolute value operation; i.e., $[\mathrm{abs}(\mathbf{C})]_{n,n} = |\mathbf{C}_{n,n}|$.

In both cases, the received signal for the $k$th user becomes

$$r_k = \gamma_k d_k + z_k. \tag{20}$$

Assume that $d_k$ is drawn from a multi-level pulse amplitude modulation (PAM) constellation, given the BER constraint $\mathrm{BER}_T$, we can obtain the achievable data rate for the $k$th user [13]

$$\zeta_k = \log_2\left(1 + \frac{\eta\gamma_k}{\sigma_k}\right), \tag{21}$$

where $\eta = \sqrt{(-\log(5\mathrm{BER}_T))^{-1}}$.

Assume that the channel matrix $\mathbf{H}$ is perfectly known at the transmitter, the precoding matrix $\mathbf{W}$ only depends on the symbol gain vector $\boldsymbol{\gamma}$ . We formulate the following optimization problem to find the solution for $\boldsymbol{\gamma}$:

$$\begin{aligned} &\underset{\boldsymbol{\gamma}}{\text{maximize}} && f(\boldsymbol{\gamma}) \\ &\text{subject to} && \mathrm{abs}\,(\mathbf{C})\,\boldsymbol{\gamma} \leq \mathbf{p} \end{aligned} \tag{22}$$

where $f(\boldsymbol{\gamma})$ is the objective function that represents the performance metric of interest. By replacing $\gamma_k$ with $\mu_k\sigma_k/\eta$ in both the objective function and the constraint, we can express Eq. (22) as

$$\begin{aligned} &\underset{\boldsymbol{\mu}}{\text{maximize}} && f(\boldsymbol{\gamma}(\boldsymbol{\mu})) \\ &\text{subject to} && \boldsymbol{A\mu} \leq \mathbf{p} \\ & && \boldsymbol{\mu} \geq \mathbf{0} \end{aligned} \tag{23}$$

where $\boldsymbol{A} = \mathrm{abs}(\mathbf{C})\mathrm{diag}(\boldsymbol{\sigma})/\eta$.

Next, we consider two widely-used objective functions and the corresponding solutions, namely, max-min fairness and throughput [14].

### 3.1. Max-min fairness

Max-min fairness criterion maximizes the minimum data rate. The objective function is given by

$$\begin{aligned} \underset{\boldsymbol{\mu}}{\text{maximize}} \quad f(\boldsymbol{\gamma}(\boldsymbol{\mu})) &= \underset{\boldsymbol{\mu}}{\text{maximize}} \quad \min_k \quad \zeta_k \\ &= \underset{\boldsymbol{\mu}}{\text{maximize}} \quad \min_k \quad \log_2\left(1 + \mu_k\right). \end{aligned} \tag{24}$$

The optimization problem (23) can be written as

$$\begin{aligned} &\underset{\boldsymbol{\mu}}{\text{maximize}} && \min_k \quad \mu_k \\ &\text{subject to} && \boldsymbol{A\mu} \leq \mathbf{p} \\ & && \boldsymbol{\mu} \geq \mathbf{0} \end{aligned} \tag{25}$$

We infer from [14] that the optimal solution is in the form

$$\boldsymbol{\mu}^* = \mu^*\mathbf{1}, \tag{26}$$

where $\mathbf{1}$ denotes a vector whose each element is 1. Thus, Eq. (25) is equivalent to

$$\begin{aligned} &\text{maximize} && \mu \\ &\text{subject to} && \mu\boldsymbol{A}\mathbf{1} \leq \mathbf{p} \\ & && \mu \geq 0 \end{aligned} \tag{27}$$

for which we obtain the close-form solution

$$\mu^* = \min_n \quad \frac{\boldsymbol{p}_n}{(\boldsymbol{A}\mathbf{1})_n}. \tag{28}$$

### 3.2. Throughput

The throughput criterion maximizes the sum of the data rates. The objective function is given by

$$\begin{aligned} \underset{\boldsymbol{\mu}}{\text{maximize}} \quad f(\boldsymbol{\gamma}(\boldsymbol{\mu})) &= \underset{\boldsymbol{\mu}}{\text{maximize}} \quad \sum_{k=1}^{K}\zeta_k \\ &= \underset{\boldsymbol{\mu}}{\text{maximize}} \quad \log_2\Big(\det\big(\boldsymbol{I} + \mathrm{diag}(\boldsymbol{\mu})\big)\Big), \end{aligned} \tag{29}$$

where $\boldsymbol{I}$ denotes the identity matrix. The optimization problem (23) can be written as

$$\begin{aligned} &\underset{\boldsymbol{\mu}}{\text{maximize}} && \det\big(\boldsymbol{I} + \mathrm{diag}(\boldsymbol{\mu})\big) \\ &\text{subject to} && \boldsymbol{A\mu} \leq \mathbf{p} \\ & && \boldsymbol{\mu} \geq \mathbf{0} \end{aligned} \tag{30}$$

Problem (30) is a standard determinant maximization (MAXDET) program subject to linear matrix inequalities, which was investigated

**Table 1**. Simulation parameters of VLC system configuration.

| | |
|---|---|
| Room size (length $\times$ width $\times$ height) | 5m $\times$ 5m $\times$ 3m |
| Transmitter 1 coordinate | [1.25 1.25 0.5] |
| Transmitter 2 coordinate | [1.25 3.75 0.5] |
| Transmitter 3 coordinate | [3.75 1.25 0.5] |
| Transmitter 4 coordinate | [3.75 3.75 0.5] |
| Lambertian emission mode number $m$ | 1 |
| PD responsivity $\rho_k$ | 0.4 A/W |
| PD area $A_{PD,k}$ | 1 $cm^2$ |
| Receiver FOV $\theta_{c,k}$ | 62 deg. |
| Refractive index of optical concentrator $q$ | 1.5 |
| Pre-amplifier noise density $i_{amp}$ | 5 $pA/HZ^{-1/2}$ |
| Ambient light photocurrent $\chi_{amb}$ | 10.93 $A/m^2/Sr$ |

**Table 2**. Coordinate combination for two users – case 1.

| | |
|---|---|
| User 1 coordinate | [2.05 1.6 2.15] |
| User 2 coordinate | [3.2 3.9 2.15] |
| Distance between two users | 2.57 m |

in [15]. In this paper, we used CVX, a package for specifying and solving convex programs [16, 17].

## 4. SIMULATION

We provide numerical examples to corroborate the preceding theoretical analysis. Table 1 lists the parameters used in the simulations. The optical device parameters are obtained from [8]. Optical devices for all the transmitters or all the users share identical configurations. We assume four transmitting units and two users in the simulations, and all transmitters and receivers are parallel to the horizontal plane. To see how the performance varies with the distance between two users, we tested two coordinate combinations for the users. Table 2 and Table 3 show the coordinate combinations for the two cases. Both linear zero forcing and ZF-DPC precoding were employed for each case. Fig. 3 shows the max-min fairness as a function of the optical power constraint. Fig. 4 shows the throughput as a function of the optical power constraint. All the transmitting units have the same optical power constraints. As seen from Fig. 3 and Fig. 4, for case 1, no difference was observed between linear zero forcing and ZF-DPC precoding , when the two users are relatively far from each other. For case 2, ZF-DPC outperformed linear zero forcing precoding for both measurements, when the distance between two users is only 0.2 m.

We give here an intuitive explanation. When two users were well separated, the channels are more decorrelated; when the two users move closer to each user, the channels become more correlated, and the symbol gains become less significant with both precoding techniques. Thus, the performances are degraded compared with case 1. However, since linear zero forcing requires diagonal matrix multiplication while DPC-ZF only requires lower triangular matrix multiplication, DPC-ZF has more freedom and therefore outperforms linear zero forcing.

**Table 3**. Coordinate combination for two users – case 2.

| | |
|---|---|
| User 1 coordinate | [2.05 1.6 2.15] |
| User 2 coordinate | [2.05 1.4 2.15] |
| Distance between two users | 0.2 m |

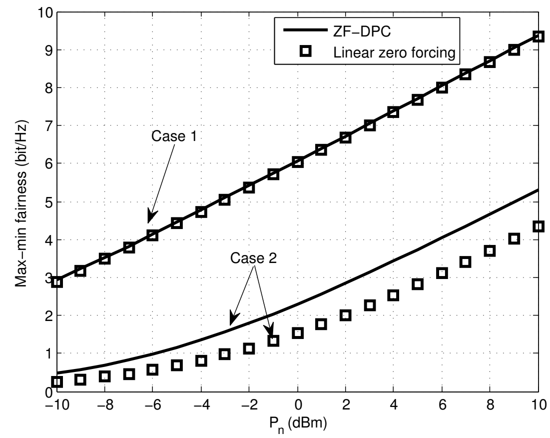


**Fig. 3**. Max-min fairness as a function of the optical power constraint.

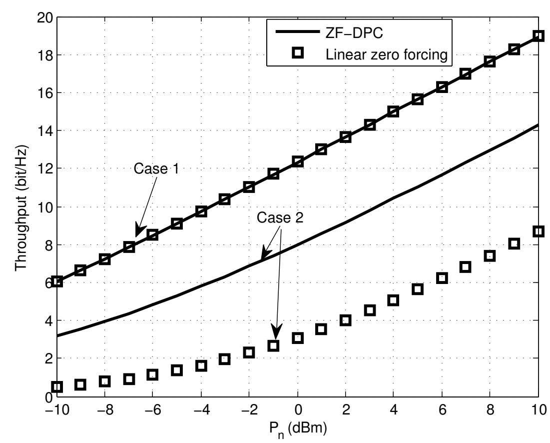


**Fig. 4**. Throughput as a function of the optical power constraint.

## 5. CONCLUSION

In this paper, we proposed the precoding and biasing model for the design of MU-MISO transmitters in indoor VLC. We utilized linear zero forcing and ZF-DPC precoding techniques to eliminate interferences at each user. We formulated and solved the optimization problem for maximizing the max-min fairness or throughput subject to the per transmitting unit optical power constraint. In the simulations, we examined two different user separation settings. The simulation results showed that when the two users are close to each other, ZF-DPC outperforms linear zero forcing; when the two users are relatively far from each other, there is little performance difference between the two precoding schemes.

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