SNDR CONSIDERATIONS FOR THE MINIMUM CLIPPING POWER LOSS SCHEME

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ABSTRACT

Orthogonal frequency division multiplexing (OFDM) is a robust wireless communications scheme but has the disadvantage that the symbols have a large dynamic range. Many proposals have been presented for suppressing the large peaks of an OFDM symbol so that higher power efficiencies can be achieved. In this paper, we will examine the recently proposed partial-transmit-sequence-based minimum clipping power loss scheme (MCPLS) and show how it can be generalized to apply to selected mapping (SLM) as well. Additionally, we will relate the clipping power metric in MPCLS to the signal to noise plus distortion ratio (SNDR) that is more commonly used to quantify clipping distortion. Finally, we will derive the SNDR-maximizing parameters that should be used in the new SLM-MPCLS scheme.

1. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) is a popular physical layer modulation format that is being widely adopted by a variety of wireless communications standards. OFDM has the advantage of a simple equalizer structure and due to the frequency division nature of OFDM, it is also easy to allocate different channels to different users. However, OFDM has the draw back that it does not perform well in peak-limited channels. Unlike single carrier signals, OFDM signals have a large dynamic range which is difficult for peak-limited devices like digital-to-analog converters and power amplifiers to accommodate. Thus, it is desirable to find methods to reduce the dynamic range of OFDM signals so that they can pass through peak limited devices without incurring too much unwanted distortion. Typically, the nonlinear distortion manifests itself as a bit error rate (BER) degradation and as out-of-band spectral distortion.

It is common in the literature to find the peak distribution of OFDM signals quantified through the peak-to-average power ratio (PAR). Accordingly, many researchers have proposed PAR reduction methods, which effectively reduced the high peaks of an OFDM signal. One popular method is known as selected mapping (SLM) [1]. In SLM, multiple signal realizations are created and the lowest-PAR realization is transmitted. Another method that can be viewed as a generalization of SLM is called partial transmit sequence (PTS) [2] which was proposed as a low-complexity alternative to SLM. However, in [3], it was demonstrated that SLM actually outperforms PTS in terms of PAR reduction (and distortion reduction) per unit of complexity.

In addition to all of the promising PAR-reduction research, it has been argued that other peak-distribution metrics, besides PAR, can be useful when considering OFDM transmission through peak-limited channels. In [4], it was suggested that, by using PTS where the minimum clipping distortion realization is selected for transmission instead of the minimum-PAR realization, the BER can be improved. In [5] a similar argument was made showing that the intermodulation distortion should be used instead of the PAR.

In this paper, we will provide a theoretical analysis of the minimum clipping power loss scheme (MCPLS) presented in [4]. Specifically, we will show how the clipping power is related to the signal-to-noise-plus-distortion ratio (SNDR). Ideally, the SNDR should be maximized in order to minimize the BER. With the link to SNDR established, we will then extend the MCPLS

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method to the SLM. Finally, we will derive the SNDR-optimal parameters that should be used with SLM-MCPLS.

2. OFDM AND SLM-MCPLS

For the purposes of examining the OFDM in peak limited channels, we can adopt a baseband OFDM model where the baseband OFDM symbol $\{x_{n/L}\}_{n=0}^{NL-1}$ is an oversampled IFFT output of the data vector $\{X_k\}_{k=0}^{N-1}$, a sequence of (complex) numbers drawn from a finite constellation. That is,

$$\begin{aligned} x_{n/L} &= \frac{1}{\sqrt{LN}}\Bigg(\sum_{k=0}^{N/2-1} X_k e^{\frac{j2\pi kn}{LN}} + \\ &\qquad \sum_{k=N/2}^{N-1} X_k e^{\frac{j2\pi (k-N)n}{LN}} \Bigg) \quad (1) \\ &= \mathrm{IFFT}\{X_k\}, \quad (2) \end{aligned}$$

where L is the oversampling factor, N is the number of subcarriers, and $\mathrm{IFFT}\{\cdot\}$ is NL-point oversampled IFFT indexed by n/L, where $n \in \{0, 1, ..., LN-1\}$. In [6] the authors demonstrated that when $L \geq 4$ the envelope of $x_{n/L}$ approximates the envelope of the continuous-time signal.

In SLM, M alternative signal mappings are created by phasing the constellation points with M N-length phase sequences, $\phi_k^{(M)}$, where $k \in \{0, 1, ..., N-1\}$ and $m \in \{1, 2, ..., M\}$. The resulting mappings

$$X_k^{(m)} = X_k e^{j\phi_k^{(m)}}, \quad (3)$$

can be used to create the time-domain mappings

$$x_{n/L}^{(m)} = \mathrm{IFFT}\{X_k^{(m)}\}. \quad (4)$$

In the original SLM [1], the transmitted signal is selected to be the lowest-PAR mapping. Contrastingly, in SLM-MCPLS, it is assumed that the channel is peak-limited by a time-invariant nonlinear function

$$g(x) = \begin{cases} Ax, & |x| \leq 1 \\ Ae^{j\angle x}, & |x| > 1, \end{cases} \quad (5)$$

where A is the peak-limiting value[1]. Additionally, as system designers we have the opportunity to digitally scale the OFDM samples $x_{n/L}$ by a time-invariant factor $1/\eta$, so that $x_{n/L}/\eta$ is sent through the peak-limiting function $g(\cdot)$. In [7], it was proven that such a scaling soft-limiter function is SNDR-optimal among all possible peak-limited functions. Define the distortion noise at each sample to be

$$B_{n/L}^{(m)} = \left| \frac{Ax_{n/L}^{(m)}}{\eta} - g\left(\frac{x_{n/L}^{(m)}}{\eta}\right)\right|^2. \quad (6)$$

Then, the index of transmitted signal is chosen according to

$$\widetilde{m} = \underset{m\in\{1,2,...,M\}}{\arg\min} \sum_{n=0}^{NL-1} B_{n/L}^{(m)}, \quad (7)$$

which means that $x_{n/L}^{(\tilde{m})}$ is transmitted. At the receiver, $\tilde{m}$ needs to be recovered so that the transmitter-side phasing can be undone. Many proposals have been made for receiver-side recovery of $\tilde{m}$ in traditional SLM [8–10]. Since the structure of SLM and SLM-MCPLS are the same except for the selection metric, any phase sequence recovery method suggested for SLM will also work for SLM-MCPLS.

In the subsequent sections we will define the SNDR and derive expressions that relate the SNDR to η. Once the relationship is established, we will determine the optimizing η, $\bar{\eta}$, in terms of the number of mappings M, such that the SNDR is maximized.

3. SIGNAL-TO-NOISE-PLUS-DISTORTION RATIO

Using Bussgang's Theorem, any memoryless nonlinearity can be decomposed into a sum of two uncorrelated parts according to

$$g(x) = \frac{A}{\eta}(\alpha x + d), \quad (8)$$

where α is chosen so that $E[x^* d] = E[d^* x] = 0$ and the indices m and n/L have been dropped from x for clarity.

In [7], it was demonstrated that, given σ_x^2, $E[x^* g(x)]$ and $E[|g(x)|^2]$, the SNDR, Ψ, can be written

$$\begin{aligned} \Psi &\triangleq \frac{\alpha^2\sigma_x^2}{\sigma_d^2 + \frac{\eta^2}{A^2}\sigma_w^2} \quad (9) \\ &= \frac{|E[x^* g(x)]|^2}{\sigma_x^2 E\left[|g(x)|^2\right] - |E[x^* g(x)]|^2 + \sigma_x^2\sigma_w^2}, \quad (10) \end{aligned}$$

[1]Digital predistortion can be applied to any monotonic peak-limiting nonlinear function to realize the soft-limiting channel characteristic in (5)

where σ_w^2 is the additive white Gaussian noise (AWGN) power. Alternatively, we can rewrite (6) as

$$B_{n/L}^{(m)} = \frac{A^2}{\eta^2}\left|q_{n/L}^{(m)}\right|^2, \tag{11}$$

where $q_{n/L}^{(m)} = (|x_{n/L}^{(m)}| - \eta)I(x_{n/L}^{(m)})$ and

$$I(x) = \begin{cases} 0, & |x| \leq \eta \\ 1, & |x| > \eta. \end{cases} \tag{12}$$

so that the expression

$$\frac{\sigma_x^2}{E\left[\left|q_{n/L}^{(\widetilde{m})}\right|^2\right] + \frac{\eta^2}{A^2}\sigma_w^2} = \frac{\sigma_x^2}{(1-\alpha)^2\sigma_x^2 + \sigma_d^2 + \frac{\eta^2}{A^2}\sigma_w^2}. \tag{13}$$

can be used to approximate the SNDR. For clipping functions that are not too harsh, $g(\cdot)$ is approximately linear over the range of x. In this case, $\alpha \approx 1$, so the term $(1-\alpha)^2 \to 0$. Thus, as $\alpha \approx 1$, the SNDR of the $\widetilde{m}$th mapping can be approximated by (13). The validity of this approximation will be verified later in the simulations. With this approximation it is possible to describe the SNDR in terms of the mean of $\left|q_{n/L}^{(\widetilde{m})}\right|^2$.

4. SNDR OPTIMIZATION

In this section we will outline the procedure for determining $\bar{\eta}$, for a given M, so that SNDR is maximized. To do this, the SNDR approximation in (13) needs to be expressed in terms of η, which involves determining an expression that relates $E\left[|q_{n/L}^{(\widetilde{m})}|^2\right]$ to η. Once the SNDR is expressed in terms of η, the SNDR-maximizing value $\bar{\eta}$ can be found with simple calculus.

When the system nonlinearity is of the soft-limiter form in (5), the pdf of $\left|q_{n/L}^{(m)}\right|^2$ can be expressed as

$$f_q(x) = \frac{\sqrt{x}+\eta}{\sigma_x^2\sqrt{x}}\exp\left(\frac{-(\sqrt{x}+\eta)^2}{\sigma_x^2}\right) + \delta(x)\left(1 - \exp\left(\frac{-\eta^2}{\sigma_x^2}\right)\right), \tag{14}$$

where $x \in [0, \infty)$. The form of this pdf makes finding a closed form for the pdf of $q_{n/L}^{(\widetilde{m})}$ difficult. Instead, define

$$C_L^{(m)} = \frac{1}{NL}\sum_{n=0}^{NL-1}\left|q_{n/L}^{(m)}\right|^2, \tag{15}$$

which is a scaled version of the metric used in (7), to determine the transmitted mapping index $\widetilde{m}$. Since $q_{n/L}$ and $q_{i/L}$ are correlated when $(i-n)/L \notin \mathbb{Z}$, here, the Nyquist total distortion $C_1^{(m)}$ is used to estimate the over-sampled total distortion $C_L^{(m)}$. Since $C_1^{(\widetilde{m})}$ is an unbiased estimator of $E\left[|q_{n/L}^{(\widetilde{m})}|^2\right]$, its mean is the same as $E\left[|q_{n/L}^{(\widetilde{m})}|^2\right]$. Accordingly, we will derive $E[C_1^{(\widetilde{m})}]$ and substitute it in to (13) for $E\left[|q_{n/L}^{(\widetilde{m})}|^2\right]$.

Next, we will find the distribution of $C_1^{(m)}$, which can be used to find the distribution of $C_1^{(\widetilde{m})}$, the minimizing $C_1^{(m)}$ among M trials. As an alternative to using multiple convolutions of (14) to derive $C_1^{(m)}$, the central limit theorem can be evoked such that $C_1^{(m)} \sim \mathcal{N}(\mu_C, \sigma_C^2)$, where

$$\mu_C = e^{-\eta^2} - \eta\sqrt{\pi}\operatorname{erfc}(\eta), \tag{16}$$

$$\sigma_C^2 = 2e^{-\eta^2}(1+\eta^2) - \eta\sqrt{\pi}(3+2\eta^2)\operatorname{erfc}(\eta) - \mu_C^2 \tag{17}$$

and $\operatorname{erfc}(x) \triangleq \frac{2}{\sqrt{\pi}}\int_x^\infty e^{-t^2}dt$.

Next, the distribution of $C_1^{(\widetilde{m})}$, which is the minimum $C_1^{(m)}$ among M trials, needs to be determined. The CDF of $C_1^{(m)}$ is

$$F_{C^{(m)}}(z) = \frac{1}{2}\left(2 - \operatorname{erfc}\left(\frac{z-\mu_C}{\sigma_C\sqrt{2}}\right)\right), \tag{18}$$

where μ_C and σ_C^2 are defined in (16) and (17). From this, it can be shown that the CDF of $C_1^{(\widetilde{m})}$ can be written as

$$F_{C^{(\widetilde{m})}}(z) = 1 - \frac{1}{2^M}\left(\operatorname{erfc}\left(\frac{z-\mu_C}{\sigma_C\sqrt{2}}\right)\right)^M. \tag{19}$$

Thus, the distribution of $C_1^{(\widetilde{m})}$, assuming that $C_1^{(m)}\ \forall\, m$ is Gaussian, is

$$f_{C^{(\widetilde{m})}}(x) = \frac{M2^{1-M}}{\sigma_C\pi\sqrt{2}}e^{-\frac{(x-\mu_C)^2}{2\sigma_C^2}}\left(\operatorname{erfc}\left(\frac{x-\mu_C}{\sigma_C\sqrt{2}}\right)\right)^{M-1} \tag{20}$$

Finally, the desired mean can be calculated numerically using

$$E[C_1^{(\widetilde{m})}] = E\left[|q_{n/L}^{(\widetilde{m})}|^2\right] = \int_0^\infty x f_{C^{(\widetilde{m})}}(x)dx. \tag{21}$$

As an alternative, we have found that for small values of M, it is possible to use the closed form approximation

$$E\left[|q_{n/L}^{(\widetilde{m})}|^2\right] \approx \sigma_C\sqrt{2N}\mathrm{erfc}^{-1}\left(2^{1-1/M}\right) + \mu_C, \quad (22)$$

which is the median of the $C_1^{(\widetilde{m})}$ distribution defined in (20). Finally, $\bar{\eta}$ can be found by solving

$$\frac{-1}{2\eta}\frac{\partial}{\partial\eta}E\left[|q_{n/L}^{(\widetilde{m})}|^2\right] = \frac{\sigma_w^2}{A^2} \quad (23)$$

for η, which can be done numerically.

5. SIMULATIONS

In this section, we will verify the approximations made in deriving $\bar{\eta}$ and show how $\bar{\eta}$ varies with other system variables. Fig. 1 is a plot of the SNDR versus η when no noise is present (i.e. the peak signal-to-noise power (PSNR), $\frac{A^2}{\sigma_w^2} \to \infty$). In the plot, the curve labelled 'exact' is the exact SNDR calculated using (10) with 10^5 Monte Carlo trials. There are also two approximations in the plot, the first, 'Approx 1' was plotted using (21) to calculate the $E\left[|q_{n/L}^{(\widetilde{m})}|^2\right]$, while 'Approx 2' was calculated using (22) to calculate the same mean value. The plot shows that both approximations are very close to the exact SNDR curve. Furthermore, the median approximation ('Approx 2') is a better estimate of the SNDR than 'Approx 1', which is convenient since (22) is easier to calculate than (21).

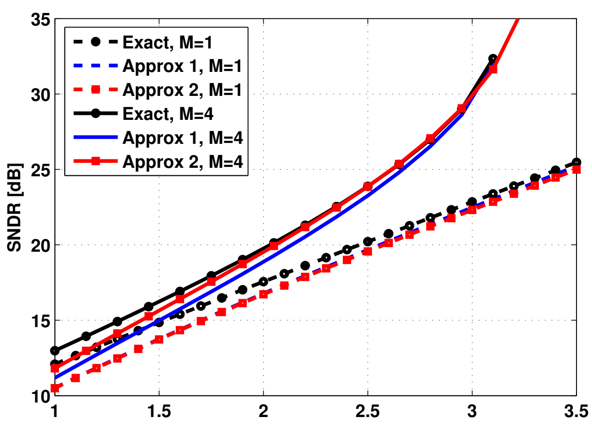

Fig. 1. SNDR with $\frac{A^2}{\sigma_w^2} \to \infty$, $N = 128$ and $L = 4$.

Fig. 2 is a plot of the SNDR with the same parameters as Fig. 1 except that $\frac{A^2}{\sigma_w^2} = 25$dB. Again, the plot shows that the approximations are very close to the exact SNDR curve. Also, the plot shows that by incorrectly choosing η, a severe SNDR penalty of more than several dBs may be incurred. Furthermore, even when the maximizing η is used, the $M = 4$ mappings case can return a 1.5dB improvement over the non-SLM $M = 1$ case.

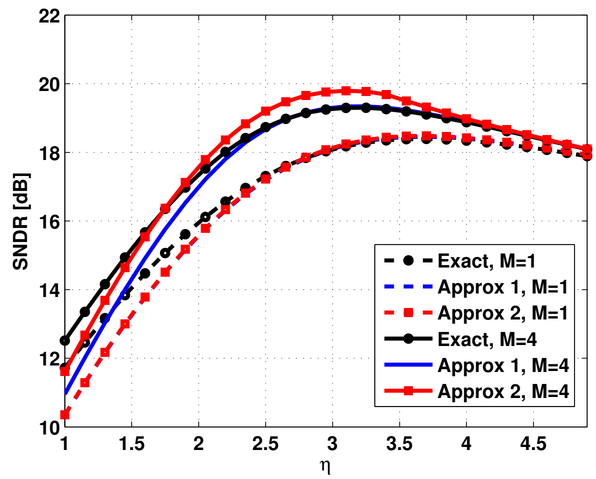

Fig. 2. SNDR with $\frac{A^2}{\sigma_w^2} = 25$dB, $N = 128$ and $L = 4$.

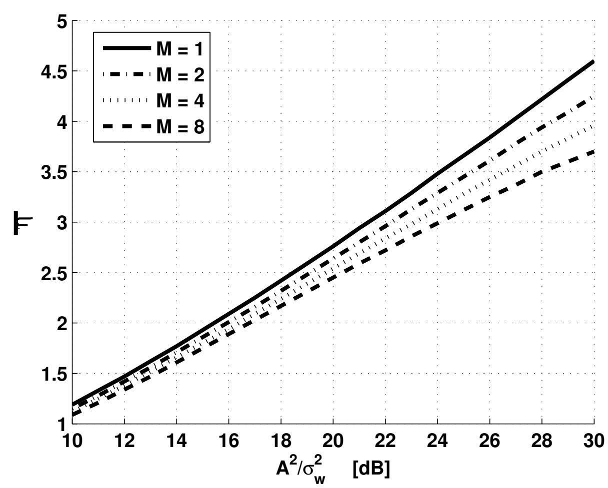

Fig. 3. Plot of SNDR-optimizing $\bar{\eta}$ versus $\frac{A^2}{\sigma_w^2}$. Where $N = 128$, $L = 4$ and $M \in \{1, 2, 4, 8\}$.

Fig. 3 is a plot of the SNDR-maximizing $\bar{\eta}$ versus $\frac{A^2}{\sigma_w^2}$ for $M \in \{1, 2, 4, 8\}$. The plot was made using the exact SNDR equation in (10), but the curves for the two approximations were indistinguishable from the exact curves. The plot can be used by system designers

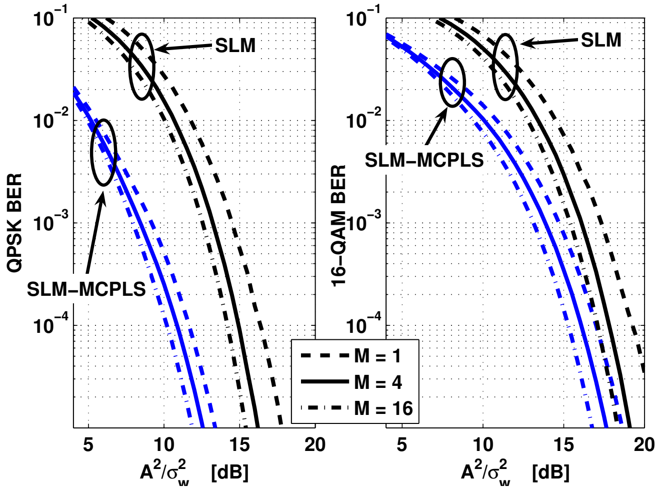

Fig. 4. Plot of BER versus $\frac{A^2}{\sigma_w^2}$. Comparison of the proposed SLM-MCPLS scheme and traditional SLM.

to choose the appropriate $\bar{\eta}$ so that the SNDR is maximized.

Finally, Fig. 4 is a plot of the bit error rate (BER) for the proposed SLM-MCPLS for both QPSK and 16-QAM. For the plot, $N = 128$, $L = 4$ and perfect detection of $\widetilde{m}$ is assumed. Also in the plot is the BER of the traditional SLM scheme [1], where the minimum-PAR mapping is selected. For the simulation, the selected signal is scaled down by its PAR so that no part of the signal is clipped [11]. By optimizing the SNDR, SLM-MCPLS significantly outperforms traditional SLM for all scenarios plotted.

6. CONCLUSIONS

In this paper we have presented the SLM-MCPLS scheme, which is based on the MCPLS scheme presented in [4]. Additionally, we provided a theoretical framework for determining the SNDR in SLM-MCPLS. Using this framework, we demonstrated how the SLM-MCPLS parameters should be chosen so that the SNDR will be maximized. Finally, we compared SLM-MCPLS to traditional SLM in terms of BER and showed that SLM-MCPLS can achieve several dBs of PSNR improvement.

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7. REFERENCES

[1] R. W. Bauml, R. F. H. Fischer, and J. B. Huber, "Reducing the peak-to-average power ratio of multicarrier modulationby selected mapping," *Electronics Letters*, vol. 32, no. 22, pp. 2056–2057, October 1996.

[2] S. H. Muller and J. B. Huber, "OFDM with reduced peak-to-average power ratio by optimum combination of partial transmit sequences," *Electronics Letters*, vol. 33, no. 5, pp. 368–369, February 1997.

[3] R. J. Baxley and G. T. Zhou, "Comparison of selected mapping and partial transmit sequence for crest factor reduction in ofdm," in *IEEE Military Communications Conference*, October 2006.

[4] X. Lei, Y. Tang, S. Li, and Y. Li, "A minimum clipping power loss scheme for mitigating the clipping noise in OFDM," in *Proc. IEEE Global Telecommunications Conference*, December 2003, vol. 1, pp. 6–9.

[5] M. R. D. Rodrigues and I. J. Wassell, "IMD reduction with SLM and PTS to improve the error-probability performance of nonlinearly distorted OFDM signals," *IEEE Transactions on Vehicular Technology*, vol. 55, no. 2, pp. 537–548, March 2006.

[6] H. Ochiai and H. Imai, "On the distribution of the peak-to-average power ratio in OFDM signals," *IEEE Transactions on Communications*, vol. 49, no. 2, pp. 282–289, February 2001.

[7] R. Raich, Hua Qian, and G. T. Zhou, "Optimization of SNDR for amplitude limited nonlinearities," *IEEE Transaction on Communications*, vol. 53, no. 11, pp. 1964–1972, November 2005.

[8] A. D. S. Jayalath and C. Tellambura, "SLM and PTS peak-power reduction of OFDM signals without side information," *IEEE Transactions on Wireless Communications*, vol. 4, no. 5, pp. 2006–2013, September 2005.

[9] R. J. Baxley and G. T. Zhou, "MAP metric for blind phase sequence detection in selected mapping," *IEEE Transactions on Broadcasting*, vol. 51, no. 4, pp. 565–570, December 2005.

[10] N. Chen and G. T. Zhou, "Peak-to-average power ratio reduction in OFDM with blind selected pilot tone modulation," *IEEE Transactions on Wireless Communications*, vol. 5, no. 8, pp. 2210–2216, August 2006.

[11] H. Ochiai, "Performance analysis of peak power and band-limited OFDM system with linear scaling," *IEEE Transactions on Wireless Communications*, vol. 2, no. 5, pp. 1055–1065, September 2003.