

GENERALIZED INTERIOR-POINT METHOD FOR CONSTRAINED PEAK POWER MINIMIZATION OF OFDM SIGNALS

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ABSTRACT

In this paper we present two results on reducing the peak power of orthogonal frequency division multiplexing (OFDM) symbols via constellation extension (CE). The first result is a derivation of the interior-point method (IPM) algorithm needed to find the optimal distortion set, where the distortion is constrained by convex functions. Next we optimize the parameters of a hybrid CE constraint set to minimize the BER. Numerical examples are provided to illustrate the findings.

Index Terms— OFDM, PAPR reduction, constellation extension, interior-point method

1. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) is a widely adopted modulation technique for wireless communications. The investigation of peak to average power ratio (PAPR) reduction techniques for OFDM signals has received much attention because PAPR reduction techniques can significantly increase the power efficiency of a communications system and reduce the system's sensitivity to nonlinear distortions. A review of the various PAPR reduction techniques can be found in [1]. Among the available methods, constellation extension (CE) implements PAPR reduction by introducing a carefully designed distortion to the constellation at the transmitter. CE algorithms are attractive because they do not require receiver-side modifications and thus are compatible with current communication standards.

Various distortion constraints (active constellation extension (ACE) [2], square boundary [3], circular boundary [3], Gaussian constellation error [4], etc.) have been proposed in the literature. To find the desired distortion signal, some authors [2, 3] have utilized repeated clipping and gradient-project method, which is not optimal. The authors in [4, 5] developed efficient interior-point method (IPM) optimization algorithms but they are only customized for Gaussian or ACE distortion constraints.

In this paper we will accomplish two goals. First, we generalize CE by deriving a generalized IPM optimization algorithm that can accommodate any convex CE constraint. Second, we show the application of this result by proposing and analyzing a mixed ACE-Gaussian CE constraint. The efficiency of the hybrid ACE-Gaussian CE technique will be demonstrated by computer simulations.

2. SYSTEM MODEL

In an OFDM system, a discrete time-domain signal is generated by applying inverse FFT (IFFT) operation to the frequency-domain signal. To approximate the peak amplitude of the continuous time-

domain signal, the L -times oversampled IFFT is utilized as

$$\mathbf{x} = \text{IFFT}_L(\mathbf{X}) = \mathbf{Q}\mathbf{X} \quad (1)$$

where $\mathbf{X} \in \mathbb{C}^N$ denotes the frequency-domain OFDM symbol and $\mathbf{x} \in \mathbb{C}^{LN}$ denotes the time-domain OFDM symbol, respectively. IFFT_L stands for L -times oversampled IFFT and $\mathbf{Q} \in \mathbb{C}^{NL \times N}$ represents the corresponding IFFT matrix. The PAPR of an OFDM symbol is defined as

$$\text{PAPR}(\mathbf{x}) = \frac{\|\mathbf{x}\|_\infty^2}{\|\mathbf{x}\|_2^2 / LN} \quad (2)$$

where $\|\cdot\|_l$ denotes the l -norm of the subject vector. According to the Central Limited Theorem, the summation over a large number of terms gives rise to an approximate complex Gaussian distribution for \mathbf{x} . As a result, the time-domain OFDM signal tends to have high a PAPR.

The basic idea of CE is to augment the original constellation \mathbf{X} with judiciously designed distortion so that the time-domain signal has reduced peak amplitude after the IFFT operation. Suppose m types of distortions can be applied. For each subcarrier set \mathcal{K}_i ($i = 1, \dots, m$), let $\mathbf{C}_i \in \mathbb{C}^{|\mathcal{K}_i|}$ denote the distortion vector and let $\mathbf{X}_i \in \mathbb{C}^{|\mathcal{K}_i|}$ denote original constellation vector. We formulate the generalized CE as a convex optimization problem

$$\begin{aligned} & \text{minimize} && p \\ & \text{subject to} && |x_k| \leq p \\ & && \mathbf{x} = \mathbf{Q}\mathbf{X} + \sum_{i=1}^m \mathbf{Q}_i \mathbf{C}_i \\ & && g_{ij}(\mathbf{X}_i, \mathbf{C}_i) \leq t_{ij} \end{aligned} \quad (3)$$

where $k = 0, \dots, LN - 1$; $j = 1, \dots, n_i$. The i^{th} type of distortion constraint can be represented by a group of n_i convex functions $g_{ij}(\cdot)$ with thresholds t_{ij} . $\mathbf{Q}_i \in \mathbb{C}^{NL \times |\mathcal{K}_i|}$ consists of columns of \mathbf{Q} that correspond the subcarrier set \mathcal{K}_i .

3. A GENERALIZED INTERIOR-POINT METHOD

To efficiently solve the optimization problem, we derive a generalized interior-point method (IPM) in this section. We use underline to denote the expansion of a complex-valued vector or a matrix to the real-valued equivalent vector or matrix. As an example, vector $\underline{\mathbf{X}} \in \mathbb{R}^{2N}$ below is the expanded form of $\mathbf{X} \in \mathbb{C}^N$

$$\underline{\mathbf{X}} = [\Re X(0), \Im X(0), \dots, \Re X(N-1), \Im X(N-1)]^T \quad (4)$$

where $\Re X$ stands for the real part of X , and $\Im X$ stands for the imaginary part of X . Matrix $\mathbf{Q} \in \mathbb{C}^{NL \times N}$ has the expanded form $\mathbf{Q} \in \mathbb{R}^{2NL \times 2N}$ where each element $Q(u, v)$ of \mathbf{Q} is expanded to a 2×2 block

$$\begin{bmatrix} \Re Q(u, v) & -\Im Q(u, v) \\ \Im Q(u, v) & \Re Q(u, v) \end{bmatrix} \quad (5)$$

3.1. Derivation

The goal of the IPM is to compute the search direction \mathbf{v}_i for different distortion vectors \mathbf{C}_i and a global step size $\alpha \in \mathbb{R}$. Solving the direction \mathbf{v}_i is equivalent to solving the following linear equation:

$$\frac{\partial^2 f}{\partial \mathbf{C}_i^2} \mathbf{v}_i = \frac{\partial^2 f}{\partial \mathbf{C}_i \partial p} - \frac{\partial f}{\partial \mathbf{C}_i} \quad (6)$$

where $f(\cdot)$ is the log-barrier function. For the task in (3), f can be written as

$$f = tp - \log(p) + f_p(p, \mathbf{x}) + f_i(X_i, \mathbf{C}_i) \quad (7)$$

where f_p denotes the log-barrier function for the peak power constraint, and f_i denotes the log-barrier function for the i^{th} distortion constraint,

$$f_p(p, \mathbf{x}) = - \sum_{k=0}^{LN-1} \log(p^2 - \underline{x}_{2k}^2 - \underline{x}_{2k+1}^2), \quad (8)$$

$$f_i(\underline{X}_i, \mathbf{C}_i) = - \sum_{j=1}^{n_i} \log(t_{ij} - g_{ij}(\underline{X}_i, \mathbf{C}_i)). \quad (9)$$

Substituting (7) into (6), we obtain

$$\left(\frac{\partial^2 f_p}{\partial \mathbf{C}_i^2} + \frac{\partial^2 f_i}{\partial \mathbf{C}_i^2} \right) \mathbf{v}_i = \frac{\partial^2 f_p}{\partial \mathbf{C}_i \partial p} - \left(\frac{\partial f_p}{\partial \mathbf{C}_i} + \frac{\partial f_i}{\partial \mathbf{C}_i} \right) \quad (10)$$

where $\partial f_p / \partial \mathbf{C}_i$, $\partial^2 f_p / \partial \mathbf{C}_i \partial p$ and $\partial^2 f_p / \partial \mathbf{C}_i^2$ are equal to $\mathbf{Q}_i^T (\partial f_p / \partial \mathbf{x})$, $\mathbf{Q}_i^T (\partial^2 f_p / \partial \mathbf{x} \partial p)$ and $\mathbf{Q}_i^T (\partial^2 f_p / \partial \mathbf{x}^2) \mathbf{Q}_i$, respectively.

The global step size α_{max} should be set as large as possible while still satisfying the constraints in (3):

$$\begin{aligned} & \text{maximize} \quad \alpha_{max} \\ & \text{subject to} \quad p - \|\mathbf{x} + \alpha_{max} \sum_{i=1}^m \mathbf{Q}_i \mathbf{v}_i\|_{\infty} \geq 0 \\ & \quad t_{ij} - g_{ij}(\mathbf{X}_i, \mathbf{C}_i + \alpha_{max} \mathbf{v}_i) \geq 0 \end{aligned} \quad (11)$$

3.2. Iteration Procedure

The procedure for the generalized IPM is described below.

Initialization: based on each type of distortion constraint, initialize \mathbf{C}_i and compute \mathbf{x} , p as follows

$$\mathbf{x} = \frac{\mathbf{Q}\mathbf{x}}{\mathbf{Q}\mathbf{x}} + \sum_{i=1}^m \mathbf{Q}_i \mathbf{C}_i \quad (12)$$

$$p = (1 + \kappa) \max_{k=1, \dots, LN} (\underline{x}_{2k-1}^2 + \underline{x}_{2k}^2) \quad (13)$$

where κ is selected as a small positive number.

Iteration:

1) Given \mathbf{x} and p , compute $\partial f_p / \partial \mathbf{x}$, $\partial^2 f_p / \partial \mathbf{x} \partial p$ and $\partial^2 f_p / \partial \mathbf{x}^2$.

2) For each type of distortion, calculate $\partial f_i / \partial \mathbf{C}_i$, $\partial^2 f_i / \partial \mathbf{C}_i^2$, $\partial f_p / \partial \mathbf{C}_i$, $\partial^2 f_p / \partial \mathbf{C}_i \partial p$ and $\partial^2 f_p / \partial \mathbf{C}_i^2$. Solve the search direction \mathbf{v}_i in (10).

3) Calculate the maximum possible global step size α_{max} from (11). The actual step size is adopted as $\alpha = (1 - \tau)\alpha_{max}$, where τ is a small positive number to keep the updated constellation satisfy the constraints in (3) more strictly.

4) Update \mathbf{C}_i , \mathbf{x} and p according to

$$\mathbf{C}_i \leftarrow \mathbf{C}_i + \alpha \mathbf{v}_i \quad (14)$$

$$\mathbf{x} \leftarrow \mathbf{x} + \alpha \sum_{i=1}^m \mathbf{Q}_i \mathbf{v}_i \quad (15)$$

$$p \leftarrow p - \alpha \quad (16)$$

5) Stop if the algorithm has converged or the maximum number of iterations has been reached, or return to step 1 and start a new iteration.

3.3. Example distortion constraints

To demonstrate the generalized IPM, we choose as examples of distortion mechanisms, ACE [2], Gaussian (GS) distortion [4] and square boundary (SQ) distortion [3]. The original constellations are denoted by \mathbf{X}_{ACE} , \mathbf{X}_{GS} , \mathbf{X}_{SQ} , and the distortion vectors are denoted by \mathbf{C}_{ACE} , \mathbf{C}_{GS} , \mathbf{C}_{SQ} for the three distortion mechanisms, respectively. The ACE constraint only allows the outer points of the M-QAM constellations to extend without reducing the constellation's minimum distance. With the Gaussian distortion, the distortion vector \mathbf{C}_{GS} is Gaussian distributed and the relative constellation error power ϵ is kept below ϵ_{max} . In this paper, ϵ is defined as

$$\epsilon = \frac{\|\mathbf{C}_{GS}\|^2 / N_{GS}}{d_{min}^2} \quad (17)$$

where N_{GS} is the number of subcarriers applied with the Gaussian distortion and d_{min} is the half minimum distance of the original constellation points. For the square distortion constraint, the distortion vector \mathbf{C}_{SQ} is constrained to lie within a square centered at the original point with half length L_{lim} . Let f_{ACE} , f_{GS} and f_{SQ} denote the corresponding log-barrier functions defined by (9). After differentiation, we obtain

$$\frac{\partial f_{ACE}}{\partial \mathbf{C}_{ACE}} = -1. / \mathbf{C}_{ACE} \quad (18)$$

$$\frac{\partial^2 f_{ACE}}{\partial \mathbf{C}_{ACE}^2} = \text{diag}\{1. / (\mathbf{C}_{ACE} * \mathbf{C}_{ACE})\} \quad (19)$$

$$\frac{\partial f_{GS}}{\partial \mathbf{C}_{GS}} = -\frac{\mathbf{X}_{GS}}{\eta/2 + \mathbf{X}_{GS}^T \mathbf{C}_{GS}} + \frac{2\mathbf{C}_{GS}}{\eta - \mathbf{C}_{GS}^T \mathbf{C}_{GS}} \quad (20)$$

$$\begin{aligned} \frac{\partial^2 f_{GS}}{\partial \mathbf{C}_{GS}^2} &= \frac{\mathbf{X}_{GS} \mathbf{X}_{GS}^T}{(\eta/2 + \mathbf{X}_{GS}^T \mathbf{C}_{GS})^2} + \frac{4\mathbf{C}_{GS} \mathbf{C}_{GS}^T}{(\eta - \mathbf{C}_{GS}^T \mathbf{C}_{GS})^2} \\ &+ \frac{2\mathbf{I}_{GS}}{\eta - \mathbf{C}_{GS}^T \mathbf{C}_{GS}} \end{aligned} \quad (21)$$

$$\frac{\partial f_{SQ}}{\partial \mathbf{C}_{SQ}} = 1. / (L_{lim} - \mathbf{C}_{SQ}) - 1. / (L_{lim} + \mathbf{C}_{SQ}) \quad (22)$$

$$\begin{aligned} \frac{\partial^2 f_{SQ}}{\partial \mathbf{C}_{SQ}^2} &= \text{diag}\{-1. / ((L_{lim} - \mathbf{C}_{SQ}) * (L_{lim} - \mathbf{C}_{SQ}))\} \\ &+ \text{diag}\{1. / ((L_{lim} + \mathbf{C}_{SQ}) * (L_{lim} + \mathbf{C}_{SQ}))\} \end{aligned} \quad (23)$$

where the dot \cdot indicates the element-wise vector or matrix operation. The error parameter $\eta = N_{GS}d_{min}^2\epsilon_{max}$.

As an example, the scatter plot of a distorted 64-QAM constellation is shown in Figure 1, where the following parameters were used in the optimization: $L = 4$, $N = 64$, $\kappa = 0.05$, $\tau = 0.02$, $\epsilon_{max} = -10\text{dB}$, $L_{lim} = 0.5$. The interior diagonal points are constrained by a square, the outer points are constrained with ACE constraints and the other points are Gaussian constrained.

4. ANALYSIS OF OPTIMAL DISTORTION

It is well known that the power amplifier (PA) is a peak power limited device. Suppose that the PA is perfectly linearized with input saturation power $\mathcal{P}_{i,sat}$ and output saturation power $\mathcal{P}_{o,sat}$ [6]. Let $\mathcal{P}_l = \|\mathbf{x}_l\|_\infty^2$ denote the peak power of the l_{th} input symbol \mathbf{x}_l . To deliver the maximum efficiency, we can linearly scale \mathcal{P}_l to $\mathcal{P}_{i,sat}$ and then amplify to $\mathcal{P}_{o,sat}$. Thus, the overall symbol-wise PA power gain for \mathbf{x}_l is $G_l = \mathcal{P}_{o,sat}/\mathcal{P}_l$. After the amplification, d_{min} for the l_{th} symbol becomes $d_l = d_{min}\sqrt{\mathcal{P}_{o,sat}/\mathcal{P}_l}$.

Obviously, decreasing \mathcal{P}_l will increase d_l and lead to improved performance. However, to decrease \mathcal{P}_l in the CE framework, we must add distortions \mathbf{C}_i which have detrimental effect. Thus, there is an optimal distortion level that corresponds to the best compromise and minimizes the bit error rate (BER). The authors in [7] computed the optimal GS distortion by maximizing the signal-to-noise ratio. The authors in [8] proposed an adaptive SQ to maximize the amplified minimum distance of distorted constellation. In this section, we focus on the optimal distortion analysis of ACE + GS and compare it with the work [7, 8]. Figure 2 is an example of the ACE + GS distortion set up. In the ACE + GS framework, distortion is generated easily by the generalized IPM. ACE is applied to the outer constellation points and GS is applied to the inner points.

Assume that the OFDM symbols pass through an additive white Gaussian noise (AWGN) channel with noise power N_0 . The instantaneous distance to noise power ratio (DNPR) for the l_{th} signal is defined as

$$\zeta_l = \frac{d_l^2}{N_0} = \frac{d_{min}^2 \mathcal{P}_{o,sat}}{N_0 \mathcal{P}_{l,\epsilon_{max}}} \quad (24)$$

where $\mathcal{P}_{l,\epsilon_{max}}$ denotes the peak power of the l_{th} input signal with distortion constraint ϵ_{max} . For the inner constellation points, since the distortion is Gaussian distributed, the instantaneous distance to noise plus distortion power ratio (DNDPR) becomes $\xi_l = 1/(\zeta_l^{-1} + \epsilon_{max})$. For the outer constellation points, Monte-Carlo experiments show that only a small amount of points are outside extended. Thus, the instantaneous BER for the l_{th} signal of M-QAM can be approximated as

$$P_b^{(M)}(\zeta_l, \epsilon_{max}) \approx \frac{2}{\log_2 M} P_0^{(M)}(\zeta_l) + \frac{\log_2 M - 2}{\log_2 M} P_e^{(M)}(\zeta_l, \epsilon_{max}) \quad (25)$$

where $P_0^{(M)}(\zeta_l)$ is the BER of undistorted constellation given by (27) in [9]. For 16-QAM, $P_e^{(16)}(\zeta_l, \epsilon_{max})$ can be shown to be

$$P_e^{(16)}(\zeta_l, \epsilon_{max}) \approx \frac{1}{8} [\text{erfc}(\sqrt{\xi_l}) + \text{erfc}(3\sqrt{\xi_l})] + \frac{1}{8} [\text{erfc}(\sqrt{\zeta_l}) + \text{erfc}(\sqrt{\xi_l}) + \text{erfc}(3\sqrt{\xi_l}) - \text{erfc}(5\sqrt{\zeta_l})] \quad (26)$$

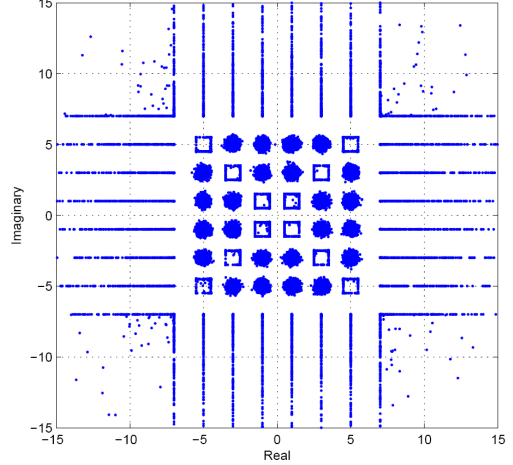


Fig. 1. Scatter plot of a distorted 64-QAM constellation with ACE, SQ and GS distortion mechanisms

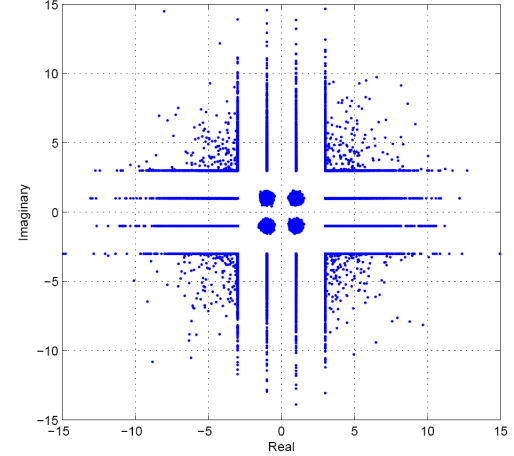


Fig. 2. Scatter plot of a distorted 16-QAM constellation with ACE and GS distortion mechanisms

For 64-QAM, $P_e^{(64)}(\zeta_l, \epsilon_{max})$ can be shown to be

$$P_e^{(64)}(\zeta_l, \epsilon_{max}) \approx \frac{1}{24} [\text{erfc}(\sqrt{\xi_l}) + \text{erfc}(3\sqrt{\xi_l}) + \text{erfc}(5\sqrt{\xi_l}) + \text{erfc}(7\sqrt{\xi_l})] + \frac{1}{24} [2\text{erfc}(\sqrt{\xi_l}) + \text{erfc}(3\sqrt{\xi_l}) + \text{erfc}(3\sqrt{\zeta_l}) + \text{erfc}(5\sqrt{\xi_l}) + \text{erfc}(7\sqrt{\xi_l}) - \text{erfc}(9\sqrt{\xi_l}) - \text{erfc}(11\sqrt{\zeta_l})] + \frac{1}{24} [\text{erfc}(\sqrt{\zeta_l}) + 3\text{erfc}(\sqrt{\xi_l}) + 3\text{erfc}(3\sqrt{\zeta_l}) - \text{erfc}(5\sqrt{\zeta_l}) - \text{erfc}(5\sqrt{\xi_l}) - 2\text{erfc}(7\sqrt{\xi_l}) + \text{erfc}(9\sqrt{\zeta_l}) + \text{erfc}(9\sqrt{\xi_l}) + \text{erfc}(11\sqrt{\xi_l}) - \text{erfc}(13\sqrt{\zeta_l})] \quad (27)$$

Let $\gamma_p = \mathcal{P}_{o,sat}/N_0$ denote the peak signal-to-noise ratio (PSNR). DNPR in (24) becomes $\zeta_l = \gamma_p d_{min}^2 / \mathcal{P}_{l,\epsilon_{max}}$. The average BER is given by

$$P_{ba}^{(M)} = \mathcal{E}_{\mathcal{P}_{l,\epsilon_{max}}} \left\{ P_b^{(M)} \left(\gamma_p \frac{d_{min}^2}{\mathcal{P}_{l,\epsilon_{max}}}, \epsilon_{max} \right) \right\} \quad (28)$$

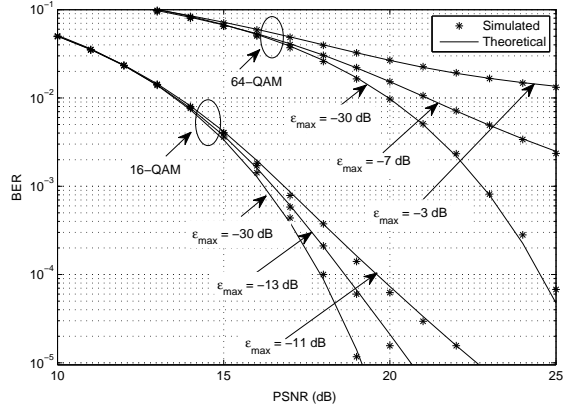


Fig. 3. BER for ACE + GS distorted OFDM signals

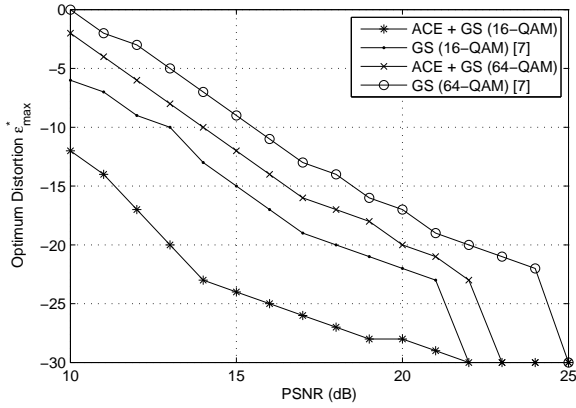


Fig. 4. Optimum amount of distortion as a function of the PSNR

where $\mathcal{E}_X\{\cdot\}$ represents the expectation with respect to the random variable X . Monte-Carlo experiments show that the variance of $\mathcal{P}_{l,\epsilon_{max}}$ is very small. To facilitate our analysis, we use $P_b^{(M)}(\gamma_p d_{min}^2 / \mathcal{E}\{\mathcal{P}_{l,\epsilon_{max}}\}, \epsilon_{max})$ to approximate $P_{ba}^{(M)}$, where the ratio $d_{min}^2 / \mathcal{E}\{\mathcal{P}_{l,\epsilon_{max}}\}$ is a function of ϵ_{max} that we approximate with a fourth-order polynomial whose parameters are extracted with least-squares techniques using data generated from Monte Carlo experiments. Figure 3 shows the average BER as a function of PSNR for 16-QAM and 64-QAM. We can see that the simulated results and the theoretical analysis matched very well. Given the PSNR γ_p , we compute the optimal distortion of ACE+GS via

$$\epsilon_{max}^* \approx \underset{\epsilon_{max}}{\operatorname{argmin}} P_b^{(M)}\left(\gamma_p \frac{d_{min}^2}{\mathcal{E}\{\mathcal{P}_{l,\epsilon_{max}}\}}, \epsilon_{max}\right) \quad (29)$$

Figure 4 shows the optimal amount of distortion for the framework ACE+GS vs. applying Gaussian distortions only. Using the optimal ϵ_{max}^* obtained from Figure 4, we plot the simulated BER as a function of the peak SNR in Figure 5. The BER of the adaptive SQ signal is also simulated and compared with the ACE + GS framework. We can see that ACE+GS performed the best among the three constraints.

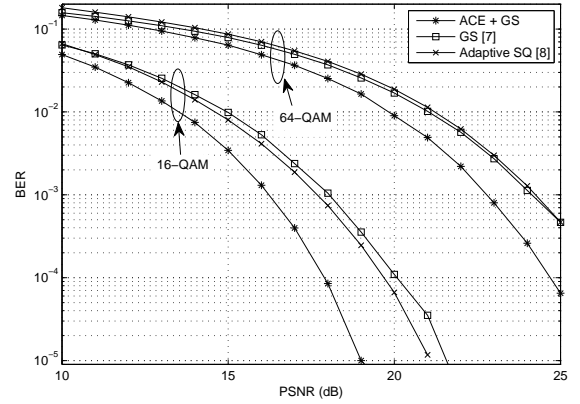


Fig. 5. BER of optimized OFDM signals over peak-limited AWGN channel

5. CONCLUSION

In this paper, we have formulated a generalized CE problem to minimize the peak power of time-domain OFDM symbols subject to arbitrary convex constraints. We have derived a generalized interior-point method to solve the problem efficiently. The ACE, GS and SQ distortion constraints are selected as examples to illustrate our approach. We optimize the distortion parameters of ACE + GS to minimize the BER. Simulation results show that the combination of ACE for outer constellation points and Gaussian distortion for inner constellation points outperformed other previously proposed CE constraints.

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