ANCHOR FREE NODE TRACKING USING RANGES, ODOMETRY, AND MULTIDIMENSIONAL SCALING

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ABSTRACT

This paper addresses the problem of estimating the positions of nodes in a mobile network over time, when pre-surveyed anchor nodes are not available. We do this by employing both inter-node range measurements and odometry data. Both types of measurement data are applied within the multidimensional scaling paradigm, which maps pairwise dissimilarity values into node coordinates. The mathematical treatment is presented, along with several advantages of our proposed approach. We demonstrate its performance through simulation across various parameter values. Further, we show the performance using real range and odometry data gathered from our CSOT mobile robot testbed.

1. INTRODUCTION

Node localization and tracking has broad interest and application in the areas of wireless sensor networks (WSNs), as well as the field of robotics. Such networks must frequently operate in GPS denied environments, or where GPS is cost or energy prohibitive, thus motivating the search for other methods [1,2]. Also, pre-surveyed node positions may not be available, creating an anchor free, or reference-less problem.

A popular approach to the cooperative relative localization problem is to use a technique known as multidimensional scaling (MDS). In MDS, high dimensional "dissimilarity" data is mapped into a lower (usually 2) dimensional set for easier visualization [3]. In the localization context, the dissimilarity data is a set of pairwise range measurements between nodes, that is, their dissimilarity in location. The MDS solution maps this set of ranges to a set of points in the 2D plane in a way that minimizes a particular *stress function*. Since only pairwise range measurements are used, in the most general case a relative map is formed without the use of anchor nodes, such that the distance between nodes is preserved as much as possible. The relative map represents the true positions up to a flip/rotation and possible translation.

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There has been significant work in the literature on using MDS as an approach to localization and tracking, particularly when anchor nodes are available [4–6]. In [7], a weighted stress function incorporating prior node position information is proposed. Other authors have emphasized distributed approaches by map stitching [8, 9] or dealing with missing data inherent in partially connected networks [10–12]. In [13], subspace tracking was used to track relative changes in node positions given their respective range measurements. However, without pre-surveyed anchor nodes, these methods either cannot be used, or will present flip/rotation ambiguities between time steps in a mobile system. This is an inherent shortcoming of MDS without anchors; the relative maps produced at different times are not comparable.

We address this shortcoming by proposing a reference-free method to track the relative positions of nodes in a network over time through fusion of range measurements and odometry data. Both the range and odometry data are placed into the MDS framework in a manner that allows solving jointly for the trajectory of all nodes. In MDS, range measurements represent dissimilarity values between nodes in a given time step, while odometry data represents dissimilarity between the *same* node at two different time steps. Thus the odometry data connects inter-node range measurements across time, allowing for a joint solution using all the available data. Other data fusion approaches to localization exist in the literature, e.g. [14–17]. However, these approaches all rely on anchor nodes to function, and do not utilize the MDS paradigm.

Our approach has several advantages, in addition to removing the need for anchor nodes. First, all available data is used to jointly estimate each node's trajectory, reducing the effects of noise on individual measurements. Second, we take a non-parametric approach of stress function minimization, which needs few assumptions about the environment in which this method is used. No state-space assumptions or statistical measurement models need be determined; these are required for extended or unscented Kalman filters, for example. Third, odometry data is commonly available on many platforms, often with high accuracy. For example, encoder wheel measurements are commonly used for dead reckoning on mobile robot platforms, and inexpensive inertial navigation systems (INS) are used on other types of mobile nodes. Fourth, our formulation can easily account for missing data, as well as differences in the relative quality of the various measurements. Finally, the joint estimation of position across multiple time steps eliminates the issue of flip/rotation ambiguities between those time steps. The relative reference frame is preserved for the time values estimated.

In Section 2, we give the mathematical treatment of our proposed method. The results of a comprehensive simulation for varying node count, time steps, and noise is found in Section 3. The results of our algorithm applied to real data gathered by our testbed follows in Section 4. Finally we offer our conclusions in Section 5.

2. MDS WITH RANGE AND ODOMETRY

We consider the problem of estimating the positions of N nodes in each of K time steps. If the position of node n at time k is given by $\mathbf{s}_n^{(k)} = \left[s_{n,x}^{(k)}\ s_{n,y}^{(k)}\right]^T$, then we can compile the NK position vectors into the matrix

$$\mathbf{S} = \left[\mathbf{s}_1^{(1)}\mathbf{s}_2^{(1)} \dots \mathbf{s}_N^{(K)}\right]^T \in \mathbb{R}^{NK\times 2}, \qquad (1)$$

where a matrix is used instead of a vector by convention in the MDS literature.

We assume that a set of $N(N-1)$ pairwise range measurements are available at each time step, arranged in matrix $\mathbf{R}^{(k)} \in \mathbb{R}^{N\times N}$ such that $\left[\mathbf{R}^{(k)}\right]_{i,j} = r_{ij}^{(k)}$ is the range estimate between nodes i and j at time step k. The main diagonal of $\mathbf{R}^{(k)}$ is equal to zero by definition. The range values are assumed to be already estimated from some method such as time-of-flight, RSSI, etc. Similarly, we assume a set of odometry distance measurements are available for each node, which represent the distance traveled by that node between time steps $k-1$ and k. These are arranged into the diagonal matrix $\mathbf{D}^{(k)} \in \mathbb{R}^{N\times N}$, such that $\left[\mathbf{D}^{(k)}\right]_{i,i} = d_i^{(k)}$ is the distance traveled by node i between times $k-1$ and k. The $d_i^{(k)}$ values are estimated from encoder wheel pulses, inertial navigation, etc. In this work we assume that the off-diagonal elements of $\mathbf{D}^{(k)}$ are unknown and set to zero.

The total data gathered are assembled into a block tridiagonal matrix $\boldsymbol{\Delta} \in \mathbb{R}^{NK\times NK}$. For example, if the network consists of 4 nodes, and data is gathered across 3 time steps, then

$$\boldsymbol{\Delta} = \begin{bmatrix} \mathbf{R}^{(3)} & \mathbf{D}^{(3)} & \mathbf{0} \\ \mathbf{D}^{(3)} & \mathbf{R}^{(2)} & \mathbf{D}^{(2)} \\ \mathbf{0} & \mathbf{D}^{(2)} & \mathbf{R}^{(1)} \end{bmatrix} \qquad (2)$$

is a 12×12 matrix of 4×4 blocks. Note that stepping between blocks in Δ is equivalent to stepping between time values k.

Taking the weighted stress function approach of [7], we seek the set of positions $\hat{\mathbf{S}}$ that minimize the following stress function:

$$\mathcal{J}(\hat{\mathbf{S}}) = \sum_{i=1}^{NK}\sum_{j=1}^{NK} [\mathbf{W}]_{i,j} \left([\Delta]_{i,j} - \left[d(\hat{\mathbf{S}})\right]_{i,j}\right)^2, \qquad (3)$$

where we have omitted the term representing prior knowledge of node positions, because none is assumed here. In (3), the matrix $\mathbf{W} \in \mathbb{R}^{NK\times NK}$ is a matrix of non-negative scalar weights, and the function $d\colon \mathbb{R}^{NK\times 2} \to \mathbb{R}^{NK\times NK}$ maps the set of coordinates $\mathbf{S}$ into a matrix of pairwise distances with the same structure as $\boldsymbol{\Delta}$. That is,

$$\left[d(\hat{\mathbf{S}})\right]_{i,j} = \left(\left([\hat{\mathbf{S}}]_{i,:} - [\hat{\mathbf{S}}]_{j,:}\right)^T \left([\hat{\mathbf{S}}]_{i,:} - [\hat{\mathbf{S}}]_{j,:}\right)\right)^{1/2}. \qquad (4)$$

The stress function (3) is both nonlinear and non-convex in the variable $\hat{\mathbf{S}}$. The most common technique for minimizing this function is known as the SMACOF algorithm [3], and involves transformation of the stress via majorizing functions. The algorithm is iterative, and usually offers quick convergence. We use the SMACOF algorithm in this paper to minimize (3), with the addition of trying multiple random initial start configurations to help avoid local minima. It should also be noted that taking the weighted approach to MDS avoids the noise amplification caused by squaring the distance values in the classical non-weighted MDS solution.

The values in the weighting matrix $\mathbf{W}$ represent our relative confidence in the data values. It conveniently allows accounting for any missing data by simply setting the corresponding weight to zero, and thus will have no influence on the stress function (3). For the remaining measurements, traditional linear weighted least squares sets $[\mathbf{W}]_{i,j} = \frac{1}{\sigma_{ij}^2}$, where σ_{ij}^2 is the assumed variance of the corresponding measurement. However, the stress function (3) is not linear due to the distance mapping $d(\cdot)$, and thus this mapping does not strictly apply. In our simulations we have obtained better results by assigning weights based on the relative standard deviations. In this paper we assign weights by $[\mathbf{W}]_{i,j} = \frac{c}{\sigma_{ij}}$, with c set so that the smallest weight value equals 1 to avoid numerical instability. In the case where no a-priori information is known about measurement quality, then all measurement weights could be set to 1.

3. SIMULATION RESULTS

For our simulations, we have assumed that nodes begin in a 20×20 meter area, and move randomly such that the straight line distance traveled in the x and y directions at each time step is distributed uniformly between -5 and 5 meters. The variance in both the range and odometry measurements is assumed proportional to the actual distance, which is consistent with both measurement types. The proportionality for range

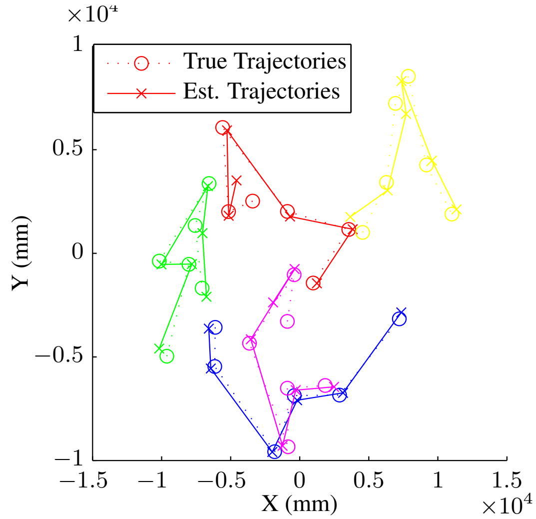

Fig. 1. Simulated reference-free tracking solution for $N = 5$, $K = 6$, $\sigma_r = 100$ mm, $\sigma_d = 10$ mm. The true trajectories are shown for comparison, aligned via the Procrustean transformation. Position RMSE $= 428$ mm.

and odometry are assumed known and denoted σ_r^2, σ_d^2 respectively. That is, σ_r^2, σ_d^2 are the variances in the range measurements *per meter*. In the results, we compute the root mean squared error (RMSE) in position estimates by first aligning the estimated reference-free map with the true trajectory map via a Procrustean transformation [3]. An example estimated trajectory for a set of 5 nodes is shown in Figure 1.

We characterize the general performance of our algorithm through comprehensive simulations of our approach for various values of node count, time steps, and noise levels. For the first simulation, we measure the average RMSE of position estimates for different values of node count N and time steps K. The results are shown in Figure 2. From these results, we see that estimation accuracy generally improves with increasing N and *decreasing* K. As N increases, more range data is available at each time index, and more odometry values are available between time steps. However, as K increases while leaving N constant, the sparsity of Δ increases. Thus, more positions must be estimated relative to the amount of data, decreasing accuracy.

For our second simulation, we set $N = 8$ and $K = 4$, with the noise standard deviations σ_r, σ_d allowed to vary. The performance generally decreases as measurement uncertainty increases. We note that across most of the range of σ_d, our algorithm performs fairly consistently at $5 - 6$ dBmm above the range noise level.

4. TRACKING RESULTS ON REAL DATA

We have also tested our proposed tracking method on real range and odometry data gathered by the CSOT testbed.

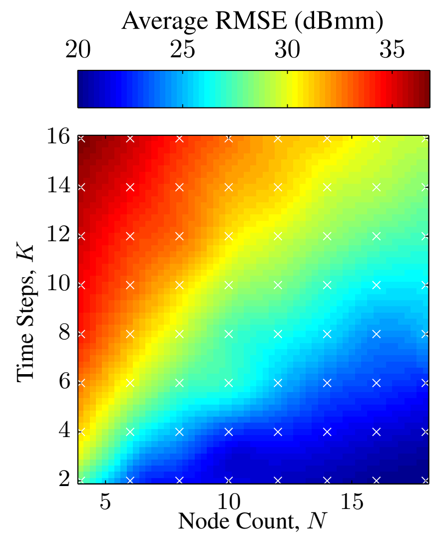

Fig. 2. First comprehensive simulation results, RMSE vs. K, N. Noise levels: $\sigma_r = 100$ mm, $\sigma_d = 10$ mm.

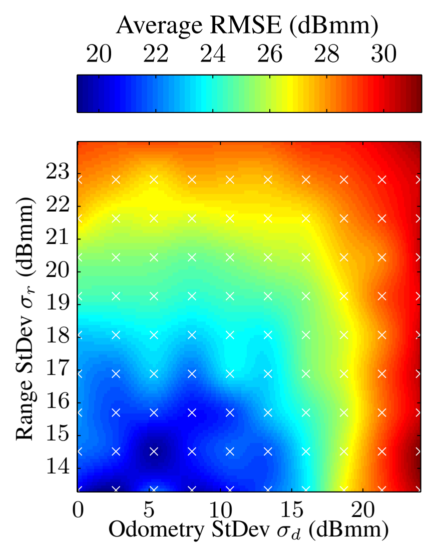

Fig. 3. Second comprehensive simulation results, RMSE vs. σ_r, σ_d, where $N = 8$, $K = 4$.

Originally introduced as RadioBOT in [18], the testbed features the ability to gather both highly accurate ultrawideband (UWB) range measurements and mobile robot odometry data. Each mobile node is equipped with two UWB antennas mounted above the robot on the left and right sides, respectively, spaced approximately 315 mm apart. The measured standard deviation of the UWB range measurement error is approximately $\sigma_r = 20$ mm, and is very stable with the distances measured. Odometry measurements are provided by the iRobot mobile base encoder wheels, with estimated standard deviation $\sigma_d = 5$ mm per meter traveled. Here

Fig. 4. Initial hallway robot configuration for Test I.

we present the results of two measurement campaigns conducted in a laboratory hallway, as a proof of concept to show our algorithm's ability to estimate node trajectories from actual measured data. The test parameters are summarized in Table1. The hallway test setup is shown in Figure 4.

	Test I	Test II
Stationary Nodes	3	7
Mobile Nodes	5	1
Surveyed Data	Stationary	Stationary + mobile
Total time steps K	19	27
RMSE vs. Survey Data	18 mm	124 mm

Table 1. CSOT measurement campaign summary

For Test I, three of the nodes remained stationary for the duration of the test, and their locations were surveyed to provide truth data. We stress that the tests are still reference-less, because the stationary node locations are not known to the algorithm. A complete set of pairwise range measurements was taken at each time step, and encoder wheel data gathered for each node's movement between time steps. The collected data were processed per our algorithm of Section 2, and the results of the tracking shown in Figure 5. The RMSE of the estimated positions is computed with the surveyed nodes after aligning the two maps, and was found to be 16 mm.

In Test II, seven of the nodes remained stationary in a cluster, while a single mobile node took a curved trajectory over a distance of approximately 10 m. Data was collected in the same manner as Test I, with the addition of surveying both the initial and final positions of the mobile node. The RMSE with all surveyed positions in this case was found to be 124 mm. We note the increase in error over Test I, as we are now comparing surveyed truth data for both stationary and mobile nodes. The test also took place over an increased number of time steps and over a larger distance.

5. CONCLUSIONS

In this paper we have proposed a novel reference-free means of estimating the trajectories of mobile nodes by adapting the popular MDS paradigm to use both inter-node range measurements and intra-node odometry data. Unlike other methods, our approach allows estimation of a node's trajectory jointly with all others, requires few assumptions, and preserves the reference frame across time steps.

Future research could expand our MDS approach in several ways. We believe position estimation accuracy could be improved by estimating the unknown values in Δ, and incorporating their uncertainty into the weighting scheme. Also, it would be beneficial to adapt our algorithm for real-time tracking, incorporating new data as it is collected. A distributed approach to computing the estimated trajectories would also be helpful for use in power restricted sensor networks.

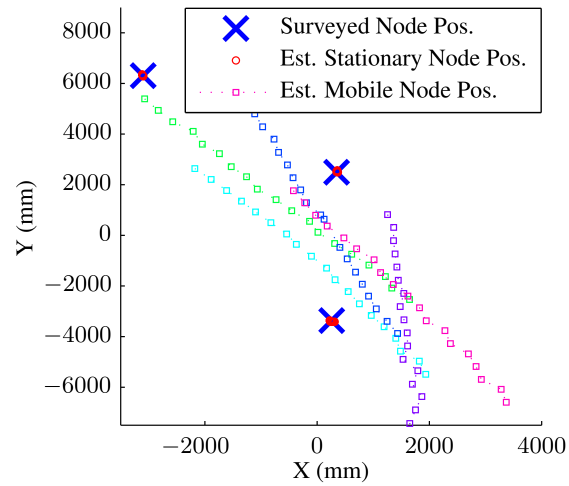

Fig. 5. Test I tracking results. RMSE = 18 mm

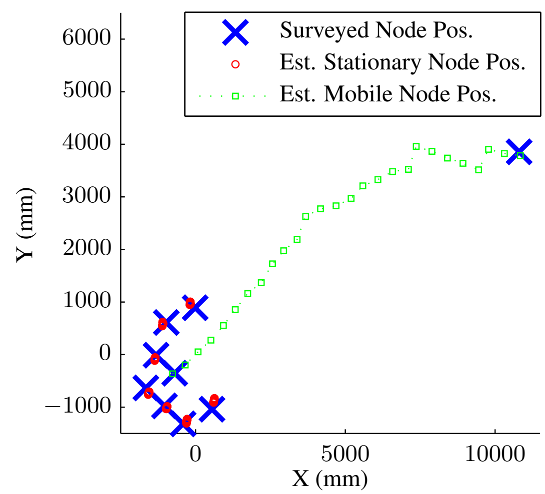

Fig. 6. Test II tracking results. RMSE = 124 mm.

6. REFERENCES

[1] N. Patwari, J.N. Ash, S. Kyperountas, A.O. Hero, R.L. Moses, and N.S. Correal, "Locating the nodes: cooperative localization in wireless sensor networks," *IEEE Signal Processing Magazine*, vol. 22, no. 4, pp. 54–69, 2005.

[2] H. Wymeersch, J. Lien, and M.Z. Win, "Cooperative localization in wireless networks," *Proceedings of the IEEE*, vol. 97, no. 2, pp. 427–450, 2009.

[3] I. Borg and P. Groenen, *Modern Multidimensional Scaling*, Springer Series in Statistics. Springer-Verlag New York, 1997.

[4] K.W. Cheung and H.C. So, "A multidimensional scaling framework for mobile location using time-of-arrival measurements," *Signal Processing, IEEE Transactions on*, vol. 53, no. 2, pp. 460–470, 2005.

[5] D. Macagnano and G.T.F. De Abreu, "Tracking multiple dynamic targets with multidimensional scaling," in *Personal, Indoor and Mobile Radio Communications (PIMRC), IEEE 18th International Symposium on*, 2007, pp. 1–5.

[6] F. Chan and H.C. So, "Efficient weighted multidimensional scaling for wireless sensor network localization," *Signal Processing, IEEE Transactions on*, vol. 57, no. 11, pp. 4548–4553, 2009.

[7] J. A. Costa, N. Patwari, and A. O. Hero, "Distributed weighted-multidimensional scaling for node localization in sensor networks," *Sensor Networks, ACM Transactions on*, vol. 2, no. 1, pp. 39–64, Feb. 2006.

[8] O.H. Kwon and H.J. Song, "Localization through map stitching in wireless sensor networks," *Parallel and Distributed Systems, IEEE Transactions on*, vol. 19, no. 1, pp. 93–105, 2008.

[9] O.H. Kwon, H.J. Song, and S. Park, "Anchor-free localization through flip-error-resistant map stitching in wireless sensor network," *Parallel and Distributed Systems, IEEE Transactions on*, vol. 21, no. 11, pp. 1644–1657, 2010.

[10] A. Amar, Y. Wang, and G. Leus, "Extending the classical multidimensional scaling algorithm given partial pairwise distance measurements," *IEEE Signal Processing Letters*, vol. 17, no. 5, pp. 473–476, 2010.

[11] P. Drineas, A. Javed, M. Magdon-Ismail, G. Pandurangan, R. Virrankoski, and A. Savvides, "Distance matrix reconstruction from incomplete distance information for sensor network localization," in *Sensor and Ad Hoc Communications and Networks (SECON), 3rd Annual IEEE Communications Society on*, 2006, vol. 2, pp. 536–544.

[12] Y. Shang and W. Ruml, "Improved mds-based localization," in *Computer and Communications Societies (INFOCOM), IEEE Joint Conference of*, 2004, vol. 4, pp. 2640–2651.

[13] H. Jamali-Rad and G. Leus, "Dynamic multidimensional scaling for low-complexity mobile network tracking," *Signal Processing, IEEE Transactions on*, vol. 60, no. 8, pp. 4485–4491, 2012.

[14] A. Al-Fuqaha, M. Elbes, and A. Rayes, "An intelligent data fusion technique based on the particle filter to perform precise outdoor localization," *Pervasive Computing and Communications, International Journal of*, vol. 9, no. 2, pp. 163–183, 2013.

[15] J.A. Corrales, F.A. Candelas, and F. Torres, "Hybrid tracking of human operators using IMU/UWB data fusion by a Kalman filter," in *Human-Robot Interaction (HRI), 3rd ACM/IEEE International Conference on*, 2008, pp. 193–200.

[16] E. North, J. Georgy, M. Tarbouchi, U. Iqbal, and A. Noureldin, "Enhanced mobile robot outdoor localization using INS/GPS integration," in *Computer Engineering Systems (ICCES), International Conference on*, 2009, pp. 127–132.

[17] H. Wang, H. Lenz, A. Szabo, J. Bamberger, and U.D. Hanebeck, "WLAN-based pedestrian tracking using particle filters and low-cost mems sensors," in *Positioning, Navigation and Communication (WPNC), 4th Workshop on*, 2007, pp. 1–7.

[18] B. M. Beck, J. H. Kim, R. J. Baxley, and B. T. Walkenhorst, "RadioBOT: A spatial cognitive radio testbed," in *IEEE Aerospace Conference*, 2013, pp. 1–9.