DSE 210: Worksheet #3 - Multiple Events, Conditioning, and Independence

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Problem 1

$\Omega = \{H,T\}^3$, so $|\Omega| = 2^3 = 8$.

(a) $p(2H \mid first\ is\ H) = \dfrac{p(2H \cap first\ is\ H)}{p(first\ is\ H)} = \dfrac{p(\{(H,H,T),(H,T,H)\}}{p(\{(H,H,H),(H,H,T),(H,T,H),(H,T,T)\}} = \dfrac{\frac{2}{8}}{\frac{4}{8}} = \boxed{\dfrac{1}{2}}$

(b) $p(2H \mid first\ is\ T) = \dfrac{p(2H \cap first\ is\ T)}{p(first\ is\ T)} = \dfrac{p(\{(T,H,H)\}}{p(\{(H,H,H),(H,H,T),(H,T,H),(H,T,T)\}} = \dfrac{\frac{1}{8}}{\frac{4}{8}} = \boxed{\dfrac{1}{4}}$

(c) $p(2H \mid first\ two\ are\ H) = p(third\ not\ H) = \boxed{\dfrac{1}{2}}$

(d) $p(2H \mid first\ two\ are\ T) = \boxed{0}$ (not possible)

(e) $p(2H \mid first\ is\ H\ and\ third\ is\ T) = p(second\ is\ H) = \boxed{\dfrac{1}{2}}$

Problem 3

Let S be the event that the bill passes the Senate, and H be the event that the bill passes the House.

The event that the bill passes both the House and the Senate is represented by $S \cap H$.

We are provided $p(H) = 0.6$, $p(S) = 0.8$, and $p(H \cup S) = 0.9$.

$p(H \cup S) = p(H) + p(S) - p(H \cap S) \implies p(H \cap S) = p(H) + p(S) - p(H \cup S)$,

so $p(H \cap S) = 0.6 + 0.8 - 0.9 = \boxed{0.5}$

Problem 5

(a) $p(heart \mid red) = \dfrac{p(heart \cap red)}{p(red)} = \dfrac{\frac{1}{4}}{\frac{1}{2}} = \boxed{\dfrac{1}{2}}$

(b) $p(> 10 \mid heart) = \dfrac{p(> 10\ \cap heart)}{p(heart)} = \dfrac{\frac{4}{52}}{\frac{1}{4}} = \boxed{\dfrac{4}{13}}$

(c) $p(Jack \mid\ > 10) = \dfrac{p(Jack \cap > 10)}{p(> 10)} = \dfrac{\frac{4}{52}}{\frac{16}{52}} = \boxed{\dfrac{1}{4}}$

Problem 7

(a) $p(\text{total is} > 7 \mid \text{first is } 4) = \dfrac{p(\text{total is} > 7 \cap \text{first is } 4)}{p(\text{first is } 4)} = \dfrac{p(\{(4,4),(4,5),(4,6)\})}{p(4)} = \dfrac{\frac{3}{36}}{\frac{1}{6}} = \boxed{\dfrac{1}{2}}$

(b) $p(\text{total is} > 7 \mid \text{first is } 1) = \boxed{0}$ (not possible)

(c) $p(\text{total is} > 7 \mid \text{first is} > 3) = \dfrac{p(\text{total is} > 7 \cap \text{first is} > 3)}{p(\text{first is} > 3)}$

$= \dfrac{p(\{(4,4),(4,5),(4,6),(5,3),(5,4),(5,5),(5,6),(6,2),(6,3),(6,4),(6,5),(6,6)\})}{p(\{4,5,6\})} = \dfrac{\frac{12}{36}}{\frac{3}{6}} = \boxed{\dfrac{2}{3}}$

(d) $p(\text{total is} > 7 \mid \text{first is} < 5) = \dfrac{p(\text{total is} > 7 \cap \text{first is} < 5)}{p(\text{first is} < 5)}$

$= \dfrac{p(\{(2,6),(3,5),(3,6),(4,4),(4,5),(4,6)\})}{p(\{1,2,3,4\})} = \dfrac{\frac{6}{36}}{\frac{4}{6}} = \boxed{\dfrac{1}{4}}$

Problem 9

We are provided the following:

$p(F_1) = 0.25$, $p(defective \mid F_1) = 0.05$,

$p(F_2) = 0.35$, $p(defective \mid F_2) = 0.04$,

$p(F_3) = 0.4$, $p(defective \mid F_3) = 0.02$

(a) $p(defective)$
$= p(defective \mid F_1) \times p(F_1) + p(defective \mid F_2) \times p(F_2) + p(defective \mid F_3) \times p(F_3)$
$= 0.05 \times 0.25 + 0.04 \times 0.35 + 0.02 \times 0.4 \approx \boxed{0.219,\ or\ 21.9\%}$

(b) $p(F_1 \mid defective) = \dfrac{p(defective \mid F_1) \times p(F_1)}{p(defective)} = \dfrac{0.05 \times 0.25}{0.219} \approx \boxed{0.057,\ or\ 5.7\%}$

Problem 11

We are provided the following:

$p(d_1) = p(d_2) = p(d_3) = \frac{1}{3},\ p(+ \mid d_1) = 0.8,\ p(+ \mid d_2) = 0.6,\ p(+ \mid d_3) = 0.4$

(a) $p(+) = p(+ \mid d_1) \times p(d_1) + p(+ \mid d_2) \times p(d_2) + p(+ \mid d_3) \times p(d_3)$
$= 0.8 \times \frac{1}{3} + 0.6 \times \frac{1}{3} + 0.4 \times \frac{1}{3} = \boxed{0.6}$

(b) $p(d_1 \mid +) = \dfrac{p(+ \mid d_1) \times p(d_1)}{p(+)} = \dfrac{0.8 \times \frac{1}{3}}{0.6} = \boxed{\dfrac{4}{9}},$

$p(d_2 \mid +) = \dfrac{p(+ \mid d_2) \times p(d_2)}{p(+)} = \dfrac{0.6 \times \frac{1}{3}}{0.6} = \boxed{\dfrac{1}{3}},$

$p(d_3 \mid +) = \dfrac{p(+ \mid d_3) \times p(d_3)}{p(+)} = \dfrac{0.4 \times \frac{1}{3}}{0.6} = \boxed{\dfrac{2}{9}}$

Problem 13

We are provided the following:

$p(tiger) = \frac{1}{3},\ p(+ \mid tiger) = \frac{5}{6}, p(mammoth) = \frac{2}{3},\ p(+ \mid mammoth) = \frac{1}{3}.$

$$p(tiger \mid -) = \frac{p(- \mid tiger) \times p(tiger)}{p(-)} = \frac{\big(1 - p(+ \mid tiger)\big) \times p(tiger)}{1 - p(+)},$$

and $p(+) = p(+ \mid tiger) \times p(tiger) + p(+ \mid mammoth) \times p(mammoth) = \frac{5}{6} \times \frac{1}{3} + \frac{1}{3} \times \frac{2}{3} = \frac{5}{18} + \frac{2}{9} = \dfrac{1}{2},$

so $p(tiger \mid -) = \dfrac{(1 - \frac{5}{6}) \times \frac{1}{3}}{1 - \frac{1}{2}} = \dfrac{\frac{1}{6} \times \frac{1}{3}}{\frac{1}{2}} = \dfrac{\frac{1}{18}}{\frac{1}{2}} = \boxed{\dfrac{1}{9}}$

Problem 15

E_1, E_2 are independent events if $p(E_1 \cap E_2) = p(E_1) + p(E_2)$.

(a) (1) $\boxed{A,\ B\ independent}$

(2) $p(A) = \frac{1}{2}, p(D) = p(\{(H,H,H),(T,T,T)\}) = \frac{2}{8} = \frac{1}{4}$,

$p(A \cap D) = p(\{(H,H,H)\}) = \frac{1}{8}$,

$p(A) \times p(D) = \frac{1}{2} \times \frac{1}{4} = \frac{1}{8} = p(A \cap D) \implies \boxed{A,\ D\ independent}$

(3) $p(A) = \frac{1}{2}, p(E) = p(\{(H,T,T),(T,H,T),(T,T,H)\}) = \frac{3}{8}$,

$p(A \cap E) = p(\{(H,T,T)\}) = \frac{1}{8}$,

$p(A) \times p(E) = \frac{1}{2} \times \frac{3}{8} = \frac{3}{16} \neq \frac{1}{8} = p(A \cap E) \implies \boxed{A,\ E\ not\ independent}$

(4) $p(D) = \frac{1}{4}, p(E) = \frac{3}{8}$,

$p(D \cap E) = 0\ (not\ possible)$,

$p(D) \times p(E) \neq p(D \cap E) \implies \boxed{D,\ E\ not\ independent}$

(b) (1) $\boxed{A, B, C\ independent}$

(2) $p(A) \times p(B) \times p(D) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{4} = \frac{1}{16}$,

$p(A \cap B \cap D) = 0\ (not\ possible)$,

$p(A) \times p(B) \times p(D) \neq p(A \cap B \cap D) \implies \boxed{A, B, D\ not\ independent}$

(3) $\boxed{C, D, E\ not\ independent}$

Problem 17

We are provided the following:

$p(UCLA) = 0.5,\ p(UCSD) = 0.3,\ p(UCLA \cap UCSD) = 0.2.$

(a) $p(UCSD \mid UCLA) = \dfrac{p(UCSD \cap UCLA)}{p(UCLA)} = \dfrac{0.2}{0.5} = \boxed{0.4}$

(b) $p(UCLA) \times p(UCSD) = 0.5 \times 0.3 = 0.15 \neq 0.2 = p(UCLA \cap UCSD) \implies \boxed{UCLA, UCSD\ not\ independent}$