DSE 210: Worksheet #2 - Probability Spaces

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Problem 1

(a) $\Omega = \{A, B\}$

(b) $\Omega = \{H, T\}$

(c) $\Omega = \{months\} \times \{days\ of\ week\} = \{(Jan,\ Mon),\ (Jan,\ Tue),\ \ldots\ ,(Dec,\ Sat),\ (Dec,\ Sun)\}$,
$|\Omega| = 12 \times 7 = 84$

(d) $\Omega = \{s1,\ s2,\ s3,\ \ldots,\ s10\}$

(e) $\Omega = \{exterior\ colors\} \times \{interior\ colors\} = \{\text{red, black, silver, green}\} \times \{\text{black, beige}\} =$
$\{$(red, black), (red, beige), (black, black), (black, beige), (silver, black), (silver, beige), (green, black), (green, beige)$\}$,
$|\Omega| = 4 \times 2 = 8$

Problem 3

(a) $A \cap B \cap C$

(b) $A \cup B \cup C$

(c) $A \cap B \cap C^c$

Problem 5

(a) $E_1 = \{H\ on\ first\ toss\},\ p(E_1) = \dfrac{|E_1|}{|\Omega|} = \dfrac{4}{8} = \boxed{\dfrac{1}{2}}$

(b) $E_2 = \{all\ outcomes\ the\ same\},\ p(E_2) = \dfrac{|E_2|}{|\Omega|} = \dfrac{2}{8} = \boxed{\dfrac{1}{4}}$

(c) $E_3 = \{exactly\ one\ T\},\ p(E_3) = \dfrac{|E_3|}{|\Omega|} = \boxed{\dfrac{3}{8}}$

Problem 7

Sample space $\Omega = \{1, 2, 3, 4, 5, 6\}^2 = \{(1, 1), (1, 2), (1, 3), \ldots, (6, 4), (6, 5), (6, 6)\}$.

Event of interest $A = \{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)\}$.

$|\Omega| = 36,\ |A| = 6$, so $p(A) = \dfrac{|A|}{|\Omega|} = \dfrac{6}{36} = \boxed{\dfrac{1}{6}}$

Problem 9

$\Omega = \{1, 2, 3, 4, 5, 6\}$, and $p(1) = p,\ p(2) = 2p,\ p(3) = 3p,\ p(4) = 4p,\ p(5) = 5p,\ p(6) = 6p$.

$p + 2p + 3p + 4p + 5p + 6p = 1 \implies p = \frac{1}{21}$.

$p(even\ number) = p(2) + p(4) + p(6) = 2p + 4p + 6p = \frac{2}{21} + \frac{4}{21} + \frac{6}{21} = \frac{12}{21} = \boxed{\frac{4}{7}}$

Problem 11

$|\Omega| = 5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$.

Since all five people are of different height, there is only one correct increasing order of height, so $|A| = 1$.

$p(correct\ order) = \frac{|A|}{|\Omega|} = \boxed{\frac{1}{120}}$

Problem 13

Assuming order matters (i.e. the first four cards dealt must be Aces and the fifth card a King):

$|\Omega| = {}_{52}P_5 = \frac{52!}{(52-5)!} = \frac{52!}{47!} = 52 \times 51 \times 50 \times 49 \times 48$,

$|A| = {}_4P_4 \times {}_4P_1 = 4! \times 4 = 24 \times 4 = 96$,

so $p = \frac{|A|}{|\Omega|} = \boxed{\frac{96}{{}_{52}P_5}\ (if\ order\ matters)}$

Assuming order does not matter:

$|\Omega| = {}_{52}C_5 = \frac{52!}{(52-5)!\ 5!} = \frac{52!}{47!\ 5!} = \frac{52 \times 51 \times 50 \times 49 \times 48}{5 \times 4 \times 3 \times 2 \times 1}$,

$|A| = {}_4C_4 \times {}_4C_1 = 1 \times 4 = 4$,

so $p = \frac{|A|}{|\Omega|} = \boxed{\frac{4}{{}_{52}C_5}\ (if\ order\ does\ not\ matter)}$

Problem 15

$|\Omega| = {}_4P_4 = 4! = 24$,

$|A| = 1$,

so $p = \frac{|A|}{|\Omega|} = \boxed{\frac{1}{24}}$

Problem 17

$\Omega = \{any\ 3\ of\ the\ 7\ dwarves\}$, so $|\Omega| = {}_7C_3 = \dfrac{7!}{(7-3)!\ 3!} = \dfrac{7!}{4!\ 3!} = \dfrac{7 \times 6 \times 5}{3 \times 2 \times 1} = \dfrac{210}{6} = 35.$

(a) A = {any 2 dwarves plus Dopey}, i.e. choose Dopey and any 2 of the remaining 6 dwarves,

so $|A| = {}_6C_2 = \dfrac{6 \times 5}{2 \times 1} = \dfrac{30}{2} = 15,$

and $p(A) = \dfrac{|A|}{|\Omega|} = \dfrac{15}{35} = \boxed{\dfrac{3}{7}}$

(b) B = {any 1 dwarf plus Dopey and Sneezy}, i.e. choose Dopey and Sneezy and any 1 of the remaining 5 dwarves,

so $|B| = {}_5C_1 = 5,$

and $p(B) = \dfrac{|A|}{|\Omega|} = \dfrac{5}{35} = \boxed{\dfrac{1}{7}}$

(c) C = {any 3 dwarves not including Dopey or Sneezy}, i.e. choose 3 dwarves from the 5 remaining after removing Dopey and Sneezy,

so $|C| = {}_5C_3 = \dfrac{5!}{(5-3)!\ 3!} = \dfrac{5!}{2!\ 3!} = \dfrac{5 \times 4 \times 3}{3 \times 2 \times 1} = \dfrac{60}{6} = 10,$

and $p(c) = \dfrac{|A|}{|\Omega|} = \dfrac{10}{35} = \boxed{\dfrac{2}{7}}$