Conical Routing For Lookahead

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Contents

1 Introduction

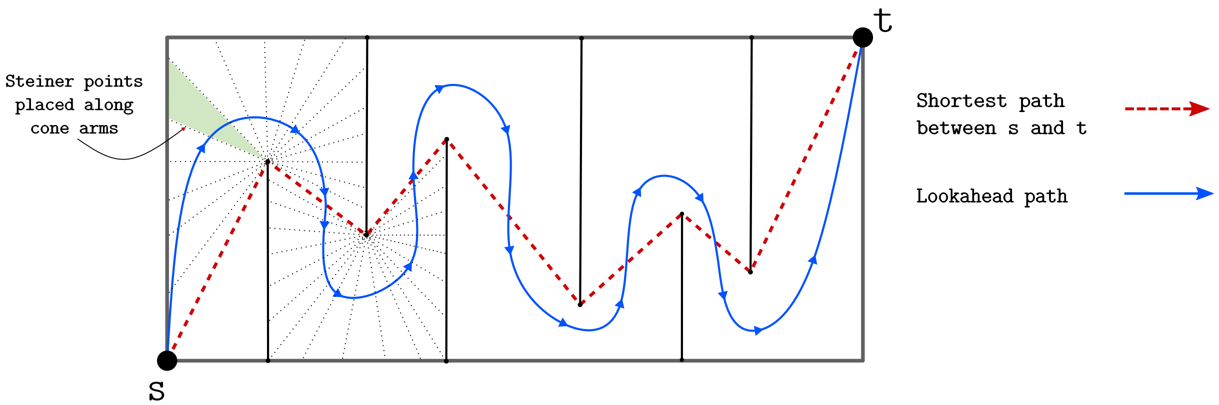


Figure 1: Conical Routing between start s and destination t

This document presents a scheme to solve the lookahead problem in the 'stalactite and stalagmite' setting using a method based on discretizing the space into a sequence of cones. The hope is that, at least experimentally, this scheme will lead to short lookahead paths where each point along the curve can see a sufficient chunk of the curve immediately ahead of it. [1]

The basic idea is captured in Figure 1:

Since this document is meant to be a proof-of-concept, for the scheme I will use the following simplifying assumptions: stalactites and stalagmites alternate in their placement and that their arrangement is such that the shortest path between s and t is taut against their tips.

The scheme is based on the intuition that if the string length upper bounded by a number [2] maximizing lookahead is equivalent to we need to either maximizing minimum amount of path string in each cone (or alternatively in each line).

[1] We are using Euclidean visibility here where a point A sees another point B iff the line segment AB is contained in the closure of the free space.

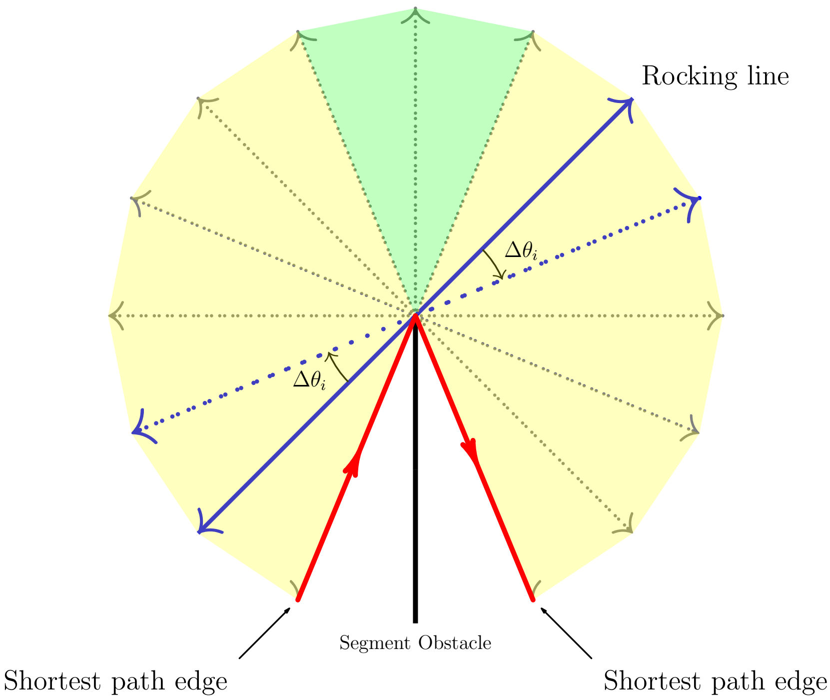
[2] say $O(1)L$, where L is the length of the shortest path between s and t, such as the red path in Figure 1

Then to get the shortest path with the specified amount of lookahead we just do a binary or parametric search over different string lengths to find the smallest string length that does not violate the given lookahead bound.

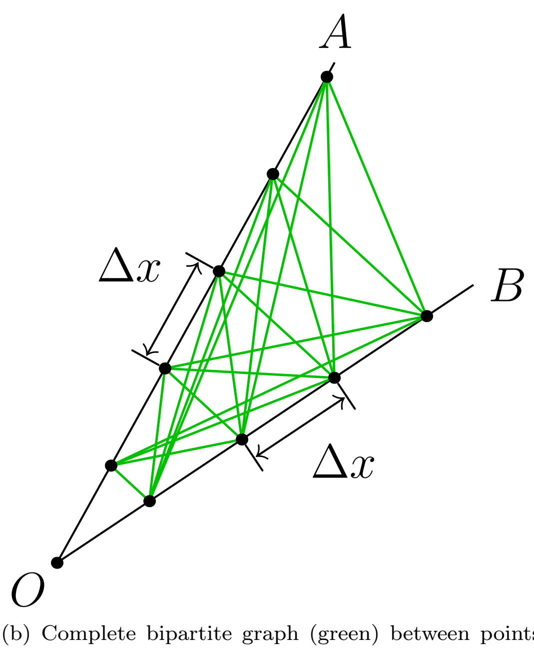
For the scheme

- ❖ Consider a finite sequence of small angles $\Delta\theta_i, 0 \leq i \leq N$, where $N \to \infty$, such that $\sum \Delta\theta_i =$ reflex angle between successive segments (red segments in Figure 2) of a shortest path incident at an obstacle tip . We draw cones of angles $\Delta\theta_i, i \in \mathbb{N}$ generated by a rocking line at each stalactite and stalagmite tip between these shortest path segments. In Figure 2 the area not swept by the line during its motion is a wedge facing upward (colored green). Note that the green wedge itself is discretized into small cones as part of the cone sequence just mentioned.
- ❖ Place Steiner points separated by a small distance Δx starting from the obstacle segment tip along each arm of every cone, for as long as the cone arm lies inside free space.
- ❖ Draw the complete bipartite graph between the Steiner points of each cone as shown in Figure 2
- ❖ Solve the following linear program, imposing any natural shape constraints if required.

Heavy use of the CGAL kernel via its Python bindings have been made in the code which implements the scheme above to ensure all geometric computations are precise.



(a) A rocking line (blue) creates a sequence of cones of angles $\Delta\theta_i$ between two successive shortest path edges



(b) Complete bipartite graph (green) between points on two arms of a cone on the Steiner points. Distance between two consecutive Steiner points along each arm is Δx. O is the tip of the stalactite/stalagmite.

Figure 2: Geometry of the discretization in the neighborhood of each stalactite/stalagmite tip