

Package Delivery with Trucks and Drones: The Horsefly Problem

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Using Trucks and Drones in Tandem



amazon



FedEx
Express

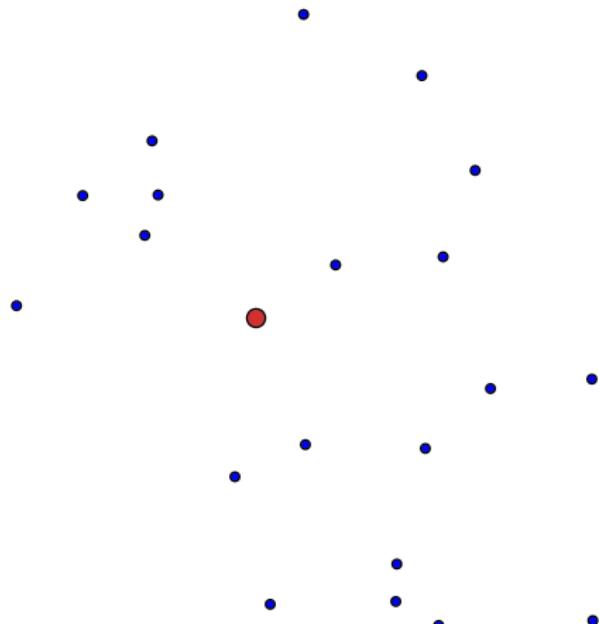
 **UNITED STATES
POSTAL SERVICE**



"UPS has estimated that cutting off just one mile for the routes of each of the company's 66,000 delivery drivers would amount to \$50 million in savings. For this reason, UPS is testing drone deliveries, using the top of its vans as a mini-helipad."¹

¹From <https://www.businessinsider.com/amazon-and-ups-are-betting-big-on-drone-delivery-2018-3>

Statement of Horsefly-PATH



Speed Ratio $\varphi = 3.0$

Truck Path :

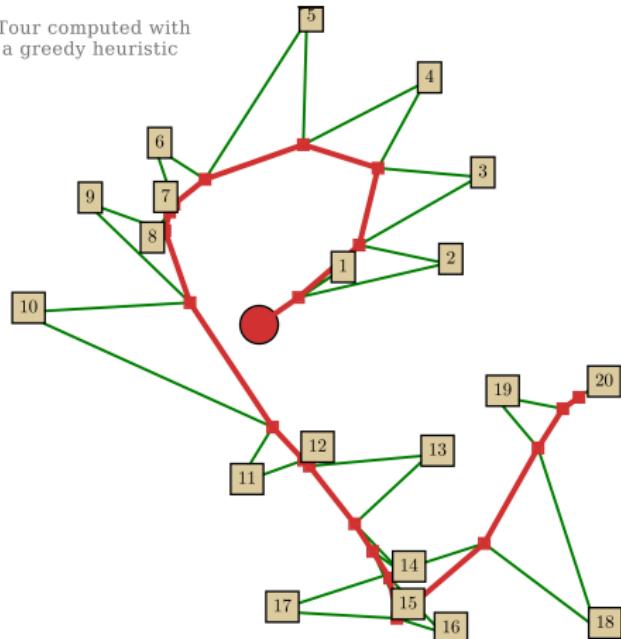
Drone Path :

Compute a route for the truck and for the drone in order to complete the delivery of all n packages (and have the drone return back to the empty truck) was soon as possible i.e., we seek to **minimize makespan** of the delivery process.

Both the **order** of visiting the sites and **Steiner points** need to be computed!

Statement of Horsefly-PATH

Tour computed with
a greedy heuristic

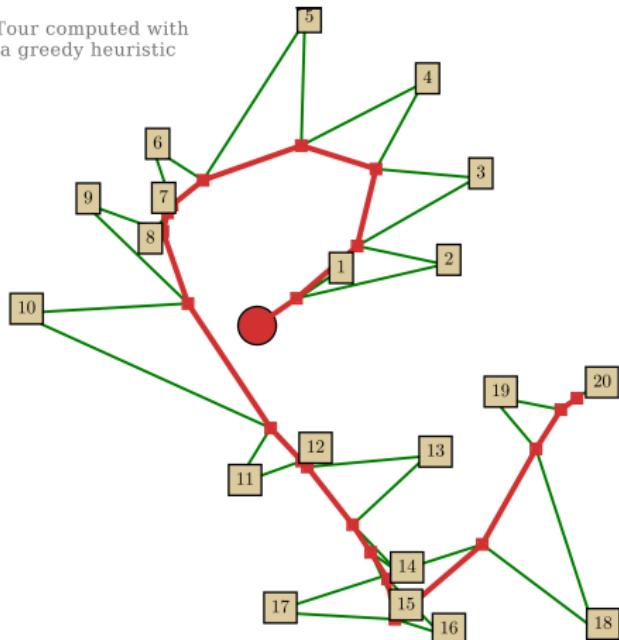


Compute a route for the truck and for the drone in order to complete the delivery of all n packages (and have the drone return back to the empty truck) was soon as possible i.e., we seek to **minimize makespan** of the delivery process.

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Preliminary Notes

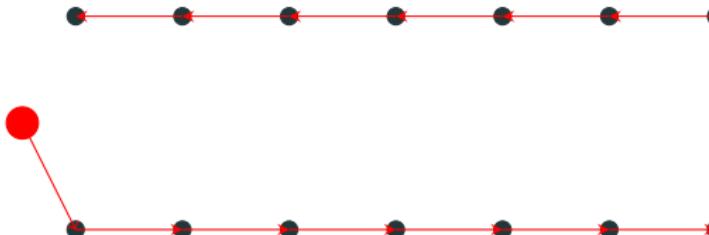
Tour computed with
a greedy heuristic



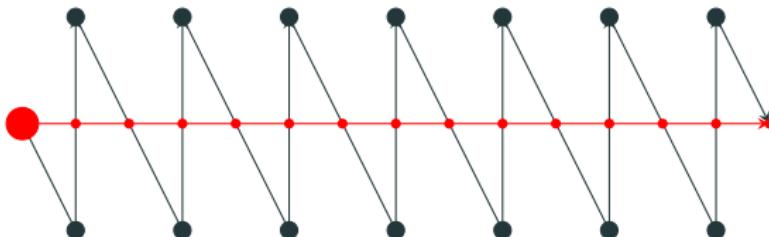
Speed Ratio $\varphi = 3.0$
Truck Path : ———
Drone Path : ———

- (1) Horsefly is **NP-hard**, by reduction from TSP ($\varphi = 1.0$)
- (2) Truck or drone **never wait** in OPT for Horsefly-PATH.
- (3) The truck route and the drone route are **polygonal**
- (4) The truck and the drone move always at their **maximum speeds** (1 and φ , respectively).

Preliminary Notes: Optimal Order depends on φ

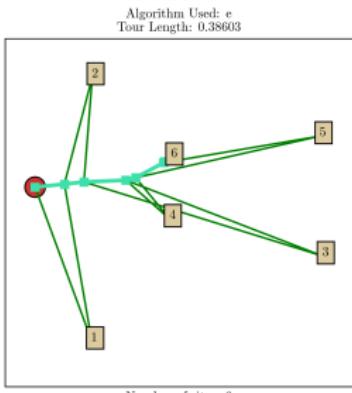
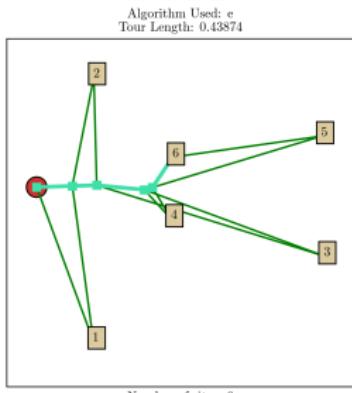
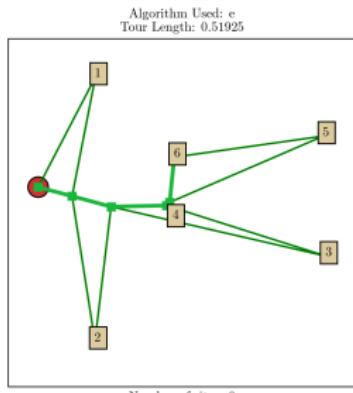
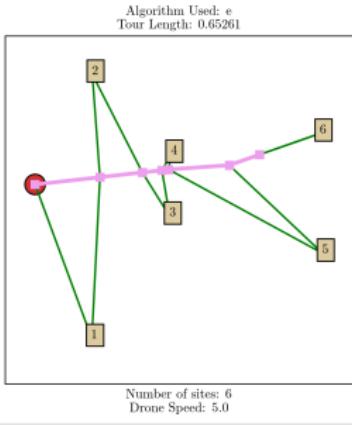
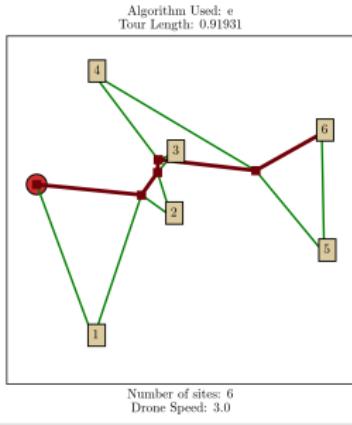
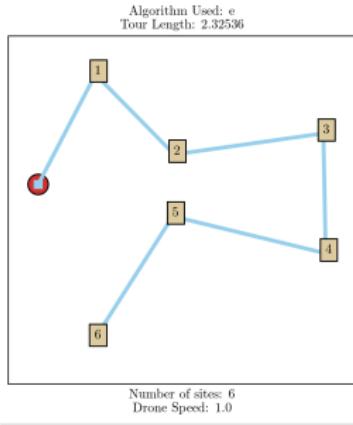


$$\varphi = 1.0$$

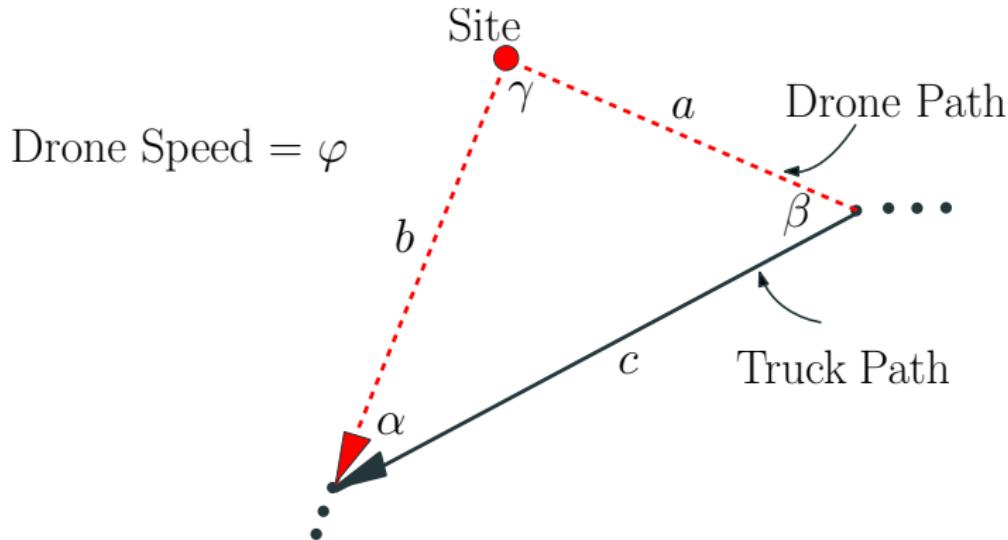


$$\varphi = 2 + \sqrt{5}$$

Preliminary Notes: Optimal Order depends on φ

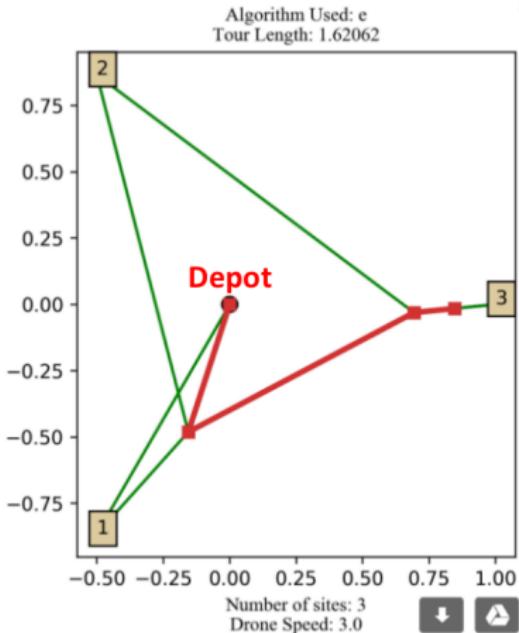
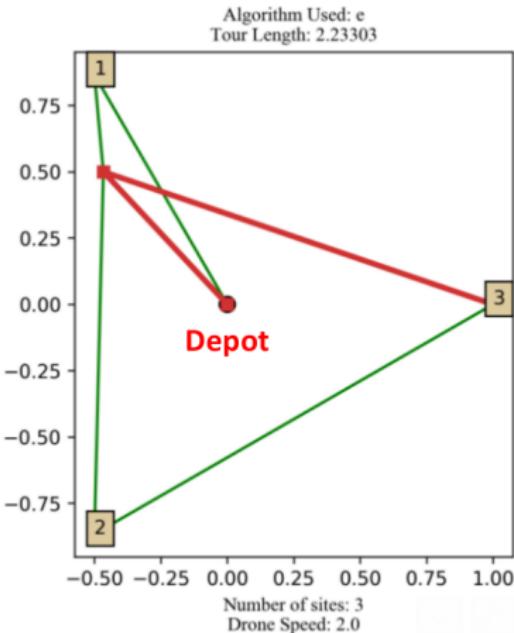


Preliminary Notes: A Local Optimality Condition



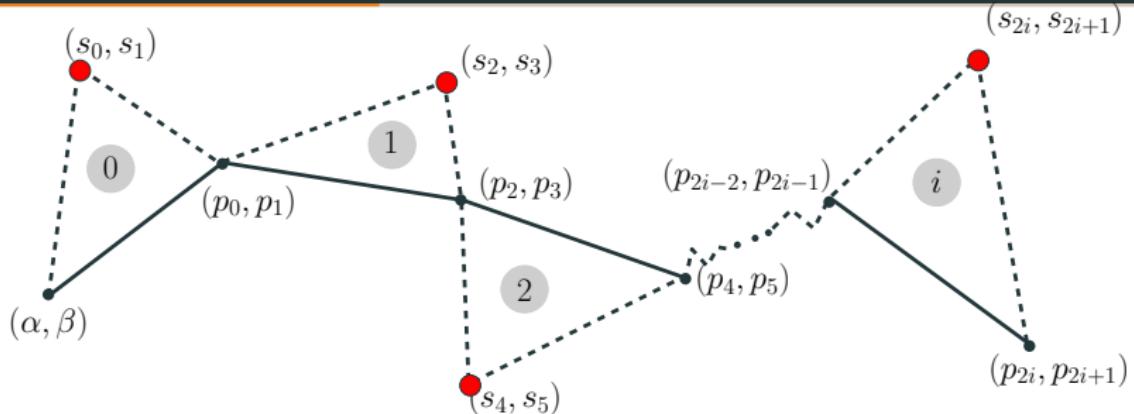
$$\varphi = \frac{\sin(\alpha) + \sin(\beta)}{\sin(\alpha + \beta)}$$

Preliminary Notes: Drone Path may self-intersect in OPT



Unsure: Might the opt truck route self-intersect? We have not seen it, but exchange argument not yet working to prove it

When Order of Visitation is Given (Exact Solution L2)



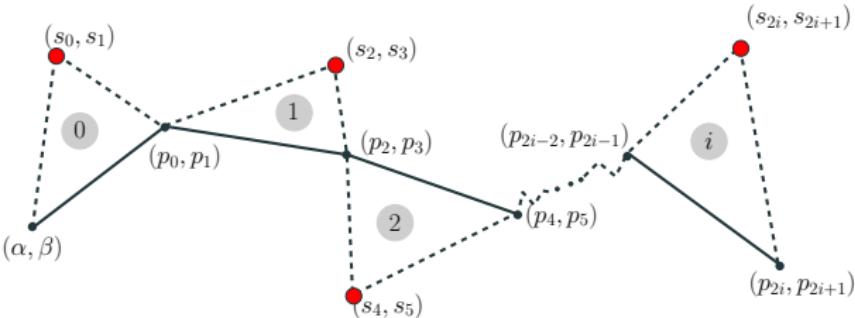
$$\min_{(p_0, p_1, \dots, p_{2n-1}) \in \mathbb{R}^{2n}} \sum_{i=1}^n \|P_i - P_{i-1}\|_2$$

subject to n constraints

$$\|P_i - P_{i-1}\|_2 = \frac{\|P_{i-1} - S_i\|_2 + \|S_i - P_i\|_2}{\varphi} \quad \text{for } i \in \{1, 2, \dots, n\}$$

This can be formulated as an SOCP (Carlsson, Song), but in experiments here we use a generic non-linear solver from SciPy.

When Order of Visitation is Given (Exact Solution L1)



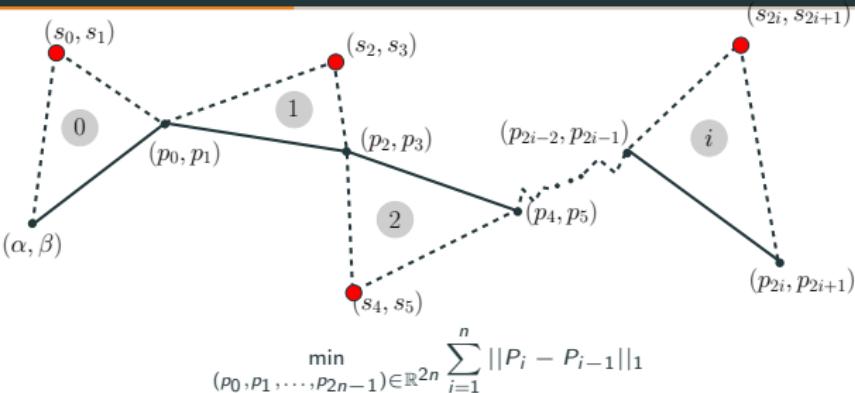
$$\min_{(p_0, p_1, \dots, p_{2n-1}) \in \mathbb{R}^{2n}} \sum_{i=1}^n ||P_i - P_{i-1}||_1$$

subject to n constraints

$$||P_i - P_{i-1}||_1 = \frac{||P_{i-1} - S_i||_1 + ||S_i - P_i||_1}{\varphi} \quad \text{for } i \in \{1, 2, \dots, n\}$$

This can be formulated as an LP. We use the MOSEK LP solver in our experiments.

When Order of Visitation is Given (Exact Solution L1)



Constraints associated with triangle 0

$$0 \leq f_0, f_1, h_0, h_1$$

$$|s_0 - \alpha| = b_0$$

$$|s_1 - \beta| = b_1$$

$$|p_0 - \alpha| \leq h_0$$

$$|p_1 - \beta| \leq h_1$$

$$|p_0 - s_0| \leq f_0$$

$$|p_1 - s_1| \leq f_1$$

$$f_0 + f_1 + b_0 + b_1 = \varphi h_0 + \varphi h_1$$

Constraints associated with triangle i

$$0 \leq b_{2i}, b_{2i+1}, f_{2i}, f_{2i+1}, h_{2i}, h_{2i+1}$$

$$|s_{2i} - p_{2i-2}| \leq b_{2i}$$

$$|s_{2i+1} - p_{2i-1}| \leq b_{2i+1}$$

$$|s_{2i} - p_{2i}| \leq f_{2i}$$

$$|s_{2i+1} - p_{2i+1}| \leq f_{2i+1}$$

$$|p_{2i} - p_{2i-2}| \leq h_{2i}$$

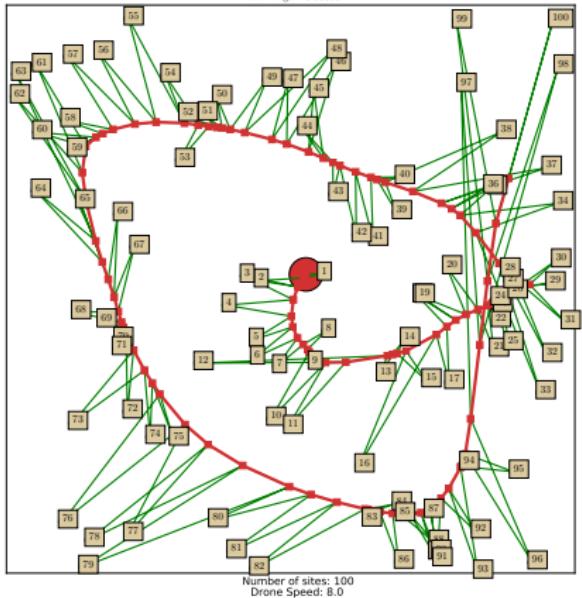
$$|p_{2i+1} - p_{2i-1}| \leq h_{2i+1}$$

$$f_{2i} + f_{2i+1} + b_{2i} + b_{2i+1} = \varphi h_{2i} + \varphi h_{2i+1}$$

L_2 Solution vs. L_1 Solution: An Example

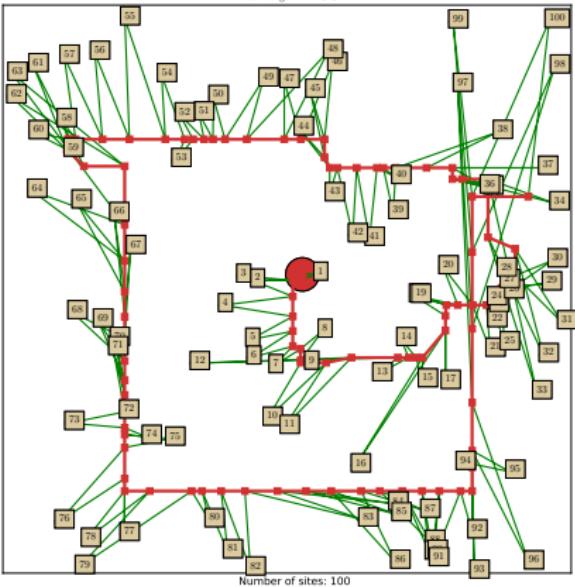
Tour Length: 3.06500

Algorithm Used: g
Tour Length: 3.06500



Tour Length: 4.21343

Algorithm Used: gl
Tour Length: 4.21343



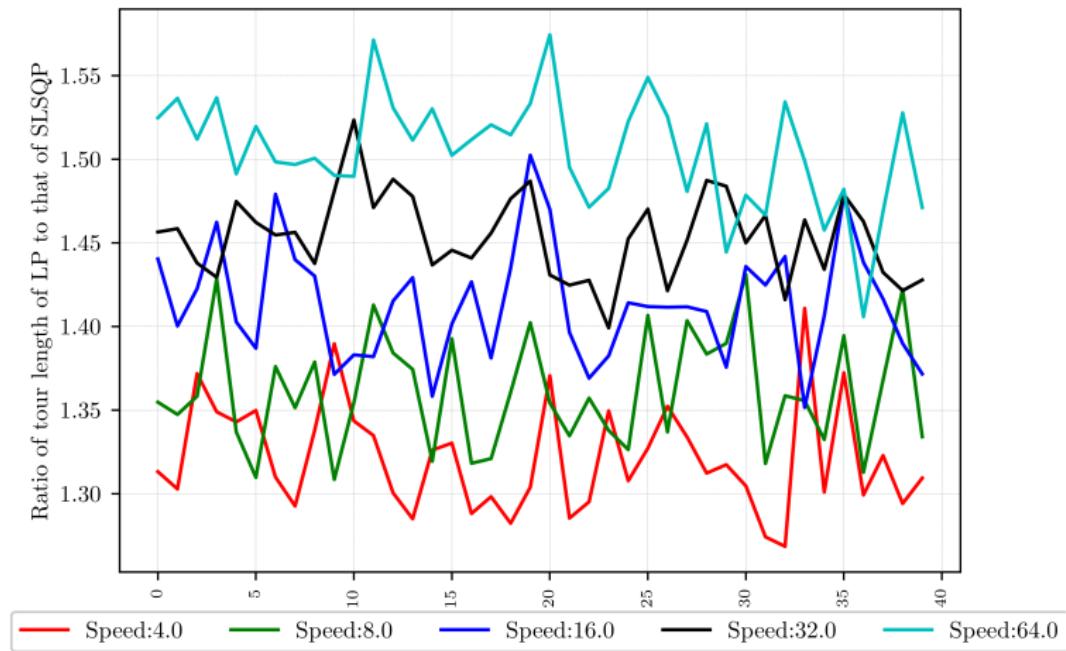
Number of Sites = 100

$$\varphi = 8.0$$

Orderings of sites for both pictures are the same.

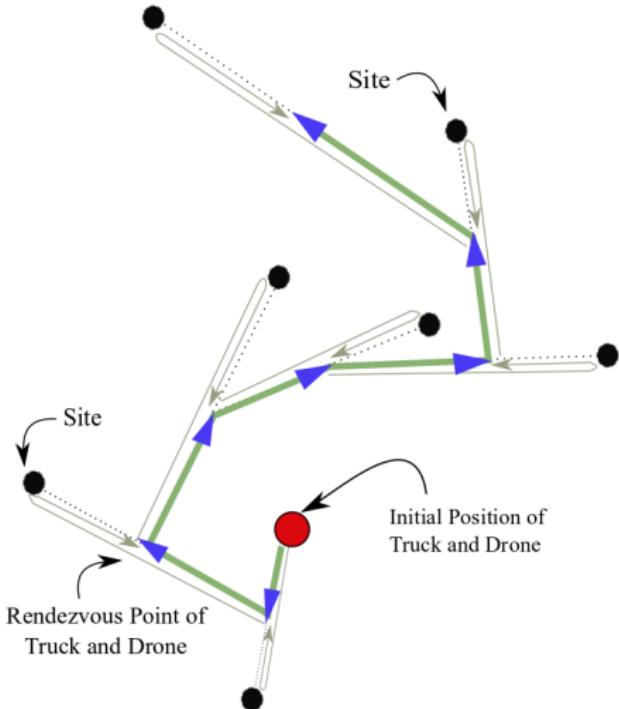
Computed with a Greedy heuristic.

$L2$ Solution vs. $L1$ Solution



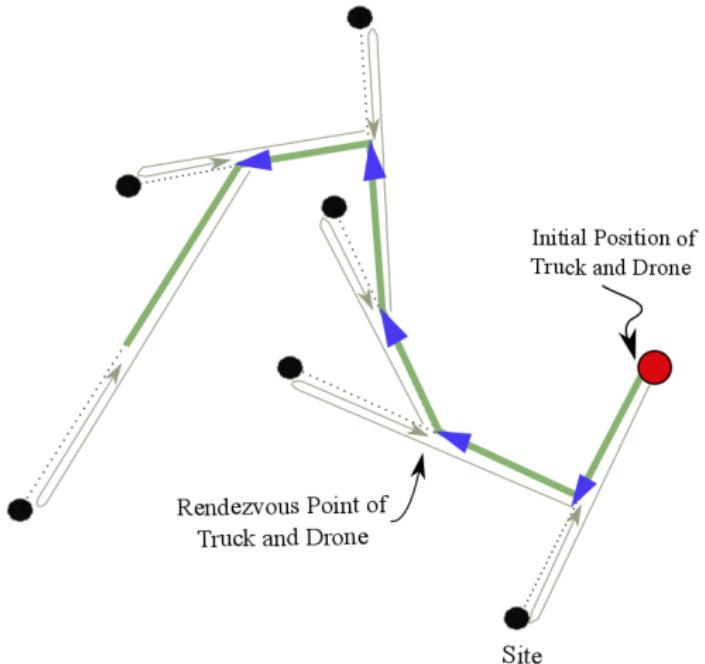
Conjectured Worst-case Approximation Ratio of $L2$ to $L1$: $\sqrt{2} o(\varphi)$

A Special Case: Collinear Horseflies



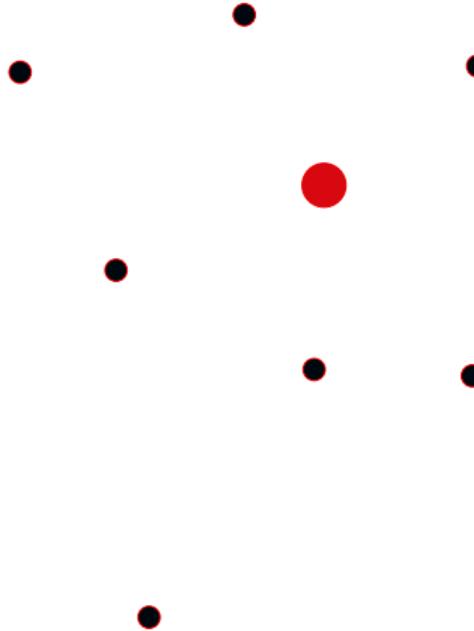
Truck and drone
always move towards a
site.

Greedy Heuristic (1) : Nearest Neighbor



Truck and drone visit
the nearest unserviced
site.

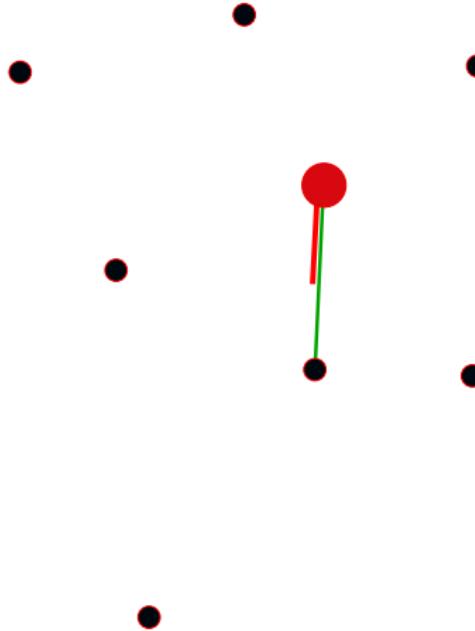
Greedy Heuristic (2) : Cheapest Insertion



In the case of TSP ($\varphi = 1$), this strategy gives us a 2 approximation.

Belief : For Horsefly the approximation is $O(1)o(\varphi)$

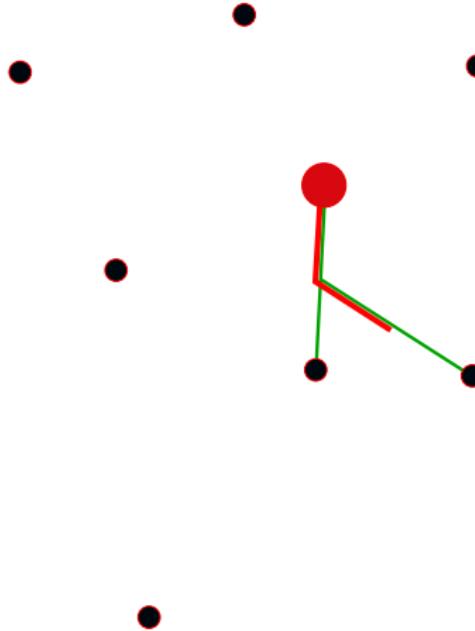
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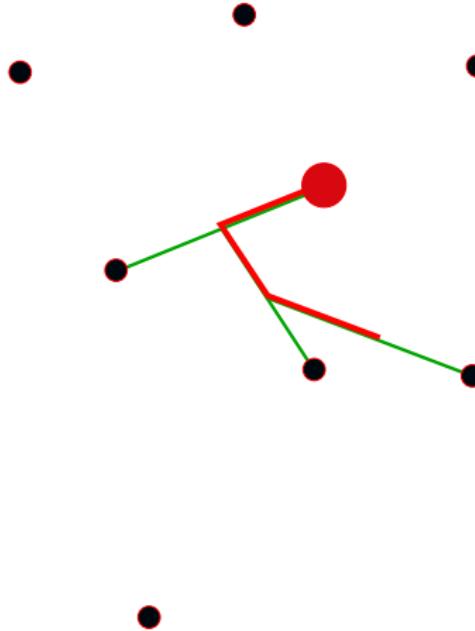
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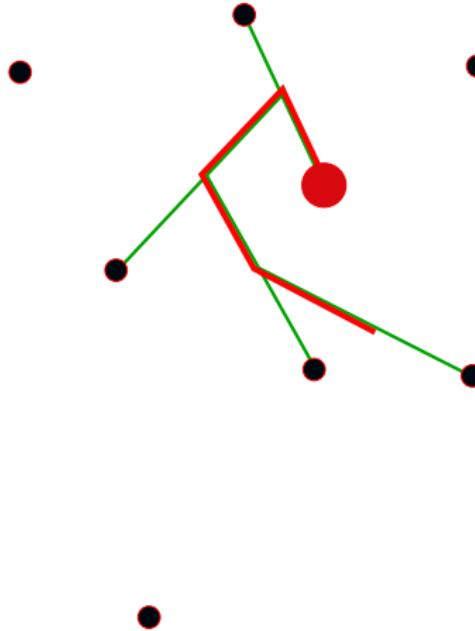
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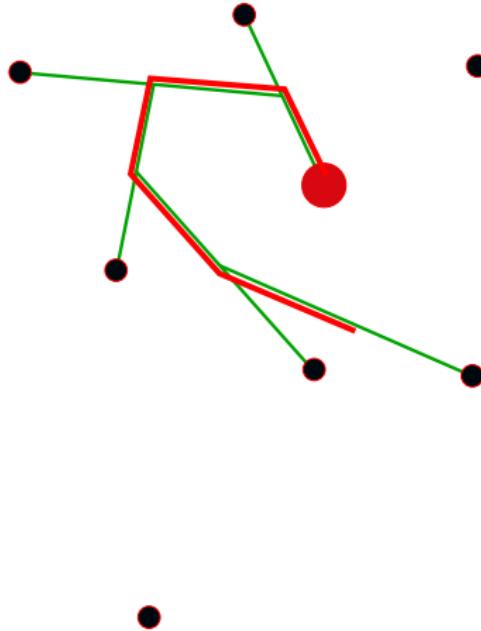
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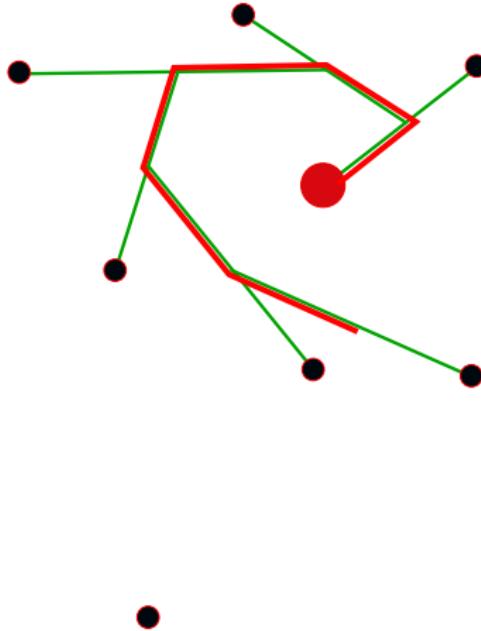
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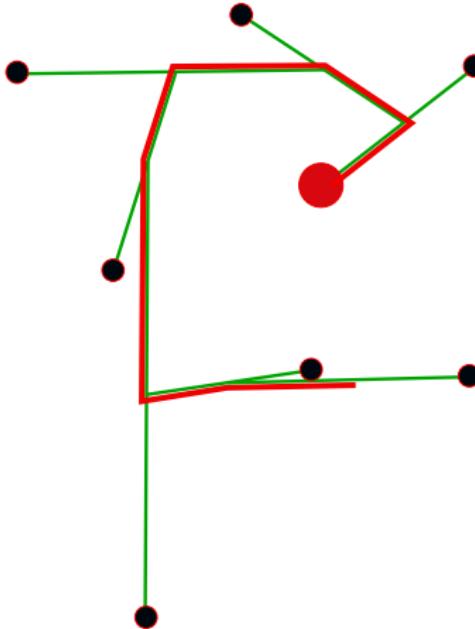
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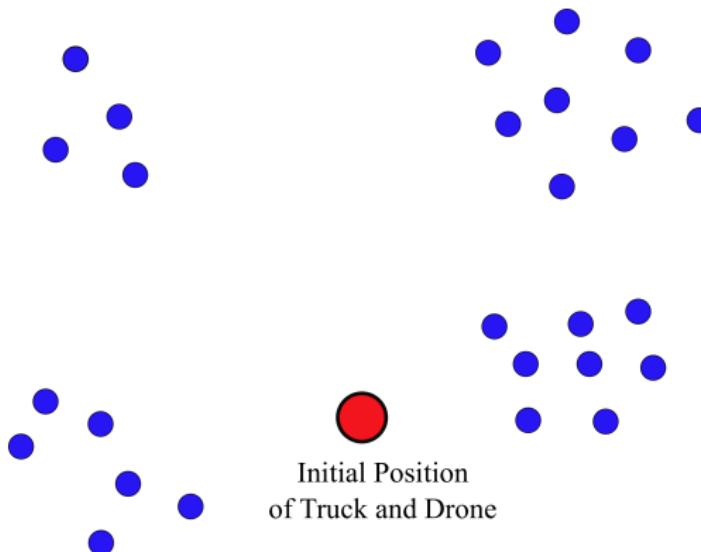
Greedy Heuristic (2) : Cheapest Insertion



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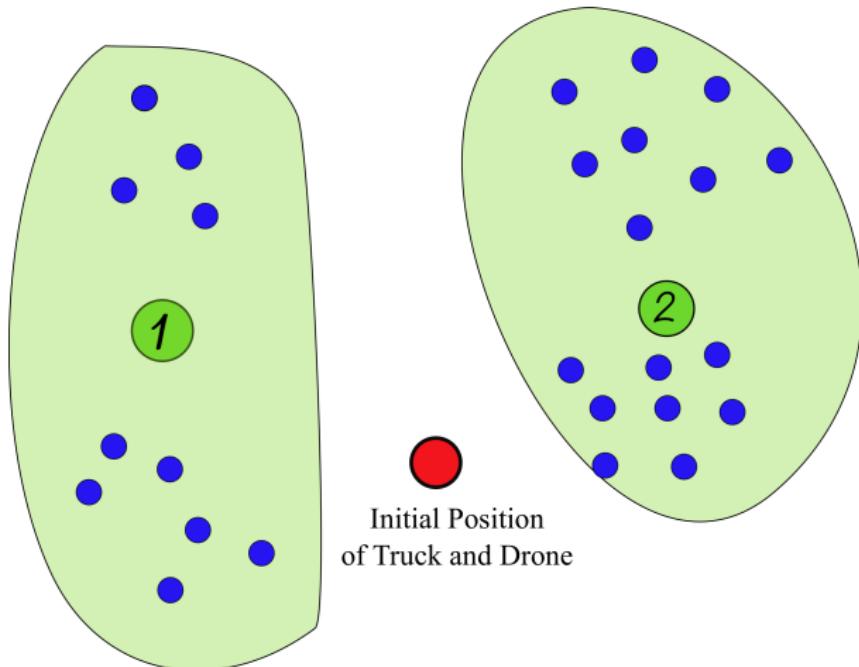
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K2means Heuristic

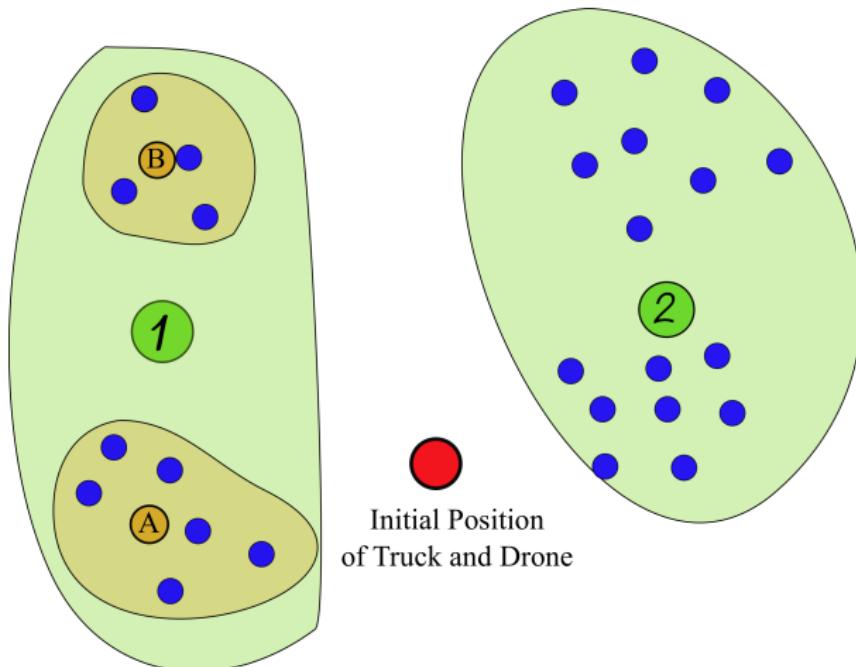


Initial Position
of Truck and Drone

K2means Heuristic



K2means Heuristic

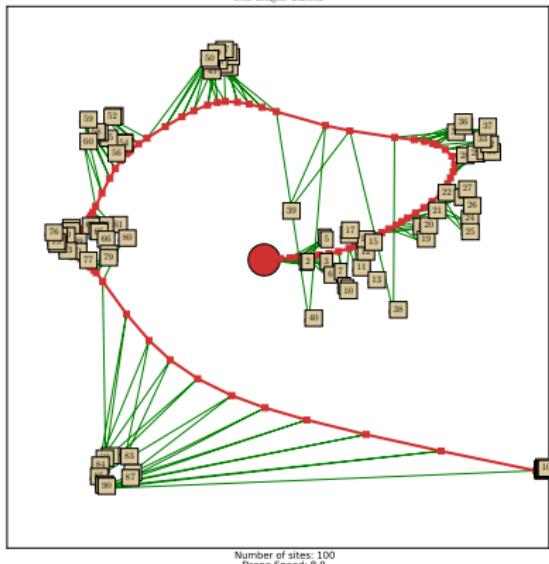


Comparing Nearest Neighbor and K2means heuristic

Tour Length: 2.27952

Greedy (1)

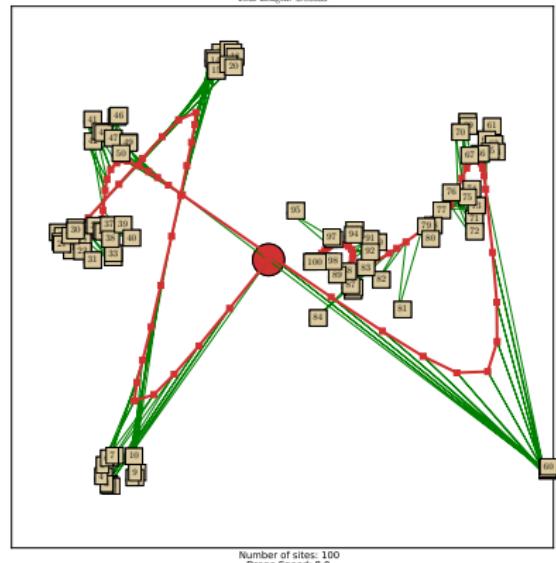
Algorithm Used: g
Tour Length: 2.27952



Tour Length: 3.06633

K2means

Algorithm Used: k
Tour Length: 3.06633

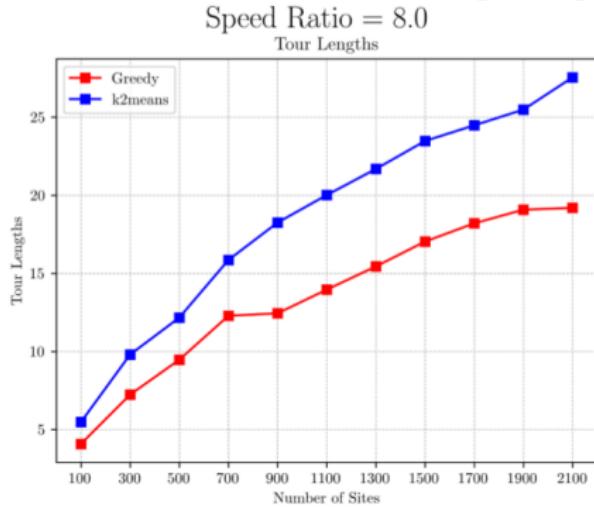
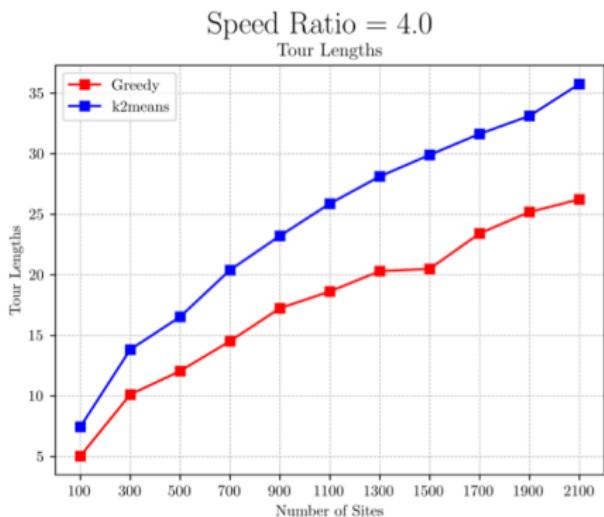


Number of Sites: 100

Drone Speed: 8.0

Comparing Nearest Neighbor and K2means heuristic

Comparing K2means and Greedy for a large number of sites uniformly distributed in $[0, 1]^2$



An $O(\log n)$ approximation

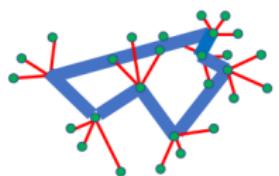
- Dynamic Program: Subproblem is a rectangle R , around which the truck travels the full perimeter.

Optimize: Find cheapest “BSP” truck network, with “spokes” to all target sites, weighting the length of the truck network by r , the speed ratio (and spokes with weight 1)



Proof of approx factor:

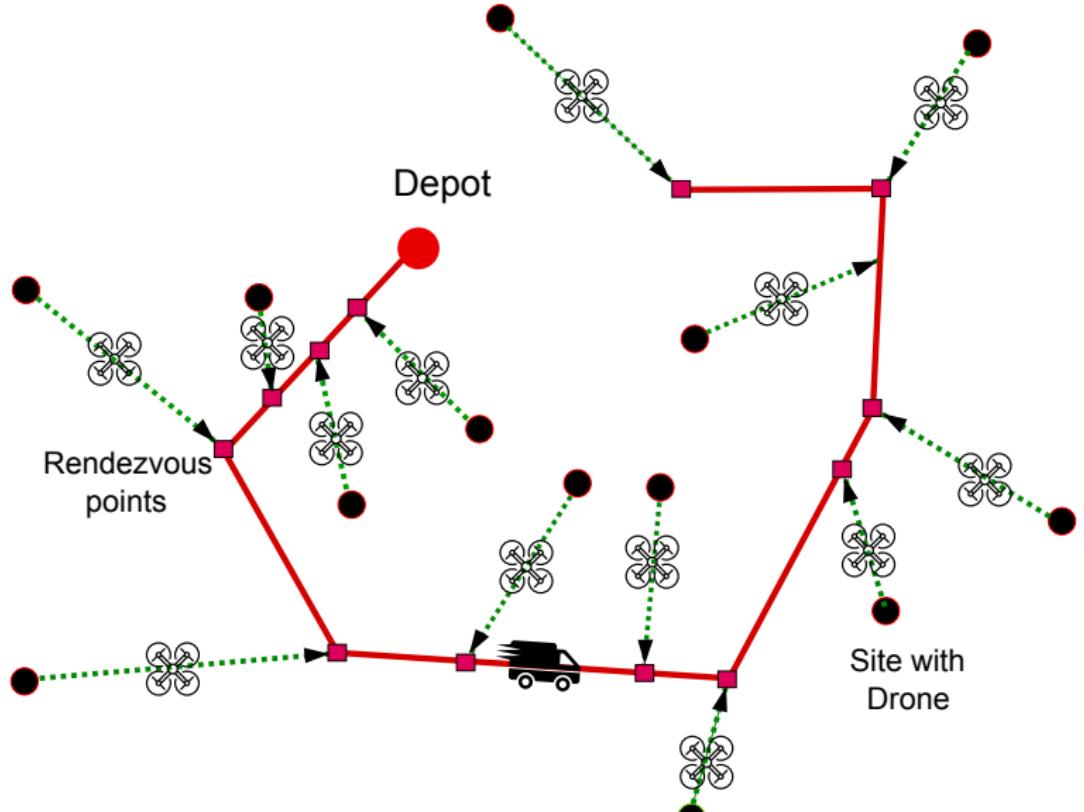
- Convert drone route to be a set of doubled “spokes” attached to truck route at pause points (truck waits) O(1) factor
- Convert Opt routes to be rectilinear (L_1) O(1) factor
- Augment truck route to be a BSP network (factor \log dilation); note that resulting solution is among those searched by DP
- Solution recovery: From DP solution, at O(1) factor can retrieve a valid solution to original problem



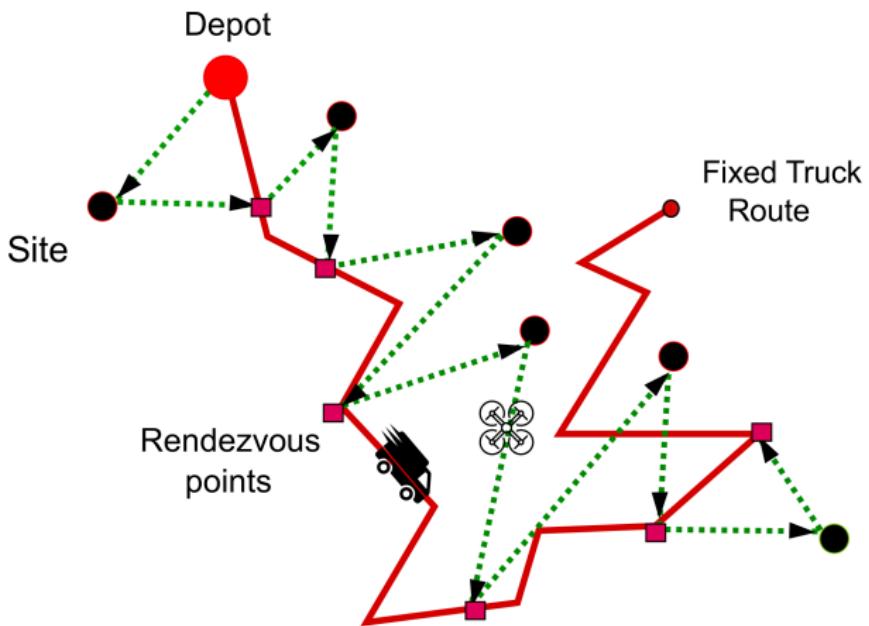
Previous Work

- John's Paper which gives an asymptotic solution
- Flying Sidekick Traveling Salesman Problem, high level results. These are all experimental, which don't allow for Steiner Point.

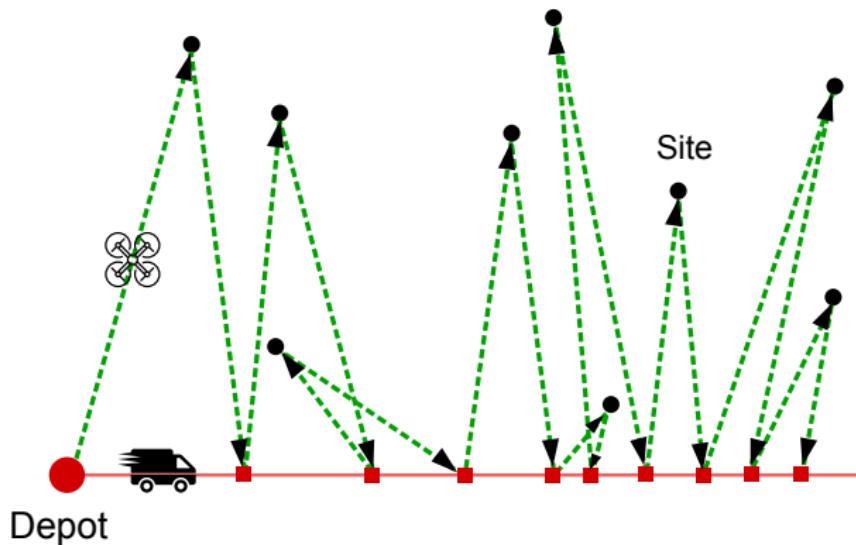
Variants: Reverse Horsefly



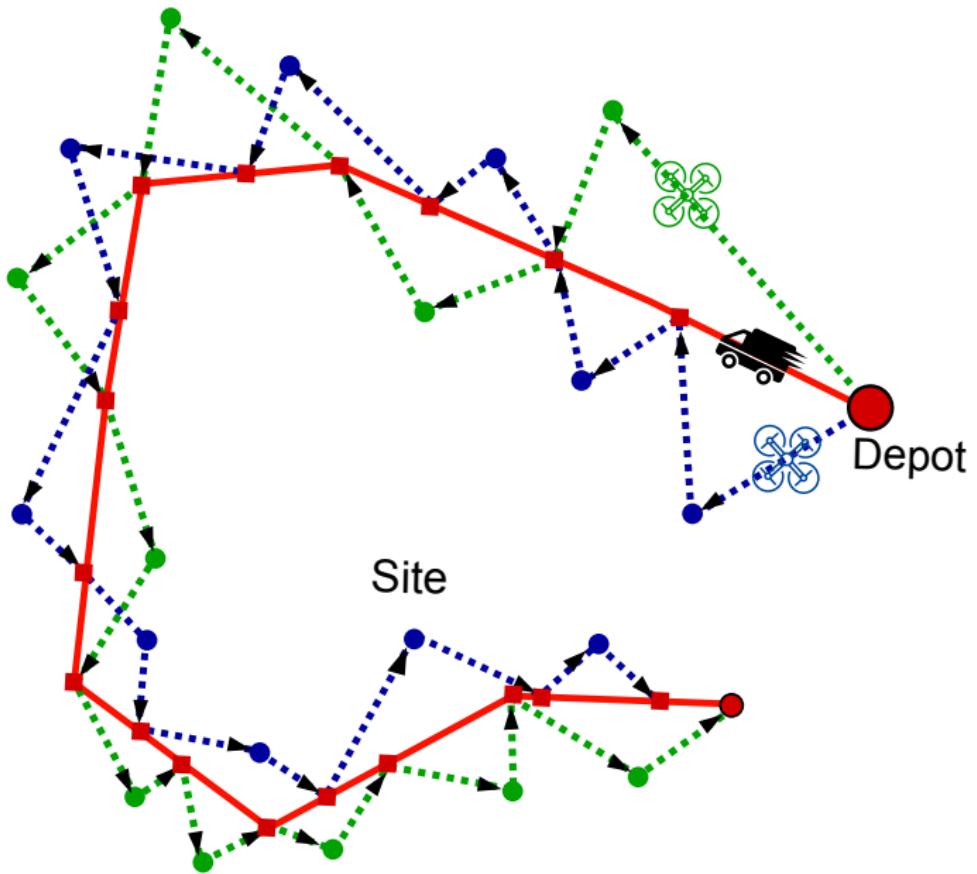
Variants: Fixed Truck Route



Variants: Segment Horsefly



Variants: Multiple Drones



Summary

- An interesting problem introduced involving 2 heterogenous vehicles. (coordinated vehicle routing).
- An implementation of some heuristics for the horsefly problem (Greedy, K2means).
- An $O(\log n)$ approximation algorithm.
- Experiments comparing various heuristics.
- Several interesting variants.