Package Delivery with Trucks and Drones: The Horsefly Problem

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Using Trucks and Drones in Tandem

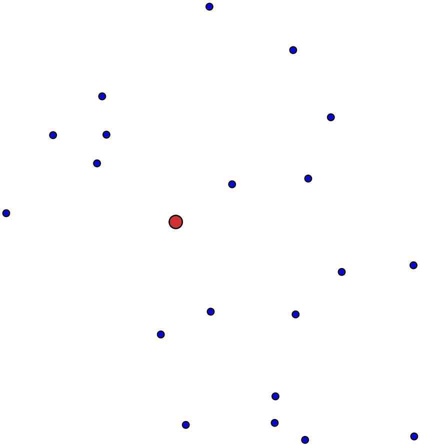
amazon ups FedEx Express

UNITED STATES POSTAL SERVICE DHL

"UPS has estimated that cutting off just one mile for the routes of each of the company's 66,000 delivery drivers would amount to $50 million in savings. For this reason, UPS is testing drone deliveries, using the top of its vans as a mini-helipad." [1]

[1] From `https://www.businessinsider.com/amazon-and-ups-are-betting-big-on-drone-delivery-2018-3`

Statement of Horsefly-PATH



Speed Ratio $\varphi = 3.0$

Truck Path :

Drone Path :

Compute a route for the truck and for the drone in order to complete the delivery of all n packages (and have the drone return back to the empty truck) was soon as possible i.e., we seek to minimize makespan of the delivery process.

Both the order of visiting the sites <u>and</u> Steiner points need to be computed!

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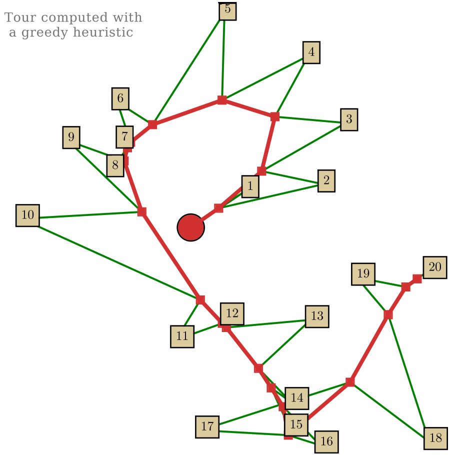

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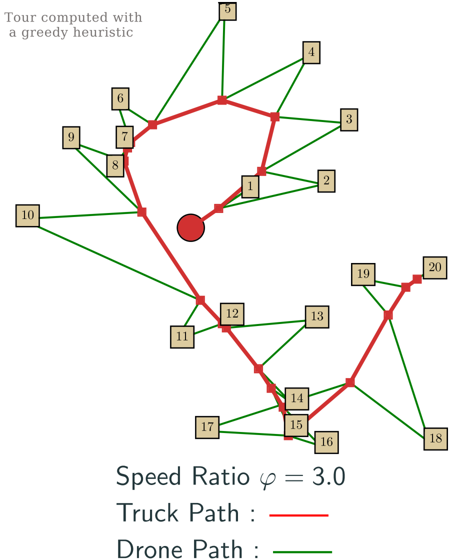
Preliminary Notes



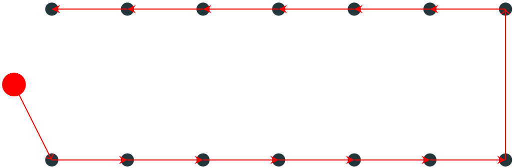
(1) Horsefly is NP-hard, by reduction from TSP ($\varphi = 1.0$)

(2) Truck or drone never wait in OPT for Horsefly-PATH.

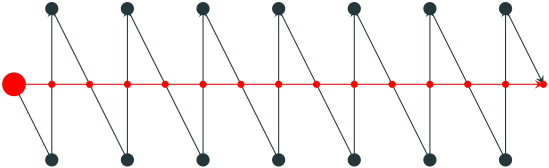
(3) The truck route and the drone route are polygonal

(4) The truck and the drone move always at their maximum speeds(1 and φ, respectively).

Preliminary Notes: Optimal Order depends on φ

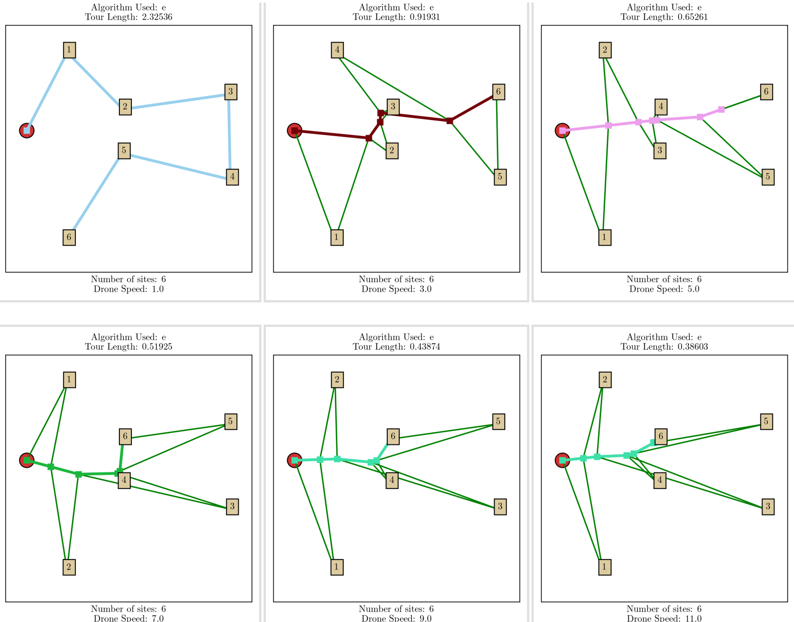


$\boxed{\varphi = 1.0}$

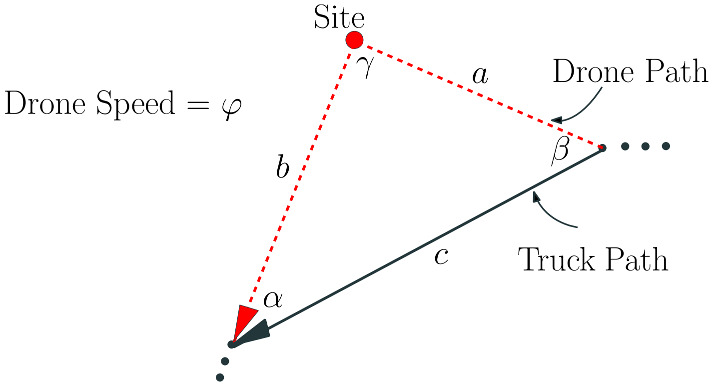


$\boxed{\varphi = 2 + \sqrt{5}}$

Preliminary Notes: Optimal Order depends on φ

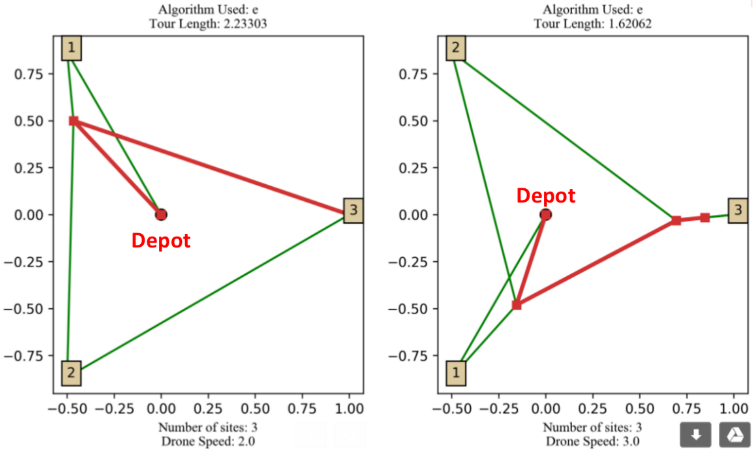


Preliminary Notes: A Local Optimality Condition



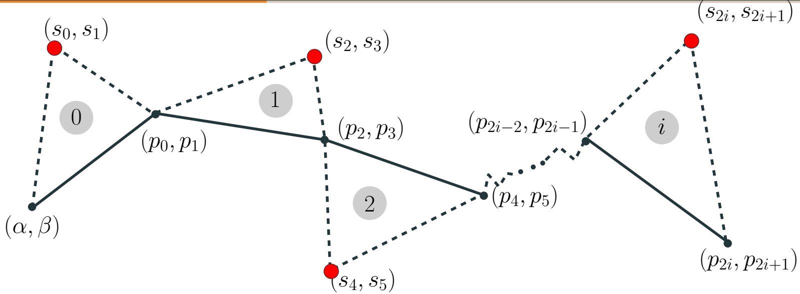
$$\boxed{\varphi = \frac{\sin(\alpha)+\sin(\beta)}{\sin(\alpha+\beta)}}$$

Preliminary Notes: Drone Path may self-intersect in *OPT*



Unsure: Might the opt truck route self-intersect? We have not seen it, but exchange argument not yet working to prove it

When Order of Visitation is Given (Exact Solution $L2$)



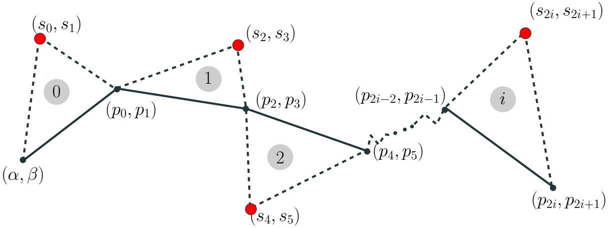
$$\min_{(p_0, p_1, \ldots, p_{2n-1}) \in \mathbb{R}^{2n}} \sum_{i=1}^{n} ||P_i - P_{i-1}||_2$$

subject to n constraints

$$||P_i - P_{i-1}||_2 = \frac{||P_{i-1} - S_i||_2 + ||S_i - P_i||_2}{\varphi} \qquad \text{for } \; i \in \{1, 2, \ldots n\}$$

This can be formulated as an SOCP (Carlsson, Song), but in experiments here we use a generic non-linear solver from SciPy.

When Order of Visitation is Given (Exact Solution $L1$)



$$\min_{(p_0, p_1, \ldots, p_{2n-1}) \in \mathbb{R}^{2n}} \sum_{i=1}^{n} ||P_i - P_{i-1}||_1$$

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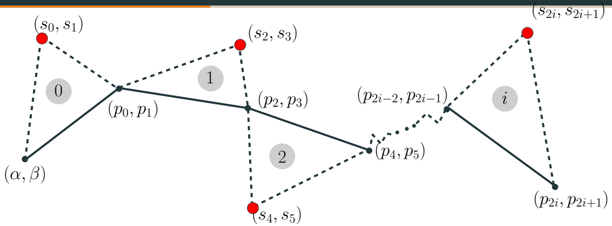
This can be formulated as an LP. We use the MOSEK LP solver in our experiments.

When Order of Visitation is Given (Exact Solution $L1$)



$$\min_{(p_0, p_1, \dots, p_{2n-1}) \in \mathbb{R}^{2n}} \sum_{i=1}^{n} ||P_i - P_{i-1}||_1$$

Constraints associated with triangle 0

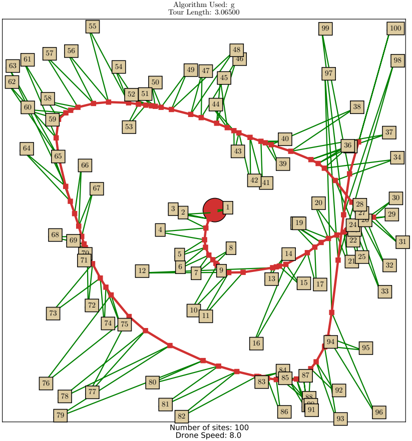
$$
\begin{aligned}
0 &\leq f_0, f_1, h_0, h_1 \\
|s_0 - \alpha| &= b_0 \\
|s_1 - \beta| &= b_1 \\
\\
|p_0 - \alpha| &\leq h_0 \\
|p_1 - \beta| &\leq h_1 \\
\\
|p_0 - s_0| &\leq f_0 \\
|p_1 - s_1| &\leq f_1 \\
\\
f_0 + f_1 + b_0 + b_1 &= \varphi h_0 + \varphi h_1
\end{aligned}
$$

Constraints associated with triangle i

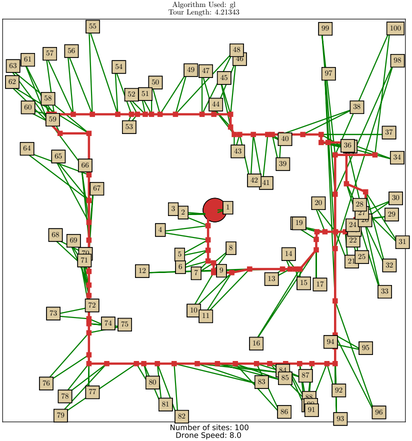
$$
\begin{aligned}
0 &\leq b_{2i}, b_{2i+1}, f_{2i}, f_{2i+1}, h_{2i}, h_{2i+1} \\
|s_{2i} - p_{2i-2}| &\leq b_{2i} \\
|s_{2i+1} - p_{2i-1}| &\leq b_{2i+1} \\
|s_{2i} - p_{2i}| &\leq f_{2i} \\
|s_{2i+1} - p_{2i+1}| &\leq f_{2i+1} \\
\\
|p_{2i} - p_{2i-2}| &\leq h_{2i} \\
|p_{2i+1} - p_{2i-1}| &\leq h_{2i+1} \\
\\
f_{2i} + f_{2i+1} + b_{2i} + b_{2i+1} &= \varphi h_{2i} + \varphi h_{2i+1}
\end{aligned}
$$

$L2$ Solution vs. $L1$ Solution: An Example

Tour Length: 3.06500



Tour Length: 4.21343



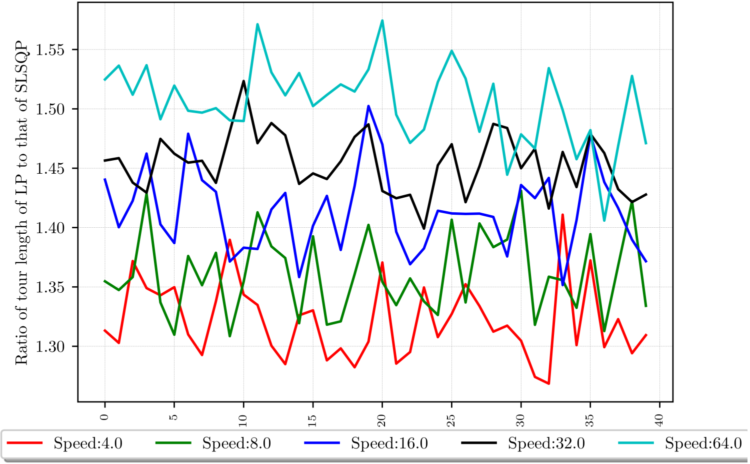
Number of Sites $= 100$

$\varphi = 8.0$

Orderings of sites for both pictures are the same.

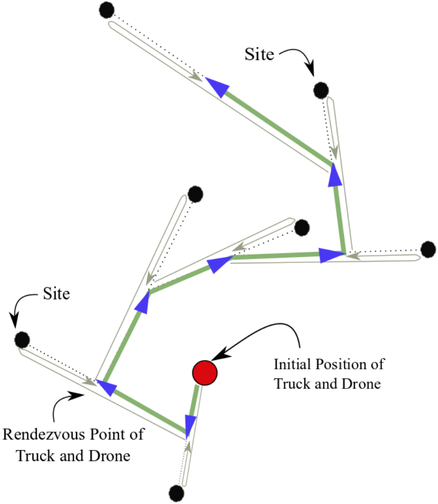
Computed with a Greedy heuristic.

$L2$ Solution vs. $L1$ Solution



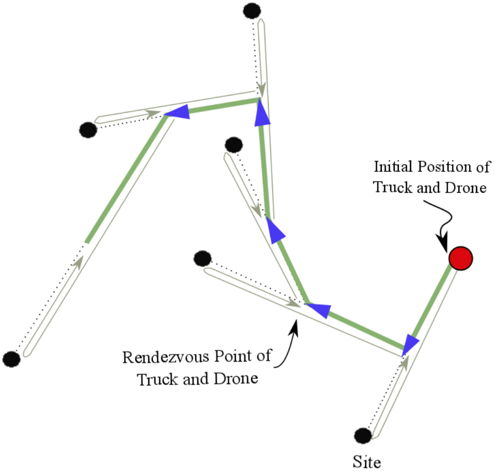
Conjectured Worst-case Approximation Ratio of $L2$ to $L1$: $\sqrt{2}\ o(\varphi)$

A Special Case: Collinear Horseflies



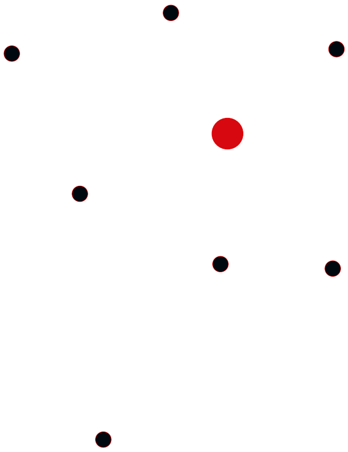
Truck and drone always move towards a site.

Greedy Heuristic (1) : Nearest Neighbor



Truck and drone visit the nearest unserviced site.

Greedy Heuristic (2) : Cheapest Insertion



In the case of TSP ($\varphi = 1$), this strategy gives us a 2 approximation.

Belief : For Horsefly the approximation is $O(1)o(\varphi)$

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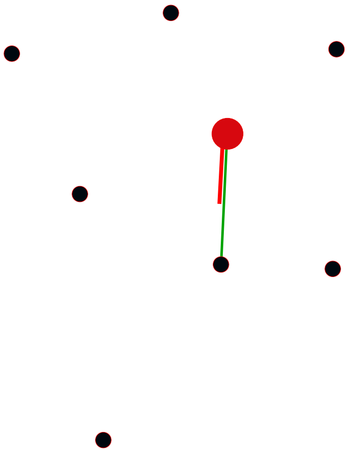

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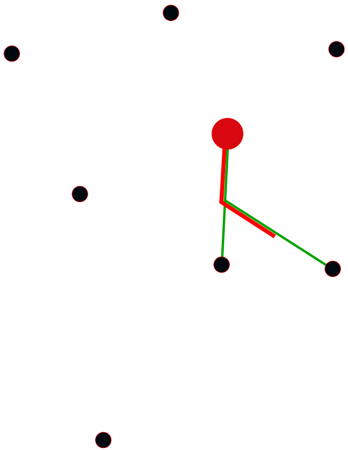

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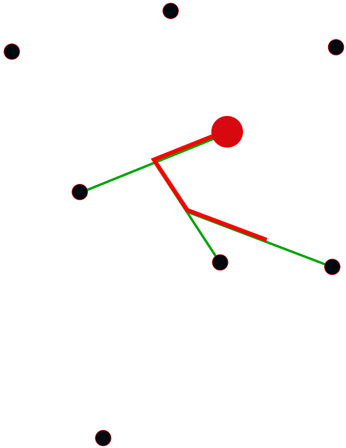

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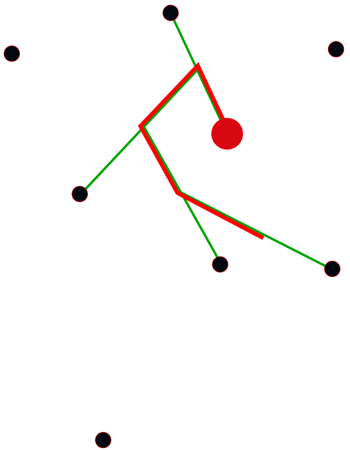

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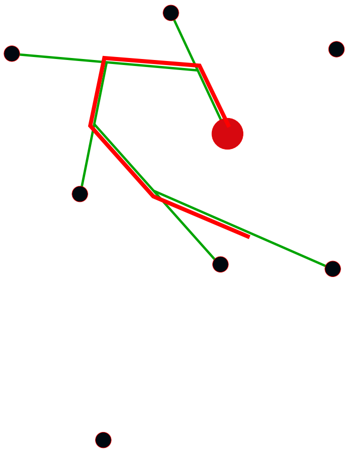

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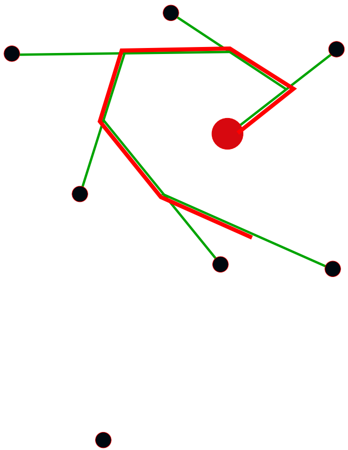

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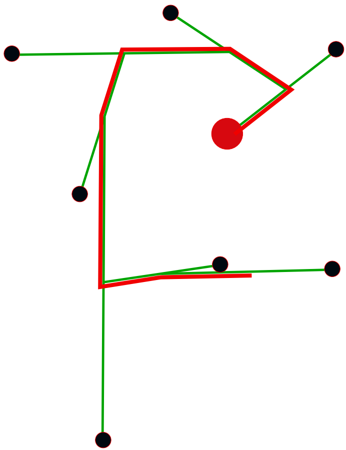

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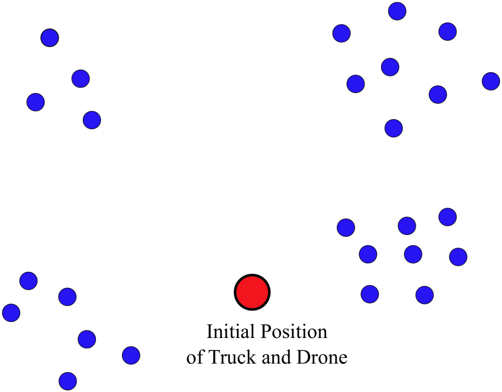
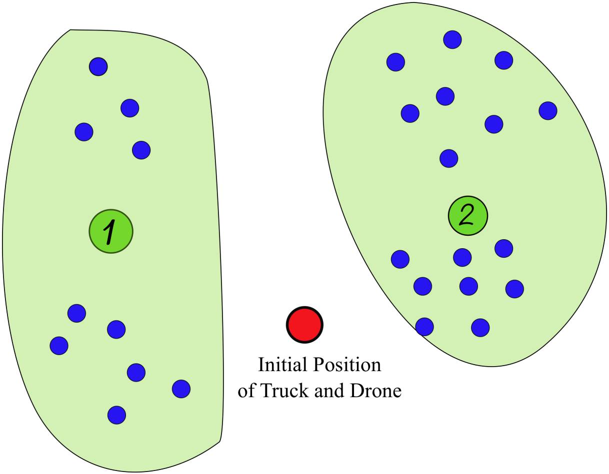
K2means Heuristic



K2means Heuristic



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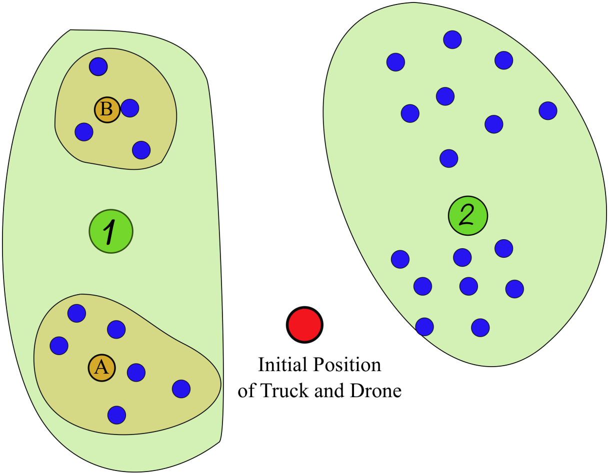
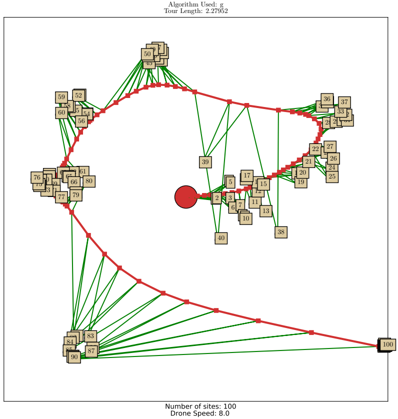


Comparing Nearest Neighbor and K2means heuristic

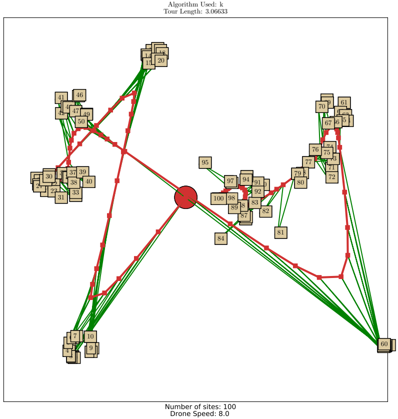
Tour Length: 2.27952

Greedy (1)



Tour Length: 3.06633

K2means

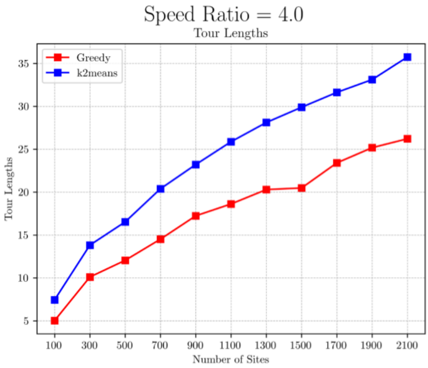


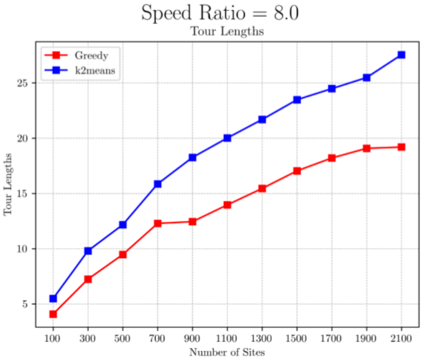
Number of Sites: 100

Drone Speed: 8.0

Comparing Nearest Neighbor and K2means heuristic

Comparing K2means and Greedy for a large number of sites uniformly distributed in $[0, 1]^2$

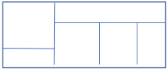




An $O(\log n)$ approximation

- Dynamic Program: Subproblem is a rectangle R, around which the truck travels the full perimeter.

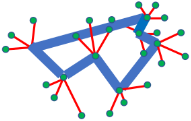
  Optimize: Find cheapest "BSP" truck network, with "spokes" to all target sites, weighting the length of the truck network by r, the speed ratio (and spokes with weight 1)



BSP

Proof of approx factor:

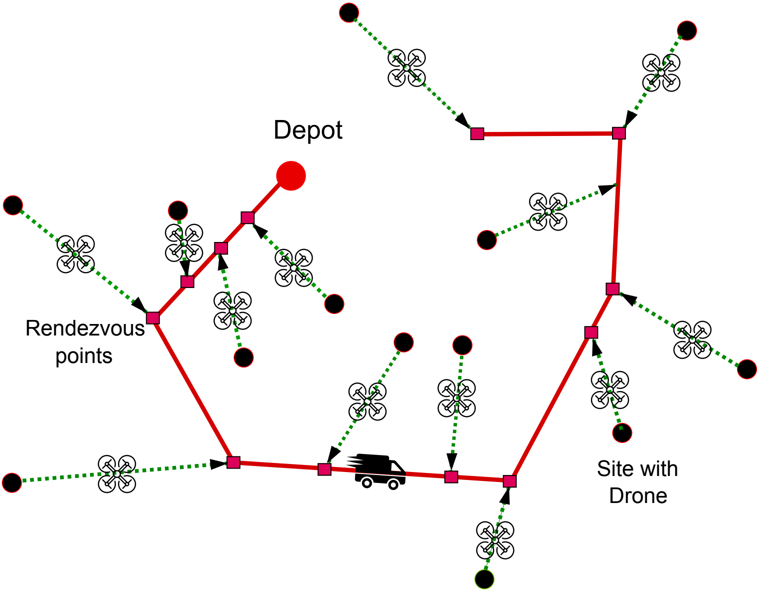
- Convert drone route to be a set of doubled "spokes" attached to truck route at pause points (truck waits) O(1) factor
- Convert Opt routes to be rectilinear (L_1) O(1) factor
- Augment truck route to be a BSP network (factor log dilation); note that resulting solution is among those searched by DP
- Solution recovery: From DP solution, at O(1) factor can retrieve a valid solution to original problem



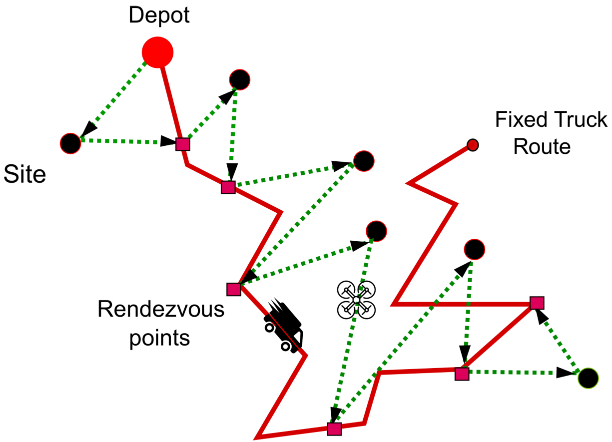
Previous Work

- **Coordinated logistics with a truck and a drone**
  John Gunnar Carlsson, Siyuan Song
  Blah Blah
- **The flying sidekick traveling salesman problem: Optimization of drone-assisted parcel delivery**
  Chase C.Murray, Amanda G.Chu
  Blah Blah

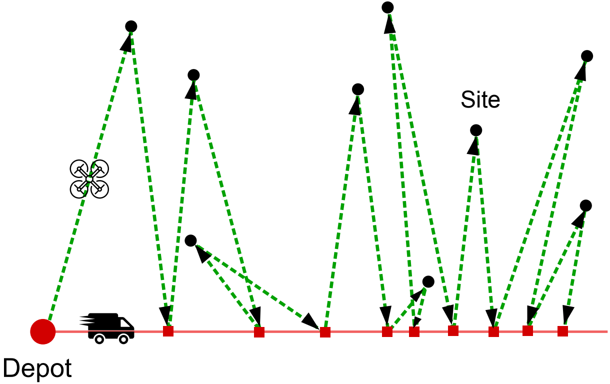
Variants: Reverse Horsefly



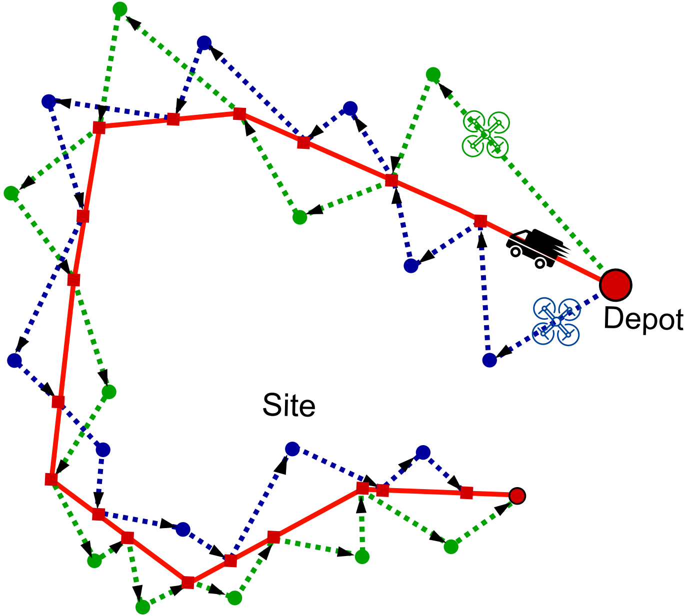
Variants: Fixed Truck Route



Variants: Segment Horsefly



Variants: Multiple Drones



- An interesting problem introduced involving 2 heterogenous vehicles. (coordinated vehicle routing).
- An implementation of some heuristics for the horsefly problem (Greedy, K2means).
- An $O(\log n)$ approximation algorithm.
- Experiments comparing various heuristics.
- Several interesting variants.