Experimental Analyses of Heuristics for Horsefly-type Problems

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Contents

Part I

Overview

Chapter 1

Descriptions of Problems

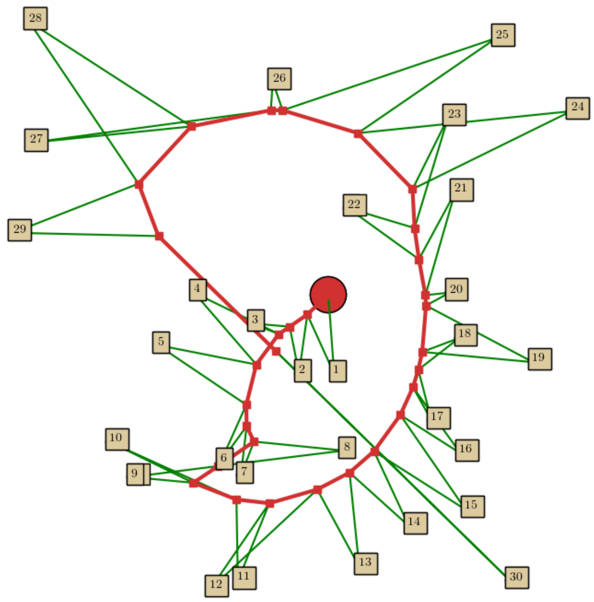


Figure 1.1: An Example of a classic Horsefly tour with $\varphi = 5$. The red dot indicates the initial position of the horse and fly, given as part of the input. The ordering of sites shown has been computed with a greedy algorithm which will be described later

The Horsefly problem is a generalization of the well-known Euclidean Traveling Salesman Problem. In the most basic version of the Horsefly problem (which we call **"Classic Horsefly"**), we are given a set of sites, the initial position of a truck(horse) with a drone(fly) mounted on top, and the speed of the drone-speed φ. [1] [2].

The goal is to compute a tour for both the truck and the drone to deliver package to sites as quickly as possible. For delivery, a drone must pick up a package from the truck, fly to the site and come back to the truck to pick up the next package for delivery to another site. [3] Both the truck and drone must coordinate their motions to minimize the time it takes for all the sites to get their packages. Figure 1.1 gives an example of such a tour computed using a greedy heuristic for $\varphi = 5$.

This suite of programs implement several experimental heuristics, to solve the above NP-hard problem and some of its variations approximately. In this short chapter, we give a description of the problem variations that we will be tackling. Each of the problems, has a corresponding chapter in Part 2, where these heuristics are described and implemented. We also give comparative analyses of their experimental performance on various problem instances.

Classic Horsefly This problem has already described in the introduction.

Segment Horsefly In this variation, the path of the truck is restricted to that of a segment, which we can consider without loss of generality to be $[0, 1]$. All sites, without loss of generality lie in the upper-half plane $\mathbb{R}^2_+$.

[1]The speed of the truck is always assumed to be 1 in any of the problem variations we will be considering in this report.
[2]φ is also called the "speed ratio".
[3]The drone is assumed to be able to carry at most one package at a time

Fixed Route Horsefly This is the obvious generalization of Segment Horsefly, where the path which the truck is restricted to travel is a piece-wise linear polygonal path. [4] Both the initial position of the truck and the drone are given. The sites to be serviced are allowed to lie anywhere in $\mathbb{R}^2$. Two further variations are possible in this setting, one in which the truck is allowed reversals and the other in which it is not.

One Horse, Two Flies The truck is now equipped with two drones. Otherwise the setting, is exactly the same as in classic horsefly. Each drone can carry only one package at a time. The drones must fly back and forth between the truck and the sites to deliver the packages. We allow the possibility that both the drones can land at the same time and place on the truck to pick up their next package. [5]

Reverse Horsefly In this model, each site (not the truck!) is equipped with a drone, which fly *towards* the truck to pick up their packages. We need to coordinate the motion of the truck and drone so that the time it takes for the last drone to pick up its package (the "makespan") is minimized.

Bounded Distance Horsefly In most real-world scenarios, the drone will not be able to (or allowed to) go more than a certain distance R from the truck. Thus with the same settings as the classic horsefly, but with the added constraint of the drone and the truck never being more than a distance R from the truck, how would one compute the truck and drone paths to minimize the makespan of the deliveries?

Watchman Horsefly In place of the TSP, we generalize the Watchman route problem here. [6] We are given as input a simple polygon and the initial position of a truck and a drone. The drone has a camera mounted on top which is assumed to have 360° vision. Both the truck and drone can move, but the drone can move at most euclidean distance [7] R from the truck.

We want every point in the polygon to be seen by the drone at least once. The goal is to minimize the time it takes for the drone to be able to see every point in the simple polygon. In other words, we want to minimize the time it takes for the drone (moving in coordinattion with the truck) to patrol the entire polygon.

[4] More generally, the truck will be restricted to travelling on a road network, which would typically be modelled as a graph embedded in the plane.

[5] In reality, one of the drones will have to wait for a small amount of time while the other is retrieving its package. In a more realisting model, we would need to take into account this "waiting time" too.

[6] although abstractly, the Watchman route problem can be viewed as a kind of TSP

[7] The version where instead geodesic distance is considered is also interesting

Chapter 2

Installation and Use

To run these programs you will need to install Docker, an open-source containerization program that is easily installable on Windows 10[1], MacOS, and almost any GNU/Linux distribution. For a quick introduction to containerization, watch the first two minutes of https://youtu.be/_dfLOzuIg2o

The nice thing about Docker is that it makes it easy to run softwares on different OS'es portably and neatly side-steps the dependency hell problem (https://en.wikipedia.org/wiki/Dependency_hell.) The headache of installing different library dependencies correctly on different machines running different OS'es, is replaced **only** by learning how to install Docker and to set up an X-windows connection between the host OS and an instantiated container running GNU/Linux.

A. [*Get Docker*] For installation instrutions watch

GNU/Linux https://youtu.be/KCckWweNSrM

Windows https://youtu.be/ymlWt1MqURY

To test your installation, run the `hello-world` container. Note that you might need administrator privileges to run docker. On Windows, you can open the Powershell as an administrator. On GNU/Linux you should use `sudo`

B. [*Download customized Ubuntu image*] `docker pull gtelang/ubuntu_customized` [2]

C. [*Clone repository*] `git clone gtelang/horseflies_literate.git`

D. [*Mount and Launch*]

If you are running GNU/Linux

- Open up your favorite terminal emulator, such as xterm, rxvt or konsole
- Copy to clipboard the output of `xauth list`
- `cd horseflies_literate`
- ```
  docker run -it --name horsefly_container --net=host \
  -e DISPLAY -v /tmp/.X11-unix \
  -v `pwd`:/horseflies_mnt gtelang/ubuntu_customized
  ```
- `cd horseflies_mnt`
- `xauth add` *<paste-from-clipboard>*

The purpose of using "`xauth`" and "`-e DISPLAY -v /tmp/.X11-unix`" is to establish an X-windows connection between your operating system and the Ubuntu container that allows you to run GUI apps e.g. the FireFox web-browser. [3]

If you are running Windows

- Follow every instruction in https://dev.to/darksmile92/run-gui-app-in-linux-Docker-container-on-windows-host-4kde. [4] Make sure you can run a gui program like the Firefox web-browser as indicated by the article before going to the next step.

[1] You might need to turn on virtualization explicitly in your BIOS, after installing Docker as I needed to while setting Docker up on Windows. Here is a snapshot of an image when turning on Intel's virtualization technology through the BIOS: https://images.techhive.com/images/article/2015/09/virtualbox_vt-x_amd-v_error04_phoenix-100612961-large.idge.jpg

[2] The customized Ubuntu image is approximately 7 GB which contains all the libraries (e.g. CGAL, VTK, numpy, and matplotlib) that I typically use to run my research codes portably.On my home internet connection downloading this Ubuntu-image typically takes about 5-10 minutes.

[3] I found the instructions for running GUI apps on containers in https://www.youtube.com/watch?v=RDg6TRwiPtg

[4] This step is necessary displaying the Matplotlib canvas as we do in the horseflies project for interactive testing of algorithms.
  ```

- To mount the horseflies folder, you need to *share* the appropriate drive (e.g. `C:\` or `D:\`) that the `horseflies` folder is in with Docker. Follow instructions here: https://rominirani.com/docker-on-windows-mounting-host-directories-d96f3f056a2c for sharing directories. [5]
- Open up a Windows Powershell (possibly as administrator)
  - `set-variable -name DISPLAY -value <your-ip-address>:0.0` [6]
  - `docker run -ti --rm -e DISPLAY=$DISPLAY -v <location-of-horseflies-folder>:/horseflies_mnt gtelang/u`

E. [*Run experiments*] If you want to run all the experiments as described in the paper again to reproduce the reported results on your machine, then run [7],
`python main.py --run-all-experiments`.

If you want to run a specific experiment, then run
`python main.py --run-experiment <experiment-name>`.

See Index for a list of all the experiments.

F. [*Test algorithms interactively*] If you want to test the algorithms in interactive mode (where you get to select the problem-type, mouse-in the sites on a canvas, set the initial position of the truck and drone and set φ), run `python main.py --<problem-name>`. The list of problems are the same as that given in the previous chapter. The problem name consists of all lower-case letters with spaces replaced by hyphens.

Thus for instance "Watchman Horsefly" becomes `watchman-horsefly` and "One Horse Two Flies" becomes `one-horse-two-flies`.

To interactively experiment with different algorithms for, say, the Watchman Horsefly problem , type at the terminal
`python main.py --watchman-horsefly`

If you want to delete the Ubuntu image and any associated containers run the command [8]

```
docker rm -f horsefly_container; docker rmi -f ubuntu_customized
```

That's it! Happy horseflying!

[5] you might need administrator privileges to perform this step, as pointed out by the article.
[6] You can find your ip-address by the output of the `ipconfig` command in the Powershell
[7] Allowing, of course, for differences between your machine's CPU and mine when it comes to reporting absolute running time
[8] the ubuntu image is 7GB afterall!

Part II

Programs

Chapter 3

Overview of the Code Base

All of the code has been written in Python 2.7 and tested using the standard CPython implementation of the language. In some cases, calls will be made to external C++ libraries (mostly CGAL and VTK) using SWIG (http://www.swig.org/) for speeding up a slow routine or to use a function that is not available in any existing Python package.

Source Tree

```
..
|-- src
|   |-- expts
|   |-- lib
|   |   |-- problem_classic_horsefly_bkp.py
|   |   |-- problem_classic_horsefly.py
|   |   |-- utils_algo.py
|   |   `-- utils_graphics.py
|   |-- tests
|   `-- Makefile
|-- tex
|   |-- directory-tree.tex
|   |-- horseflies.pdf
|   |-- horseflies.tdo
|   |-- horseflies.tex
|   `-- standard_settings.tex
|-- webs
|   |-- problem-classic-horsefly
|   |   |-- algo-bottom-up-split.web
|   |   |-- algo-doubling-phi-mst.web
|   |   |-- algo-dumb.web
|   |   |-- algo-greedy-incremental-insertion.web
|   |   |-- algo-greedy-nn.web
|   |   |-- algo-k2-means.web
|   |   |-- algo-local-search-swap.web
|   |   |-- algo-tsp-ordering.web
|   |   |-- lower-bound-phi-mst.web
|   |   `-- problem-classic-horsefly.web
|   |-- problem-fixed-route-horsefly
|   |   `-- problem-fixed-route-horsefly.web
|   |-- problem-one-horse-two-flies
|   |   `-- problem-one-horse-two-flies.web
|   |-- problem-reverse-horsefly
|   |   `-- problem-reverse-horsefly.web
|   |-- problem-segment-horsefly
|   |   `-- problem-segment-horsefly.web
|   |-- problem-watchman-horsefly
|   |   `-- problem-watchman-horsefly.web
|   |-- descriptions-of-problems.web
```

```
|   |-- horseflies.web
|   |-- installation-and-use.web
|   |-- overview-of-code-base.web
|   `-- utility-functions.web
|-- main.py
`-- weave-tangle.sh

12 directories, 32 files
```

There are three principal directories

`webs/` This contains the source code for the entire project written in the nuweb format along with documents (mostly images) needed during the compilation of the LaTeX files which will be extracted from the `.web` files.

`src/` This contains the source code for the entire project "tangled" (i.e. extracted) from the `.web` files.

`tex/` This contains the monolithic `horseflies.tex` extracted from the `.web` files and a bunch of other supporing LaTeX files. It also contains the final compiled `horseflies.pdf` (the current document) which contains the documentation of the project, interwoven with code-chunks and cross-references between them along with the experimental results.

The files in `src` and `tex` should not be touched. Any editing required should be done directly to the `.web` files which should then be weaved and tangled using `weave-tangle.sh`.

The Main Files

3.2.1

A. [`main.py`] The file `main.py` in the top-level folder is the *entry-point* for running code. Its only job is to parse the command-line arguments and pass relevant information to the handler functions for each problem and experiment.

B. [*Algorithmic Code*] All such files are in the directory `src/lib/`. Each of the files with prefix "`problem_*`" contain implmentations of algorithms for one specific problem. For instance `problem_watchman_horsefly.py` contains algorithms for approximately solving the Watchman Horsefly problem.

Since Horsefly-type problems are typically NP-hard, an important factor in the subsequent experimental analysis will require, comparing an algorithm's output against good lower bounds. Each such file, will also have routines for efficiently computing or approximating various lower-bounds for the corresponding problem's OPT.

C. [*Experiments*] All such files are in the directory `src/expt/` Each of the files with prefix "`expt_*`" contain code for testing hypotheses regarding a problem, generating counter-examples or comparing the experimental performance of the algorithm implementations for each of the problems. Thus `expt_watchman_horsefly.py` contains code for performing experiments related to the Watchman Horsefly problem.

If you need to edit the source-code for algorithms or experiment you should do so to the `.web` files in the web directory. Every problem has a dedicated *folder* containing source-code for algorithms and experiments pertaining to that problem. Every algorithm and experiment has a dedicated `.web` file in these problem directories. Such files are all "tied" together using the file with prefix problem-<problem-name> in that same directory (i.e. the file acts as a kind of handler for each problem, that includes the algorithms and experiment `web` files with the `@i` macro.)

3.2.2 Let's define the `main.py` file now.

Each problem or experiment has a handler routine that effectively acts as a kind of "main" function for that module that does house-keeping duties by parsing the command-line arguments passed by main, setting up the canvas by calling the appropriate graphics routines and calling the algorithms on the input specified through the canvas.

"../main.py" 12a≡

```
⟨ Turn off Matplotlibs irritating DEBUG messages 12b ⟩
⟨ Import problem module files 12c ⟩

if __name__=="__main__":
     # Select algorithm or experiment
     if (len(sys.argv)==1):
          print "Specify the problem or experiment you want to run"
          sys.exit()

     elif sys.argv[1] == "--problem-classic-horsefly":
          chf.run_handler()

     elif sys.argv[1] == "--problem-segment-horsefly":
          shf.run_handler()

     elif sys.argv[1] == "--problem-one-horse-two-flies":
          oh2f.run_handler()

     else:
          print "Option not recognized"
          sys.exit()
◇
```

3.2.3

On my customized Ubuntu container, Matplotlib produces tons of DEBUG log messages because it recently switched to the `logging` library for...well...logging. The lines in this chunk were suggested by the link `http://matplotlib.1069221.n5.nabble.com/How-to-turn-off-matplotlib-DEBUG-msgs-td48822.html` for quietening down Matplotlib.

⟨ *Turn off Matplotlibs irritating DEBUG messages* 12b ⟩ ≡

```
import logging
mpl_logger = logging.getLogger('matplotlib')
mpl_logger.setLevel(logging.WARNING)
◇
```

Fragment referenced in 12a.

⟨ *Import problem module files* 12c ⟩ ≡

```
import sys
sys.path.append('src/lib')
import problem_classic_horsefly as chf
#import problem_segment_horsefly as shf
#import problem_one_horse_two_flies as oh2f
◇
```

Fragment referenced in 12a.

Support Files

A. [*Utility Files*] All such utility files are in the directory `src/lib/`. These files contain common utility functions for manipulating data-structures, plotting and graphics routines common to all horsefly-type problems. All such files have the prefix `utils_*`. These Python files are generated from the single `.web` file `utils.web` in the `web` subdirectory.

B. [*Tests*] All such files are in the directory `src/test/` To automate testing of code during implementations, tests for various routines across the entire code-base have been written in files with prefix `test_*`.

Every problem, utility, and experimental files in `src/lib` and `src/expts` has a corresponding test-file in this folder.

Chapter 4

Some (Boring) Utility Functions

We will be needing some utility functions, for drawing and manipulating data-structures which will be implemented in files separate from `problem_classic_horsefly.py`. All such files will be prefixed with the work `utils_`. Many of the important common utility functions are defined here; others will be defined on the fly throughout the rest of the report. This chapter just collects the most important of the functions for the sake of clarity of exposition in the later chapters.

Graphical Utilities

Here we will develop routines to interactively insert points onto a Matplotlib canvas and clear the canvas. Almost all variants of the horsefly problem will involve mousing in sites and the initial position of the horse and fly. These points will typically be represented by small circular patches. The type of the point will be indicated by its color and size e.g. intial position of truck and drone will typically be represented by a large red dot while and the sites by smaller blue dots.

Matplotlib has extensive support for inserting such circular patches onto its canvas with mouse-clicks. Each such graphical canvas corresponds (roughly) to Matplotlib figure object instance. Each figure consists of several Axes objects which contains most of the figure elements i.e. the Axes objects correspond to the "drawing area" of the canvas.

4.1.1 First we set up the axes limits, dimensions and other configuration quantities which will correspond to the "without loss of generality" assumptions made in the statements of the horsefly problems. We also need to set up the axes limits, dimensions, and other fluff. The following fragment defines a function which "normalizes" a drawing area by setting up the x and y limits and making the aspect ratio of the axes object the same i.e. 1.0. Since Matplotlib is principally a plotting software, this is not the default behavior, since scales on the x and y axes are adjusted according to the data to be plotted.

`"../src/lib/utils_graphics.py"` 14≡

```
from matplotlib import rc
from colorama import Fore
from colorama import Style
from scipy.optimize import minimize
from sklearn.cluster import KMeans
import argparse
import itertools
import math
import matplotlib as mpl
import matplotlib.pyplot as plt
import numpy as np
import os
import pprint as pp
import randomcolor
import sys
import time

xlim, ylim = [0,1], [0,1]

def applyAxCorrection(ax):
      ax.set_xlim([xlim[0], xlim[1]])
      ax.set_ylim([ylim[0], ylim[1]])
```

```
    ax.set_aspect(1.0)
◇
```

File defined by 14, 15abc.

4.1.2

Next, given an axes object (i.e. a drawing area on a figure object) we need a function to delete and remove all the graphical objects drawn on it.

"../src/lib/utils_graphics.py" 15a≡

```

def clearPatches(ax):
    # Get indices cooresponding to the polygon patches
    for index , patch in zip(range(len(ax.patches)), ax.patches):
        if isinstance(patch, mpl.patches.Polygon) == True:
            patch.remove()
    ax.lines[:]=[]
    applyAxCorrection(ax)
◇
```

File defined by 14, 15abc.

4.1.3

Now remove the patches which were rendered for each cluster Unfortunately, this step has to be done manually, the canvas patch of a cluster and the corresponding object in memory are not reactively connected.I presume, this behavioue can be achieved by sub-classing.

"../src/lib/utils_graphics.py" 15b≡

```

def clearAxPolygonPatches(ax):

    # Get indices cooresponding to the polygon patches
    for index , patch in zip(range(len(ax.patches)), ax.patches):
        if isinstance(patch, mpl.patches.Polygon) == True:
            patch.remove()
    ax.lines[:]=[]
    applyAxCorrection(ax)
◇
```

File defined by 14, 15abc.

4.1.4

Now for one of the most important routines for drawing on the canvas! To insert the sites, we double-click the left mouse button and to insert the initial position of the horse and fly we double-click the right mouse-button.

The following chunk defines a function that creates a closure for a mouseclick even on the matplotlib canvas.

Note that the left mouse-button corresponds to button 1 and right mouse button to button 3 in the code-fragment below.

Remove ous red which c the old of the h fly. Doi slightly hence k for late

"../src/lib/utils_graphics.py" 15c≡

```
def wrapperEnterRunPoints(fig, ax, run):
    def _enterPoints(event):
        if event.name      == 'button_press_event'          and \
           (event.button   == 1 or event.button == 3)       and \
            event.dblclick == True and event.xdata  != None and event.ydata  != None:

             if event.button == 1:
                 ⟨ Insert blue circle representing a site 16a ⟩

             elif event.button == 3:
```

⟨ *Insert big red circle representing initial position of horse and fly* 16b ⟩

⟨ *Clear polygon patches and set up last minute* `ax` *tweaks* 16c ⟩

```
    return _enterPoints
```
◇

File defined by 14, 15abc.

4.1.5

⟨ *Insert blue circle representing a site* 16a ⟩ ≡

```
newPoint = (event.xdata, event.ydata)
run.sites.append( newPoint  )
patchSize  = (xlim[1]-xlim[0])/140.0

ax.add_patch( mpl.patches.Circle( newPoint, radius = patchSize,
                                  facecolor='blue', edgecolor='black'  ))
ax.set_title('Points Inserted: ' + str(len(run.sites)), \
             fontdict={'fontsize':40})
```
◇

Fragment referenced in 15c.

4.1.6

⟨ *Insert big red circle representing initial position of horse and fly* 16b ⟩ ≡

```
inithorseposn     = (event.xdata, event.ydata)
run.inithorseposn = inithorseposn
patchSize         = (xlim[1]-xlim[0])/70.0

ax.add_patch( mpl.patches.Circle( inithorseposn,radius = patchSize,
                                  facecolor= '#D13131', edgecolor='black' ))
```
◇

Fragment referenced in 15c.

4.1.7 It is inefficient to clear the polygon patches *inside* the `enterRunpoints` event loop as done here. However, this has just been done for simplicity: the intended behaviour at any rate, is to clear all the polygon patches from the axes object, once the user starts entering in more points to the cloud for which the clustering was just computed and rendered. The moment the user starts entering new points, the previous polygon patches are garbage collected.

⟨ *Clear polygon patches and set up last minute* `ax` *tweaks* 16c ⟩ ≡

```
clearAxPolygonPatches(ax)
applyAxCorrection(ax)
fig.canvas.draw()
```
◇

Fragment referenced in 15c.

Algorithmic Utilities

4.2.1 Given a list of points $[p_0, p_1, p_2, ...p_{n-1}]$. the following function returns, $[p_1 - p_0, p_2 - p_1, ..., p_{n-1} - p_{n-2}]$ i.e. it converts the list of points into a consecutive list of numpy vectors. Points should be lists or tuples of length 2

`"../src/lib/utils_algo.py"` 16d≡

```
import numpy as np
import random
from colorama import Fore
from colorama import Style


def vector_chain_from_point_list(pts):
    vec_chain = []
    for pair in zip(pts, pts[1:]):
        tail= np.array (pair[0])
        head= np.array (pair[1])
        vec_chain.append(head-tail)

    return vec_chain
◇
```

File defined by 16d, 17ab, 18abcde.

4.2.2 Given a polygonal chain in the form of successive points $[p_0, p_1, p_2, \ldots. p_{n-1}]$, an important computation is to calculate its length. Points should be lists or tuples of length 2 If no points or just one point is given in the list of points, then 0 is returned.

Typically used for computing the length of the horse's and fly's tours.

"../src/lib/utils_algo.py" 17a≡

```
def length_polygonal_chain(pts):
    vec_chain = vector_chain_from_point_list(pts)

    acc = 0
    for vec in vec_chain:
        acc = acc + np.linalg.norm(vec)
    return acc
◇
```

File defined by 16d, 17ab, 18abcde.

4.2.3 The following routine is useful on long lists returned from external solvers. Often point-data is given to and returned from these external routines in flattened form. The following routines are needed to convert such a "flattened" list into a list of points and vice versa.

Convert a vector of even length into a vector of points. i.e. $[x_0, x_1, x_2, \ldots x_{2n}] \rightarrow [[x_0, x_1], [x_2, x_3], , ..[x_{2n-1}, x_{2n}]]$

"../src/lib/utils_algo.py" 17b≡

```
def pointify_vector (x):
    if len(x) % 2 == 0:
        pts = []
        for i in range(len(x))[::2]:
            pts.append( [x[i],x[i+1]] )
        return pts
    else :
        sys.exit('List of items does not have an even length to be able to be pointifyed')
◇
```

File defined by 16d, 17ab, 18abcde.

The next chunk peforms the opposite process i.e. it flatten's the vector e.g. $[[0,1],[2,3],[4,5]] \rightarrow [0,1,2,3,4,5]$

"../src/lib/utils_algo.py" 18a≡
```
def flatten_list_of_lists(l):
       return [item for sublist in l for item in sublist]
```
◇

File defined by 16d, 17ab, 18abcde.

4.2.4 Python's default print function prints each list on a single line. For debugging purposes, it helps to print a list with one item per line.

"../src/lib/utils_algo.py" 18b≡
```
def print_list(xs):
    for x in xs:
        print x
```
◇

File defined by 16d, 17ab, 18abcde.

4.2.5 This chunk just calculates the list of partial sums e.g. $[4,2,3] \rightarrow [4,6,9]$"

"../src/lib/utils_algo.py" 18c≡
```
def partial_sums( xs ):
    psum = 0
    acc = []
    for x in xs:
        psum = psum+x
        acc.append( psum )
    return acc
```
◇

File defined by 16d, 17ab, 18abcde.

4.2.6 For two given lists of points test if they are equal or not. We do this by checking the L^∞ norm.

"../src/lib/utils_algo.py" 18d≡
```
def are_site_orderings_equal(sites1, sites2):

    for (x1,y1), (x2,y2) in zip(sites1, sites2):
        if (x1-x2)**2 + (y1-y2)**2 > 1e-8:
            return False
    return True
```
◇

File defined by 16d, 17ab, 18abcde.

4.2.7 This function just generates a bunch of non-uniformly distributed random points inside the unit-square. According to this scheme, you will often notice clusters clumped near the border of the unit-square.

"../src/lib/utils_algo.py" 18e≡
```
def bunch_of_non_uniform_random_points(numpts):
    cluster_size = int(np.sqrt(numpts))
    numcenters   = cluster_size

    import scipy
    import random
    centers = scipy.rand(numcenters,2).tolist()

    scale, points = 4.0, []
    for c in centers:
        cx, cy = c[0], c[1]
        ⟨For current center c of this loop, generate cluster_size points uniformly in a square centered at it 19a⟩
```

⟨ *Whatever number of points are left to be generated, generate them uniformly inside the unit-square* 19b ⟩

```
    return points
```
◇

File defined by 16d, 17ab, 18abcde.
Defines: `cluster_size` 19ab, `scale`, 19a.

4.2.8

Note that the smaller square around a center, inside which the points are generated is made to lie in the unit-square. This is reflected in the assignment to `sq_size` below.

⟨ *For current center c of this loop, generate* `cluster_size` *points uniformly in a square centered at it* 19a ⟩ ≡

```
sq_size      = min(cx,1-cx,cy, 1-cy)
loc_pts_x    = np.random.uniform(low=cx-sq_size/scale, high=cx+sq_size/scale, size=(cluster_size,))
loc_pts_y    = np.random.uniform(low=cy-sq_size/scale, high=cy+sq_size/scale, size=(cluster_size,))
points.extend(zip(loc_pts_x, loc_pts_y))
```
◇

Fragment referenced in 18e.
Uses: `cluster_size` 18e, `scale`, 18e.

4.2.9

⟨ *Whatever number of points are left to be generated, generate them uniformly inside the unit-square* 19b ⟩ ≡

```
num_remaining_pts = numpts - cluster_size * numcenters
remaining_pts = scipy.rand(num_remaining_pts, 2).tolist()
points.extend(remaining_pts)
```
◇

Fragment referenced in 18e.
Uses: `cluster_size` 18e.

Chapter 5

Classic Horsefly

Module Overview

5.1.1 All algorithms to solve the classic horsefly problems have been implemented in `problem_classic_horsefly.py`. The `run_handler` function acts as a kind of main function for this module. It is called from `main.py` to process the command-line arguments and run the experimental or interactive sections of the code.

"../src/lib/problem_classic_horsefly.py" 20a≡

```
⟨ Relevant imports for classic horsefly 20b ⟩
⟨ Set up logging information relevant to this module 21a ⟩
def run_handler():
    ⟨ Define key-press handler 21b ⟩
    ⟨ Set up interactive canvas 24b ⟩

⟨ Local data-structures for classic horsefly 25a ⟩
⟨ Local utility functions for classic horsefly 57a, ... ⟩
⟨ Algorithms for classic horsefly 27, ... ⟩
⟨ Lower bounds for classic horsefly 49a ⟩
⟨ Plotting routines for classic horsefly 59a, ... ⟩
⟨ Animation routines for classic horsefly 62 ⟩
◇
```

Module Details

5.2.1

⟨ *Relevant imports for classic horsefly* 20b ⟩ ≡

```
from colorama import Fore, Style
from matplotlib import rc
from scipy.optimize import minimize
from sklearn.cluster import KMeans
import argparse
import inspect
import itertools
import logging
import math
import matplotlib as mpl
import matplotlib.pyplot as plt
# plt.style.use('seaborn-poster')
import numpy as np
import os
```

```
import pprint as pp
import randomcolor
import sys
import time
import utils_algo
import utils_graphics
◇
```

Fragment referenced in 20a.

5.2.2 The logger variable becomes becomes global in scope to this module. This allows me to write customized `debug` and `info` functions that let's me format the log messages according to the frame level. I learned this trick from the following Stack Overflow post https://stackoverflow.com/a/5500099/505306.

⟨ *Set up logging information relevant to this module* 21a ⟩ ≡

```
logger=logging.getLogger(__name__)
logging.basicConfig(level=logging.DEBUG)

def debug(msg):
    frame,filename,line_number,function_name,lines,index=inspect.getouterframes(
        inspect.currentframe())[1]
    line=lines[0]
    indentation_level=line.find(line.lstrip())
    logger.debug('{i} [{m}]'.format(
        i='.'*indentation_level, m=msg))

def info(msg):
    frame,filename,line_number,function_name,lines,index=inspect.getouterframes(
        inspect.currentframe())[1]
    line=lines[0]
    indentation_level=line.find(line.lstrip())
    logger.info('{i} [{m}]'.format(
        i='.'*indentation_level, m=msg))
◇
```

Fragment referenced in 20a.
Uses: `logger` 37b.

5.2.3 The key-press handler function detects the keys pressed by the user when the canvas is in active focus. This function allows you to set some of the input parameters like speed ratio φ, or selecting an algorithm interactively at the command-line, generating a bunch of uniform or non-uniformly distributed points on the canvas, or just plain clearing the canvas for inserting a fresh input set of points.

⟨ *Define key-press handler* 21b ⟩ ≡

```

# The key-stack argument is mutable! I am using this hack to my advantage.
def wrapperkeyPressHandler(fig,ax, run):
       def _keyPressHandler(event):
           if event.key in ['i', 'I']:
                ⟨ Start entering input from the command-line 22 ⟩
           elif event.key in ['n', 'N', 'u', 'U']:
                ⟨ Generate a bunch of uniform or non-uniform random points on the canvas 23 ⟩
           elif event.key in ['c', 'C']:
                ⟨ Clear canvas and states of all objects 24a ⟩
       return _keyPressHandler
◇
```

Fragment referenced in 20a.
Defines: `wrapperkeyPressHandler` 24b.

5.2.4 Before running an algorithm, the user needs to select through a menu displayed at the terminal, which one to run. Each algorithm itself, may be run under different conditions, so depending on the key-pressed(and thus algorithm chosen) further sub-menus will be generated at the command-line.

After running the appropriate algorithm, we render the structure computed to a matplotlib canvas/window along with possibly some meta data about the run at the terminal.

This code-chunk is long, but just has brain-dead code. Nothing really needs to be explained about it any further, nor does it need to be broken down.

⟨ *Start entering input from the command-line* 22 ⟩ ≡

```
phi_str = raw_input(Fore.YELLOW + "Enter speed of fly (should be >1): " + Style.RESET_ALL)
phi = float(phi_str)

input_str = raw_input(Fore.YELLOW                                            +\
          "Enter algorithm to be used to compute the tour:\n Options are:\n" +\
        "  (e)    Exact \n"                                                  +\
        "  (t)    TSP   \n"                                                  +\
        "  (tl)   TSP   (using approximate L1 ordering)\n"                   +\
        "  (k)    k2-center   \n"                                            +\
        "  (kl)   k2-center (using approximate L1 ordering)\n"               +\
        "  (g)    Greedy\n"                                                  +\
        "  (gl)   Greedy (using approximate L1 ordering])\n"                 +\
        "  (ginc) Greedy Incremental\n"                                      +\
        "  (phi-mst) Compute the phi-prim-mst "                              +\
        Style.RESET_ALL)

input_str = input_str.lstrip()

# Incase there are patches present from the previous clustering, just clear them
utils_graphics.clearAxPolygonPatches(ax)

if   input_str == 'e':
      horseflytour = \
             run.getTour( algo_dumb,
                          phi )
elif input_str == 'k':
      horseflytour = \
             run.getTour( algo_kmeans,
                          phi,
                          k=2,
                          post_optimizer=algo_exact_given_specific_ordering)
      print " "
      print Fore.GREEN, horseflytour['tour_points'], Style.RESET_ALL
elif input_str == 'kl':
      horseflytour = \
             run.getTour( algo_kmeans,
                          phi,
                          k=2,
                          post_optimizer=algo_approximate_L1_given_specific_ordering)
elif input_str == 't':
      horseflytour = \
             run.getTour( algo_tsp_ordering,
                          phi,
                          post_optimizer=algo_exact_given_specific_ordering)
elif input_str == 'tl':
      horseflytour = \
             run.getTour( algo_tsp_ordering,
                          phi,
                          post_optimizer= algo_approximate_L1_given_specific_ordering)
elif input_str == 'g':
      horseflytour = \
             run.getTour( algo_greedy,
                          phi,
                          post_optimizer= algo_exact_given_specific_ordering)
elif input_str == 'gl':
```

```
            horseflytour = \
                   run.getTour( algo_greedy,
                                phi,
                                post_optimizer= algo_approximate_L1_given_specific_ordering)

      elif input_str == 'ginc':
            horseflytour = \
                   run.getTour( algo_greedy_incremental_insertion,
                                phi )

      elif input_str == 'phi-mst':
            phi_mst = \
                   run.computeStructure(compute_phi_prim_mst ,phi)
      else:
            print "Unknown option. No horsefly for you! ;-D "
            sys.exit()

      #print horseflytour['tour_points']

      if input_str not in ['phi-mst']:
           plotTour(ax,horseflytour, run.inithorseposn, phi, input_str)
      elif input_str == 'phi-mst':
           draw_phi_mst(ax, phi_mst, run.inithorseposn, phi)

      utils_graphics.applyAxCorrection(ax)
      fig.canvas.draw()
      ◇
```

Fragment referenced in 21b.
Uses: `algo_exact_given_specific_ordering` 30a, `algo_greedy_incremental_insertion`, 37a, `computeStructure` 25d, `draw_phi_mst` 61b, `getTour` 25c, `plotTour` 59a.

5.2.5 This chunk generates points uniformly or non-uniformly distributed in the unit square $[0,1]^2$ in the Matplotlib canvas. I will document the schemes used for generating the non-uniformly distributed points later. These schemes are important to test the effectiveness of the horsefly algorithms. Uniform point clouds do no highlight the weaknesses of sequencing algorithms as David Johnson implies in his article on how to write experimental algorithm papers when he talks about algorithms for the TSP.

Note that the option keys `'n'` or `'N'` for entering in non-uniform random-points is just incase the caps-lock key has been pressed on by the user accidentally. Similarly for the `'u'` and `'U'` keys.

⟨ *Generate a bunch of uniform or non-uniform random points on the canvas* 23 ⟩ ≡

```
      numpts = int(raw_input("\n" + Fore.YELLOW+\
                             "How many points should I generate?: "+\
                             Style.RESET_ALL))
      run.clearAllStates()
      ax.cla()

      utils_graphics.applyAxCorrection(ax)
      ax.set_xticks([])
      ax.set_yticks([])

      fig.texts = []

      import scipy
      if event.key in ['n', 'N']:
              run.sites = utils_algo.bunch_of_non_uniform_random_points(numpts)
      else :
              run.sites = scipy.rand(numpts,2).tolist()

      patchSize  = (utils_graphics.xlim[1]-utils_graphics.xlim[0])/140.0
```

```
    for site in run.sites:
        ax.add_patch(mpl.patches.Circle(site, radius = patchSize, \
                     facecolor='blue',edgecolor='black' ))

    ax.set_title('Points : ' + str(len(run.sites)), fontdict={'fontsize':40})
    fig.canvas.draw()
    ◇
```

Fragment referenced in 21b.
Uses: `clearAllStates` 25b.

5.2.6

Clearing the canvas and states of all objects is essential when we want to test out the algorithm on a fresh new point-set; the program need not be shut-down and rerun.

⟨ *Clear canvas and states of all objects* 24a ⟩ ≡

```
run.clearAllStates()
ax.cla()

utils_graphics.applyAxCorrection(ax)
ax.set_xticks([])
ax.set_yticks([])

fig.texts = []
fig.canvas.draw()
◇
```

Fragment referenced in 21b.
Uses: `clearAllStates` 25b.

5.2.7

⟨ *Set up interactive canvas* 24b ⟩ ≡

```
fig, ax =  plt.subplots()
run = HorseFlyInput()
#print run

ax.set_xlim([utils_graphics.xlim[0], utils_graphics.xlim[1]])
ax.set_ylim([utils_graphics.ylim[0], utils_graphics.ylim[1]])
ax.set_aspect(1.0)
ax.set_xticks([])
ax.set_yticks([])

mouseClick   = utils_graphics.wrapperEnterRunPoints (fig,ax, run)
fig.canvas.mpl_connect('button_press_event' , mouseClick )

keyPress     = wrapperkeyPressHandler(fig,ax, run)
fig.canvas.mpl_connect('key_press_event', keyPress   )
plt.show()
◇
```

Fragment referenced in 20a.
Uses: `HorseFlyInput` 25a, `wrapperkeyPressHandler` 21b.

Local Data Structures

5.3.1 This class manages the input and the output of the result of calling various horsefly algorithms.

⟨ *Local data-structures for classic horsefly* 25a ⟩ ≡
```
class HorseFlyInput:
      def __init__(self, sites=[], inithorseposn=()):
           self.sites         = sites
           self.inithorseposn = inithorseposn

      ⟨ Methods for HorseFlyInput 25b, ... ⟩
```
◇

Fragment referenced in 20a.
Defines: `HorseFlyInput` 24b.

5.3.2 Set the sites to an empty list and initial horse position to the empty tuple.

⟨ *Methods for* `HorseFlyInput` 25b ⟩ ≡
```
def clearAllStates (self):
   self.sites = []
   self.inithorseposn = ()
```
◇

Fragment defined by 25bcd, 26.
Fragment referenced in 25a.
Defines: `clearAllStates` 23, 24a.

5.3.3 This method sets an algorithm for calculating a horsefly tour. The name of the algorithm is passed as a command-line argument. The list of possible algorithms are typically prefixed with `algo_`.

The output is a dictionary of size 2, containing two lists:

1. Contains the vertices of the polygonal path taken by the horse
2. The list of sites in the order in which they are serviced by the tour, i.e. the order in which the sites are serviced by the fly.

⟨ *Methods for* `HorseFlyInput` 25c ⟩ ≡
```
def getTour(self, algo, speedratio, k=None, post_optimizer=None):

    if k==None and post_optimizer==None:
          return algo(self.sites, self.inithorseposn, speedratio)
    elif k == None:
          return algo(self.sites, self.inithorseposn, speedratio, post_optimizer)
    else:
          return algo(self.sites, self.inithorseposn, speedratio, k, post_optimizer)
```
◇

Fragment defined by 25bcd, 26.
Fragment referenced in 25a.
Defines: `getTour` 22.
Uses: `self.inithorseposn`, 45a, `self.sites`, 45a.

5.3.4

⟨ *Methods for* `HorseFlyInput` 25d ⟩ ≡
```
def  computeStructure(self, structure_func, phi):
   print Fore.RED, "Computing the phi-mst", Style.RESET_ALL
   return structure_func(self.sites, self.inithorseposn, phi)
```
◇

Fragment defined by 25bcd, 26.
Fragment referenced in 25a.
Defines: `computeStructure` 22.
Uses: `self.inithorseposn`, 45a, `self.sites`, 45a.

5.3.5 This chunk prints a customized representation of the `HorseFlyInput` class

⟨ *Methods for* `HorseFlyInput` 26 ⟩ ≡

```
def __repr__(self):

  if self.sites != []:
     tmp = ''
     for site in self.sites:
         tmp = tmp + '\n' + str(site)
     sites = "The list of sites to be serviced are " + tmp
  else:
     sites = "The list of sites is empty"

  if self.inithorseposn != ():
     inithorseposn = "\nThe initial position of the horse is " + str(self.inithorseposn)
  else:
     inithorseposn = "\nThe initial position of the horse has not been specified"

  return sites + inithorseposn
```
◇

Fragment defined by 25bcd, 26.
Fragment referenced in 25a.

Now that all the boring boiler-plate and handler codes have been written, its finally time for algorithmic ideas and implementations! Every algorithm is given an algorithmic overview followed by the detailed steps woven together with the source code.

Any local utility functions, needed for algorithmic or graphing purposes are collected at the end of this chapter.

Algorithm: Dumb Brute force

5.4.1 Algorithmic Overview

For each of the $n!$ ordering of sites find the ordering which gives the smallest horsefly tour length. Note that given a particular order of visitation, the optimal tour for the horse can be computed optimally using convex optimization methods or by using the SLSQP solver as I do here.

This method is practical only for a very small number of sites, like say 6 or 7. However, it is useful in generating small counter-examples for various conjectures and as a benchmark for the quality of other algorithms for a small number of sites.

5.4.2 Algorithmic Details

⟨ *Algorithms for classic horsefly* 27 ⟩ ≡

```
def algo_dumb(sites, horseflyinit, phi):

    tour_length_fn = tour_length(horseflyinit)
    best_tour      = algo_exact_given_specific_ordering(sites, horseflyinit, phi)
    i              = 0

    for sites_perm in list(itertools.permutations(sites)):

        print "Testing a new permutation ", i, " of the sites"; i = i + 1
        tour_for_current_perm = algo_exact_given_specific_ordering (sites_perm, horseflyinit, phi)

        if tour_length_fn(utils_algo.flatten_list_of_lists(tour_for_current_perm ['tour_points']) ) \
         < tour_length_fn(utils_algo.flatten_list_of_lists(            best_tour ['tour_points']) ):

                best_tour = tour_for_current_perm
                print Fore.RED + "Found better tour!" + Style.RESET_ALL

    #print Fore.RED + "\nHorse Waiting times are ", best_tour['horse_waiting_times'] , Style.RESET_ALL
    return best_tour
```
◇

Fragment defined by 27, 28, 30a, 32, 37a, 51, 54, 56.
Fragment referenced in 20a.
Uses: `algo_exact_given_specific_ordering` 30a, `tour_length` 57a.

Algorithm: Greedy—Nearest Neighbor

5.5.1 Algorithmic Overview

Before proceeding we give a special case of the classical horseflies problem, which we term "collinear-horsefly". Here the objective function is again to minimize the tour-length of the drone with the additional restriction that the truck must always be moving in a straight line towards the site on the line-segment joining itself and the site, while the drone is also restricted to travelling along the same line segment.

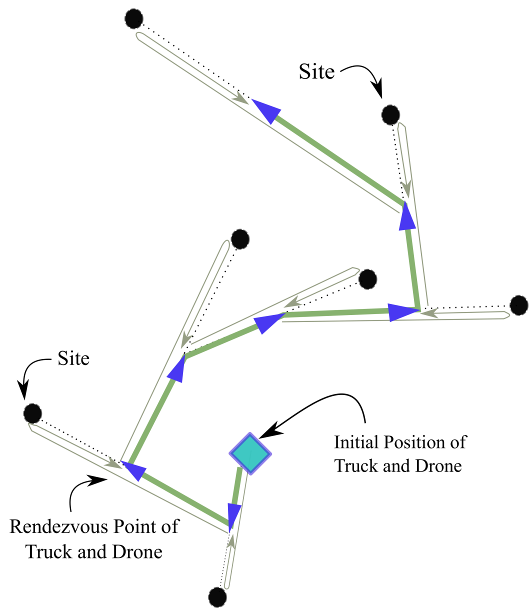


Figure 5.1: The Collinear Horsefly Problem

We can show that an optimal (unrestricted) horsfly solution can be converted to a collinear-horsefly solution at a constant factor increase in the makespan.

5.5.2 Algorithmic Details

5.5.3 This implements the greedy algorithm for the canonical greedy algorithm for collinear horsefly, and then uses the ordering obtained to get the exact tour for that given ordering. Many variations on this are possible. However, this algorithm is simple and may be more amenable to theoretical analysis. We will need an inequality for collapsing chains however.

After extracting the ordering. we use exact/approximate solver for getting a horse-tour that is optimal/approximately optimal for the computed ordering of sites by greedy.

⟨ *Algorithms for classic horsefly* 28 ⟩ ≡

```
def algo_greedy(sites, inithorseposn, phi, post_optimizer):

    ⟨ Define function next_rendezvous_point_for_horse_and_fly 29a ⟩
    ⟨ Define function greedy 29b ⟩

    sites1                 = sites[:]
    sites_ordered_by_greedy = greedy(inithorseposn, remaining_sites=sites1)
    answer                 = post_optimizer(sites_ordered_by_greedy, inithorseposn, phi)
    return answer
◇
```

Fragment defined by 27, 28, 30a, 32, 37a, 51, 54, 56.
Fragment referenced in 20a.
Uses: `greedy` 29b.

5.5.4 When there is a single site, the meeting point of horse and fly can be computed exactly (A simple formula is trivial to derive too, which I do so later)/

Here I just use the exact solver for computing the horse tour when the ordering is given foir a single site.

⟨ *Define function* `next_rendezvous_point_for_horse_and_fly` 29a ⟩ ≡

```
def next_rendezvous_point_for_horse_and_fly(horseposn, site):

     horseflytour = algo_exact_given_specific_ordering([site], horseposn, phi)
     return horseflytour['tour_points'][-1]
```
◇

Fragment referenced in 28.
Uses: `algo_exact_given_specific_ordering` 30a.

5.5.5 Begin the recursion process where for a given initial position of horse and fly and a given collection of sites you find the nearst neighbor proceed according to segment horsefly formula for just and one site, and for the new position repeat the process for the remaining list of sites. The greedy approach can be extended to by finding the k nearest neighbors, constructing the exact horsefly tour there, at the exit point, you repeat by taking k nearest neighbors and so on.

For reference see this link on how nn queries are performed. https://docs.scipy.org/doc/scipy/reference/generated/scipy.spatial.KDTree.query.html Warning this is inefficient!!! I am rebuilding the kd-tree at each step. Right now, I am only doing this for convenience.

The next site to get serviced by the drone and horse after they meet-up is the one which is closest to the current position of the horse.

⟨ *Define function* `greedy` 29b ⟩ ≡

```
def greedy(current_horse_posn, remaining_sites):

    if len(remaining_sites) == 1:
          return remaining_sites
    else:
          from scipy import spatial
          tree          = spatial.KDTree(remaining_sites)
          pts           = np.array([current_horse_posn])
          query_result  = tree.query(pts)
          next_site_idx = query_result[1][0]
          next_site     = remaining_sites[next_site_idx]

          next_horse_posn = next_rendezvous_point_for_horse_and_fly(current_horse_posn, next_site)
          remaining_sites.pop(next_site_idx) # the pop method modifies the list in place.

          return [next_site] + greedy(current_horse_posn = next_horse_posn, remaining_sites = remaining_sites)
```
◇

Fragment referenced in 28.
Defines: `greedy` 28, 37b.

5.5.6 Many of the heuristics, such as the two above that we just implemented, we compute an ordering of sites to visit and then compute the tour-points for the horse. For a given order of visitation calcualting the horse-tour can be done by convex optimization. We give one such routine below, that uses the SLSQP non-linear solver from scipy for computing this horse-tour. I will implement the convex optimization routine from John's paper in a later section. Having two such independent routines for doing the same computation can help in benchmarking.

Later, we will also study approximation algorithms for methods to compute horse-tours for a given order of visitation. For these I will need to benchmark the speed of solving SOCP's versus LP's to see what interesting questions can be studies in this regard.

Since the horsely tour lies inside the square, the bounds for each coordinate for the initial guess is between 0 and 1. Many options are possible, Below I try two possibilities

⟨ *Algorithms for classic horsefly* 30a ⟩ ≡

```
def algo_exact_given_specific_ordering (sites, horseflyinit, phi):

    ⟨ Useful functions for algo_exact_given_specific_ordering 30b, ... ⟩

    cons = generate_constraints(horseflyinit, phi, sites)

    # Initial guess for the non-linear solver.
    #x0 = np.empty(2*len(sites)); x0.fill(0.5) # choice of filling vector with 0.5 is arbitrary
    x0 = utils_algo.flatten_list_of_lists(sites) # the initial choice is just the sites

    assert(len(x0) == 2*len(sites))

    x0                 = np.array(x0)
    sol                = minimize(tour_length(horseflyinit), x0, method= 'SLSQP', \
                                  constraints=cons         , options={'maxiter':500})
    tour_points        = utils_algo.pointify_vector(sol.x)
    numsites           = len(sites)
    alpha              = horseflyinit[0]
    beta               = horseflyinit[1]
    s                  = utils_algo.flatten_list_of_lists(sites)
    horse_waiting_times = np.zeros(numsites)
    ps                 = sol.x

    for i in range(numsites):

        if i == 0 :
            horse_time         = np.sqrt((ps[0]-alpha)**2 + (ps[1]-beta)**2)
            fly_time_to_site   = 1.0/phi * np.sqrt((s[0]-alpha)**2 + (s[1]-beta)**2 )
            fly_time_from_site = 1.0/phi * np.sqrt((s[0]-ps[1])**2 + (s[1]-ps[1])**2)
        else:
            horse_time         = np.sqrt((ps[2*i]-ps[2*i-2])**2 + (ps[2*i+1]-ps[2*i-1])**2)
            fly_time_to_site   = 1.0/phi * np.sqrt(( (s[2*i]-ps[2*i-2])**2 + (s[2*i+1]-ps[2*i-1])**2 ))
            fly_time_from_site = 1.0/phi * np.sqrt(( (s[2*i]-ps[2*i])**2   + (s[2*i+1]-ps[2*i+1])**2 ))

        horse_waiting_times[i] = horse_time - (fly_time_to_site + fly_time_from_site)

    return {'tour_points'                : tour_points,
            'horse_waiting_times'        : horse_waiting_times,
            'site_ordering'              : sites,
            'tour_length_with_waiting_time_included': \
                                        tour_length_with_waiting_time_included(\
                                                     tour_points, \
                                                     horse_waiting_times,
                                                     horseflyinit)}
```

◇

Fragment defined by 27, 28, 30a, 32, 37a, 51, 54, 56.
Fragment referenced in 20a.
Defines: `algo_exact_given_specific_ordering` 22, 27, 29a.
Uses: `generate_constraints` 31, `tour_length` 57a, `tour_length_with_waiting_time_included` 57b.

5.5.7 For the ith segment of the horsefly tour this function returns a constraint function which models the fact that the time taken by the fly is equal to the time taken by the horse along that particular segment.

⟨ *Useful functions for* `algo_exact_given_specific_ordering` 30b ⟩ ≡

```
def ith_leg_constraint(i, horseflyinit, phi, sites):
        if i == 0 :
            def _constraint_function(x):

                #print "Constraint  ", i
```

```
            start = np.array (horseflyinit)
            site  = np.array (sites[0])
            stop  = np.array ([x[0],x[1]])

            horsetime = np.linalg.norm( stop - start )

            flytime_to_site   = 1/phi * np.linalg.norm( site - start )
            flytime_from_site = 1/phi * np.linalg.norm( stop - site  )
            flytime           = flytime_to_site + flytime_from_site
            return horsetime-flytime

        return _constraint_function
    else :

        def _constraint_function(x):

           #print "Constraint  ", i
           start = np.array (  [x[2*i-2], x[2*i-1]]  )
           site  = np.array (  sites[i])
           stop  = np.array (  [x[2*i]  , x[2*i+1]]  )

           horsetime = np.linalg.norm( stop - start )

           flytime_to_site   = 1/phi * np.linalg.norm( site - start )
           flytime_from_site = 1/phi * np.linalg.norm( stop - site  )
           flytime           = flytime_to_site + flytime_from_site
           return horsetime-flytime

        return _constraint_function
◇
```

Fragment defined by 30b, 31.
Fragment referenced in 30a.
Defines: `ith_leg_constraint` 31.

5.5.8 Given input data, of the problem generate the constraint list for each leg of the tour. The number of legs is equal to the number of sites for the case of single horse, single drone

⟨ *Useful functions for* `algo_exact_given_specific_ordering` 31 ⟩ ≡

```
def generate_constraints(horseflyinit, phi, sites):
   cons = []
   for i in range(len(sites)):
        cons.append({'type':'eq','fun': ith_leg_constraint(i,horseflyinit,phi,sites)})
   return cons
◇
```

Fragment defined by 30b, 31.
Fragment referenced in 30a.
Defines: `generate_constraints` 30a, 54.
Uses: `ith_leg_constraint` 30b.

5.5.9 Another useful post-optimizer is one using the $L1$ metric and linear programming. This solves a Linear program using MOSEK and tries to solve the $L1$ version of the equations, with some modifications as outlined in the notebook.

The hope is that solving this is more scalable even if approximate than using the SLSQP solver which chokes on $>=$ 70-80 sites.

I followed the MOSEK tutorial given here to set up the linear system `https://docs.mosek.com/8.1/pythonapi/tutorial-lo-shared.html`

Note that MOSEK has been optimized to solve large sparse systems of LPs. The LP that I set up here is extremely sparse! And hence a perfect fit for MOSEK.

⟨ *Algorithms for classic horsefly* 32 ⟩ ≡

```
def  algo_approximate_L1_given_specific_ordering(sites, horseflyinit, phi):
    import mosek
    numsites = len(sites)

    def p(idx):
        return idx + 0*numsites

    def b(idx):
        return idx + 2*numsites

    def f(idx):
        return idx + 4*numsites

    def h(idx):
        return idx + 6*numsites

    # Define a stream printer to grab output from MOSEK
    def streamprinter(text):
        sys.stdout.write(text)
        sys.stdout.flush()

   numcon = 9 + 13*(numsites-1) # the first site has 9 constraints while the remaining n-1 sites have 13 constraints each
    numvar = 8 * numsites # Each ``L1 triangle'' has 8 variables associated with it

    alpha = horseflyinit[0]
    beta  = horseflyinit[1]

    s = utils_algo.flatten_list_of_lists(sites)

    # Make mosek environment
    with mosek.Env() as env:
        # Create a task object
        with env.Task(0, 0) as task:
            # Attach a log stream printer to the task
            task.set_Stream(mosek.streamtype.log, streamprinter)
            # Append 'numcon' empty constraints.
            # The constraints will initially have no bounds.
            task.appendcons(numcon)
            # Append 'numvar' variables.
            # The variables will initially be fixed at zero (x=0).
            task.appendvars(numvar)

            for idx in range(numvar):
                if (0   <= idx) and (idx < 2*numsites): # free variables (p section of the vector)
                    task.putvarbound(idx, mosek.boundkey.fr, -np.inf, np.inf)

                elif  idx == 2*numsites : # b_0 is a known variable
                    val = abs(s[0]-alpha)
                    task.putvarbound(idx, mosek.boundkey.fx, val, val)

                elif  idx == 2*numsites +1 : # b_1 is a known variable
                    val = abs(s[1]-beta)
                    task.putvarbound(idx, mosek.boundkey.fx, val, val)

                else : # b_2, onwards and the f and h sections of the vector
                    task.putvarbound(idx, mosek.boundkey.lo, 0.0, np.inf)

            # All the coefficients corresponding to the h's are 1's
            # and for the others the coefficients are 0.
```

```
    for i in range(numvar):
        if i >= 6*numsites: # the h-section
            task.putcj(i,1)
        else: # the p,b,f sections of x
            task.putcj(i,0)

    # Constraints for the zeroth triangle corresponding to the zeroth site
    row = -1
  row += 1; task.putconbound(row, mosek.boundkey.up, -np.inf, alpha ) ; task.putarow(row, [p(0), h(0)],[1.0, -1.0])
  row += 1; task.putconbound(row, mosek.boundkey.lo, alpha  , np.inf) ; task.putarow(row, [p(0), h(0)],[1.0,  1.0])

  row += 1; task.putconbound(row, mosek.boundkey.up, -np.inf, beta ) ; task.putarow(row, [p(1), h(1)],[1.0, -1.0])
  row += 1; task.putconbound(row, mosek.boundkey.lo, beta  , np.inf) ; task.putarow(row, [p(1), h(1)],[1.0,  1.0])

  row += 1; task.putconbound(row, mosek.boundkey.up, -np.inf, s[0]  ) ; task.putarow(row, [p(0), f(0)],[1.0, -1.0])
  row += 1; task.putconbound(row, mosek.boundkey.lo,  s[0]  , np.inf) ; task.putarow(row, [p(0), f(0)],[1.0,  1.0])

  row += 1; task.putconbound(row, mosek.boundkey.up, -np.inf, s[1]  ) ; task.putarow(row, [p(1), f(1)],[1.0, -1.0])
  row += 1; task.putconbound(row, mosek.boundkey.lo,  s[1]  , np.inf) ; task.putarow(row, [p(1), f(1)],[1.0,  1.0])

    # The most important constraint of all! On the ``L1 triangle''
    # time for drone to start from the truck reach site and get back to truck
    # = time for truck between the two successive rendezvous points
    # The way I have modelled the following constraint it is not exactly
    # the same as the previous statement of equality of times of truck
    # and drone, but for initial experiments it looks like this gives
    # waiting times to be automatically close to 0 (1e-9 close to machine-epsilon)
    # Theorem in the making??
    row += 1; task.putconbound(row, mosek.boundkey.fx, 0.0 , 0.0 ) ;
    task.putarow(row, [b(0), b(1), f(0), f(1), h(0), h(1)], [1.0,1.0,1.0,1.0,-phi, -phi])

    # Constraints beginning from the 1st triangle
    for  i in range(1,numsites):
     row+=1 ;  task.putconbound(row, mosek.boundkey.lo, -s[2*i]  , np.inf) ; task.putarow(row, [b(2*i),   p(2*i-2)],[1
     row+=1 ;  task.putconbound(row, mosek.boundkey.lo,  s[2*i]  , np.inf) ; task.putarow(row, [b(2*i),   p(2*i-2)],[1
     row+=1 ;  task.putconbound(row, mosek.boundkey.lo, -s[2*i+1], np.inf) ; task.putarow(row, [b(2*i+1), p(2*i-1)],[
     row+=1 ;  task.putconbound(row, mosek.boundkey.lo,  s[2*i+1], np.inf) ; task.putarow(row, [b(2*i+1), p(2*i-1)],[

     row+=1 ;  task.putconbound(row, mosek.boundkey.lo, -s[2*i]  , np.inf) ; task.putarow(row, [f(2*i),    p(2*i)]  ,[
     row+=1 ;  task.putconbound(row, mosek.boundkey.lo,  s[2*i]  , np.inf) ; task.putarow(row, [f(2*i),    p(2*i)]  ,[1
     row+=1 ;  task.putconbound(row, mosek.boundkey.lo, -s[2*i+1], np.inf) ; task.putarow(row, [f(2*i+1),  p(2*i+1)],
     row+=1 ;  task.putconbound(row, mosek.boundkey.lo,  s[2*i+1], np.inf) ; task.putarow(row, [f(2*i+1),  p(2*i+1)],

     row+=1 ;  task.putconbound(row, mosek.boundkey.lo, 0.0    , np.inf); task.putarow(row, [p(2*i)  , p(2*i-2), h(2*
     row+=1 ;  task.putconbound(row, mosek.boundkey.up, -np.inf , 0.0  ); task.putarow(row, [p(2*i)  , p(2*i-2), h(2*
     row+=1 ;  task.putconbound(row, mosek.boundkey.lo, 0.0    , np.inf); task.putarow(row, [p(2*i+1), p(2*i-1), h(2*
     row+=1 ;  task.putconbound(row, mosek.boundkey.up, -np.inf , 0.0  ); task.putarow(row, [p(2*i+1), p(2*i-1), h(2*
        # The most important constraint of all! On the ``L1 triangle''
        # time for drone to start from the truck reach site and get back to truck
        # = time for truck between the two successive rendezvous points
        row+=1; task.putconbound(row, mosek.boundkey.fx, 0.0 , 0.0 ) ;
     task.putarow(row, [b(2*i), b(2*i+1), f(2*i), f(2*i+1), h(2*i), h(2*i+1)], [1.0,1.0,1.0,1.0,-phi, -phi])

    # Input the objective sense (minimize/maximize)
    task.putobjsense(mosek.objsense.minimize)
    task.optimize()
    # Print a summary containing information
    # about the solution for debugging purposes
    #task.solutionsummary(mosek.streamtype.msg)

    # Get status information about the solution
    solsta = task.getsolsta(mosek.soltype.bas)
```

```
if (solsta == mosek.solsta.optimal or
            solsta == mosek.solsta.near_optimal):
        xx = [0.] * numvar
            # Request the basic solution.
        task.getxx(mosek.soltype.bas, xx)
        #print("Optimal solution: ")
        #for i in range(numvar):
        #    print("x[" + str(i) + "]=" + str(xx[i]))
elif (solsta == mosek.solsta.dual_infeas_cer or
        solsta == mosek.solsta.prim_infeas_cer or
        solsta == mosek.solsta.near_dual_infeas_cer or
        solsta == mosek.solsta.near_prim_infeas_cer):
        print("Primal or dual infeasibility certificate found.\n")
elif solsta == mosek.solsta.unknown:
        print("Unknown solution status")
else:
        print("Other solution status")

# Now that we have solved the LP
# We need to extract the ``p'' section of the vector
ps = xx[:2*numsites]
bs = xx[2*numsites:4*numsites]
fs = xx[4*numsites:6*numsites]
hs = xx[6*numsites:]

#######################################################################################
# This commented out section is important to check how close to zero the waiting times
# are as calculated by the LP. To understand this, comment in this section and comment
# out the part using tghe L2 metric below it
#######################################################################################
# horse_waiting_times = np.zeros(numsites)
# for i in range(numsites):
#     if i == 0 :
#         horse_time         = abs(ps[0]-alpha) + abs(ps[1]-beta)
#         fly_time_to_site   = 1.0/phi * (abs(s[0]-alpha) + abs(s[1]-beta))
#         fly_time_from_site = 1.0/phi * (abs(s[0]-ps[1]) + abs(s[1]-ps[1]))
#     else:
#         horse_time         = abs(ps[2*i]-ps[2*i-2]) + abs(ps[2*i+1]-ps[2*i-1])
#         fly_time_to_site   = 1.0/phi * ( abs(s[2*i]-ps[2*i-2]) + abs(s[2*i+1]-ps[2*i-1]) )
#         fly_time_from_site = 1.0/phi * ( abs(s[2*i]-ps[2*i])   + abs(s[2*i+1]-ps[2*i+1]) )
#     horse_waiting_times[i] = horse_time - (fly_time_to_site + fly_time_from_site)

horse_waiting_times = np.zeros(numsites)
for i in range(numsites):
    if i == 0 :
        horse_time         = np.sqrt((ps[0]-alpha)**2 + (ps[1]-beta)**2)
        fly_time_to_site   = 1.0/phi * np.sqrt((s[0]-alpha)**2 + (s[1]-beta)**2)
        fly_time_from_site = 1.0/phi * np.sqrt((s[0]-ps[1])**2 + (s[1]-ps[1])**2)
    else:
        horse_time         = np.sqrt((ps[2*i]-ps[2*i-2])**2 + (ps[2*i+1]-ps[2*i-1])**2)
        fly_time_to_site   = 1.0/phi * np.sqrt( (s[2*i]-ps[2*i-2])**2 + (s[2*i+1]-ps[2*i-1])**2 )
        fly_time_from_site = 1.0/phi * np.sqrt( (s[2*i]-ps[2*i])**2   + (s[2*i+1]-ps[2*i+1])**2 )

    horse_waiting_times[i] = horse_time - (fly_time_to_site + fly_time_from_site)

tour_points = utils_algo.pointify_vector(ps)
return {'tour_points'        : tour_points,
        'horse_waiting_times': horse_waiting_times,
        'site_ordering'      : sites,
    'tour_length_with_waiting_time_included': tour_length_with_waiting_time_included(tour_points, horse_waiting
```

◇

Fragment defined by 27, 28, 30a, 32, 37a, 51, 54, 56.
Fragment referenced in 20a.
Uses: `tour_length_with_waiting_time_included` 57b.

Algorithm: Greedy—Incremental Insertion

Algorithmic Overview

5.6.1 The greedy nearest neighbor heuristic described in section 5.5 gives an $O(\log n)$ approximation for n sites in the plane. However, there exists an alternative greedy incremental insertion algorithm for the TSP that yields a 2-approximation. Similar to the greedy-nn algorithm we can generalize the greedy-incremental approach to the collinear-horseflies setting (cf: Figure 5.1).

5.6.2 In this approach, we maintain a list of visited sites V (along with the order of visitation $\mathcal{O}$) and the unvisited sites U. For the given collinear-horsefly tour serving V pick a site s from U along with a position in $\mathcal{O}$ (calling the resulting ordering $\mathcal{O}^{'}$) that minimizes the cost of the horsefly tour serving the sites $V \cup \{s\}$ in the order $\mathcal{O}^{'}$.

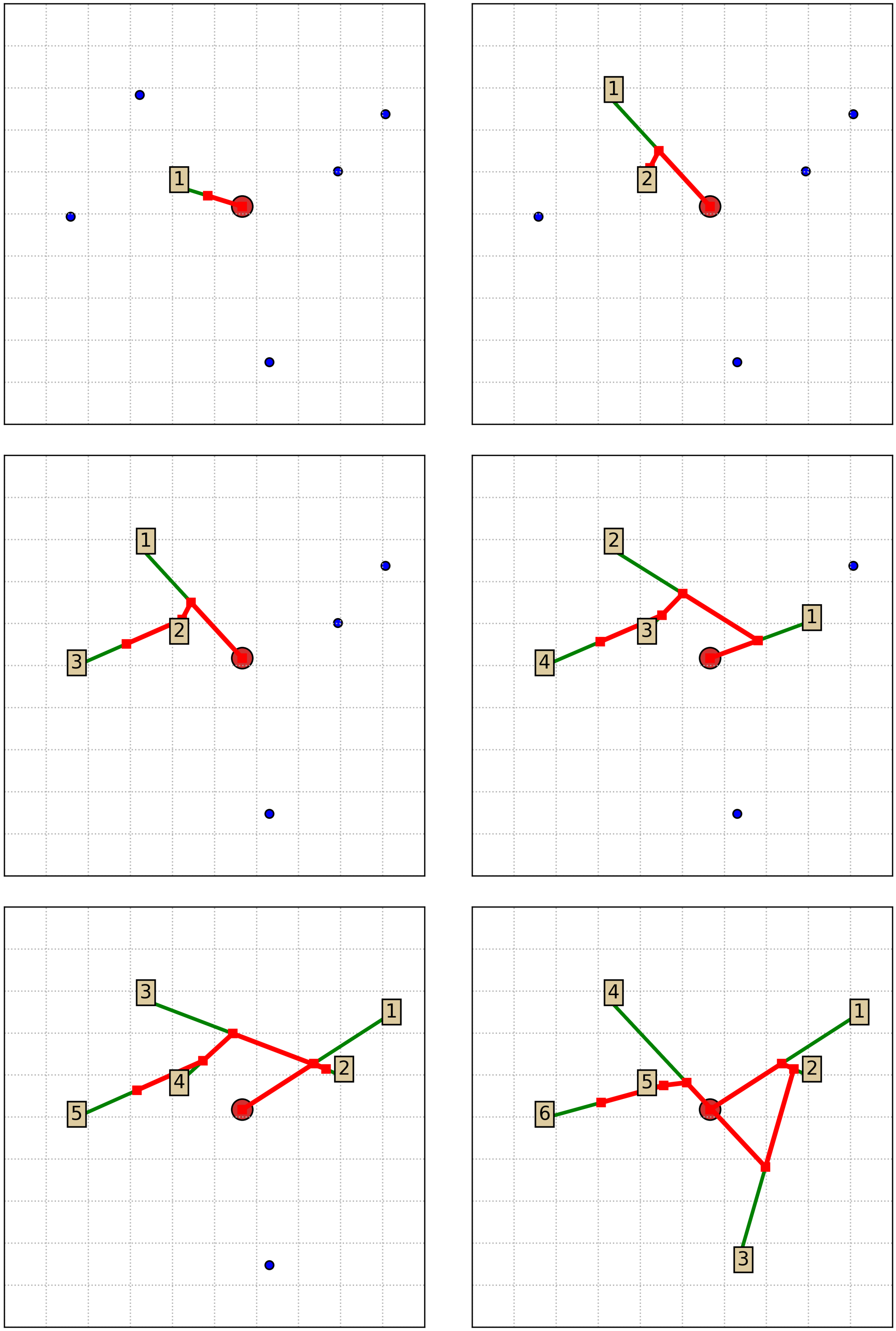


Figure 5.2: Greedy incremental insertion for collinear horseflies. $\varphi = 3.0$. Notice that the ordering of the visited sites keep changing based on where we decide to insert an unvisited site.

Figure 5.2 depicts the incremental insertion process for the case of 4 sites and $\varphi = 3$. Notice that the ordering of the visited sites keep changing based on where we decide to insert an unvisited site.

The implementation of this algorithm for collinear-horseflies raises several interesting non-trivial data-structural questions in their own right: how to quickly find the site from U to insert into V, and keep track the changing length of the horsefly tour. Note that inserting a site causes the length of the tour of the truck to change, for all the sites after s.

Algorithmic Details

5.6.3 The implementation of the algorithm is "parametrized" over various strategies for insertion. i.e. we treat each insertion policy as a black-box argument to the function.

Efficient policies for detecting the exact or approximate point for cheapest insertion will be described in section 5.7. We also implement a "naive" policy as a way benchmark the quality and speed of implementation of future insertion policies.

⟨ *Algorithms for classic horsefly* 37a ⟩ ≡

```
⟨ Define auxiliary helper functions 43c, ... ⟩
⟨ Define various insertion policy classes 45a ⟩
def algo_greedy_incremental_insertion(sites, inithorseposn, phi,
                                      insertion_policy_name       = "naive",
                                      write_algo_states_to_disk_p = True   ,
                                      animate_schedule_p          = True   ,
                                      post_optimizer              = None):
      ⟨ Set log, algo-state and input-output files config 37b ⟩
      ⟨ Set insertion policy class for current run 38a ⟩

      while insertion_policy.unvisited_sites_idxs:
          ⟨ Use insertion policy to find the cheapest site to insert into current tour 38b ⟩
          ⟨ Write algorithms current state to file 39a ⟩

      ⟨ Write input and output to file 42a ⟩
      ⟨ Make an animation of the schedule, if animate_schedule_p == True 43a ⟩
      sys.exit()
      ⟨ Make an animation of algorithm states, if write_algo_states_to_disk_p == True 42b ⟩
      ⟨ Return horsefly tour, along with additional information 43b ⟩
◇
```

Fragment defined by 27, 28, 30a, 32, 37a, 51, 54, 56.
Fragment referenced in 20a.
Defines: `algo_greedy_incremental_insertion`, 22, `write_algo_states_to_disk_p` 39ab, 42b.

5.6.4 Note that for each run of the algorithm, we create a dedicated directory and use a corresponding log file in that directory. It will typically containe detailed information on the progress of the algorithm and the steps executed.

For algorithm analysis, and verification of correctness,on the other hand, we will typically be interested in the states of the data-structures at the end of the while loop; each such state will be written out as a YAML file. Such files can be useful for animating the progress of the algorithm.

Finally, just before returning the answer, we write the input and output to a separate YAML file. All in all, there are three "types" of output files within each directory that corresponds to an algorithm's run: a log file, algorithm states files, and finally an input-output file.

⟨ *Set log, algo-state and input-output files config* 37b ⟩ ≡

```
import sys, logging, datetime, os, errno

algo_name     = 'algo-greedy-incremental-insertion'
time_stamp    = datetime.datetime.now().strftime('Day-%Y-%m-%d_ClockTime-%H:%M:%S')
dir_name      = algo_name + '---' + time_stamp
log_file_name = dir_name + '/' + 'run.log'
io_file_name  = 'input_and_output.yml'
```

```
# Create directory for writing data-files and logs to for
# current run of this algorithm
try:
    os.makedirs(dir_name)
except OSError as e:
    if e.errno != errno.EEXIST:
        raise

logging.basicConfig( filename = log_file_name,
                     level    = logging.DEBUG,
                     format   = '%(asctime)s: %(levelname)s: %(message)s',
                     filemode = 'w' )
#logger = logging.getLogger()
info("Started running greedy_incremental_insertion for classic horsefly")

algo_state_counter = 0
◇
```

Fragment referenced in 37a.
Defines: `io_file_name`, 42a, `logger` 21a.
Uses: `greedy` 29b.

5.6.5 This fragment merely sets the variable `insertion_policy` to the appropriate function. This will later help us in studying the speed of the algorithm and quality of the solution for various insertion policies during the experimental analysis.

⟨ *Set insertion policy class for current run* 38a ⟩ ≡

```
if insertion_policy_name == "naive":
     insertion_policy = PolicyBestInsertionNaive(sites, inithorseposn, phi)
else:
     print insertion_policy_name
     sys.exit("Unknown insertion policy: ")
debug("Finished setting insertion policy: " + insertion_policy_name)
◇
```

Fragment referenced in 37a.

5.6.6 Note that while defining the body of the algorithm, we treat the insertion policy (whose name has already been passed as an string argument) as a kind of black-box, since all policy classes have the same interface. The detailed implementation for the various insertion policies are given later.

⟨ *Use insertion policy to find the cheapest site to insert into current tour* 38b ⟩ ≡

```
insertion_policy.insert_another_unvisited_site()
debug(Fore.GREEN + "Inserted another unvisited site" + Style.RESET_ALL)
◇
```

Fragment referenced in 37a.

5.6.7 When using Python 2.7 (as I am doing with this suite of programs), you should have the `pyyaml` module version 3.12 installed. Version 4.1 breaks for some weird reason; it can't seem to serialized Numpy objects. See https://github.com/kevin1024/vcrpy/issues/366 for a brief discussion on this topic.

The version of `pyyaml` on your machine can be checked by printing the value of `yaml.__version__`. To install the correct version of pyyaml (if you get errors) use

```
sudo pip uninstall pyyaml && sudo pip install pyyaml=3.12
```

5.6.8 We use the `write_algo_states_to_disk_p` boolean argument to explicitly specify whether to write the current algorithm state along with its image to disk or not. This is because Matplotlib and PyYaml is very slow when writing image files to disk. Later on, I will probably switch to Asymptote for all my plotting, but for the moment I will stick to Matplotlib because I don't want to have to switch languages right now.

And much of my plots will be of a reasonably high-quality for the purpose of presentations. This will naturally affect timing/benchmarking results.

⟨ *Write algorithms current state to file* 39a ⟩ ≡

```
if write_algo_states_to_disk_p:
     import yaml
     algo_state_file_name = 'algo_state_'                    + \
                      str(algo_state_counter).zfill(5) + \
                      '.yml'

     data = {'insertion_policy_name' : insertion_policy_name                       ,
             'unvisited_sites'       : [insertion_policy.sites[u] \
                                           for u in insertion_policy.unvisited_sites_idxs],
             'visited_sites'         : insertion_policy.visited_sites                    ,
             'horse_tour'            : insertion_policy.horse_tour }

     with open(dir_name + '/' + algo_state_file_name, 'w') as outfile:
          yaml.dump( data   , \
                     outfile, \
                     default_flow_style = False)
          ⟨ Render current algorithm state to image file 39b ⟩

     algo_state_counter = algo_state_counter + 1
     debug("Dumped algorithm state to " + algo_state_file_name)
◇
```

Fragment referenced in 37a.
Uses: `write_algo_states_to_disk_p` 37a.

⟨ *Render current algorithm state to image file* 39b ⟩ ≡

```
import utils_algo
if write_algo_states_to_disk_p:
     ⟨ Set up plotting area and canvas, fig, ax, and other configs 39c ⟩
     ⟨ Extract x and y coordinates of the points on the horse, fly tours, visited and unvisited sites 40a ⟩
     ⟨ Mark initial position of horse and fly boldly on canvas 40b ⟩
     ⟨ Place numbered markers on visited sites to mark the order of visitation explicitly 41b ⟩
     ⟨ Draw horse and fly-tours 41a ⟩
     ⟨ Draw unvisited sites as filled blue circles 41c ⟩
     ⟨ Give metainformation about current picture as headers and footers 41d ⟩
     ⟨ Write image file 41e ⟩
◇
```

Fragment referenced in 39a.
Uses: `write_algo_states_to_disk_p` 37a.

5.6.9

⟨ *Set up plotting area and canvas, fig, ax, and other configs* 39c ⟩ ≡

```
from matplotlib import rc
rc('font', **{'family': 'serif', \
           'serif': ['Computer Modern']})
rc('text', usetex=True)
fig,ax = plt.subplots()
ax.set_xlim([0,1])
ax.set_ylim([0,1])
ax.set_aspect(1.0)
```

```
ax = fig.gca()
ax.set_xticks(np.arange(0, 1, 0.1))
ax.set_yticks(np.arange(0, 1., 0.1))
plt.grid(linestyle='dotted')
ax.set_xticklabels([]) # to remove those numbers at the bottom
ax.set_yticklabels([])

ax.tick_params(
    bottom=False,      # ticks along the bottom edge are off
    left=False,        # ticks along the top edge are off
    labelbottom=False) # labels along the bottom edge are off
◇
```

Fragment referenced in 39b.

5.6.10 Matplotlib typically plots points using x and y coordinates of the points in separate points.

⟨ *Extract x and y coordinates of the points on the horse, fly tours, visited and unvisited sites* 40a ⟩ ≡

```
# Route for the horse
xhs = [ data['horse_tour'][i][0] \
          for i in range(len(data['horse_tour']))  ]
yhs = [ data['horse_tour'][i][1] \
          for i in range(len(data['horse_tour']))  ]

# Route for the fly. The fly keeps alternating between the site and the horse
xfs , yfs = [xhs[0]], [yhs[0]]
for site, pt in zip (data['visited_sites'],
                     data['horse_tour'][1:]):
    xfs.extend([site[0], pt[0]])
    yfs.extend([site[1], pt[1]])

xvisited = [ data['visited_sites'][i][0] \
               for i in range(len(data['visited_sites']))  ]
yvisited = [ data['visited_sites'][i][1] \
               for i in range(len(data['visited_sites']))  ]

xunvisited = [ data['unvisited_sites'][i][0] \
                 for i in range(len(data['unvisited_sites']))  ]
yunvisited = [ data['unvisited_sites'][i][1]
                 for i in range(len(data['unvisited_sites'])) ]
debug("Extracted x and y coordinates for route of horse, fly, visited and unvisited sites")
◇
```

Fragment referenced in 39b.

5.6.11

⟨ *Mark initial position of horse and fly boldly on canvas* 40b ⟩ ≡

```
ax.add_patch( mpl.patches.Circle( inithorseposn, \
                                  radius = 1/55.0,\
                                  facecolor= '#D13131', #'red',\
                                  edgecolor='black')  )
debug("Marked the initial position of horse and fly on canvas")
◇
```

Fragment referenced in 39b.

⟨ *Draw horse and fly-tours* 41a ⟩ ≡

```
ax.plot(xfs,yfs,'g-',linewidth=1.1)
ax.plot(xhs, yhs, color='r', \
        marker='s', markersize=3, \
        linewidth=1.6)
debug("Plotted the horse and fly tours")
◇
```

Fragment referenced in 39b.

⟨ *Place numbered markers on visited sites to mark the order of visitation explicitly* 41b ⟩ ≡

```
for x,y,i in zip(xvisited, yvisited, range(len(xvisited))):
     ax.text(x, y, str(i+1),  fontsize=8, \
             bbox=dict(facecolor='#ddcba0', alpha=1.0, pad=2.0))
debug("Placed numbered markers on visited sites")
◇
```

Fragment referenced in 39b.

⟨ *Draw unvisited sites as filled blue circles* 41c ⟩ ≡

```
for x, y in zip(xunvisited, yunvisited):
     ax.add_patch( mpl.patches.Circle( (x,y),\
                                     radius    = 1/100.0,\
                                     facecolor = 'blue',\
                                     edgecolor = 'black')  )
debug("Drew univisted sites")
◇
```

Fragment referenced in 39b.

5.6.12

⟨ *Give metainformation about current picture as headers and footers* 41d ⟩ ≡

```
fontsize = 15
ax.set_title( r'Number of sites visited so far: ' +\
               str(len(data['visited_sites']))   +\
               '/' + str(len(sites))           ,  \
                    fontdict={'fontsize':fontsize})
ax.set_xlabel(r'$\varphi=$'+str(phi), fontdict={'fontsize':fontsize})
debug("Setting title, headers, footers, etc...")
◇
```

Fragment referenced in 39b.

Note that after writing image files, you should close the current figure. Otherwise the collection of all the open figures starts hogging the RAM. Matplotlib throws a a warning to this effect (if you don't close to the figures) after writing about 20 figures:

```
/usr/local/lib/python2.7/dist-packages/matplotlib/pyplot.py:528: RuntimeWarning:
More than 20 figures have been opened. Figures created through the pyplot interface
(`matplotlib.pyplot.figure`) are retained until explicitly closed and may consume
too much memory. (To control this warning, see the rcParam `figure.max_open_warning`).
max_open_warning, RuntimeWarning)
```

There is a Stack Overflow answer (https://stackoverflow.com/a/21884375/505306) which advises to call `plt.close()` after writing out a file that closes the *current* figure to avoid the above warning.

⟨ *Write image file* 41e ⟩ ≡

```
image_file_name = 'algo_state_'                    +\
                  str(algo_state_counter).zfill(5) +\
                     '.png'
plt.savefig(dir_name + '/' + image_file_name,  \
            bbox_inches='tight', dpi=250)
print "Wrote " + image_file_name + " to disk"
plt.close()
debug(Fore.BLUE+"Rendered algorithm state to image file"+Style.RESET_ALL)
◇
```

Fragment referenced in 39b.

5.6.13 The final answer is written to disk in the form of a YAML file. It lists the input sites in the order of visitation computed by the algorithm and gives the tour of the horse. Note that the number of points on the horse's tour is 1 more than the number of given sites.

⟨ *Write input and output to file* 42a ⟩ ≡

```
# ASSERT: `inithorseposn` is included as first point of the tour
assert(len(insertion_policy.horse_tour) == len(insertion_policy.visited_sites) + 1)

# ASSERT: All sites have been visited. Simple sanity check
assert(len(insertion_policy.sites)   == len(insertion_policy.visited_sites))

data = {'insertion_policy_name' : insertion_policy_name   ,
        'visited_sites'  : insertion_policy.visited_sites ,
        'horse_tour'     : insertion_policy.horse_tour    ,
        'phi'            : insertion_policy.phi           ,
        'inithorseposn'  : insertion_policy.inithorseposn}

import yaml
with open(dir_name + '/' + io_file_name, 'w') as outfile:     yaml.dump( data, \
                outfile, \
                default_flow_style=False)
debug("Dumped input and output to " + io_file_name)
◇
```

Fragment referenced in 37a.
Uses: `io_file_name`, 37b.

5.6.14 If algorithm states have been rendered to files in the run-folder, we stitch them together using `ffmpeg` and make an `.mp4` animation of the changing states of the algorithms. The `.mp4` file will be in the algorithm's run folder. I used the tutorial given on https://en.wikibooks.org/wiki/FFMPEG_An_Intermediate_Guide/image_sequence for choosing the particular command-line options to `ffmpeg` below. The options `-hide_banner -loglevel panic` to quieten `ffmpeg`'s output were suggested by https://superuser.com/a/1045060/102371

⟨ *Make an animation of algorithm states, if* `write_algo_states_to_disk_p == True` 42b ⟩ ≡

```
if write_algo_states_to_disk_p:
     import subprocess, os
     os.chdir(dir_name)
     subprocess.call( ['ffmpeg',  '-hide_banner', '-loglevel', 'verbose', \
                       '-r', '1',  '-i', 'algo_state_%05d.png', \
                       '-vcodec', 'mpeg4', '-r', '10' , \
                       'algo_state_animation.avi']  )
     os.chdir('../')
◇
```

Fragment referenced in 37a.
Uses: `write_algo_states_to_disk_p` 37a.

5.6.15 This chunks reads the information in the input-output file just written out as a YAML file in the run-folder and then renders the process of the horse and fly moving around the plane delivering packages to sites.

⟨ *Make an animation of the schedule, if* `animate_schedule_p == True` 43a ⟩ ≡

```
if animate_schedule_p :
     animateSchedule(dir_name + '/' + io_file_name)
```
◇

Fragment referenced in 37a.

5.6.16

⟨ *Return horsefly tour, along with additional information* 43b ⟩ ≡

```
debug("Returning answer")
horse_waiting_times = np.zeros(len(sites)) # TODO write this to file later
return {'tour_points'                : insertion_policy.horse_tour[1:],
        'horse_waiting_times'        : horse_waiting_times,
        'site_ordering'              : insertion_policy.visited_sites,
        'tour_length_with_waiting_time_included': \
                                       tour_length_with_waiting_time_included(\
                                                    insertion_policy.horse_tour[1:], \
                                                    horse_waiting_times, \
                                                    inithorseposn)}
```
◇

Fragment referenced in 37a.
Uses: `tour_length_with_waiting_time_included` 57b.

5.6.17 We now define some of the functions that were referred to in the above chunks. Given the intial position of the truck and drone, and a list of sites, we need to compute the collinear horsefly tour length for the given ordering. This is the function that is used in every policy class while deciding which is the cheapest unvisited site to insert into the current ordering of visited sites.

Note that the order in which sites are passed to this function matters. It assumes that you want to compute the collinear horseflies tour length for the sites *in the given order.*

For this, we use the formula for computing the rendezvous point when there is only a single site, given by the code-chunk below.

⟨ *Define auxiliary helper functions* 43c ⟩ ≡

```
def single_site_solution(site, horseposn, phi):

     h = np.asarray(horseposn)
     s = np.asarray(site)

     hs_mag  = 1.0/np.linalg.norm(s-h)
     hs_unit = 1.0/hs_mag * (s-h)

     r      = h +  2*hs_mag/(1+phi) * hs_unit # Rendezvous point
     hr_mag = np.linalg.norm(r-h)

     return (tuple(r), hr_mag)
```
◇

Fragment defined by 43c, 44ab.
Fragment referenced in 37a.
Defines: `single_site_solution` 44ab, 50b.

With that the tour length functions for collinear horseflies can be implemented as an elementary instance of the fold pattern of functional programming. [1]

[1] Python has folds tucked away in some corner of its standard library. But I am not using it during the first hacky portion of this draft. Also Shane mentioned it has performance issues? Double-check this later!

⟨ *Define auxiliary helper functions* 44a ⟩ ≡

```
def compute_collinear_horseflies_tour_length(sites, horseposn, phi):

     if not sites: # No more sites, left to visit!
          return 0
     else:         # Some sites are still left on the itinerary

          (rendezvous_pt, horse_travel_length) = single_site_solution(sites[0], horseposn, phi )
          return horse_travel_length  + \
                 compute_collinear_horseflies_tour_length( sites[1:], rendezvous_pt, phi )
```
◇

Fragment defined by 43c, 44ab.
Fragment referenced in 37a.
Defines: `compute_collinear_horseflies_tour_length` 45c, 46b.
Uses: `single_site_solution` 43c.

⟨ *Define auxiliary helper functions* 44b ⟩ ≡

```
def compute_collinear_horseflies_tour(sites, inithorseposn, phi):

      horseposn         = inithorseposn
      horse_tour_points = [inithorseposn]

      for site in sites:
          (rendezvous_pt, _) = single_site_solution(site, horseposn, phi )

          horse_tour_points.append(rendezvous_pt)
          horseposn = rendezvous_pt

      return horse_tour_points
```
◇

Fragment defined by 43c, 44ab.
Fragment referenced in 37a.
Defines: `compute_collinear_horseflies_tour` 47.
Uses: `single_site_solution` 43c.

Insertion Policies

We have finished implemented the entire algorithm, except for the implementation of the various insertion policy classes.

The main job of an insertion policy class is to keep track of the unvisited sites, the order of the visited sites and the horsefly tour itself. Every time, the method `.get_next_site(...)` is called, it chooses an appropriate (i.e. cheapest) unvisited site to insert into the current ordering, and update the set of visited and unvisited sites and details of the horsefly tour.

To do this quickly it will typically need auxiliary data-structures whose specifics will depend on the details of the policy chosen.

5.7.1 Naive Insertion

First, a naive implementation of the cheapest insertion heuristic, that will be useful in future benchmarking of running times and solution quality for implementations that are quicker but make more sophisticated uses of data-structures.

In this policy for each unvisited site we first find the position in the current tour, which after insertion into that position amongst the visited sites yields the smallest increase in the collinear-horseflies tour-length.

Then we pick the unvisited site which yields the overall smallest increase in tour-length and insert it into its computed position from its previous paragraph.

Clearly this implementation and has at least quadratic running time. Later on, we will be investigating algorithms and data-structures for speeding up this operation.

The hope is to be able to find a dynamic data-structure to perform this insertion in logarithmic time. Variations on tools such as the well-separated pair decomposition might help achieve this goal. Jon Bentley used kd-trees to perform the insertion in his experimental TSP paper, but he wasn't dealing with the shifting tour structure as we have in horseflies. Also he did not deal with the question of finding an approximate point for insertion. These

5.7.2 Since the interface for all policy classes will be the same, it is best, if have a base class for such classes. Since the details of the interface may change, I'll probably do this later. For now, I'll just keep all the policy classes completely separate while keeping the interface of the constructors and methods the same. I'll refactor things later.

The plan in that case should be to make an abstract class that has an abstract method called `insert_unvisited_site` and three data-fields made from the base-constructor named `sites`, `inithorseposn` and `phi`. Classes which inherit this abstract base class, will add their own local data-members and methods for keeping track of data for insertion.

⟨ *Define various insertion policy classes* 45a ⟩ ≡

```
class PolicyBestInsertionNaive:

    def __init__(self, sites, inithorseposn, phi):

         self.sites          = sites
         self.inithorseposn  = inithorseposn
         self.phi            = phi

         self.visited_sites       = []                # The actual list of visited sites (not indices)
         self.unvisited_sites_idxs = range(len(sites)) # This indexes into self.sites
         self.horse_tour          = [self.inithorseposn]

    ⟨ Methods for PolicyBestInsertionNaive 45b ⟩
◇
```

Fragment referenced in 37a.
Defines: `self.horse_tour` 47, `self.inithorseposn`, 25cd, 45c, 46b, 47, `self.sites`, 25cd, `self.visited_sites`, 45c, 47.

5.7.3

⟨ *Methods for* `PolicyBestInsertionNaive` 45b ⟩ ≡

```
def insert_another_unvisited_site(self):
   ⟨ Compute the length of the tour that currently services the visited sites 45c ⟩
   delta_increase_least_table = [] # tracking variable updated in for loop below

   for u in self.unvisited_sites_idxs:
      ⟨ Set up tracking variables local to this iteration 46a ⟩
      ⟨ If self.sites[u] is chosen for insertion, find best insertion position and update delta_increase_least_table 46b ⟩

   ⟨ Find the unvisited site which on insertion increases tour-length by the least amount 46c ⟩
   ⟨ Update states for PolicyBestInsertionNaive 47 ⟩
◇
```

Fragment referenced in 45a.
Defines: `delta_increase_least_table` 46bc.

5.7.4

⟨ *Compute the length of the tour that currently services the visited sites* 45c ⟩ ≡

```
current_tour_length    = \
         compute_collinear_horseflies_tour_length(\
                    self.visited_sites,\
                    self.inithorseposn,\
                    self.phi)
◇
```

Fragment referenced in 45b.
Defines: `current_tour_length` 46b.
Uses: `compute_collinear_horseflies_tour_length` 44a, `self.inithorseposn`, 45a, `self.visited_sites`, 45a.

5.7.5

⟨ *Set up tracking variables local to this iteration* 46a ⟩ ≡

```
ibest                = 0
delta_increase_least = float("inf")
◇
```

Fragment referenced in 45b.
Defines: `delta_increase_least` 46b, `ibest`, 46b.

5.7.6

⟨ *If* `self.sites[u]` *is chosen for insertion, find best insertion position and update* `delta_increase_least_table` 46b ⟩ ≡

```
for i in range(len(self.sites)):

            visited_sites_test = self.visited_sites[:i] +\
                                 [ self.sites[u] ]      +\
                                 self.visited_sites[i:]

            tour_length_on_insertion = \
                       compute_collinear_horseflies_tour_length(\
                                  visited_sites_test,\
                                  self.inithorseposn,\
                                  self.phi)

            delta_increase = tour_length_on_insertion - current_tour_length
            assert(delta_increase >= 0)

            if delta_increase < delta_increase_least:
                  delta_increase_least = delta_increase
                  ibest                = i

delta_increase_least_table.append({'unvisited_site_idx'      : u    , \
                                   'best_insertion_position' : ibest, \
                                   'delta_increase'          : delta_increase_least})
◇
```

Fragment referenced in 45b.
Uses: `compute_collinear_horseflies_tour_length` 44a, `current_tour_length` 45c, `delta_increase_least` 46a, `delta_increase_least_table` 45b, `ibest`, 46a, `self.inithorseposn`, 45a.

5.7.7

⟨ *Find the unvisited site which on insertion increases tour-length by the least amount* 46c ⟩ ≡

```
best_table_entry = min(delta_increase_least_table, \
                         key = lambda x: x['delta_increase'])

unvisited_site_idx_for_insertion = best_table_entry['unvisited_site_idx']
insertion_position               = best_table_entry['best_insertion_position']
delta_increase                   = best_table_entry['delta_increase']
◇
```

Fragment referenced in 45b.
Uses: `delta_increase_least_table` 45b.

5.7.8

⟨ *Update states for* `PolicyBestInsertionNaive` 47 ⟩ ≡

```
# Update visited and univisted sites info
self.visited_sites = self.visited_sites[:insertion_position]      +\
                     [self.sites[unvisited_site_idx_for_insertion]] +\
                     self.visited_sites[insertion_position:]

self.unvisited_sites_idxs = filter( lambda elt: elt != unvisited_site_idx_for_insertion, \
                                    self.unvisited_sites_idxs )

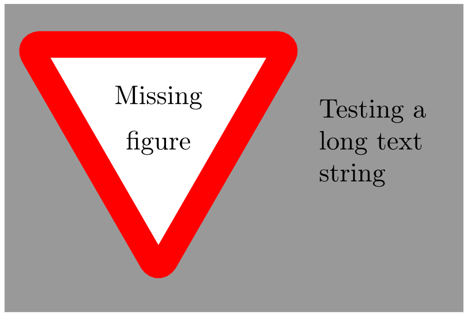
# Update the tour of the horse
self.horse_tour = compute_collinear_horseflies_tour(\
                           self.visited_sites,         \
                           self.inithorseposn, \
                           self.phi)
◇
```

Fragment referenced in 45b.
Uses: `compute_collinear_horseflies_tour` 44b, `self.horse_tour` 45a, `self.inithorseposn`, 45a, `self.visited_sites`, 45a.

Lower Bound: The φ-Prim-MST

Overview To compare the experimental performance of algorithms for NP-hard optimization problems wrt solution quality, it helps to have a cheaply computable lower bound that acts as a proxy for OPT. In the case of the TSP, a lower bound is the weight of the minimum spanning tree on the set of input sites.

To compute the MST on a set of points, on typically uses greedy algorithms such as those by Prim, Kruskal or Boruvka. To get a lower-bound for Horsefly, we define a network that we call the φ-Prim-MST by a simple generalization of Prim. Currently, we don't have a natural interpretation of this structure means in terms of the sites. This is something we need to add to our TODO list.



This is clearly a lower-bound on the weight of OPT for Collinear Horsefly. However, I believe that the stronger statement is also true

Conjecture 1. *The weight of the φ-MST is a lower-bound on the length of the horse's tour in OPT for the classic horsefly problem.*

The proof of this conjecture seems to be non-trivial off-hand. I'll put a hold on all my attempts so far to prove this, since I want the experiments to guide my intuition here.

It is possible that there could be other lower bounds based on generalizing the steps in Kruskal's and Boruvka's algorithms. Based on the experimental success of the φ-MST's, I will think of the appropriate generalizations for them later.

One particular experiment that I would be interested would be how bad is to check the crossing structure of the edges. In the MST edges never cross. What is the structure of the crossing in φ-MSTs? That might help me in designing a local search operation for the Horsefly problem.

Also note, that the construction of this φ-Prim MST can be generalized to two or more flies (single horse) we buld two separate trees; with two or more drones since we are interested in minimizing the makespan, probably we greedily them so that the trees are well-balanced.....?????? dunno doesn't strike as clean now that I think of it. It certainly isn't as clean as my node-splitting horsefly framework. Hopefully, I can prove some sort of theorems on those later?

As I type this, a separate question strikes me to be of independent interest: *Given a point-cloud in the plane, preprocess the points such that for a query φ we can compute the φ-MST in linear time.* Perhaps the MST, itself could be useful for this augmented with some data-structures for performing ray-shooting in an arrangement of segments. One can use such a data-structure, for making a quick animation of the evolution of the φ-MST as we keep changing the φ-parameter, as one often does while playing with Mathematica's `Manipulate` function. Can we motivate this by saying φ might be uncertain? I don't know, people would only find this interesting if the particular data-structure helps in the computation of horsefly like tours.

Computing the φ-Prim-MST

5.8.1 For the purposes of this section we define the notion of a rendezvous point for an edge. Given a directed segment $\overrightarrow{XY}$ and a speed ratio φ, assume a horse and a fly are positioned at X and there is a site that needs to be serviced at Y. The rendezvous point of $\overrightarrow{XY}$ is that point along R at which the horse and fly meet up at the earliest after the fly leaves X.

Explicit formulae for computing this point have already been implemented in `single_site_solution`, in one of the previous sections.

5.8.2

Prim's algorithm for computing MSTs is essentially a greedy incremental insertion process. The same structure is visible in the code fragment below. The only essential change from Prim's original algorithm is that we "grow" the tree only from the rendezvous points computed while inserting a new edge into the existing partial tree on the set of sites. This process is animated in **??**

I have will be using the `NetworkX` library (https://networkx.github.io/) for storing and manipulating graphs. For performing efficient nearest-neighbor searches for each rendezvous point in the partially constructed MST, I will use the `scikit-learn` library (https://scikit-learn.org/stable/modules/neighbors.html). When porting my codes to C++, I will probably have to switch over to the Boost Graph library and David Mount's ANN for the same purposes(both these libraries have been optmized for speed).

In the `while` loop below, `node_site_info` stores a tuple for each node in the tree consisting of

1. a node-id (this corresponds to a rendezvous point in the tree)
2. the index of the closest site in the array sites for the node (the site)
3. distance of the node to the site with the above index.

⟨ *Lower bounds for classic horsefly* 49a ⟩ ≡

```
def compute_phi_prim_mst(sites, inithorseposn,phi):

    import networkx as nx
    from sklearn.neighbors import NearestNeighbors

    ⟨ Create singleton graph, with node at inithorseposn 49b ⟩

    unmarked_sites_idxs = range(len(sites))
    while unmarked_sites_idxs:
        node_site_info = []

        ⟨ For each node, find the closest site 50a ⟩
        ⟨ Find the node with the closest site, and generate the next node and edge for the φ-MST 50b ⟩

        # Marking means removing from unmarked list :-D
        unmarked_sites_idxs.remove(next_site_to_mark_idx)

    utils_algo.print_list(G.nodes.data())
    utils_algo.print_list(G.edges.data())
    return G
◇
```

Fragment referenced in 20a.
Defines: `compute_phi_prim_mst`, Never used, `unmarked_sites_idxs` 50a.

5.8.3

Every node in the tree stores its own id as an integer along with its X-Y coordinates and the X-Y coordinates of the sites that it will be joined to with a straight-line segment. At the beginning the single node of the tree at the initial position of the horse and fly has not been joined to any sites, and hence is empty.

⟨ *Create singleton graph, with node at* `inithorseposn` 49b ⟩ ≡

```
G = nx.Graph()
G.add_node(0, mycoordinates=inithorseposn, joined_site_coords=[])
◇
```

Fragment referenced in 49a.

5.8.4

⟨ *For each node, find the closest site* 50a ⟩ ≡

```
for nodeid, nodeval in G.nodes.data():
    current_node_coordinates = np.asarray(nodeval['mycoordinates'])
    distances_of_current_node_to_sites = []

    # The following loop finds the nearest unmarked site. So far, I am
    # using brute force for this, later, I will use sklearn.neighbors.
    for j in unmarked_sites_idxs:
        site_coordinates = np.asarray(sites[j])
        dist             =  np.linalg.norm( site_coordinates - current_node_coordinates )

        distances_of_current_node_to_sites.append( (j, dist) )

        nearest_site_idx, distance_of_current_node_to_nearest_site = \
                        min(distances_of_current_node_to_sites, key=lambda (_, d): d)

        node_site_info.append((nodeid, \
                                  nearest_site_idx, \
                                  distance_of_current_node_to_nearest_site))
```
◇

Fragment referenced in 49a.
Uses: unmarked_sites_idxs 49a.

5.8.5

⟨ *Find the node with the closest site, and generate the next node and edge for the* φ*-MST* 50b ⟩ ≡

```
opt_node_idx,          \
next_site_to_mark_idx, \
distance_to_next_site_to_mark = min(node_site_info, key=lambda (h,k,d) : d)

tmp = sites[next_site_to_mark_idx]
G.nodes[opt_node_idx]['joined_site_coords'].append(  tmp   )
(r, h) = single_site_solution(tmp, G.nodes[opt_node_idx]['mycoordinates'], phi)

# Remember! indexing of nodes started at 0, thats why you set
# numnodes to the index of the newly inserted node.
newnodeid = len(list(G.nodes))

# joined_site_coords will be updated in the future iterations of while :
G.add_node(newnodeid, mycoordinates=r, joined_site_coords=[])

# insert the edge weight, will be useful later when
# computing sum total of all the edges.
G.add_edge(opt_node_idx, newnodeid, weight=h )
```
◇

Fragment referenced in 49a.
Uses: single_site_solution 43c.

$\boxed{\text{Algorithm}}$: Doubling the φ-MST

5.9.1 Algorithmic Overview

5.9.2 Algorithmic Details

$\boxed{\text{Algorithm}}$: Bottom-Up Split

5.10.1 Algorithmic Overview

5.10.2 Algorithmic Details

$\boxed{\text{Algorithm}}$: Local Search—Swap

5.11.1 Algorithmic Overview

5.11.2 Algorithmic Details

$\boxed{\text{Algorithm}}$: K2 Means

5.12.1 Algorithmic Overview

5.12.2 Algorithmic Details

5.12.3

⟨ *Algorithms for classic horsefly* 51 ⟩ ≡

```
def algo_kmeans(sites, inithorseposn, phi, k, post_optimizer):
     """
     type Point   (Double, Double)
     type Site    Point
     type Cluster (Point, [Site])
     type Tour    {'site_ordering':[Site],
                   'tour_points'  :[Point]}
     algo_kmeans :: [Site] -> Point -> Double -> Int
     """
     def get_clusters(site_list):
           """
           get_clusters :: [Site] -> [Cluster]
           For the given list of sites, perform k-means clustering
           and return the list of k-centers, along with a list of sites
           assigned to each center.
           """
           X      = np.array(site_list)
           kmeans = KMeans(n_clusters=k, random_state=0).fit(X)

           accum = [ (center, [])  for center in kmeans.cluster_centers_ ]
           for label, site in zip(kmeans.labels_, site_list):
                 accum [label][1].append(site)

           return accum

     def extract_cluster_sites_for_each_cluster(clusters):
         """
         extract_cluster_sites_for_each_cluster :: [Cluster] -> [[Site]]
         """
```

```
    return [ cluster_sites for (_, cluster_sites) in clusters  ]

def fuse_tours(tours):
    """
     fuse_tours :: [Tour] -> Tour
    """
    fused_tour = {'site_ordering':[], 'tour_points':[]}
    for tour, i in zip(tours, range(len(tours))):
          fused_tour['site_ordering'].extend(tour['site_ordering'])
          if i != len(tours)-1:
                # Remember! last point of previous tour is first point of
                # this tour, which is why we need to avoid duplication
                # Hence the [:-1]
                fused_tour['tour_points'].extend(tour['tour_points'][:-1])
          else:
                # Because this is the last tour in the iteration, we include
                # its end point also, hence no [:-1] here
                fused_tour['tour_points'].extend(tour['tour_points'])
    return fused_tour

def  weighted_center_tour(clusters, horseflyinit):
    """
    weighted_center_tour :: [Cluster] -> Point -> [Cluster]

    Just return a permutation of the clusters.
    need to return actual weighted tour
    since we are only interested in the order
    in which the weighted center tour is performed
    on k weighted points, where k is the clustering
    number used here
    """

    #print Fore.CYAN, " Clusters: "    , clusters, Style.RESET_ALL
    #print " "
    #print Fore.CYAN, " Horseflyinit: ", horseflyinit, Style.RESET_ALL

    assert( k == len(clusters) )
    tour_length_fn = tour_length(horseflyinit)

    #-------------------------------------------------
    # For each of the k! permutations of the weighted sites
    # give the permutation with the smallest weighted tour
    # Note that k is typically small, say 2,3 or 4
    #-------------------------------------------------
    # But first we initialize the accumulator variables prefixed with best_

    #print Fore.YELLOW , " Computing Weighted Center Tour ", Style.RESET_ALL
    clustering_centers = [ center          for (center, _)    in clusters]
    centers_weights   =  [ len(site_list)  for (_, site_list) in clusters]

    #utils_algo.print_list(clustering_centers)
    #utils_algo.print_list(centers_weights)
    #time.sleep(5000)

    best_perm = clusters
    best_perm_tour = algo_weighted_sites_given_specific_ordering(clustering_centers, \
                                                               centers_weights, \
                                                               horseflyinit, \
                                                               phi)

    i = 1
    for clusters_perm in list(itertools.permutations(clusters)):
```

```
            #print Fore.YELLOW , "......Testing a new cluster permutation [ ", i ,  \
            #                    "/", math.factorial(k) , " ] of the sites", \
            #                   Style.RESET_ALL

            i = i + 1
            # cluster_centers_and_weights ::  [(Point, Int)]
            # This is what is used for the weighted tour
            clustering_centers = [ center          for (center, _)    in clusters_perm]
            centers_weights    = [ len(site_list)  for (_, site_list) in clusters_perm]

            tour_current_perm = \
                algo_weighted_sites_given_specific_ordering(clustering_centers, \
                                                            centers_weights, \
                                                            horseflyinit, \
                                                            phi)

            if tour_length_fn( utils_algo.flatten_list_of_lists(tour_current_perm ['tour_points']) ) \
             < tour_length_fn( utils_algo.flatten_list_of_lists(   best_perm_tour ['tour_points']) ):

                print Fore.RED + "................Found better cluster order" + Style.RESET_ALL
                best_perm = clusters_perm

      return best_perm

  def get_tour (site_list, horseflyinit):
     """
     get_tour :: [Site] -> Point -> Tour

     A recursive function which does the job
     of extracting a tour
     """

     if len (site_list) <= k: # Base-case for the recursion
           #print Fore.CYAN + ".....Reached Recursion Base case" + Style.RESET_ALL
           result = algo_dumb(site_list, horseflyinit, phi)
           return result
     else: # The main recursion
        # Perform k-means clustering and get the clusters
        clusters = get_clusters(site_list)

        #utils_algo.print_list(clusters)

        ####################################################################
        # Permute the clusters depending on which is better to visit first
        clusters_perm = weighted_center_tour(clusters, horseflyinit)
        #####################################################################

        # Extract cluster sites for each cluster
        cluster_sites_for_each_cluster  = \
               extract_cluster_sites_for_each_cluster(clusters_perm)

        # Apply the get_tour function on each chunk while folding across
        # using the last point of the tour of the previous cluster
        # as the first point of this current one. This is a kind of recursion
        # that pays forward.
        tours = []
        for site_list, i in zip(cluster_sites_for_each_cluster,
                                range(len(cluster_sites_for_each_cluster))):

              if i == 0:# first point is horseflyinit. The starting fold value!!
                    tours.append( get_tour(site_list, inithorseposn)  )
```

```
            else: # use the last point of the previous tour (i-1 index)
                  # as the first point of this one !!
                  prev_tour  = tours[i-1]
                  tours.append( get_tour(site_list, prev_tour['tour_points'][-1]))
      # Fuse the tours you obtained above to get a site ordering
      return fuse_tours(tours)

print Fore.MAGENTA + "RUNNING algo_kmeans......." + Style.RESET_ALL
sites1 = get_tour(sites, inithorseposn)['site_ordering']
return  post_optimizer(sites1, inithorseposn, phi )
```

◇

Fragment defined by 27, 28, 30a, 32, 37a, 51, 54, 56.
Fragment referenced in 20a.
Uses: `tour_length` 57a.

5.12.4

⟨ *Algorithms for classic horsefly* 54 ⟩ ≡

```
def algo_weighted_sites_given_specific_ordering (sites, weights, horseflyinit, phi):

     def site_constraints(i, sites, weights):
          """
          site_constraints :: Int -> [Site] -> [Double]
                          -> [ [Double] -> Double  ]

          Generate a list of constraint functions for the ith site
          The number of constraint functions is equal to the weight
          of the site!
          """

          #print Fore.RED, sites, Style.RESET_ALL

          psum_weights = utils_algo.partial_sums( weights ) # partial sum of ALL the site-weights
          accum        = [ ]
          site_weight  = weights[i]

          for j in range(site_weight):

              if i == 0 and j == 0:

                    #print "i= ", i, " j= ", j
                    def _constraint_function(x):
                        """
                        constraint_function :: [Double] -> Double
                        """
                        start = np.array (horseflyinit)
                        site  = np.array (sites[0])
                        stop  = np.array ([x[0],x[1]])

                        horsetime = np.linalg.norm( stop - start )

                        flytime_to_site   = 1/phi * np.linalg.norm( site - start )
                        flytime_from_site = 1/phi * np.linalg.norm( stop - site  )
                        flytime           = flytime_to_site + flytime_from_site
                        return horsetime-flytime

                    accum.append( _constraint_function )
```

```
        elif  i == 0 and j != 0 :

              #print "i= ", i, " j= ", j
              def _constraint_function(x):
                    """
                    constraint_function :: [Double] -> Double
                    """
                    start = np.array( [x[2*j-2], x[2*j-1]] )
                    site  = np.array(sites[0])
                    stop  = np.array( [x[2*j]  , x[2*j+1]] )

                    horsetime = np.linalg.norm( stop - start )

                    flytime_to_site   = 1/phi * np.linalg.norm( site - start )
                    flytime_from_site = 1/phi * np.linalg.norm( stop - site  )
                    flytime           = flytime_to_site + flytime_from_site
                    return horsetime-flytime

              accum.append( _constraint_function )
        else:

              #print "i= ", i, " j= ", j
              def _constraint_function(x):
                    """
                    constraint_function :: [Double] -> Double
                    """

                    offset = 2 * psum_weights[i-1]

                    start  = np.array( [ x[offset + 2*j-2 ], x[offset + 2*j-1 ] ] )
                    site   = np.array(sites[i])
                    stop   = np.array( [ x[offset + 2*j ]  , x[offset + 2*j+1 ] ] )

                    horsetime = np.linalg.norm( stop - start )

                    flytime_to_site   = 1/phi * np.linalg.norm( site - start )
                    flytime_from_site = 1/phi * np.linalg.norm( stop - site  )
                    flytime           = flytime_to_site + flytime_from_site
                    return horsetime-flytime

              accum.append( _constraint_function )

    return accum

def generate_constraints(sites, weights):
    return [site_constraints(i, sites, weights) for i in range(len(sites))]

#####
#print weights
#### For debugging
weights = [1 for wt in weights]
####

cons = utils_algo.flatten_list_of_lists (generate_constraints(sites, weights))
cons1 = [  {'type':'eq', 'fun':f}  for f in cons]

# Since the horsely tour lies inside the square,
# the bounds for each coordinate is 0 and 1
x0 = np.empty(2*sum(weights))
x0.fill(0.5) # choice of filling vector with 0.5 is arbitrary

# Run scipy's minimization solver
```

```
    sol = minimize(tour_length(horseflyinit), x0, method= 'SLSQP', constraints=cons1)
    tour_points = utils_algo.pointify_vector(sol.x)

    #print sol

    #time.sleep(5000)
    return {'tour_points'  : tour_points,
            'site_ordering': sites}
◇
```

Fragment defined by 27, 28, 30a, 32, 37a, 51, 54, 56.
Fragment referenced in 20a.
Uses: `generate_constraints` 31, `tour_length` 57a.

Algorithm: TSP ordering

5.13.1 Algorithmic Overview

5.13.2 Algorithmic Details

5.13.3 Use the TSP ordering for the horsefly tour, irrespective of the speedratio. Useful to see the benefit obtained from the various heurtiustics you will be designing.

This will be especially useful for larger ratios of speeds

I use the tsp package for this: https://pypi.org/project/tsp/#files If the tsp ordering has already been pre-computed, then use it.

⟨ *Algorithms for classic horsefly* 56 ⟩ ≡

```
def algo_tsp_ordering(sites, inithorseposn, phi, post_optimizer):
    import tsp
    horseinit_and_sites = [inithorseposn] + sites

    _, tsp_idxs = tsp.tsp(horseinit_and_sites)


    # Get the position of the horse in tsp_idxss
    h = tsp_idxs.index(0) # 0 because the horse was placed first in the above vector

    if h != len(tsp_idxs)-1:
        idx_vec = tsp_idxs[h+1:] + tsp_idxs[:h]
    else:
        idx_vec = tsp_idxs[:h]

    # idx-1 because all the indexes of the sites were pushed forward
    # by 1 when we tacked on inithorseposn at the very beginning
    # of horseinit_and_sites, hence we auto-correct for that
    sites_tsp = [sites[idx-1] for idx in idx_vec]

    tour0    = post_optimizer (sites_tsp                , inithorseposn, phi)
    tour1    = post_optimizer (list(reversed(sites_tsp)), inithorseposn, phi)

    tour0_length = utils_algo.length_polygonal_chain([inithorseposn] + tour0['site_ordering'])
    tour1_length = utils_algo.length_polygonal_chain([inithorseposn] + tour1['site_ordering'])
```

```
    print Fore.RED, " TSP paths in either direction are ", tour0_length, " ", tour1_length, Style.RESET_ALL

    if tour0_length < tour1_length:
        print Fore.RED, "Selecting tour0 ", Style.RESET_ALL
        return tour0
    else:
        print Fore.RED, "Selecting tour1 ", Style.RESET_ALL
        return tour1
◇
```

Fragment defined by 27, 28, 30a, 32, 37a, 51, 54, 56.
Fragment referenced in 20a.

Local Utility Functions

5.14.1 For a given initial position of horse and fly return a function computing the tour length. The returned function computes the tour length in the order of the list of stops provided beginning with the initial position of horse and fly. Since the horse speed = 1, the tour length = time taken by horse to traverse the route.

This is in other words the objective function.

⟨ *Local utility functions for classic horsefly* 57a ⟩ ≡

```
def tour_length(horseflyinit):
   def _tourlength (x):

        # the first point on the tour is the
        # initial position of horse and fly
        # Append this to the solution x = [x0,x1,x2,....]
        # at the front
        htour = np.append(horseflyinit, x)
        length = 0

        for i in range(len(htour))[:-3:2]:
                length = length + \
                         np.linalg.norm([htour[i+2] - htour[i], \
                                         htour[i+3] - htour[i+1]])
        return length

   return _tourlength
◇
```

Fragment defined by 57ab.
Fragment referenced in 20a.
Defines: `tour_length` 27, 30a, 51, 54, 60b, 61a.

5.14.2 It is possible that some heuristics might return non-negligible waiting times. Hence I am writing a separate function which adds the waiting time (if it is positive) to the length of each link of the tour. Again note that because speed of horse = 1, we can add "time" to "distance".

⟨ *Local utility functions for classic horsefly* 57b ⟩ ≡

```
def tour_length_with_waiting_time_included(tour_points, horse_waiting_times, horseflyinit):
      tour_points   = np.asarray([horseflyinit] + tour_points)
      tour_links    = zip(tour_points, tour_points[1:])

      # the +1 because the inital position has been tacked on at the beginning
      # the solvers written the tour points except for the starting position
      # because that is known and part of the input. For this function
```

```
        # I need to tack it on for tour length
        assert(len(tour_points) == len(horse_waiting_times)+1)

        sum = 0
        for i in range(len(horse_waiting_times)):

            # Negative waiting times means drone/fly was waiting
            # at rendezvous point
            if horse_waiting_times[i] >= 0:
                wait = horse_waiting_times[i]
            else:
                wait = 0

            sum += wait + np.linalg.norm(tour_links[i][0] - tour_links[i][1], ord=2) #
        return sum
    ◇
```

Fragment defined by 57ab.
Fragment referenced in 20a.
Defines: `tour_length_with_waiting_time_included` 30a, 32, 43b, 60b.

Plotting Routines

5.15.1

⟨ *Plotting routines for classic horsefly* 59a ⟩ ≡

```
def plotTour(ax,horseflytour, horseflyinit, phi, algo_str, tour_color='#d13131'):

    ⟨ Get x and y coordinates of the endpoints of segments on the horse-tour 59b ⟩
    ⟨ Get x and y coordinates of the sites 59c ⟩
    ⟨ Construct the fly-tour from the information about horse tour and sites 59d ⟩
    ⟨ Print information about the horse tour 59e ⟩
    ⟨ Print information about the fly tour 60a ⟩
    ⟨ Print meta-data about the algorithm run 60b ⟩
    ⟨ Plot everything 61a ⟩
    ◇
```

Fragment defined by 59a, 61b.
Fragment referenced in 20a.
Defines: `plotTour` 22.

5.15.2

⟨ *Get x and y coordinates of the endpoints of segments on the horse-tour* 59b ⟩ ≡

```
xhs, yhs = [horseflyinit[0]], [horseflyinit[1]]
for pt in horseflytour['tour_points']:
    xhs.append(pt[0])
    yhs.append(pt[1])
◇
```

Fragment referenced in 59a.

5.15.3

⟨ *Get x and y coordinates of the sites* 59c ⟩ ≡

```
xsites, ysites = [], []
for pt in horseflytour['site_ordering']:
    xsites.append(pt[0])
    ysites.append(pt[1])
◇
```

Fragment referenced in 59a.

5.15.4 Route for the fly keeps alternating between the site and the horse

⟨ *Construct the fly-tour from the information about horse tour and sites* 59d ⟩ ≡

```
xfs , yfs = [xhs[0]], [yhs[0]]
for site, pt in zip (horseflytour['site_ordering'],
                     horseflytour['tour_points']):
   xfs.extend([site[0], pt[0]])
   yfs.extend([site[1], pt[1]])
◇
```

Fragment referenced in 59a.

5.15.5 Note that the waiting time at the starting point is 0

⟨ *Print information about the horse tour* 59e ⟩ ≡

```
print "\n----------", "\nHorse Tour", "\n-----------"
```

```
waiting_times = [0.0] + horseflytour['horse_waiting_times'].tolist()
#print waiting_times
for pt, time in zip(zip(xhs,yhs), waiting_times) :
   print pt, Fore.GREEN, " ---> Horse Waited ", time, Style.RESET_ALL
◇
```

Fragment referenced in 59a.

5.15.6

⟨ *Print information about the fly tour* 60a ⟩ ≡

```
print "\n----------", "\nFly Tour", "\n----------"
for item, i in zip(zip(xfs,yfs), range(len(xfs))):
   if i%2 == 0:
       print item
   else :
       print Fore.RED + str(item) + "----> Site" +  Style.RESET_ALL

◇
```

Fragment referenced in 59a.

5.15.7

⟨ *Print meta-data about the algorithm run* 60b ⟩ ≡

```
print "----------------------------------"
print Fore.GREEN, "\nSpeed of the drone was set to be", phi
#tour_length = utils_algo.length_polygonal_chain( zip(xhs, yhs))
tour_length = horseflytour['tour_length_with_waiting_time_included']
print "Tour length of the horse is ",  tour_length
print "Algorithm code-Key used "    , algo_str, Style.RESET_ALL
print "----------------------------------\n"
◇
```

Fragment referenced in 59a.
Uses: `tour_length` 57a, `tour_length_with_waiting_time_included` 57b.

5.15.8

⟨ *Plot everything* 61a ⟩ ≡

```
#kwargs = {'size':'large'}
for x,y,i in zip(xsites, ysites, range(len(xsites))):
    ax.text(x, y, str(i+1), bbox=dict(facecolor='#ddcba0', alpha=1.0))

ax.plot(xfs,yfs,'g-')
ax.plot(xhs, yhs, color=tour_color, marker='s', linewidth=3.0)

ax.add_patch( mpl.patches.Circle( horseflyinit, radius = 1/140.0,
                                  facecolor= '#D13131', edgecolor='black'   )  )
fontsize = 20


plt.rc('text', usetex=True)
plt.rc('font', family='serif')
ax.set_title( r'Algorithm Used: ' + algo_str +  '\nTour Length: ' \
               + str(tour_length)[:7], fontdict={'fontsize':fontsize})
ax.set_xlabel(r'Number of sites: ' + str(len(xsites)) + '\nDrone Speed: ' + str(phi) ,
                  fontdict={'fontsize':fontsize})
```
◇

Fragment referenced in 59a.
Uses: `tour_length` 57a.

5.15.9

⟨ *Plotting routines for classic horsefly* 61b ⟩ ≡

```
def draw_phi_mst(ax, phi_mst, inithorseposn, phi):

     # for each tree node draw segments joining to sites (green segs)
     for (nodeidx, nodeinfo) in list(phi_mst.nodes.data()):
         mycoords           = nodeinfo['mycoordinates']
         joined_site_coords = nodeinfo['joined_site_coords']

         for site in joined_site_coords:
               ax.plot([mycoords[0],site[0]], [mycoords[1], site[1]], 'g-', linewidth=1.5)
               ax.add_patch( mpl.patches.Circle( [site[0],site[1]], radius = 0.007, \
                                                  facecolor='blue', edgecolor='black'))

     # draw each tree edge (red segs)
     edges = list(phi_mst.edges.data())
     for (idx1, idx2, edgeinfo) in edges:
          (xn1, yn1) =  phi_mst.nodes[idx1]['mycoordinates']
          (xn2, yn2) =  phi_mst.nodes[idx2]['mycoordinates']
          ax.plot([xn1,xn2],[yn1,yn2], 'ro-' ,linewidth=1.7)

     ax.set_title(r'$\varphi$-MST', fontdict={'fontsize':30})
```
◇

Fragment defined by 59a, 61b.
Fragment referenced in 20a.
Defines: `draw_phi_mst` 22.

Animation routines

5.16.1 After writing out the schedule, it would be nice to have a function that animates the delivery process of the schedule. Every problem will have animation features unique to its features. Any abstraction will reveal itself only after I design the various algorithms and extract the various features, which is why I will develop these animation routines on the fly.

In general, all algorithms for a problem will write out a YAML file containing the schedule in the outputted run-folder. To animate a schedule and write the resulting movie to disk we just pass the name of the file containing the schedule. Since the output file-format of the schedule is identical for all algorithms of a problem, it is sufficient to have just one animation function.

Schedules will typically be animated iff there is a `animate_schedule_p` boolean flag set to `True` in the arguments of every algorithm's function.

Here we render the Horse and Fly moving according to their assigned tours at their respective speeds, we don't need to "coordinate" the plotting since that has already been done by the scheudle itself.

A site that has been unserviced is represented by a blue dot. A site that has been serviced is represented by a yellow dot.

⟨ *Animation routines for classic horsefly* 62 ⟩ ≡

```
def animateSchedule(schedule_file_name):
    import yaml
    import numpy as np
    import matplotlib.animation as animation
    from matplotlib.patches import Circle
    import matplotlib.pyplot as plt

    ⟨ Set up configurations and parameters for animation and plotting 63a ⟩
    ⟨ Parse input-output file and set up required data-structures 63b ⟩
    ⟨ Construct and store every frame of the animation in ims 64a ⟩
    ⟨ Write animation of schedule to disk and display in live window 66b ⟩
◇
```

Fragment referenced in 20a.

5.16.2 In the animation, we are going to show the process of the fly delivering packages to the sites according to the pre-computed schedule. Thus the canvas must reflect the underlying euclidean space. For this, we need to set the bounding box of the `Axes` object to an axis-parallel unit-square whose lower-left corner is at the origin.

While displaying the animation it also helps to have a major and minor grid lightly visible to get a rough sense of distances between the sites. The settings for setting up these grids were done following the tutorial on http://jonathansoma.com/lede/data-studio/matplotlib/adding-grid-lines-to-a-matplotlib-chart/

We also use LaTeX for typesetiing symbols and equations and the Computer Modern font for text on the plot canvas. Unfortunately, Matplotlib's present default font for text seems to be DejaVu Sans Mono, which isn't pretty for publications.

⟨ *Set up configurations and parameters for animation and plotting* 63a ⟩ ≡

```
plt.rc('text', usetex=True)
plt.rc('font', family='serif')

fig, ax = plt.subplots()
ax.set_xlim([0,1])
ax.set_ylim([0,1])
ax.set_aspect('equal')

ax.set_xticks(np.arange(0, 1, 0.1))
ax.set_yticks(np.arange(0, 1, 0.1))

# Turn on the minor TICKS, which are required for the minor GRID
ax.minorticks_on()

# customize the major grid
ax.grid(which='major', linestyle='--', linewidth='0.3', color='red')

# Customize the minor grid
ax.grid(which='minor', linestyle=':', linewidth='0.3', color='black')

ax.get_xaxis().set_ticklabels([])
ax.get_yaxis().set_ticklabels([])
◇
```

Fragment referenced in 62.

5.16.3 In this chunk, by `horse_leg` we mean the segment of a horse's tour between two successive rendezvous points with a fly while a `fly_leg` stands for the part of a fly tour when the fly leaves the horse, reaches a site, and returns back to the horse. These concepts are illustrated in the diagram below. The frames of the animation are constructed by first extracting the `horse_legs` and `fly_legs` of the horse and fly-tours and then animating the horse and fly moving along each of their respective legs.

⟨ *Parse input-output file and set up required data-structures* 63b ⟩ ≡

```
with open(schedule_file_name, 'r') as stream:
      schedule = yaml.load(stream)

phi           = float(schedule['phi'])
inithorseposn = schedule['inithorseposn']

# Get legs of the horse and fly tours
horse_tour  = map(np.asarray, schedule['horse_tour']   )
sites       = map(np.asarray, schedule['visited_sites'])

# set important meta-data for plot
ax.set_title("Number of sites: " + str(len(sites)), fontsize=25)
ax.set_xlabel(r"$\varphi$ = " + str(phi), fontsize=20)

xhs = [ horse_tour[i][0] for i in range(len(horse_tour))]
yhs = [ horse_tour[i][1] for i in range(len(horse_tour))]
xfs , yfs = [xhs[0]], [yhs[0]]
for site, pt in zip (sites,horse_tour[1:]):
         xfs.extend([site[0], pt[0]])
         yfs.extend([site[1], pt[1]])
fly_tour = map(np.asarray,zip(xfs,yfs))

horse_legs = zip(horse_tour, horse_tour[1:])
fly_legs   = zip(fly_tour, fly_tour[1:], fly_tour[2:]) [0::2]

assert(len(horse_legs) == len(fly_legs))
```

◇

Fragment referenced in 62.

5.16.4

The `ims` array stores each frame of the animation. Every frame consists of various "artist" objects [2] (e.g. circles and segments) which change dynamically as the positions of the horse and flies change.

⟨ *Construct and store every frame of the animation in* `ims` 64a ⟩ ≡

```
ims = []
for horse_leg, fly_leg, leg_idx in zip(horse_legs, \
                                       fly_legs,   \
                                       range(len(horse_legs))):
     debug(Fore.YELLOW + "Animating leg: "+ str(leg_idx) + Style.RESET_ALL)

     ⟨ Define function to place points along a leg 66a ⟩

     horse_posns = discretize_leg(horse_leg)
     fly_posns   = discretize_leg(fly_leg)
     assert(len(horse_posns) == len(fly_posns))

     hxs = [xhs[i] for i in range(0,leg_idx+1) ]
     hys = [yhs[i] for i in range(0,leg_idx+1) ]

     fxs , fys = [hxs[0]], [hys[0]]
     for site, pt in zip (sites,(zip(hxs,hys))[1:]):
          fxs.extend([site[0], pt[0]])
          fys.extend([site[1], pt[1]])

     number_of_sites_serviced = leg_idx
     for horse_posn, fly_posn, subleg_idx in zip(horse_posns, \
                                                 fly_posns,   \
                                                 range(len(horse_posns))):
          ⟨ Render frame and append it to ims 64b ⟩
```

◇

Fragment referenced in 62.
Defines: `number_of_sites_serviced` 64b.

5.16.5

While rendering the horse and fly tours we need to keep track of the horse and fly-legs and sites that have been serviced so far.

- The path covered by the horse from the initial point till its current position is colored red
- The path covered by the fly from the initial point till its current position is colored green
- Unserviced sites are marked blue.
- When sites get serviced, they are marked yellow.

While iterating through all the sublegs of the current fly-leg, we need to keep track if the fly has serviced the site or not. That is the job of the `if subleg_idx==9` block in the code-fragment below. The magic-number "9" is related to the 10 and 19 constants from the `discretize_leg` function defined later in subsection 5.16.6.

⟨ *Render frame and append it to* `ims` 64b ⟩ ≡

```
debug(Fore.RED + "Rendering subleg "+ str(subleg_idx) + Style.RESET_ALL)
hxs1 = hxs + [horse_posn[0]]
hys1 = hys + [horse_posn[1]]

fxs1 = fxs + [fly_posn[0]]
fys1 = fys + [fly_posn[1]]

# There is a midway update for new site check is site
```

[2] This is Matplotlib terminology

```
    # has been serviced. If so, update fxs and fys
    if subleg_idx == 9:
        fxs.append(sites[leg_idx][0])
        fys.append(sites[leg_idx][1])
        number_of_sites_serviced += 1

    horseline, = ax.plot(hxs1,hys1,'ro-', linewidth=5.0, markersize=6, alpha=1.00)
    flyline,   = ax.plot(fxs1,fys1,'go-', linewidth=1.0, markersize=3)

    objs = [flyline,horseline]

    # Mark serviced and unserviced sites with different colors.
    # Use https://htmlcolorcodes.com/ for choosing good colors along with their hex-codes.

    for site, j in zip(sites, range(len(sites))):
        if j < number_of_sites_serviced:       # site has been serviced
            sitecolor = '#DBC657' # yellowish
        else:                                  # site has not been serviced
            sitecolor = 'blue'

        circle = Circle((site[0], site[1]), 0.02, \
                        facecolor = sitecolor   , \
                        edgecolor = 'black'     , \
                        linewidth=1.4)
        sitepatch = ax.add_patch(circle)
        objs.append(sitepatch)

    debug(Fore.CYAN + "Appending to ims "+ Style.RESET_ALL)
    ims.append(objs[::-1])
    ◇
```

Fragment referenced in 64a.
Uses: `number_of_sites_serviced` 64a.

5.16.6 The numbers 19 and 10 to discretize the horse and fly legs have been arbitrarily chosen. These seem to work well for giving smooth real-time animation. However, you will notice both the horse and fly seem to speed up or sometimes slow down.

That's why ideally, these discretization params should actually depend on the length of the legs, and the speeds of the horse and fly. However, just using constants is good enough for now. I just want a working animation.

A leg consists of either one segment (for horse) or two segments(for fly).

For a horse-leg, we must make sure that the leg-end points are part of the discretization of the leg.

For a fly-leg, we must ensure that the leg-end points <u>and</u> the site being serviced during the leg are in its discretization. Note that in this case, since each of the two segments are being discretized with `np.linspace`, we need to make sure that the site corresponding to the fly-leg is not counted twice, which explains the odd-looking `subleg_pts.extend(tmp[:-1])` statement in the code-fragment below.

⟨ *Define function to place points along a leg* 66a ⟩ ≡

```
def discretize_leg(pts):
   subleg_pts = []
   numpts     = len(pts)

   if numpts == 2:
       k  = 19 # horse
   elif numpts == 3:
       k  = 10 # fly

   for p,q in zip(pts, pts[1:]):
       tmp = []
       for t in np.linspace(0,1,k):
           tmp.append( (1-t)*p + t*q )
       subleg_pts.extend(tmp[:-1])

   subleg_pts.append(pts[-1])
   return subleg_pts
```
◇

Fragment referenced in 64a.

5.16.7

⟨ *Write animation of schedule to disk and display in live window* 66b ⟩ ≡

```
from colorama import Back

debug(Fore.BLACK + Back.WHITE + "\nStarted constructing ani object"+ Style.RESET_ALL)
ani = animation.ArtistAnimation(fig, ims, interval=80, blit=True, repeat_delay=1000)
debug(Fore.BLACK + Back.WHITE + "\nFinished constructing ani object"+ Style.RESET_ALL)

plt.show() # For displaying the animation in a live window.

debug(Fore.MAGENTA + "\nStarted writing animation to disk"+ Style.RESET_ALL)
ani.save(schedule_file_name+'.avi', dpi=250)
debug(Fore.MAGENTA + "\nFinished writing animation to disk"+ Style.RESET_ALL)
```
◇

Fragment referenced in 62.

Chapter Index of Fragments

Chapter Index of Identifiers

Chapter 6

Fixed Route Horsefly

Chapter 7

One Horse, Two Flies

Chapter 8

Reverse Horsefly

Chapter 9

Watchman Horsefly

Appendices

Appendix A

Index of Files

Appendix B

Man-page for `main.py`

Bucketlist of TODOS