The r-Gather Problem in Euclidean Space

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October 24, 2015

Outline

1. Intro
2. Hardness
   - $r \geq 4$ and cost $=$ max bounding disk diameter
3. Algorithms
4. Conclusion

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Solution cost: $\max_i d_i$

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The r is **requirement**, not radius.

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Privacy motivation r-anonymity

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In this application, the space is arguably (?) Euclidean

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Reductions from Planar 3SAT and Planar Circuit 3SAT.

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	$r = 2$	$r = 3$	$r = 4$	$r \geq 5$
disk diam	(in P)	$\sqrt{13}/2 \approx 1.802$	$\frac{\sqrt{35}+\sqrt{3}}{4} \approx 1.912$	same
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- Vars = cyclic chain of unit-diam disks.
- Clauses: meeting point of three chains — one of them must cover it.

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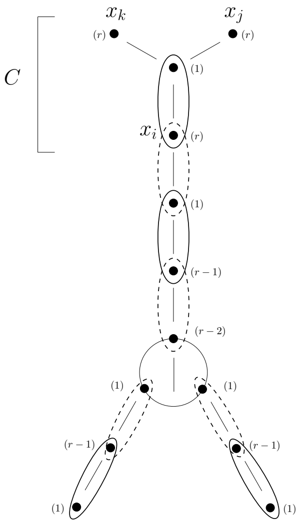

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Only other option:

- correcting at a splitter

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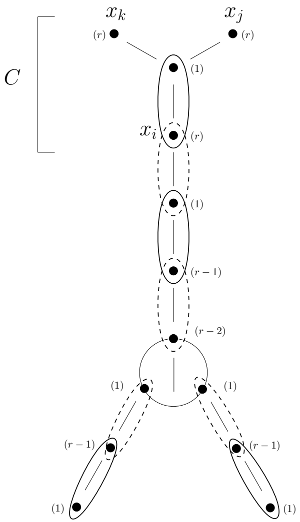

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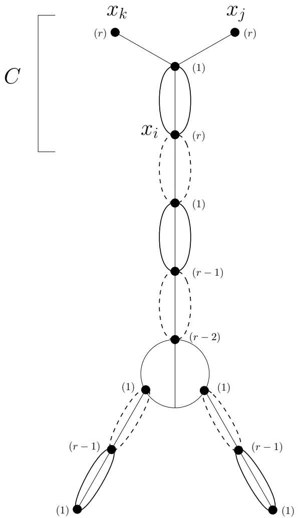

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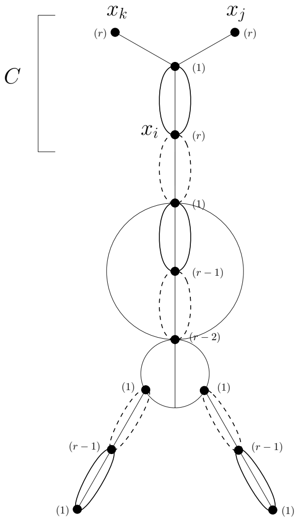

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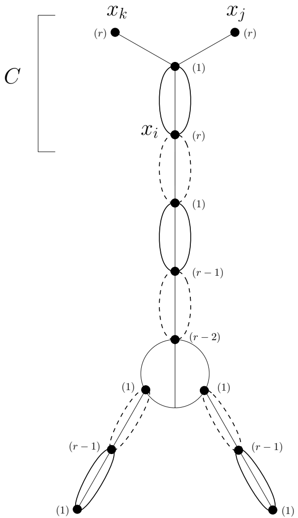

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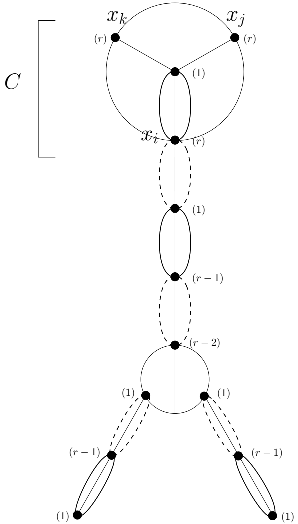

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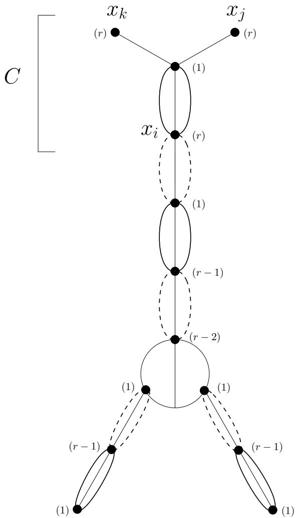

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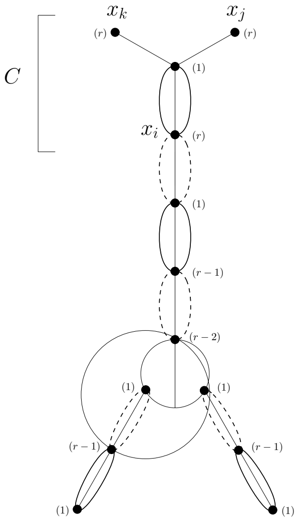

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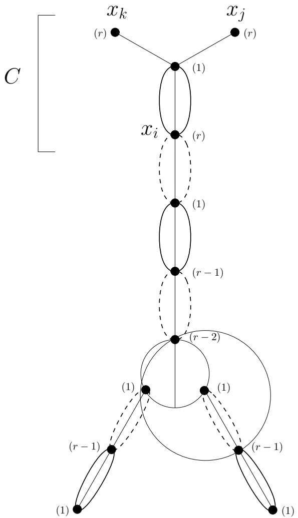

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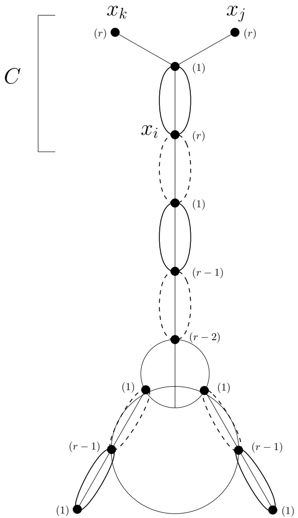

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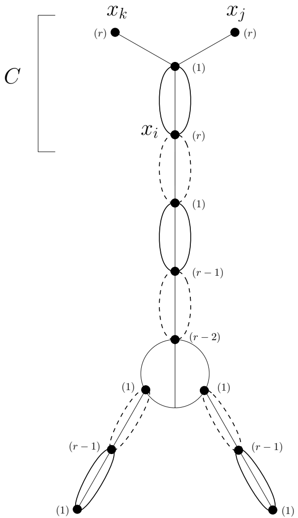

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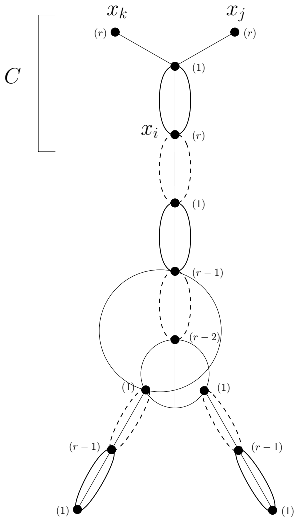

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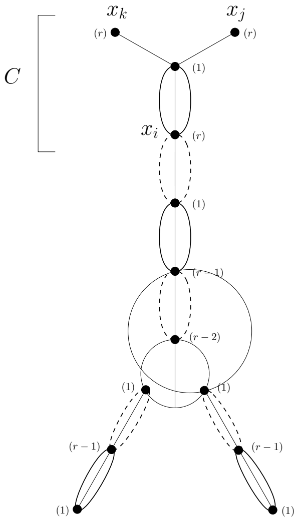

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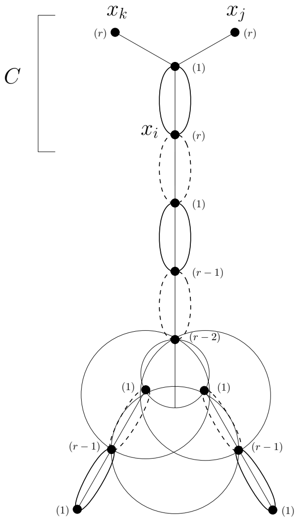

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Then we compute optimal way for them to touch it $\rightarrow$ the 1.912 factor.

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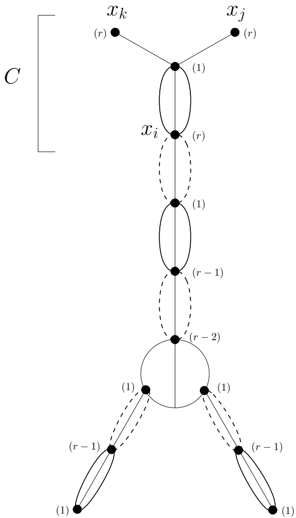

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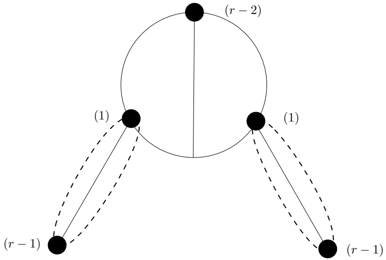

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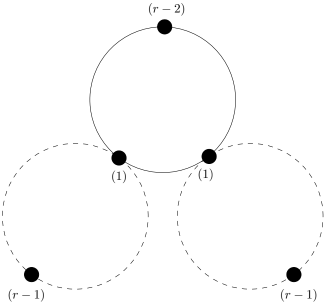

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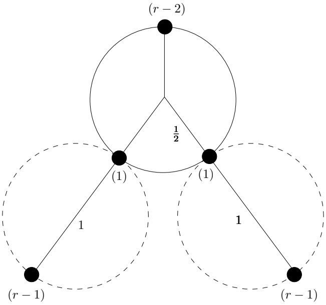

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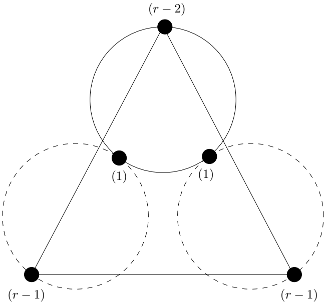

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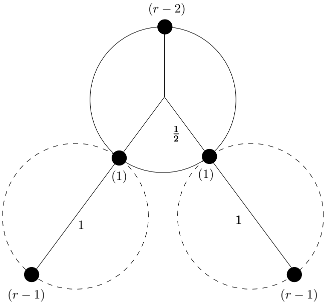

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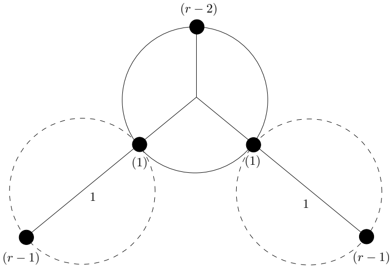

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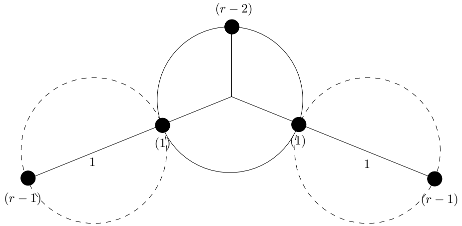

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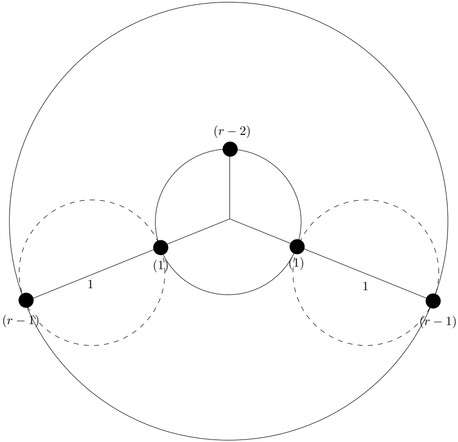

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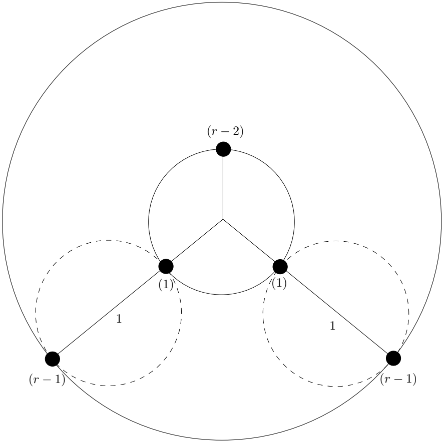

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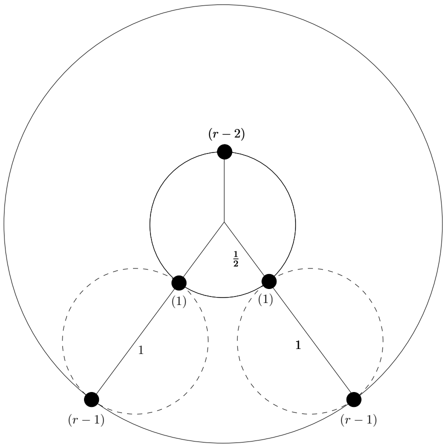

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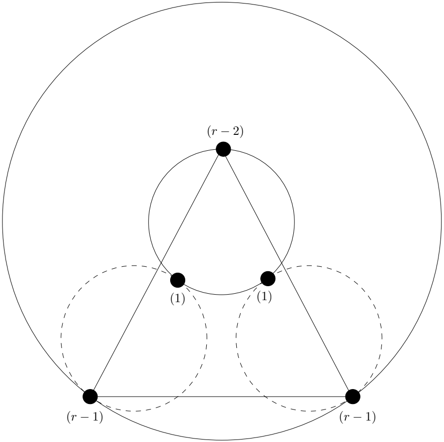

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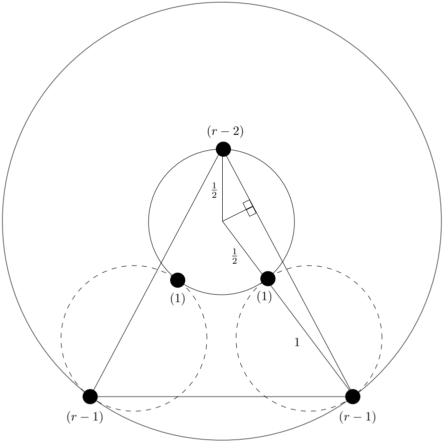

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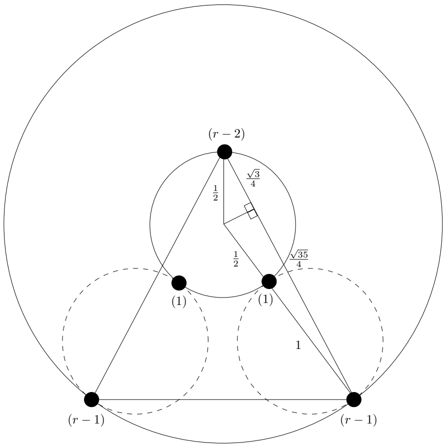

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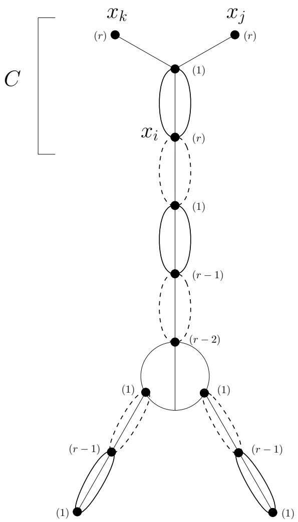

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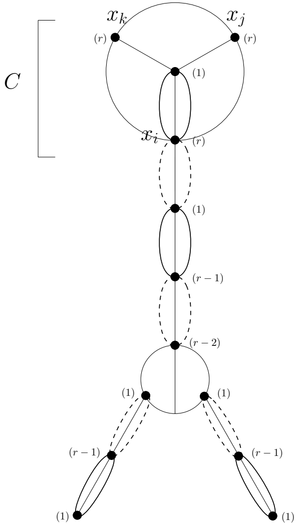

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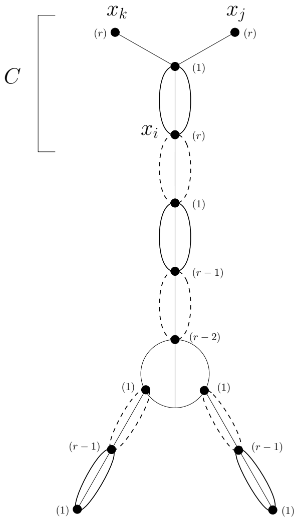

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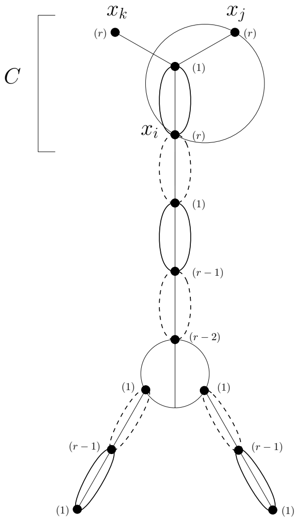

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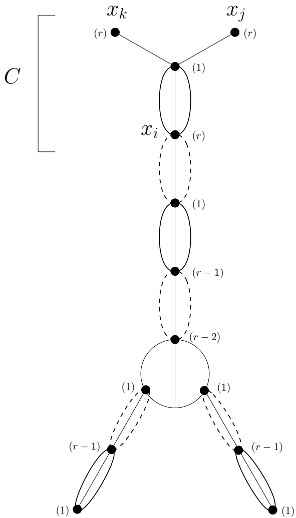

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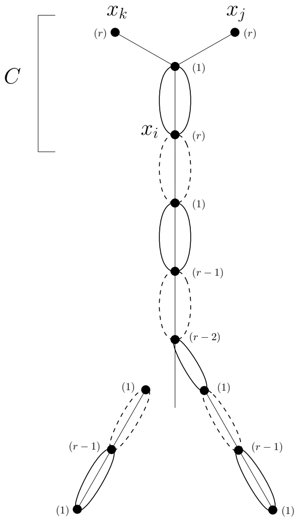

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