For most of these problems, all known algorithms require computing times that grow exponentially with n. (Recent work in complexity theory[7] indicates that problems like the traveling-salesman problem very probably are inherently exponential.) Heuristic methods appear to be the only feasible line of attack. From a theoretical standpoint, although we cannot generally prove optimality of solutions, we can obtain statistical confidence; for practical applications, frequently all that matters is that good answers are obtained in feasible running times.

One basic approach to heuristics for combinatorial optimization problems is iterative improvement of a set of randomly selected feasible solutions:

1. Generate a pseudorandom feasible solution, that is, a set T that satisfies C.

2. Attempt to find an improved feasible solution T' by some transformation of T.

3. If an improved solution is found, i.e., $f(T') < f(T)$, then replace T by T' and repeat from Step 2.

4. If no improved solution can be found, T is a locally optimum solution. Repeat from Step 1 until computation time runs out, or the answers are satisfactory.

The actual heuristic procedure (the transformation of Step 2) maps the random starting solutions of Step 1 into locally optimum solutions, among which the global optimum will hopefully appear. The better the heuristic is, the smaller the set of local optima will be, and the higher will be the fraction of random starts that lead to the global optimum. Random, uniformly distributed starting solutions are chosen in Step 1 (rather than, say, good solutions), unless we know in advance that a particular kind of starting solution leads to better answers. There are two reasons for this. First, a worthwhile heuristic should produce 'good' starting solutions just as fast as any other starting procedure—this is certainly our experience. Second, constructive solutions are usually deterministic, so that it may not be possible to get more than one initial solution.

The heart of the iterative procedure is, of course, Step 2, the process that tries to improve upon a given solution. One transformation that has been applied to a variety of problems[2,10–12] is the exchange of a fixed number k of elements from T with k elements from S-T, such that the resulting solution T' is feasible and better. This is repeated as long as such groups can be found. Eventually it will not be possible to improve T further by such exchanges, at which time we have a locally optimum solution. Naturally enough, the whole problem is finding the right elements to exchange, for one can always find optimum solutions by exchanging the correct groups.

This interchange strategy was applied to the traveling-salesman problem by Croes,[2] with k fixed at 2, and by Lin,[11] with $k=3$, with considerable success.