Local Search for the Direct Aperture Optimisation in IMRT

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***Abstract*—Radiotherapy (also called Radiation therapy) is a cancer treatment that uses high doses of radiation to destroy cancer cells and shrink tumors. Into external radiotherapy, there is the Intensity Modulated Radiation Therapy (known as IMRT), where it is taken a specific part of the body through the deliver the dose from different angles to damage the tumor, avoiding surrounding organs.**

When IMRT is approached as a sequential problem, we first need to establish a set of beam angles from which radiation will be released. Then, the radiation intensities for each selected beam angles are computed. Finally, the sequence of apertures we need to deliver the computed treatment plan is generated. Unlike this sequential approach, in the Direct Aperture Optimization (DAO) problems, constraints associated with the number of deliverable aperture shapes, just as some physical constraints, are taken into consideration while the intensities optimisation process is taking place. According to some authors, DAO generates better treatments with fewer apertures for IMRT.

In this work, we propose a heuristic algorithm, mixing a local search algorithm and mathematical programming to solve the DAO problem. We apply our algorithm on a prostate cancer case and compare ours results with those obtained in the sequential approach. Results show that our algorithms can find treatment plans in competitive time when considering the number of deliverable aperture shapes.

***Index Terms*—intensity modulated radiation therapy, direct aperture optimisation, local search.**

I. Introduction

Cancer produces millions of deaths a year [1], generating both economic and social costs. In Chile, more than 25.000 people died because of cancer last year. Furthermore, cancer is the second cause of death in the country, and in some regions, it is already the first cause of death [2]. Although the prevalence of cancer has increased in the last four decades, cancer survival has doubled in the same period [3]. Improvements in cancer survival rates are mainly explained by the great efforts that have been made by researchers in this area, developing new techniques and drugs to fight the disease.

One of the most common therapy for these diseases is radiotherapy. Radiation therapy is a collective term for medical treatments where the patient is exposed to ionising radiation, the primary application of which is to treat malignant disease. Radiation therapy gives the highest chance of curing or shrinking cancer while reducing the risk of side effects. Radiotherapy can be internal or external. The external radiotherapy delivering ionizing radiation from an external source, usually generated by a linear accelerator (linac) as shown in 1.

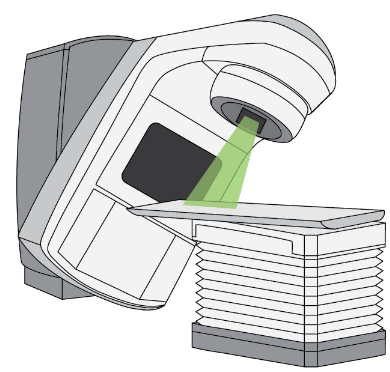

Fig. 1. linear accelerator

Intensity modulated radiation therapy (IMRT) is the most common technique in external radiotherapy, as it allows us to obtain good dose coverage in the tumour while avoiding surrounding healthy organs. The problem of finding a treatment plan that maximises the radiation delivered to the tumour while minimising the radiation delivered to surrounding organs is quite complex.

Due to the complexity of generating a treatment plan for IMRT, the IMRT planning problem is usually divided into three sequential subproblem called, beam angle optimisation (BAO), fluence map optimisation (FMO), and multi-leaf collimator sequencing (MLC) [4]. In the BAO problem, the aim is to find the optimal beam angle configuration (BAC), the BAC that leads to the optimal treatment plan. To find this optimal treatment plan, the intensities for the give BAC, need to be computed FMO. Finally, the MCL sequencing problem must be solved to find the set of deliverable aperture shapes and intensities of the MLC during the delivery process.

One problem of this sequential approach is that, once we have computed the optimal intensities for a given BAC, we need to determine a set of apertures for the MLC such that it can be delivered to the patient. Usually, a solution for the

Mauricio Moyano is a beneficiary of "Beca Postgrado PUCV 2019".

978-1-7281-8328-2/20/$31.00 ©2020 IEEE

MLC sequencing problem involves a large set of deliverable aperture shapes and their associated intensities. Having too many apertures, as well as, larger intensity values per aperture mean longer treatment times. Unfortunately, longer treatment times is something that we want to avoid, as it means patients should stay longer on the treatment couch and, therefore, fewer patients can be treated per day. In order to reduce the number of apertures and the beam-on time, treatment planners usually simplify the treatment plan by rounding the intensities at each beam angle to some predefined values. This simplification of the optimal plan allows us to deliver a treatment plan using fewer deliverable aperture shapes and shorter beam-on times, with a compromise on the treatment quality, though. Thus, it seems reasonable to incorporate some MLC sequencing considerations into the FMO problem such that the optimal intensities found during its optimisation process can be directly delivered to the patient without needing any adjustment process. To address this problem, direct aperture optimisation (DAO) is proposed by shepard [5].

The DAO problem aims to optimise the intensities and shapes of the apertures simultaneously, for each beam in a BAC. In other words, we aim to solve the FMO problem taking into account a constraint on the number of deliverable apertures and the physical constraints associated with the MLC sequencing. Thus, we do not longer need any leaves sequencing procedure after solving the DAO problem. DAO involves two steps: aperture shape optimisation and aperture weight optimization [6]. In the first step, the deliverable aperture that provides the most considerable potential improvement in the objective function is identified and added to the treatment plan. In the second step, gradient-based optimisation methods are typically used to optimise the aperture weights. Thus, compared to the three sequential approach, the IMRT plan quality is considerably improved with DAO [7].

In this paper, we propose new algorithms that solve the DAO problem, combining heuristic algorithms with mathematical programing within clinically acceptable times. It is important to note that, although quite a few matheuristic methods have been proposed for other problems in single objective optimisation, and matheuristics have not yet been studied in-depth. Experiments in this study will consider data coming from a public dataset which means that no patients are involved in this study.

The remaining of this paper is organised as follows: Sect. 2 introduces the general concepts of IMRT as well as the mathematical models we will consider in this study. In Sect. 3, the algorithms we implement in this paper are presented. Section 4 presents the obtained results of our algorithm applied to the prostate case. A discussion on these results is also included in this section. Finally, in Sect. 5, we draw the main conclusions of our work as well as outline future work.

II. IMRT: An overview

The radiation dose distribution deposited in the patient, measured in Gray (Gy), needs to be assessed accurately in order to solve the DAO problem, i.e., to determine optimal fluence maps. Each beam angle is discretised into sub-beams or beamlets, with (N_b) being the total number of beamlets summed over all possible beam angles. Moreover, Each structure's volume is discretised into voxels (small volume elements), and the dose is computed for each voxel considering the contribution of each beamlet. Let vector b be the fluence map, where b_j correspond to the fluence of beamlet j. Typically, for each volume, a dose matrix D is constructed from the collection of all beamlet weights, by indexing its rows to each voxel and its columns to each beamlet. That is, the number of rows of D equals the number of voxels (N_v). Similarly, the number of columns of D equals the number of beamlets (N_b) from all beam angles considered in the BAC. Therefore, using matrix format, we can say that the total dose received by the voxel i is given by equation(1), with b_j the fluence of beamlet j.

$$\sum_{j=1}^{N^b} (D_{ij} b_j) \tag{1}$$

In this study, we use a convex penalty function, voxel-based, nonlinear model [8], [9]. In this model, each voxel is penalized according to the squared difference of the amount of dose received by the voxel and the amount of dose desiredby the voxel. This formulation yields a quadratic programming problem with only linear non-negativity constraints on the fluence values [8].

$$\min_{b} {}_{z(b)} = \sum_{i=1}^{N_v} \frac{1}{v_s} \left[\underline{\lambda}_i \left(T_i - \sum_{j=1}^{N^b} (D_{ij} b_j) \right)_+^2 + \overline{\lambda}_i \left(\sum_{j=1}^{N^b} (D_{ij} b_j) - T_i \right)_+^2 \right] \tag{2}$$

s.t. $\quad b_j \geq 0, j = 1, \ldots, N_b$

Where T_i is the desired dose for voxel i, $\underline{\lambda}_i$ and $\overline{\lambda}_i$ are the penalty weights of underdose and overdose of voxel i, and $(\cdot)_+ = max\{0, \cdot\}$. T_i, $\underline{\lambda}_i$ and $\overline{\lambda}_i$ considered for each structure included in the optimization are displayed in the following table I, where consider a planning target volume(PTV), the rectum and bladder :

Organ	T_i	$\underline{\lambda}_i$	$\overline{\lambda}_i$
PTV	76 Gy	5	5
Rectum	65 Gy	0	1
Bladder	65 Gy	0	1

TABLE I
VALUE OF T_i,$\underline{\lambda}_i$ AND $\overline{\lambda}_i$ FOR FUNCTION Z(B).

A. DAO Search Space Representation

Based on the representation in [10] and [11], we proposed to represent the fluence maps of each beam angle, through the linear combination of a set of apertures and their associated intensities, as shown in figure 2.Where each element in the linear combination matrix represent a beamlet for the fluence map of a beam angle. The values of each aperture matrix range from 0 to 1, where 0 represents that the leaf is closed in that space, and 1 where the leaf is open and the radiation beam passes.

$$\begin{bmatrix} 2 & 2 & 1 & 1 \\ 2 & 5 & 1 & 0 \\ 0 & 4 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = 2 \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} + 1 \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} + 3 \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} + 1 \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Fig. 2. Intensity matrix composed of the linear combination of apertures and intensities.

III. Local Search

In this section, we present a local search algorithm to solve DAO. For a given BAC K, and subject to a maximum number of aperture for each beam angle, the algorithms attempt to find a set of aperture shapes for each angle in K such that the corresponding fluence map b minimises $z(b)$. The local search algorithm implemented in this work(1), starts with an initial solution generated for BAC K on line 1. This solution is assigned to S^*. Then the neighbourhood of S^* is generated on line 4. The neighbour with the best objective function value is chosen on line 5 and assigned to $S^{'}$. We then compare our best solution S^* to its best neighbour $S^{'}$ on line 6 to 8. If $S^{'}$ is better than S^*, then the algorithm updates the best solution found on line 7, and repeats lines 2 to 11. If $S^{'}$ is not better than S^*, then the algorithm stops and returns S^* on line 12.

Algorithm 1: Local Search

```
   Result: S*
1  S* = initialSolution(K);
2  localOptimum = false;
3  while localOptimum == false do
4  |  N = generateBestNeighbourhood(S*);
5  |  S'=BestNeighbour(N);
6  |  if S' ≤ S* then
7  |  |  S*= S';
8  |  else
9  |  |  localOptimum=true;
10 |  end
11 end
12 return S*;
```

A. Initial Solution

The proposed algorithm uses a two-step solution generation. The first step is to set the apertures to their initial configuration. To this end, we try either to open all apertures, i.e., the apertures have all leafs completely open or to use geometric shapes that somehow mimics the shape of the tumour. In this work, we choose the former, that is, we use an ad-hoc set of aperture shapes for each angle as it is shown in figure 3. As we can see, the first shape is half-open at the bottom, the second shape is half-open at the top, the third shape is half-open at the right, the fourth shape is half-open at the left, and the last shape is entirely open. The second step is to assign intensities for each aperture shape. To do this, we use Gurobi solver [12] and, thus, we obtain the optimal intensities for the aperture shapes generated in the previous step.

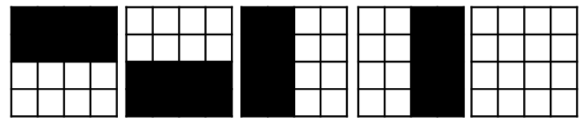

Fig. 3. Ad-hoc aperture shapes

B. Local Search Operator

For the neighbour's generation, two movements have been developed. The first movement generates neighbours by modifying the aperture shapes, as shown in figure 4. Consider a set of apertures A where $A = \{A_1, ..., A_n\}$, from which a random aperture of the treatment plan is chosen. Then for each pair of leaf four different neighbours are generated. Were one neighbor opening the leaf to the right, second neighbor opening the leaf to the left, other neighbor closing the leaf to the right, and last neighbor closing the leaf to the left.

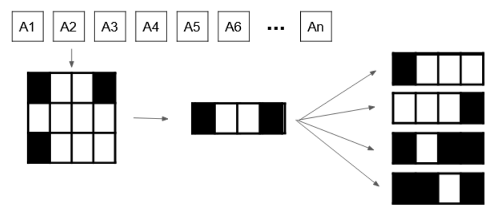

Fig. 4. First neighborhood movement

The movements explained previously generate small changes in the solutions. If this change in a leaf position produces an improvement in the objective function, it makes sense to go further in the same direction with the aim of obtaining better results. We can do this through a method that we call intensification. Intensifying a movement means that we will keep moving the leaf for each solution generated by the first neighbourhood movement, that produced an improvement in the objective function, in the same direction until no more improvements are generated. As shown in Figure 5, we present two case. In the first case we have an aperture that close the first leaf to the right, we generate a neighbour closing the same leaf in the same direction, if the neighbour improve the solution, we repeat the process until that cannot close more the leaf. In the second case we have an aperture that open the last leaf to the right, we generate a neighbour opening the same leaf in the same direction. For two case we keep with the last neighbour that improve the solution.

IV. Experiments

A clinical prostate case obtained from the *CERRpackage* [13] has been considered in this study. In this case, the tumour

First case

Second case

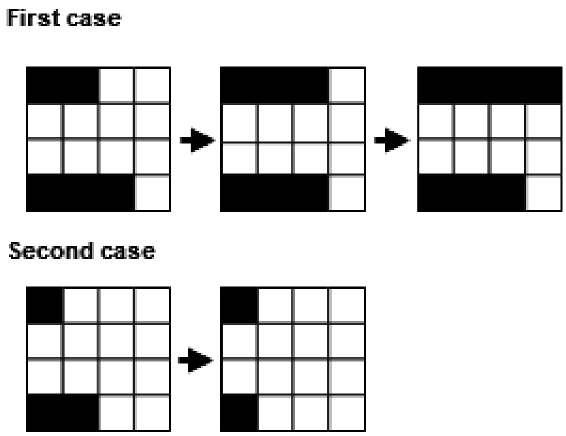

Fig. 5. intensification movement

is in prostate like shows Figure 6. We label the rectum and the bladder as organs at risk (OARs). The total number of voxels is about $56,000$. The number of decision variables (beamlets) depends on the BAC and ranges between 320 and 380. The number of beam angles considered in a BAC is equal to 5. The dose deposition matrix D is given for each BAC. We consider 72 beam angles, all of which are on the same plane. Just as in [14] [15] [11], we consider a set of 14 equidistant BACs to make our experiments. we run each configuration 30 times per experiment, being 30 a widely accepted value for statistical analysis [16]. Note that our implementation uses the Java platform and the tests are performed on a core Ryzen 2700x ,with 16 GB of RAM, running on Windows 10.

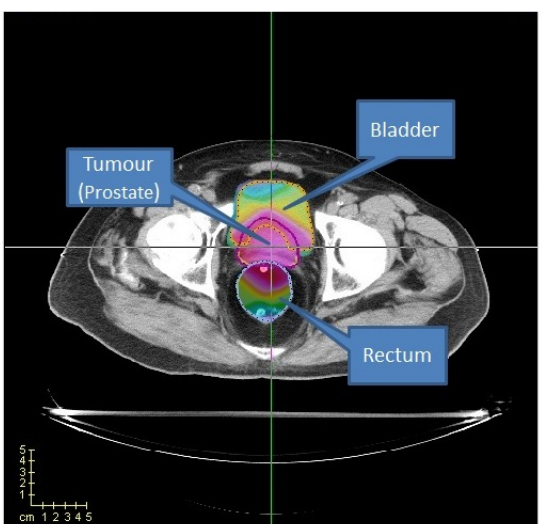

Fig. 6. Prostate case

We compare the algorithms described before to the sequential approach commonly used in clinical practice. That is, we first solve the FMO problem (with a given BAC), and then, we find the set of deliverable apertures corresponding to the optimal intensities. To solve the FMO problem, we use the Gurobi solver [12] while to solve the MLC sequencing problem we use the algorithm proposed in [17], which can find a set of apertures delivering the optimal intensities while minimising the beam-on time.

A. Initial Solution

In the following experiments, we evaluate the effect of the initialisation strategy on the performance of the proposed algorithms. As explained in previous sections, for the aperture shapes, we define two types of initial solutions, namely open and ad-hoc. We show in Table II the difference between the initial value and the final value of $z(b)$ of the two initialisation methods. As we can see, the ad-hoc method gets a better initial solution. Also, the objective function value of the final treatment plan we obtain with the ad-hoc configuration is better than the one we obtain when using the "open" initial configuration.

Initial solution	Initial Value	Final Value
Open	147.75	145.18
Ad-hoc	78.38	69.92

TABLE II
MEAN VALUE OF Z(B) USING DIFFERENT INITIALIZATION METHOD ACROSS THE SET OF 14 BACS.

B. Comparison of Performance

In the first test, we compere the use of neighborhood movements. The first neighborhood, define as N_1, use a movement that opens or closes the leafs of a selected aperture. For the second neighborhood, define as N_2, we use two the movement. The first movement is the same used in N_1, and the second is intensification movement. This seek to improve the neighbors obtained by the first movement moving the leaf that in the same direction until no more improvements are generated. For example, if a leaf was opened from the right, open again the leaf for the right. Table III compare the results of using N_1 and N_2. We can see that the use of both movements in N_2 achieves better values of the treatment generated.

Instances	Neigborhood Movements	
	N_1	N_2
0-70-140-210-280	70.20	68.88
5-75-145-215-285	63.58	62.96
10-80-150-220-290	72.59	70.56
15-85-155-225-295	63.41	61.74
20-90-160-230-300	75.18	71.81
25-95-165-235-305	68.76	64.99
30-100-170-240-310	69.58	68.78
35-105-175-245-315	89.09	82.87
40-110-180-250-320	74.65	71.24
45-115-185-255-325	74.12	68.56
50-120-190-260-330	70.15	69.04
55-125-195-265-335	72.70	64.00
60-130-200-270-340	73.82	71.16
65-135-205-275-345	86.38	82.32

TABLE III
MEAN RESULTS FOR EACH BAC USING ONLY THE FIRST MOVEMENT(N_1), AND BOTH MOVEMENTS(N_2).

Table IV shows a comparison between the proposed local search(LS) and the Gurobi solver, which solves the FMO problem to optimality. The first column shows the BAC for which the intensities and deliverable apertures are computed. Columns 2–4 show the mean value(z(b)), the beam-on time (BOT) and the number of apertures used (#ap) found by LS Gurobi solver, considering a maximum of 5 apertures per beam angle, that is, a maximum of 25 deliverable aperture shapes per BAC. Each heuristic algorithm run 30 times. Columns 5–7 show the optimal z(b) for the FMO problem found by the Gurobi solver, the minimal beam-on time of the optimal solution found and a lower bound for the number of the aperture shapes required. The last two values were obtained by using the MLC sequencing algorithm proposed in [17]. The results obtained by the proposed techniques are worst, in terms of the evaluation function, when compared to the optimal solution of the FMO problem. Nevertheless, the average beam-on time and number of aperture shapes required by this optimal solution are considerably larger. As mentioned before, larger number of apertures and larger beam-on times result in longer treatment times which are undesirable from a practical perspective [18].

Instances	Local Search			Gurobi		
	z(b)	BOT	#ap	z(b)	BOT	#ap
0-70-140-210-280	68.88	92	16	42.98	198	140
5-75-145-215-285	62.96	95	17	43.41	215	140
10-80-150-220-290	70.56	96	15	43.71	203	144
15-85-155-225-295	61.74	96	16	43.53	206	145
20-90-160-230-300	71.81	88	16	43.23	200	142
25-95-165-235-305	64.99	100	16	43.05	212	149
30-100-170-240-310	68.78	99	15	42.86	212	152
35-105-175-245-315	82.87	94	15	43.06	197	146
40-110-180-250-320	71.24	88	15	43.66	186	141
45-115-185-255-325	68.56	95	15	44.15	200	144
50-120-190-260-330	69.04	99	17	43.83	190	138
55-125-195-265-335	64.00	95	16	43.32	214	144
60-130-200-270-340	71.16	89	16	42.84	229	157
65-135-205-275-345	82.32	99	16	42.85	217	142

TABLE IV
MEAN RESULTS FOR EACH EQUIDISTANT BAC FOR LOCAL SEARCH AND TWO-STEP APPROACH.

V. CONCLUSION

In this paper, we introduce two movements for the local search algorithm to solve the DAO problem in radiation therapy for cancer treatment. Proposed heuristics can find a set of deliverable aperture shapes and intensities for each beam angle of a given BAC within a clinically acceptable time. Further, even though the heuristic algorithms were allowed to use only five aperture shapes per beam angle, they were able to find very competitive treatment plans.

We compare the heuristic algorithms to the traditional two-step approach where the optimisation of the intensities is performed first and, then, the MLC sequencing problem is solved given an optimal intensity map.

The results obtained by the local search algorithm is worst in the evaluate function, when compared with the two-step approach. However, the beam-on time and number of apertures required by our method are considerable less compared with two-step approach.

We can observe that the use of ad-hoc geometric figures on the initial aperture shapes improved the initial and final values of the obtained treatment plans. As future work, we proposed to use a heuristic that, based on a set of aperture shapes, based on geometric figures, select the best apertures for each angle, in order to improve the initial solution. Also, we propose to use other neighbourhood movements that generate new apertures, like for example, crossing the apertures of a given angle so we can increase the diversity of the obtained treatment plans. Also, better techniques can be implemented when selecting the apertures in the neighbourhood movements, replacing current random selection. For instance, we might select apertures with probabilities, where the unchanged apertures get more probabilities to be selected. Moreover, we also want to extend the local search algorithm to the multiobjective DAO problem.

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