Title: Enriched duality in double categories

Abstract: Back in the late ’60s, Moss Sweedler introduced the concept of a “measuring k-coalgebra” as a space of generalized k-algebra maps. A particular case is that of the finite dual of a k-algebra, namely a coalgebra with the property that coalgebra maps into it naturally correspond to algebra maps into the classical linear dual of a coalgebra. Gavin Wraith was the first to observe that measuring coalgebras induce an enrichment of the category of k-algebras in k-coalgebras. Interestingly, in modern terms, this renders the category of k-algebras an example of a semi-Hopf linear category [2]. Anel and Joyal first referred to the (tensored and cotensored) enrichment of dg-algebras in dg-coalgebras along with involved structures related to the bar-cobar construction as “Sweedler theory”. In this talk, we will investigate how this fact of an enrichment of monoids in comonoids, established in a broader context of locally presentable and braided monoidal closed categories [4], can lead to a many-object generalization in the setting of monoidal double categories [5, 1]. In the process of extending such results in other double categories of interest, it turns out that the structure of an “oplax” monoidal double category [3] is required. Analogous results therein are envisioned to provide insight to further cases of interest, for example that of symmetric coloured operads and cooperads.

References

[1] Aravantinos-Sotiropoulos, V.; Vasilakopoulou, C. Enriched duality in double categories II: V-modules and V-comodules, preprint arXiv:2408.03180.
[2] Buckley, M.; Fieremans, T.; Vasilakopoulou, C.; Vercruysse, J. Oplax Hopf algebras. Higher structures 5 (2021), no. 1, 71–120.
[3] Gambino, N.; Garner, C.; Vasilakopoulou, C. Monoidal Kleisli bicategories and the arithmetic product of symmetric sequences. Documenta Mathematica (3) 29, 627-702,
[4] Hyland, J.M.E.; Lopez Franco, I.; Vasilakopoulou, C. Hopf measuring comonoids and enrichment. Proceedings of the London Mathematical Society 115 (2017), no. 5, 1118–1148.
[5] Vasilakopoulou, C. Enriched duality in double categories: V-categories and V-cocategories. Journal of Pure and Applied Algebra 223 (2019), no. 7, 2889–2947.