Ejercicio 6

Aproximación para e^{-5} mediante expansión por polinomio de Taylor de grado 9, y con aritmética de 3 dígitos por truncamiento.

Polinomio de Taylor para $f(x) = e^x$,

$$e^x = \sum_{i=0}^{n} \frac{x^i}{i!} = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots, \qquad -\infty < x < \infty$$

a)

$$e^{-5} \approx \sum_{i=0}^{9} \frac{(-5)^i}{i!} = \sum_{i=0}^{9} \frac{(-1)^i(5)^i}{i!}$$

$$= 1 - 5 + \frac{25}{2} - \frac{125}{6} + \frac{625}{24} - \frac{3125}{120} + \frac{15625}{720} - \frac{78125}{5040} + \frac{390625}{40320} - \frac{1953125}{362880}$$

Aplicando aritmética de 3 dígitos por truncamiento a cada valor:

$$1 = 0,1 * 10^1 \to 1$$
$$5 = 0,5 * 10^1 \to 5$$
$$25 = 0,25 * 10^2 \to 25$$
$$2 = 0,2 * 10^1 \to 2$$
$$125 = 0,125 * 10^3 \to 125$$
$$6 = 0,6 * 10^1 \to 6$$
$$625 = 0,625 * 10^3 \to 625$$
$$24 = 0,24 * 10^2 \to 24$$
$$3125 = 0,3125 * 10^4 \to 3120$$
$$120 = 0,120 * 10^3 \to 120$$
$$15625 = 0,15625 * 10^5 \to 15600$$
$$720 = 0,720 * 10^3 \to 720$$
$$78125 = 0,78125 * 10^5 \to 78100$$
$$5040 = 0,5040 * 10^4 \to 5040$$
$$390625 = 0,390625 * 10^6 \to 390000$$
$$40320 = 0,40320 * 10^5 \to 40300$$
$$1953125 = 0,1953125 * 10^7 \to 15600$$
$$362880 = 0,362880 * 10^6 \to 362000$$

Aplicando aritmética de 3 dígitos por truncamiento a cada término:

$$\frac{1}{1} = 1$$
$$\frac{5}{1} = 5$$
$$\frac{25}{2} = 0,125 * 10^2 \to 12,5$$

$$\frac{125}{6} = 0{,}208\hat{3} * 10^2 \rightarrow 20{,}8$$
$$\frac{625}{24} = 0{,}26041\hat{6} * 10^2 \rightarrow 26{,}0$$
$$\frac{3120}{120} = 0{,}26 * 10^2 \rightarrow 26{,}0$$
$$\frac{15600}{720} = 0{,}21\hat{6} * 10^2 \rightarrow 21{,}6$$
$$\frac{78100}{5040} \approx 0{,}15496 * 10^2 \rightarrow 15{,}4$$
$$\frac{390000}{40300} \approx 0{,}96774 * 10^2 \rightarrow 9{,}67$$
$$\frac{1950000}{362000} \approx 0{,}53867 * 10^2 \rightarrow 5{,}38$$

$$e^{-5} \approx 1 - 5 + 12{,}5 - 20{,}8 + 26{,}0 - 26{,}0 + 21{,}6 - 15{,}4 + 9{,}67 - 5{,}38 = -1{,}81$$

b)

$$e^{-5} = \frac{1}{e^5} \approx \frac{1}{\sum_{i=0}^{9} \frac{5^i}{i!}} =$$

$$= \frac{1}{1 + 5 + \frac{25}{2} + \frac{125}{6} + \frac{625}{24} + \frac{3125}{120} + \frac{15625}{720} + \frac{78125}{5040} + \frac{390625}{40320} + \frac{1953125}{362880}}$$

Aplicando aritmética de 3 dígitos por truncamiento a cada valor:

$$1 = 0{,}1 * 10^1 \rightarrow 1$$
$$5 = 0{,}5 * 10^1 \rightarrow 5$$
$$25 = 0{,}25 * 10^2 \rightarrow 25$$
$$2 = 0{,}2 * 10^1 \rightarrow 2$$
$$125 = 0{,}125 * 10^3 \rightarrow 125$$
$$6 = 0{,}6 * 10^1 \rightarrow 6$$
$$625 = 0{,}625 * 10^3 \rightarrow 625$$
$$24 = 0{,}24 * 10^2 \rightarrow 24$$
$$3125 = 0{,}3125 * 10^4 \rightarrow 3120$$
$$120 = 0{,}120 * 10^3 \rightarrow 120$$
$$15625 = 0{,}15625 * 10^5 \rightarrow 15600$$
$$720 = 0{,}720 * 10^3 \rightarrow 720$$
$$78125 = 0{,}78125 * 10^5 \rightarrow 78100$$
$$5040 = 0{,}5040 * 10^4 \rightarrow 5040$$
$$390625 = 0{,}390625 * 10^6 \rightarrow 390000$$
$$40320 = 0{,}40320 * 10^5 \rightarrow 40300$$
$$1953125 = 0{,}1953125 * 10^7 \rightarrow 15600$$
$$362880 = 0{,}362880 * 10^6 \rightarrow 362000$$

Aplicando aritmética de 3 dígitos por truncamiento a cada término:

$$\frac{1}{1} = 1$$
$$\frac{5}{1} = 5$$
$$\frac{25}{2} = 0{,}125 * 10^2 \to 12{,}5$$
$$\frac{125}{6} = 0{,}208\hat{3} * 10^2 \to 20{,}8$$
$$\frac{625}{24} = 0{,}26041\hat{6} * 10^2 \to 26{,}0$$
$$\frac{3120}{120} = 0{,}26 * 10^2 \to 26{,}0$$
$$\frac{15600}{720} = 0{,}21\hat{6} * 10^2 \to 21{,}6$$
$$\frac{78100}{5040} \approx 0{,}15496 * 10^2 \to 15{,}4$$
$$\frac{390000}{40300} \approx 0{,}96774 * 10^2 \to 9{,}67$$
$$\frac{1950000}{362000} \approx 0{,}53867 * 10^2 \to 5{,}38$$

Aplicando aritmética de 3 dígitos por truncamiento a cada suma parcial:

$$1 + 5 = 6$$
$$6 + 12{,}5 = 18{,}5$$
$$18{,}5 + 20{,}8 = 39{,}3$$
$$39{,}3 + 26{,}0 = 65{,}3$$
$$65{,}3 + 26{,}0 = 91{,}3$$
$$91{,}3 + 21{,}6 = 112{,}9 = 0{,}1129 * 10^2 \to 112$$
$$112 + 15{,}4 = 127{,}4 = 0{,}1274 * 10^2 \to 127$$
$$127 + 9{,}67 = 136{,}67 = 0{,}13667 * 10^2 \to 136$$
$$136 + 5{,}38 = 141{,}38 = 0{,}14138 * 10^2 \to 141$$

$$e^{-5} = \frac{1}{e^5} \approx \frac{1}{141} \approx 0{,}0070921 * 10^0 = 7{,}09 * 10^{-3}$$

c) La fórmula en el punto b) es más precisa, debido a que son todas adiciones y luego un cociente; nunca intervienen sustracciones.