a)

$$z^2 = (x+iy)(x+iy) = x^2 + xiy + xiy + i^2y^2$$
$$= x^2 + 2xiy - y^2$$

$$Re[x^2 - y^2 + 2xiy] = x^2 - y^2$$
$$Im[x^2 - y^2 + 2xiy] = 2xy$$

b)

$$(x-iy)(x+iy) = x^2 + \cancel{xiy} - \cancel{xiy} - \underbrace{i^2}_{=-1}y^2$$
$$= x^2 + y^2$$

c)

$$\frac{1}{z} = \frac{1}{x+iy}$$

$$e^{ix} = \cos(x) + i\sin(x)$$

d)

$$e^{x+iy} = e^x \cdot e^{iy}$$
$$= e^x(\cos(y) + i\sin(y))$$

e)

$$\sinh(z) = -i\sin i(z)$$

$$\sin(a-b) = \sin a \cos b - \cos a \sin b$$

$$\sinh(x+iy)$$
$$-i\sin i(x+iy)$$
$$= -i\sin(xi + i^2y)$$
$$= -i\sin(xi - y)$$
$$-i[\sin(xi)\cos(y) - \cos(xi)\sin(y)]$$

$$\sin iz = \frac{\sinh z}{-i}$$
$$= \frac{i\sinh z}{-i^2}$$
$$= i\sinh z$$

$$= -i[i\sinh z\cos y - \cosh x \sin y]$$
$$= -i^2\sinh z \cos y + i\cosh x \sin y)$$
$$= \underline{\sinh z \cos y + i\cosh x \sin y}$$

f)

$$z^n = r^n(\cos n\phi + i\sin n\phi)$$

$$z^n = (re^{i\phi})^n = r^n(e^{i\phi})^n = r^n(\cos\phi + i\sin\phi)^n$$
$$= r^n(\cos n\phi + i\sin n\phi)$$

g) $\ln(z) = \ln r + i(\phi + 2\pi n)$ $\quad z = re^{i\phi}$ $\quad \ln(re^{i\phi})$

$\ln(re^{i\phi}) = \ln(r) + \ln(e^{i\phi})$

$= \ln(r) + i\phi$

$= \ln(r) + i\phi + i2\pi n = \ln(r) + i(\phi + 2\pi n)$

$z = x + iy$

$z^3 - 1$

$(x+iy)^3 = x^3 + 3x^2yi - 3xy^2 - iy^3 - 1$

$Re = x^3 - 3xy^2 - 1$

$Im = 3x^2y - y^3$

$\frac{\partial Re}{\partial x} = 3x^2 - 3y^2$

$\frac{\partial Re}{\partial y} = -6xy$

$J = \begin{bmatrix} 3x^2 - 3y^2 & -6xy \\ 6xy & 3x^2 - 3y^2 \end{bmatrix}$

$\frac{\partial Im}{\partial x} = 6xy$

$\frac{\partial Im}{\partial y} = 3x^2 - 3y^2$

$J^{-1} = \frac{1}{(3x-3y)^2 + 36x^2y^2} \begin{bmatrix} 3x^2 - 3y^2 & 6xy \\ -6xy & 3x^2 - 3y^2 \end{bmatrix}$

$= \frac{1}{9x^4 + 18x^2y^2 + 9y^4}$

Problem 3

$f(x) = x^4 - 20x^2 + x$

$= x(x^3 - 20x + 1)$

a) I expect 4 roots since this is a 4^{th} degree polynomial

b) $x^3 - 20x + 1 = 0$

$20x = x^3 + 1 \qquad x = \frac{x^3+1}{20}$

$1 = 20x - x^3$

$1 = x(20 - x^2)$

$x = \frac{1}{20 - x^2}$

$g_1(x) = \frac{x^3+1}{20}$

$g_2(x) = (20x-1)^{1/3}$

$g_3(x) = \frac{1}{20-x^2}$

$g_1'(x) = \frac{3x^2}{20} \qquad g_2'(x) = \frac{20}{3}(20x-1)^{-2/3}$

$g_3'(x) = 2(20-x^2)^{-2}$

$g_1'(x)$ region : $\left(-\sqrt{\frac{20}{3}}, \sqrt{\frac{20}{3}}\right)$

$\frac{3}{20} = \frac{1}{(20x-1)}^{-2/3}$

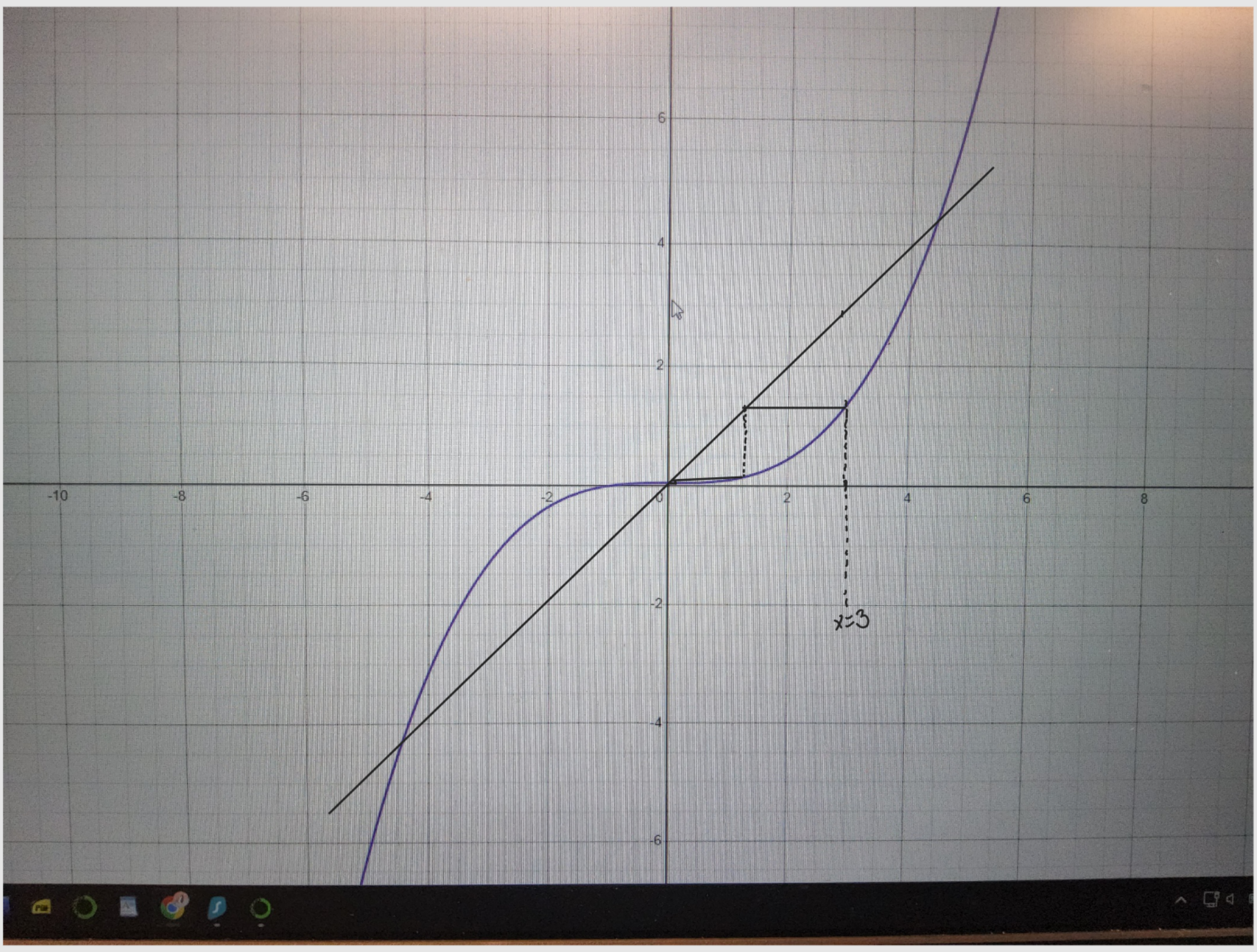
$\frac{20}{3} = 3\sqrt{(20x-1)^2}$

$g_2'(x)$ region : $\left(-\infty, -\frac{\sqrt{(\frac{20}{3})^3} - 1}{20}\right)$

$\left(\frac{3\cdot\sqrt{3} + 40\cdot\sqrt{3}}{60\sqrt{3}}, \infty\right)$

$g_3'(x)$ region : $(-\infty, -4.628)\ (-4.311, 4.311)\ (4.628, \infty)$

$|g'(x)| < 1 \quad = \quad \left(-\sqrt{\frac{20}{3}}, \sqrt{\frac{20}{3}}\right)$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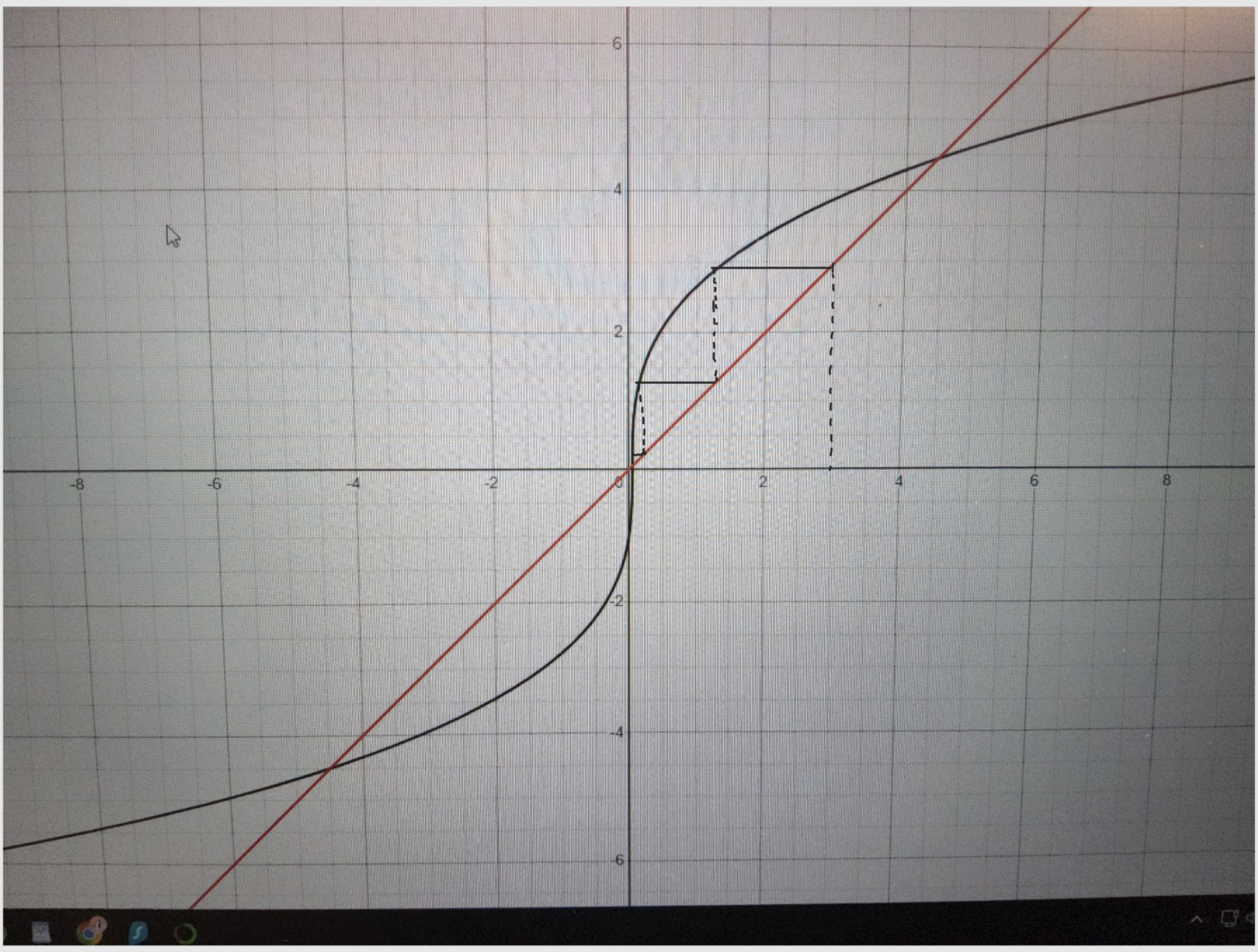

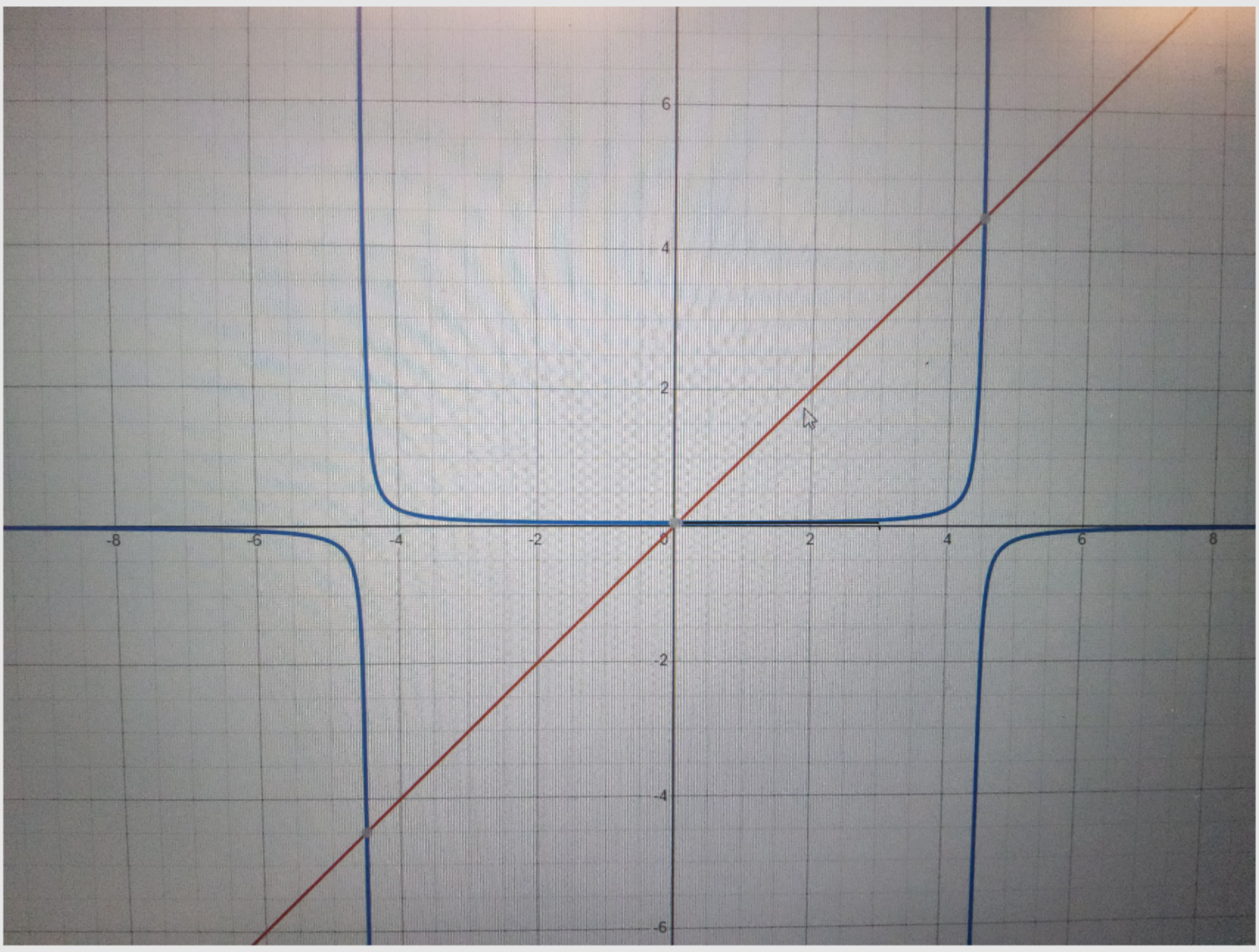

I don't think this method will find this root.