$$\frac{dy}{dx} = -2x - y \quad ; \quad y(0) = -1$$

$$\frac{dy}{dx} + y = -2x$$

$$P(x) = 1$$

$$Q(x) = -2x$$

$$\mu(x) = e^{\int 1\,dx}$$

$$= e^{x}$$

$$e^{x}\frac{dy}{dx} + e^{x}y = -2xe^{x}$$

product rule

$$\frac{d}{dx}\left(e^{x}y\right) = -2xe^{x}$$

$$\int \frac{d}{dx} e^{x}y\,dx = -2\int xe^{x}\,dx$$

$$e^{x}y = -2\left(xe^{x} - e^{x} + C\right)$$

$$e^{x}y = -2xe^{x} + 2e^{x} + C$$

$$e^{0}(-1) = 0 + 2(e^{0})$$

$$-1 - 2 = -3 = C$$

$$\frac{e^{x}y}{e^{x}} = \frac{-2xe^{x}}{e^{x}} + \frac{2e^{x}}{e^{x}} + \frac{C}{e^{x}}$$

$$y(x) = -2x + 2 - \frac{3}{e^{x}}$$

$$y(x) = -2x + 2 - 3e^{-x}$$