ECON 7710 Homework 4: Potential Outcomes and Causality

Part A: Definitions and Derivations

A.1. Selection bias (10 points)

a. Why won't the difference in average outcomes between treated and untreated groups identify the average treatment effect?

b. Consider the "switching equation", where y_i is expressed in terms of potential outcomes y_{0i} and y_{1i} and D_i is a treatment indicator (or the "switch"):

$$y_i = y_{0i} + (y_{1i} - y_{0i})D_i.$$

Use this relationship to show that in general the difference in mean outcomes for the treated and untreated groups will not produce the ATE.

c. How does randomization solve the selection problem?

Answers

a. The difference in average outcomes between treated and untreated groups won't identify the average treatment effect due to the issue of selection bias. For example, if individuals who are more likely to benefit from treatment are also more likely to select into it, this positive selection will lead to an overestimation of the treatment's effect when comparing treated to untreated. Furthermore, if individuals who are less likely to benefit are more likely to receive treatment, it can lead to an underestimation. The observed difference between the average outcomes of treated and untreated groups can be expressed as:

$$E[Y_1|D=1] - E[Y_0|D=0] = (E[Y_1|D=1] - E[Y_0|D=1]) + (E[Y_0|D=1] - E[Y_0|D=0])$$

Where:

- $E[Y_1|D=1] - E[Y_0|D=0]$: Observed difference in outcomes between treated and untreated groups.
- $E[Y_1|D=1] - E[Y_0|D=1]$: True effect of the treatment on those who received it.
- $E[Y_0|D=1] - E[Y_0|D=0]$: Selection bias, reflecting differences between those selected for treatment and those not.

b. The switching equation illustrates that the observed outcome y_i for an individual depends on their treatment status D_i. This relationship shows that the mean outcomes for the treated and untreated groups generally will not produce the ATE because it only considers the observed states and is susceptible to selection bias. The ATE, $E[y_{1i} - y_{0i}]$, reflects the average effect across the entire population, independent of treatment assignment. This means the simple comparison is conditioned on being treated or untreated, which can differ due to underlying characteristics. That is why, unless D_i is randomized, the difference will typically not equal ATE and will probably misrepresent the causal effect.

c. Randomization helps to ensure that the estimate of the average treatment effect (ATE) is unbiased and reliable by eliminating the influence of confounding variables through equal distribution across treatment conditions. In other words, observed difference in outcomes between the groups can be attributed solely to the treatment itself, not to pre-existing differences between the participants.

A.2. Provide the meaning of each term (10 points)

In the context of the potential outcomes model, explain what these terms mean:

a. Independence assumption

b. Conditional independence assumption

c. Common support assumption

d. Stable unit treatment value assumption

Answers

a. The independence assumption states that the assignment of the treatment is independent of the potential outcomes. In other words, it implies that receiving the treatment does not depend on any characteristics of the units. This assumption is usually given to us through randomization where each unit has an equal chance of receiving treatment. This ensures that the groups are statistically similar.

b. The conditional independence assumption states that the treatment assignment is independent of the potential outcomes, conditional on a set of observed covariates. More specifically, given a set of covariates Xi, the treatment assigment Di is independent of the potential outcomes. This is used more in observatinal studies where randomization is not possible.

c. The common support assumption ensures that for every unit within the range of observed covariates Xi, there is a positive probability of receiving each treatment condition. This assumption is critical for methods like matching and weighting. because it guarantees that comparisons are made between units that are comparable in terms of their covariates.

d. SUTVA essentially just means the treatment of one unit does not affect the outcome of other units. The potential outcome for any unit i under a specific treatment is the same regardless of the treatment assignment of other units. It assumes that there is only one version of the treatment, meaning the treatment effect is uniform across all units.

A.3. Ignorability, overlap and selection bias in Project STAR (10 points)

a. Did Project STAR, as described in Section II of Krueger 1999, satisfy the conditional independence assumption?

b. To what extent, if any, is there a concern about selection bias? Explain.

Answers

a. Project STAR randomly assigned students and teachers to different class sizes. Due to randomization, the conditional independence assumption (CIA) should be satisfied. Randomization ensures that on average, treatment (i.e. class size placement) is independent of other factors. If randomization was implemented correctly, without systematic attrition or non-compliance, Project STAR would satisfy the CIA.

b. While randomization is supposed to eliminate selection bias, other factors such as attrition or non-compliance could cause bias. For example, if certain students were to opt out of their assigned class and move into the other class size, or students were to leave their assigned class, this could hinder the randomization and cause positive/negative bias, either overstating or understating the effect that small class sizes have on test scores.

A.4. The CEF for the ATEs in Project Star (10 points)

a. Write an equation for the CEF that is consistent with the ATEs presented in Table V, Column (1) in Krueger (1999).

Answers

a.
$$Y_i = \beta_0 + \beta_1 \text{SmallClass}_i + \beta_2 \text{RegularAide}_i + \epsilon_i$$

Part B: Empirical Analysis

Using the Project Star data provided in the `AER` package, estimate the effect of class size on student achievement. Substantively, you will be replicating the Column (1) results in Table 5 of Krueger (1999), except with testing data reported in raw scores, not percentiles. Thus, the scale of your estimated effects will be different.

a. Estimate the class-size effects for each grade, K through 3, using OLS and report your results in a table using `modelsummary` analogously to Column (1) in the OLS panels of Table V in Krueger (1999).

b. Write a sentence that describes the statistical significance of small-class effect for kindergartners. Write a sentence explaining whether adding an aide to a regular class has a statistically significant effect on kindergartner test scores.

c. Repeat the sentences in (b) for the first, second and third-grade results.

d. Use the `scale` function to create standardized (z-score) version of the test-score variables. Repeat the exercise in (a).

e. Write a sentence that interprets the results for kindergartners. Write a sentence that compares the results for first, second and third-graders with those for kindergartners.

Results and answers

a. The OLS regression results indicate that small class size has a strong positive effect on test scores across all grades, with the largest impact in Grade 1 (29.8 points) and the smallest in Grade 3 (15.6 points). This suggests that the benefits of small class size are most pronounced in early elementary years but decline over time.The presence of a teacher's aide has no significant effect in Kindergarten, Grade 2, or Grade 3, but it increases test scores in Grade 1 by 11.96 points. This implies that aides may be most effective in early elementary education but not beyond first grade.

b. The effect of being in a small class in kindergarten is statistically significant at $p < 0.001$, with an estimated increase of 13.9 points in test scores. This suggests that reducing class size in kindergarten has a meaningful and significant impact on student achievement.

The presence of a teacher's aide in a regular kindergarten class does not have a statistically significant effect on test scores, with an estimated coefficient of 0.314 and a high p-value, indicating no meaningful impact.

c. The effect of being in a small class in first grade is statistically significant at $p < 0.001$, with an estimated increase of 29.8 points in test scores. Unlike in kindergarten, the presence of a teacher's aide has a significant positive effect, increasing test scores by 11.96 points at $p < 0.001$.

The effect of being in a small class in second grade is statistically significant at $p < 0.001$, with an estimated increase of 19.4 points in test scores, though smaller than the first-grade effect. However, the presence of a teacher's aide does not have a statistically significant impact on test scores

The effect of being in a small class in third grade is statistically significant at $p < 0.001$, with an estimated increase of 15.6 points in test scores, continuing the trend of decreasing impact over time. The presence of a teacher's aide does not have a statistically significant effect and may even have a negligible or slightly negative impact.

d. After standardizing test scores, the effect of being in a small class remains statistically significant across all grades. In kindergarten, being in a small class increases test scores by 0.188 standard deviations ($p < 0.001$). The effect is strongest in first grade, increasing scores by 0.326 standard deviations, before decreasing in second (0.231) and third grade (0.213). The presence of a teacher's aide is only significant in first grade, where it increases scores by 0.131 standard deviations ($p < 0.001$), but has no effect in other grades. These results suggest that small class size has the greatest impact in early elementary education, particularly in first grade, and that teacher's aides may only provide benefits in the early school years.

e. In kindergarten, being in a small class increases test scores by 0.188 standard deviations, however the presence of a teacher's aide has no significant impact on kindergarten test scores.

Compared to kindergarten, the effect of being in a small class is strongest in first grade (0.326 standard deviations) and then decreases in second grade (0.231) and third grade (0.213). The presence of a teacher's aide has a significant impact only in first grade (0.131 standard deviations, $p < 0.001$), but has no meaningful effect in kindergarten, second, or third grade.