

# New Results in Bounded-Suboptimal Search

Maximilian Fickert<sup>1</sup> and Tianyi Gu<sup>2</sup> and Wheeler Ruml<sup>2</sup>



# Planning as Heuristic Graph Search

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heuristic search: a planning approach

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heuristic search: a planning approach

planning is a model-based AI method, it models the environment as a state space and finds a sequence of actions that accomplishes some objective

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heuristic search: a planning approach

planning is a model-based AI method, it models the environment as a state space and finds a sequence of actions that accomplishes some objective

**heuristic search:**

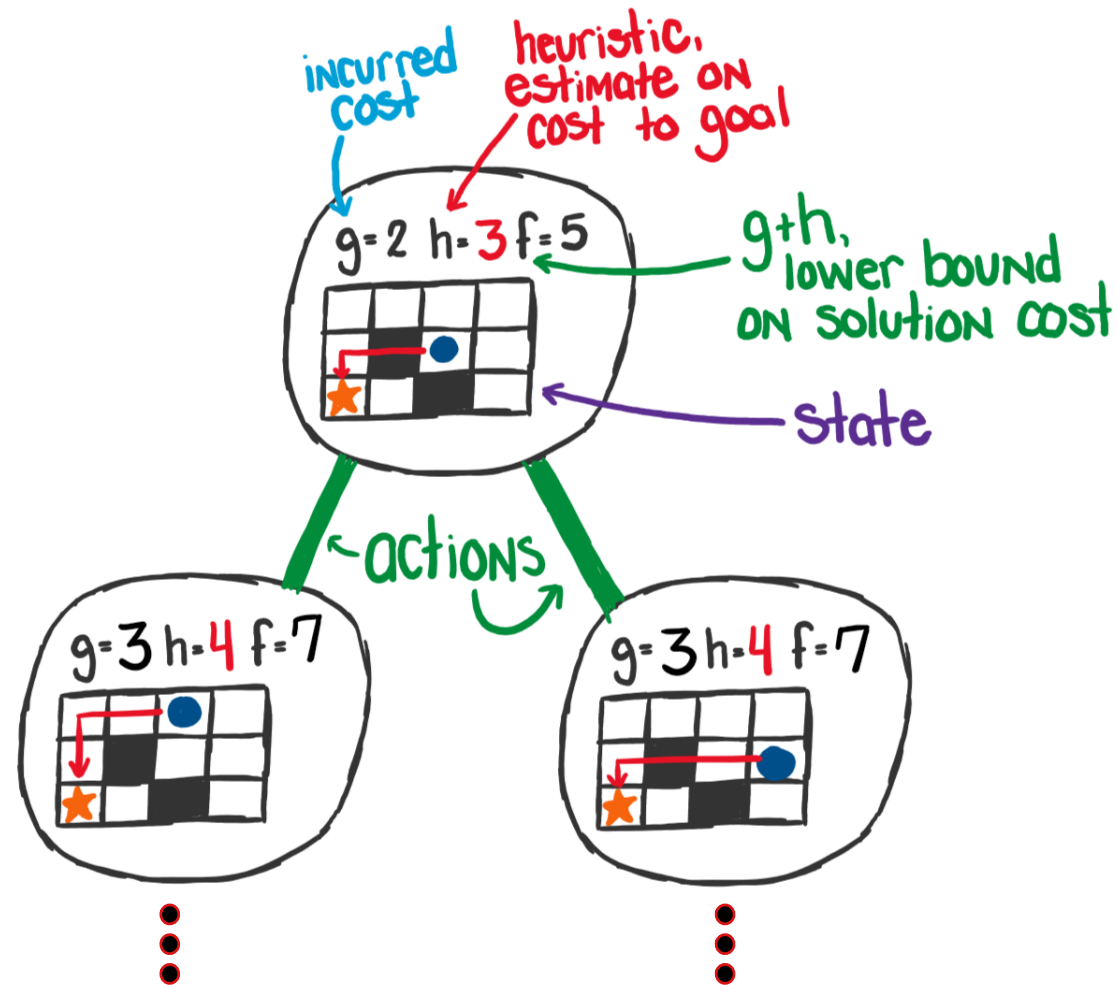
$\{\text{states, actions}\} \rightarrow \{V, E\}$

planning problem  $\rightarrow$  find a path from  $s_{init}$  to  $\{s_{goal}\}$

guide graph search by a heuristic estimate of cost-to-goal

# Planning as Heuristic Graph Search

heuristic search associates costs with states,  
used to guide search



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**A\***: expands the node with minimal  $f$  value  
returns optimal path  
**optimal search can take too long!**  
because it must expand every node with  $f < C^*$ <sup>1</sup>

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<sup>1</sup>How Good is Almost Perfect, Malte Helmert and Gabriele Roger, AAAI, 2008.

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because it must expand every node with  $f < C^*$ <sup>1</sup>

What if we don't have time?

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<sup>1</sup>How Good is Almost Perfect, Malte Helmert and Gabriele Roger, AAAI, 2008.

# Alternatives to Optimal Search: Problem Settings

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**optimal:** minimize solution cost  
expand every node with  $f < C^*$

**greedy:** minimize solving time

**anytime:** incrementally converge to optimal

**bounded-suboptimal:** minimize time subject to relative cost  
bound (factor of optimal)

**bounded-cost:** minimize time subject to absolute cost bound

**contract:** minimize cost subject to absolute time bound

**utility function:** minimize utility function of cost and time



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# Bounded-Suboptimal Search

# Bounded-Suboptimal Search: The Problem Setting

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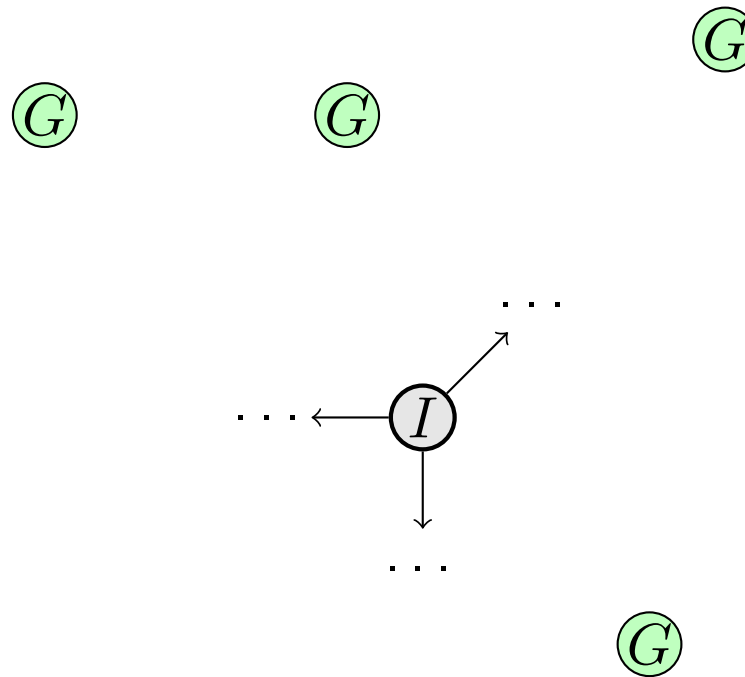
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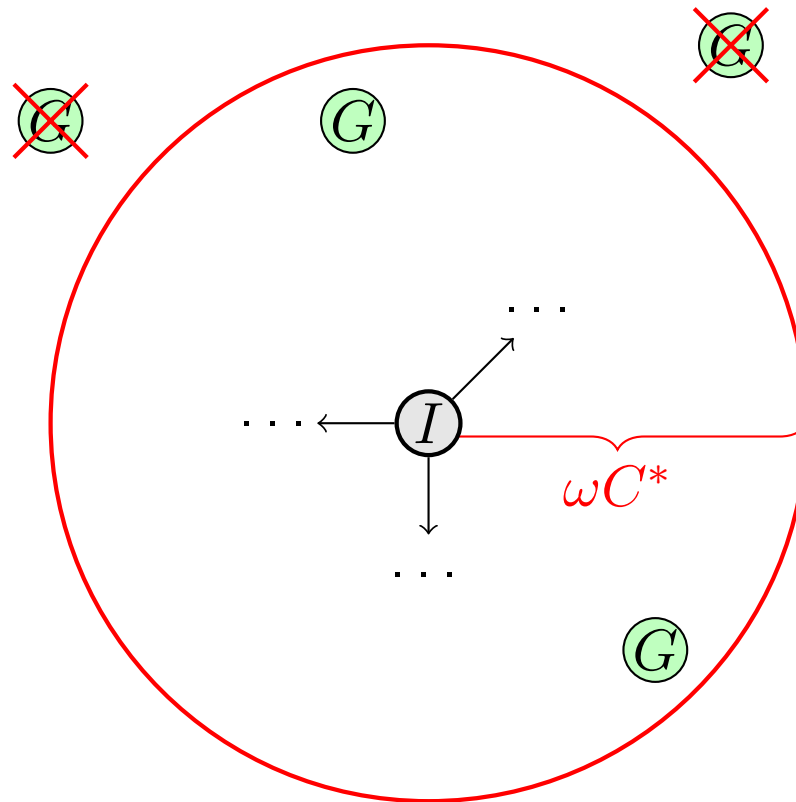
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*Objective: Find a plan with cost at most  $\omega C^*$  as fast as possible.*

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Three source of heuristic information:

$h$ : a lower bound on cost-to-go

$$f(n) = g(n) + h(n)$$

traditional A\* lower bound

$\hat{h}$ : an estimate of cost-to-go

$$\hat{f} = g(n) + \hat{h}(n)$$

unbiased estimates can be more informed

$\hat{d}$ : an estimate of distance-to-go

nearest goal is the easiest to find

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Three source of heuristic information:  $h, \hat{h}, \hat{d}$

EES search strategy:

$best_f$ : open node giving lower bound on cost

$best_{\hat{f}}$ : open node giving estimated optimal cost

$best_{\hat{d}}$ : estimated  $\omega$ -suboptimal node with minimum  $\hat{d}$

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node to expand next:

1. pursue the nearest goal estimated to lie within the bound
- 2.
- 3.

in other words:

1. **if**  $\hat{f}(best_{\hat{d}}) < \omega \cdot f(best_f)$  **then**  $best_{\hat{d}}$
- 2.
- 3.



Three source of heuristic information:  $h, \hat{h}, \hat{d}$

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2. pursue the estimated optimal solution
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in other words:

1. **if**  $\hat{f}(best_{\hat{d}}) < \omega \cdot f(best_f)$  **then**  $best_{\hat{d}}$
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Three source of heuristic information:  $h, \hat{h}, \hat{d}$

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node to expand next:

1. pursue the nearest goal estimated to lie within the bound
2. pursue the estimated optimal solution
3. raise the lower bound on optimal solution cost

in other words:

1. **if**  $\hat{f}(best_{\hat{d}}) < \omega \cdot f(best_f)$  **then**  $best_{\hat{d}}$
2. **else if**  $\hat{f}(best_{\hat{f}}) < \omega \cdot f(best_f)$  **then**  $best_{\hat{f}}$
3. **else**  $best_f$

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Other EES variants:

1. **if**  $\hat{f}(best_{\hat{d}}) < \omega \cdot f(best_f)$  **then**  $best_{\hat{d}}$
2. ~~**else if**  $\hat{f}(best_{\hat{f}}) < \omega \cdot f(best_f)$  **then**  $best_{\hat{f}}$  ?~~
3. **else**  $best_f$

see paper for more details.

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Problem:

- EES does not consider the uncertainty of its estimates (brittle)
- EES does not spend enough effort on estimating the bound

# State-of-The-Art: 2/2 DPS (Gilon, Felner, and Stern, 2016)

Best first search on “potential”:

$$ud(n) = \frac{\omega \cdot f_{min} - g(n)}{h(n)}$$

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# State-of-The-Art: 2/2 DPS (Gilon, Felner, and Stern, 2016)

Best first search on “potential”:

$$ud(n) = \frac{\omega \cdot f_{min} - g(n)}{h(n)}$$

does not explicitly optimize search time

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# Bounded-Cost: XES (Fickert, Gu, and Ruml, 2021)

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Best first search on **expected search effort**:

$$xe(n) = \frac{T_n}{p(n)}$$

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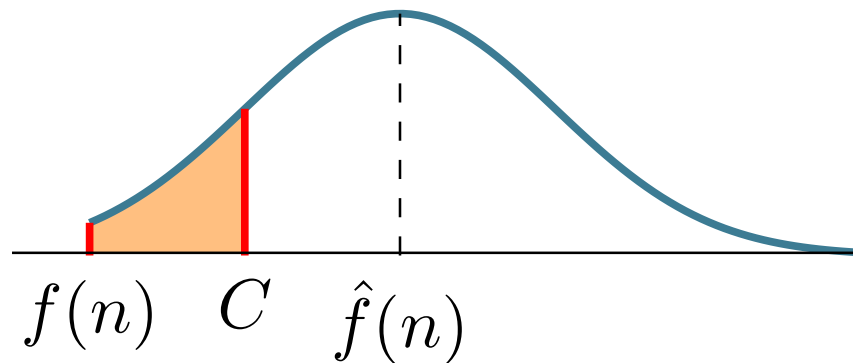
Conclusions

Best first search on **expected search effort**:

$$xe(n) = \frac{T_n}{p(n)}$$

$T(n)$ : total search effort, estimated by  $d(n)$

$p(n)$ : the probability of finding a solution within the bound, estimated by:



# Bounded-Cost: XES (Fickert, Gu, and Ruml, 2021)

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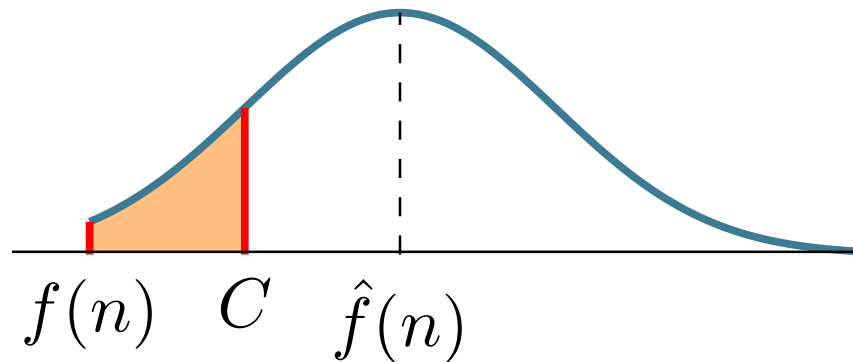
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**XES performs better than BEES and PS.**

**Can we adapt XES to bounded-suboptimal setting?**

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# New Algorithms

# Our Approach: 1/2 Dynamic XES

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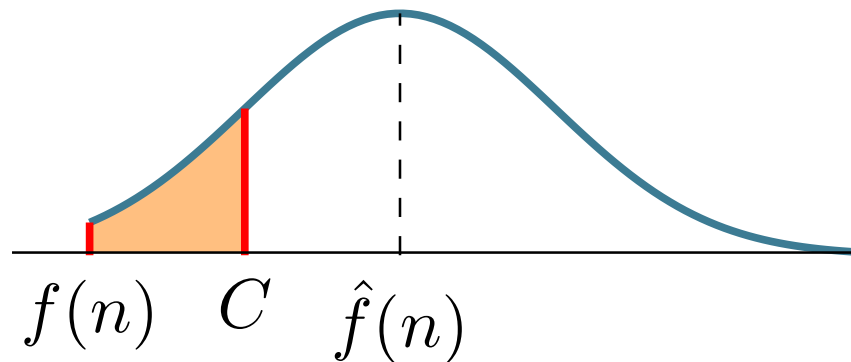
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**hard to estimate when raising the bound is useful!**

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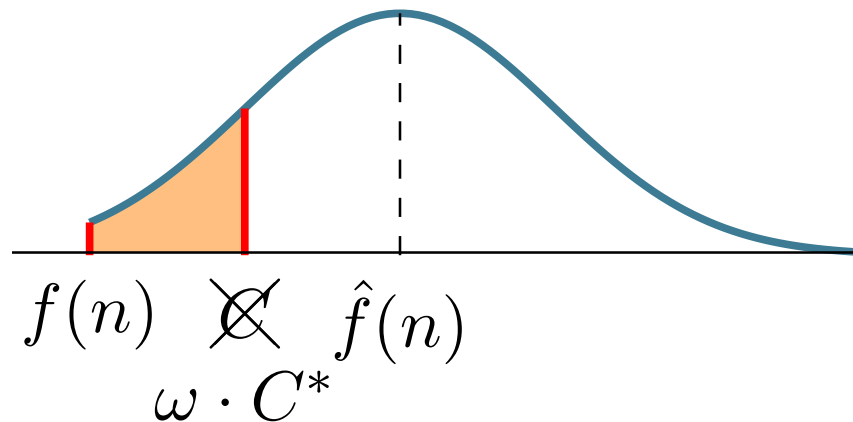
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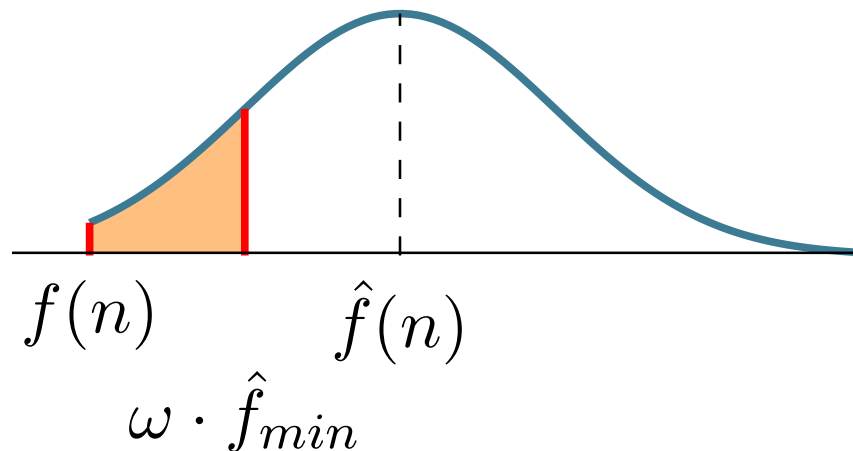
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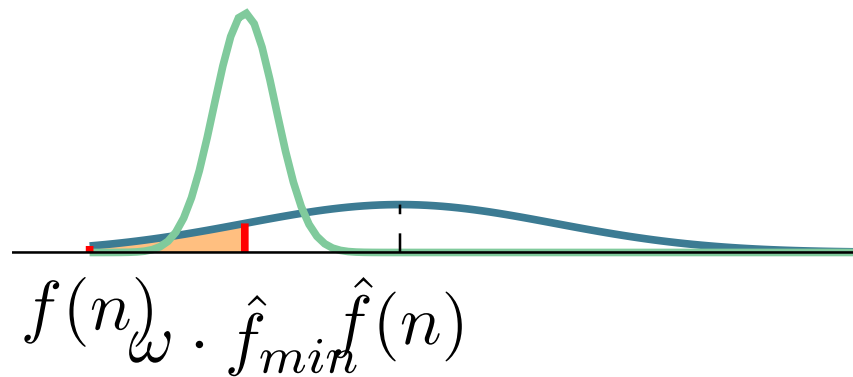
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# Our Approach: 2/2 A Round-Robin Scheme

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Round-Robin on:

**focal list:** sorted by  $d(\text{EES})$  or  $ud(\text{DPS})$  or  $xe(\text{DXES})$

**open list:** sorted by  $\hat{f}$

**cleanup list:** sorted by  $f$

focal and open condition:  $f(n) < \omega \cdot f_{min}$

Simple but work well empirically!



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## Planning Domains:

- Implementation in Fast Downward<sup>2</sup>
- Benchmarks:  
IPS optimal tracks (48 domains)

## Search Domains:

- Sliding-Tile Puzzle, Vacuum World, Pancake, Racetrack

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<sup>2</sup>Helmert 2006.

# IPC Coverage ( $\omega = 1.5$ )

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Coverage	WA*	EES	DPS	DXES	RR- <i>d</i>	RR-DPS	RR-DXES
<b>Sum (1652)</b>	995	967	1012	894	1025	982	<b>1052</b>
Normalized(%)	58.7	57.0	60.0	51.5	60.7	57.9	<b>62.5</b>
Expansions	569	558	472	734	383	665	<b>371</b>

→ RR-DXES and RR-*d* perform best overall.

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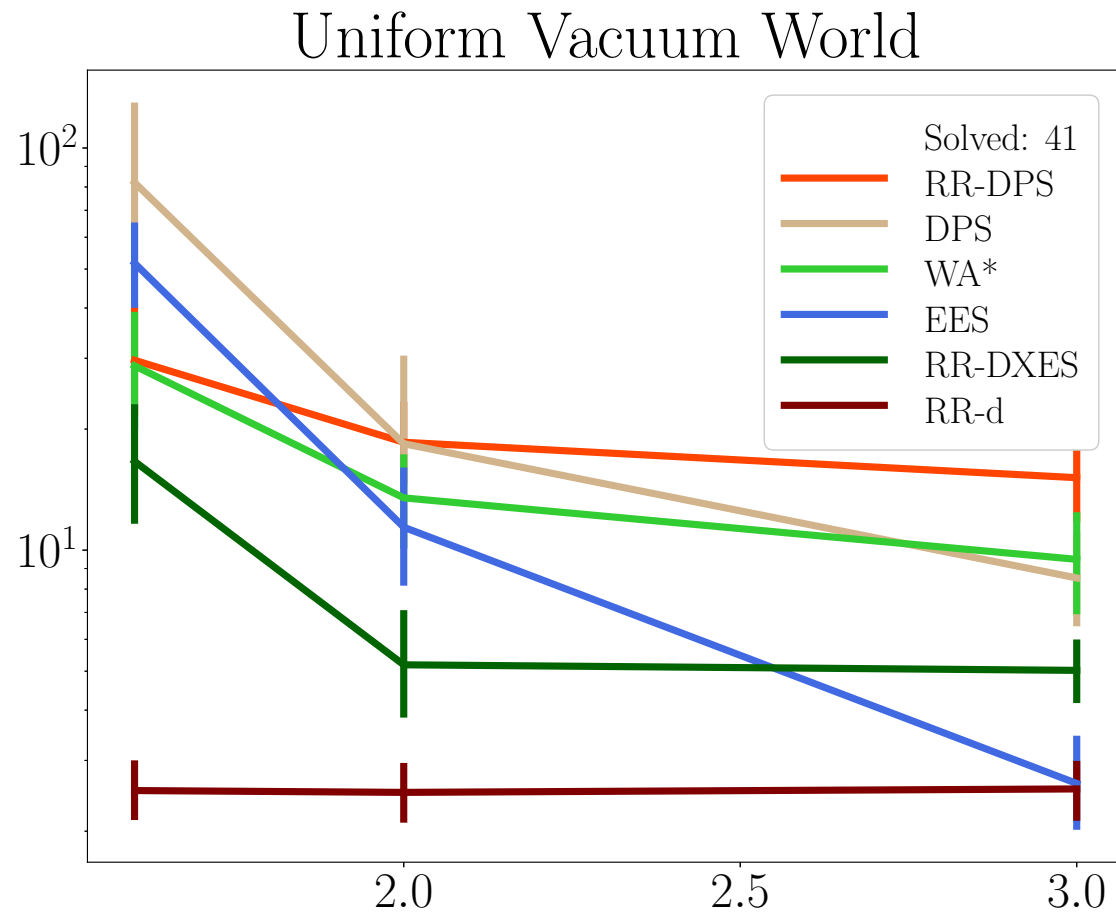
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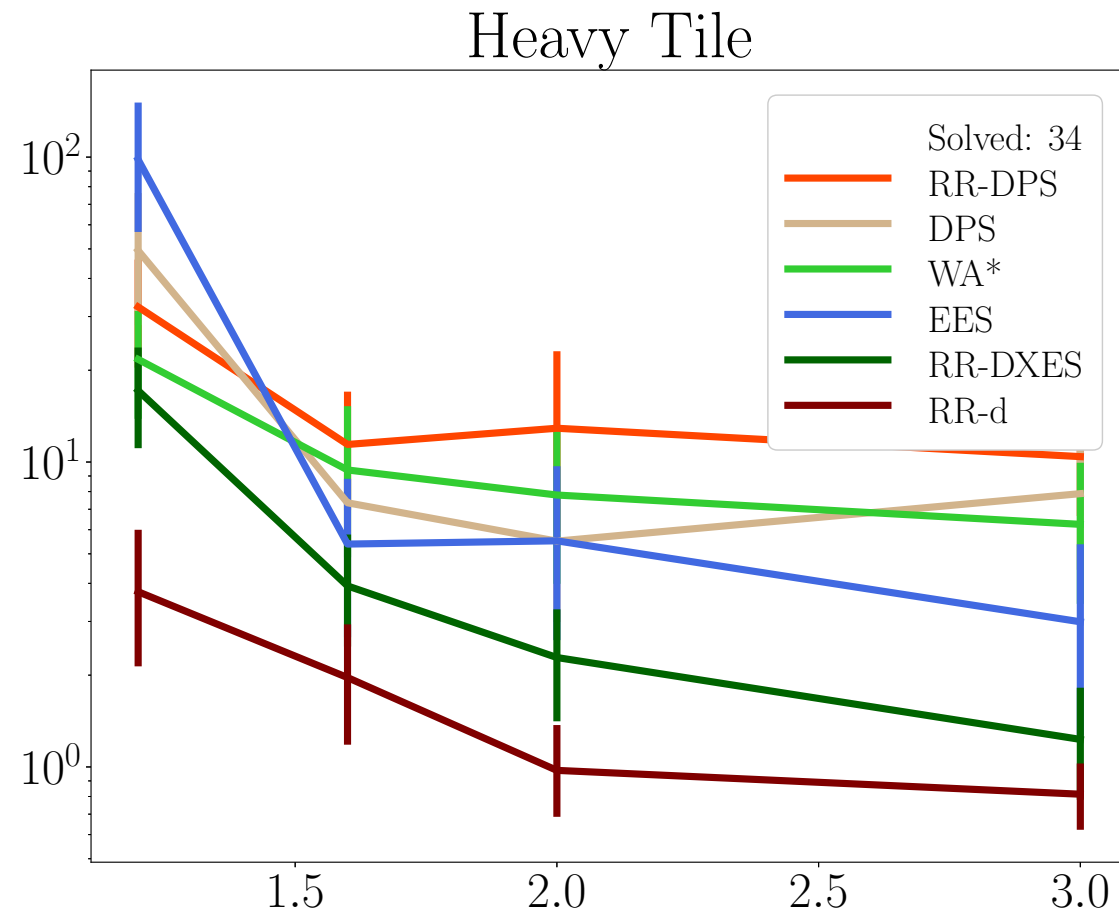
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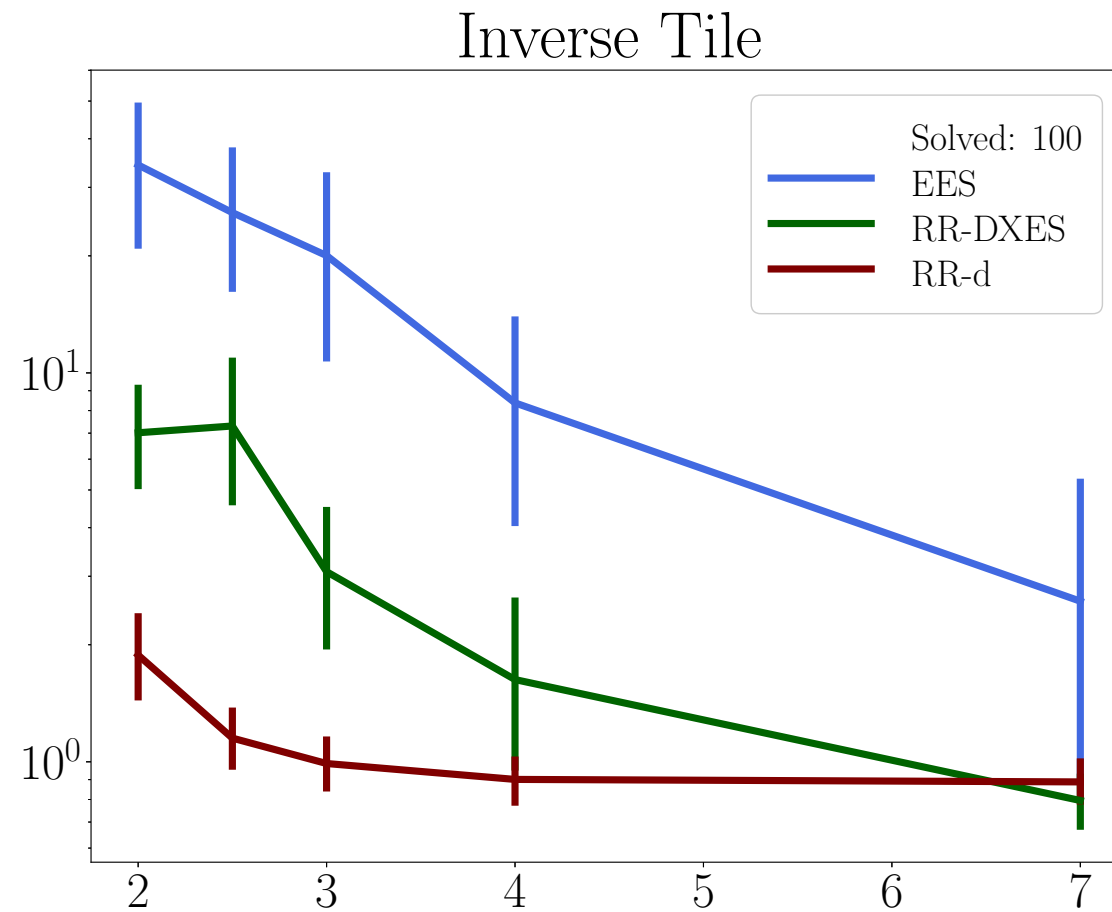
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What to do for bounded-suboptimal search:

- Weighted-A\* is the first thing to try
- Round-Robin on  $d$ ,  $\hat{f}$ ,  $f$  is the next to try



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- Weighted-A\* is the first thing to try
- Round-Robin on  $d$ ,  $\hat{f}$ ,  $f$  is the next to try

What's next:

- When to raise bound, and when to pursue solution?
- DXES should help, but not quite, why?

# Questions?

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