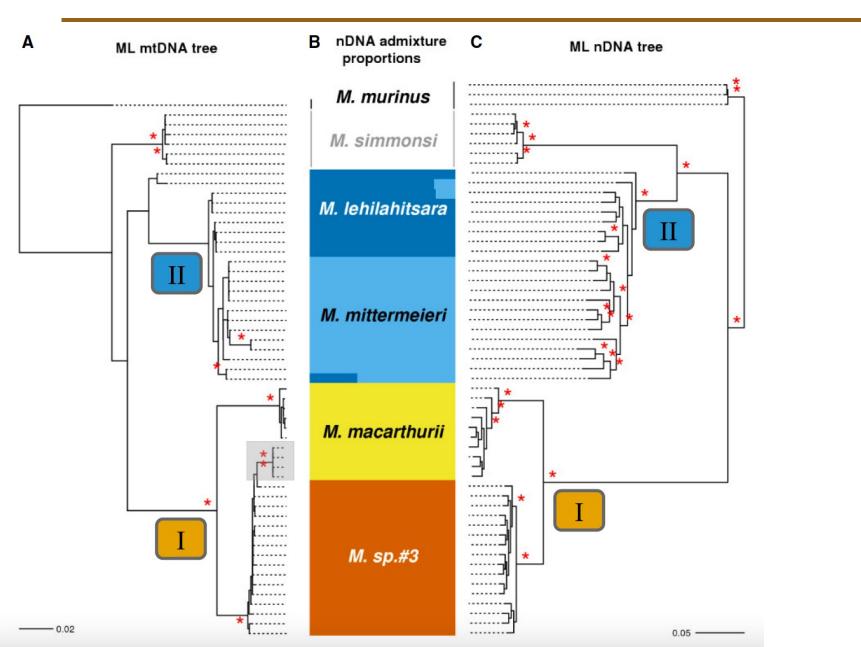
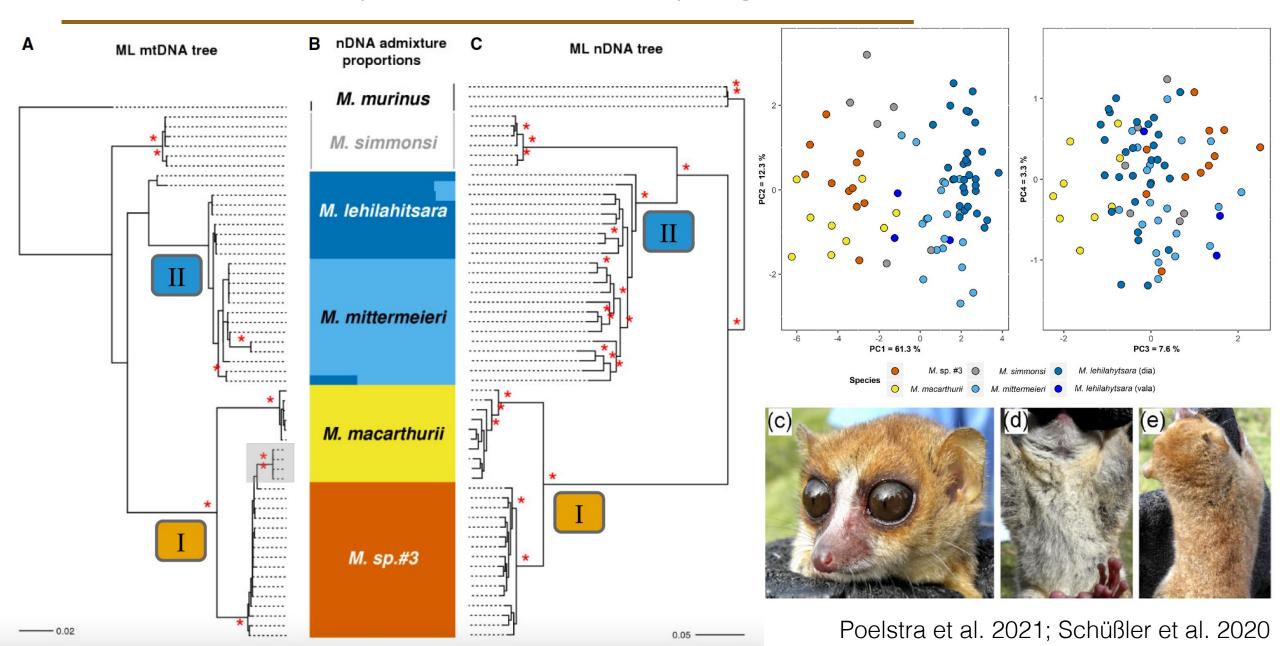
Phylogenetics: A Maximum Likelihood Approach

George P. Tiley
University of Antananarivo
DBEV Phylogenomics Workshop
7 March 2022

Motivation: Why Phylogenies?



Motivation: Why Molecular Phylogenies?



Motivation: Why Molecular Phylogenies

Valuable for taxonomy

Informative about trait evolution

Used for divergence time estimation

Understand biogeographic patterns

Many More!

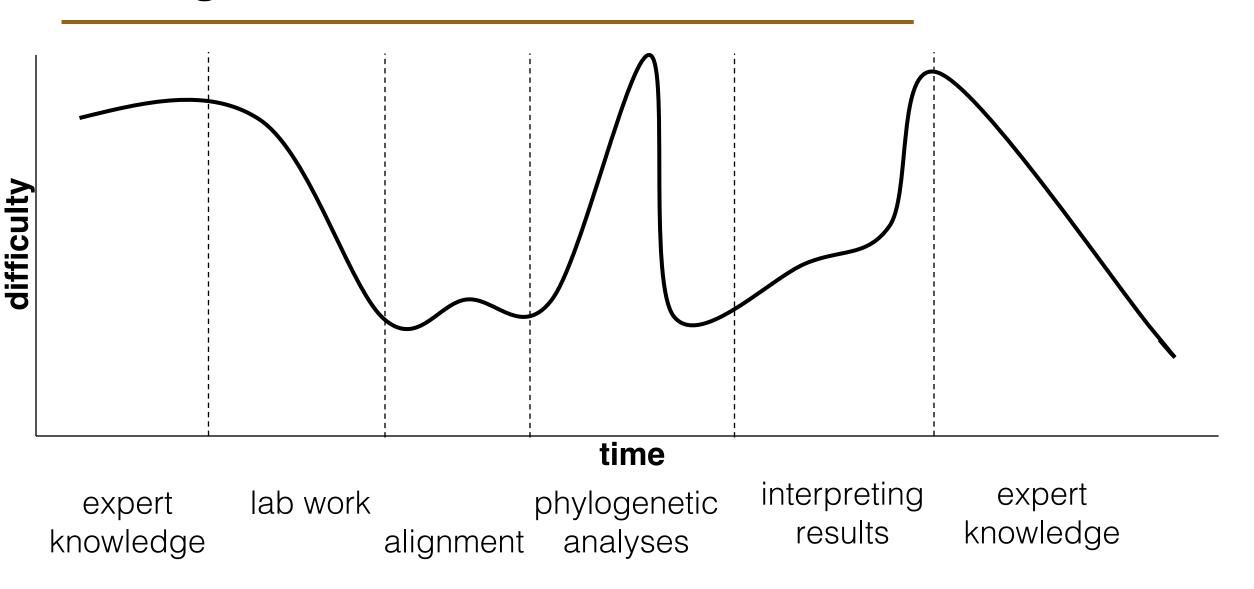
Explain terminology

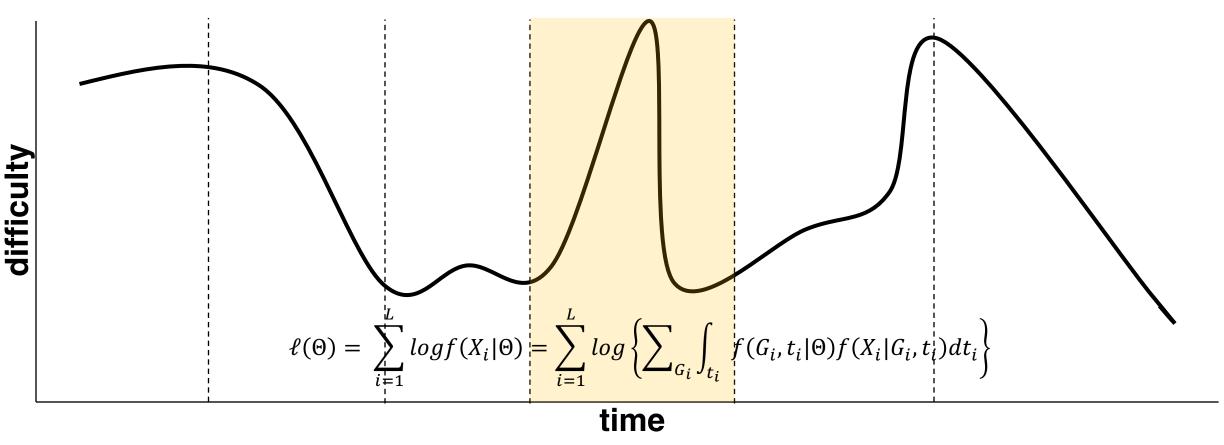
Primer on probability and likelihood

Models of molecular evolution

How to select a model

Application of models for phylogenetic estimation





expert knowledge lab work

alignment

phylogenetic analyses

interpreting results

expert knowledge

Explain terminology

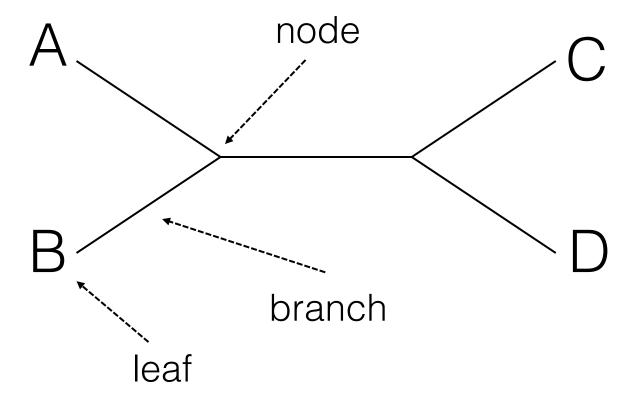
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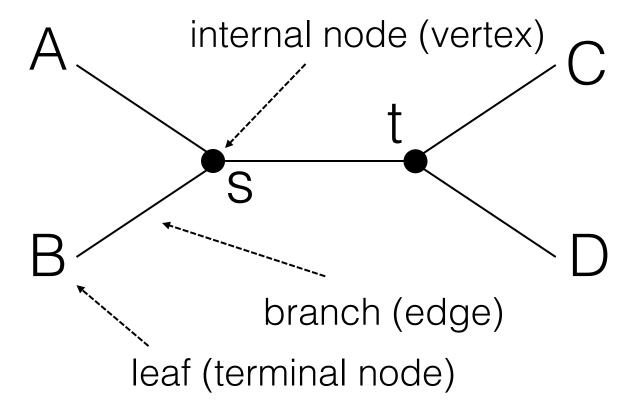
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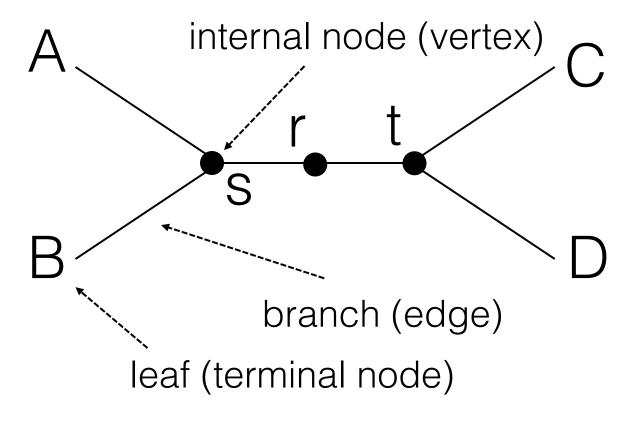
Unrooted Tree



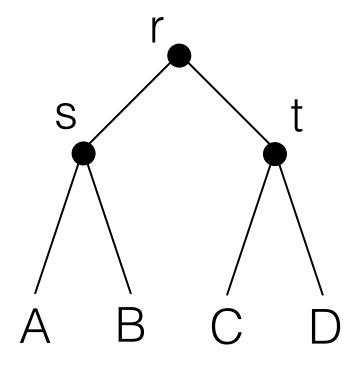
Unrooted Tree



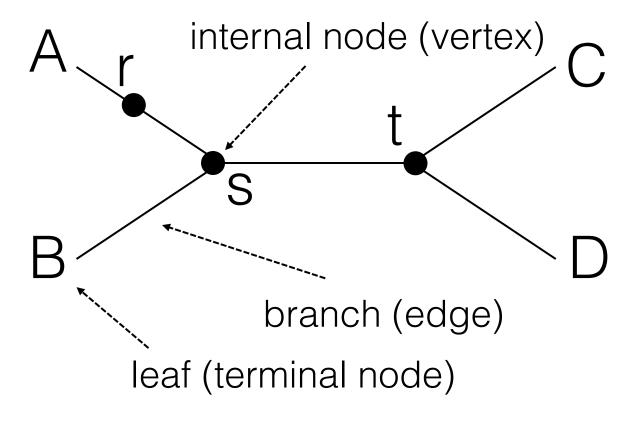
Unrooted Tree



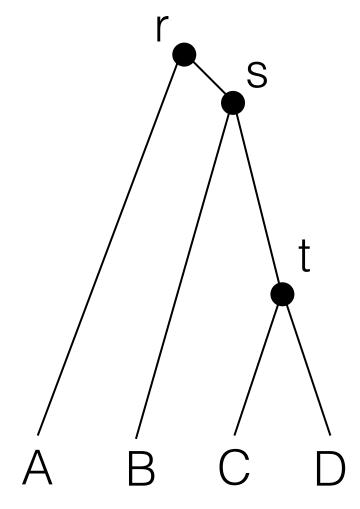
Rooted Tree



Unrooted Tree



Rooted Tree



Explain terminology

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Models of molecular evolution

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Rules of probability

Combining multiple *independent* events

AND OR (x) (+)

Rules of probability

Combining multiple *independent* events

AND (x)

Probability I roll a 1 and 2?

OR (+)

Probability I roll a 1 or 2?

Rules of probability

Combining multiple independent events

AND (x)

Probability I roll a 1 and 2?

OR (+)

Probability I roll a 1 or 2?

 $Pr\{1 \text{ and } 2\} = 1/10 \times 1/10 = 1/100$

 $Pr{1 \text{ or } 2} = 1/10 + 1/10 = 2/10 = 1/5$

Difference between probability and likelihood?

Consider I rolled a 2 and 1 and 1 and 4. Is this a surprising result?

 $Pr\{2,1,1,4 \mid 10\text{-sided die}\} = 1/10 \times 1/10 \times 1/10 \times 1/10 = 1/10000$

Difference between probability and likelihood?

Consider I rolled a 2 and 1 and 1 and 4. Is this a surprising result?

 $Pr\{2,1,1,4 \mid 10\text{-sided die}\} = 1/10 \times 1/10 \times 1/10 \times 1/10 = 1/10000$

Maybe I used a 4-sided die. Is this less surprising?

 $Pr\{2,1,1,4 \mid 4\text{-sided die}\} = 1/4 \times 1/4 \times 1/4 \times 1/4 = 1/256$

Difference between probability and likelihood?

Consider I rolled a 2 and 1 and 1 and 4. Is this a surprising result?

$$Pr\{2,1,1,4 \mid 10\text{-sided die}\} = 1/10 \times 1/10 \times 1/10 \times 1/10 = 1/10000$$

Maybe I used a 4-sided die. Is this less surprising?

$$Pr\{2,1,1,4 \mid \frac{4-\text{sided die}}{4-\text{sided die}}\} = \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} = \frac{1}{256}$$

These are models!

Likelihood – probability of observed data with respect to a particular model

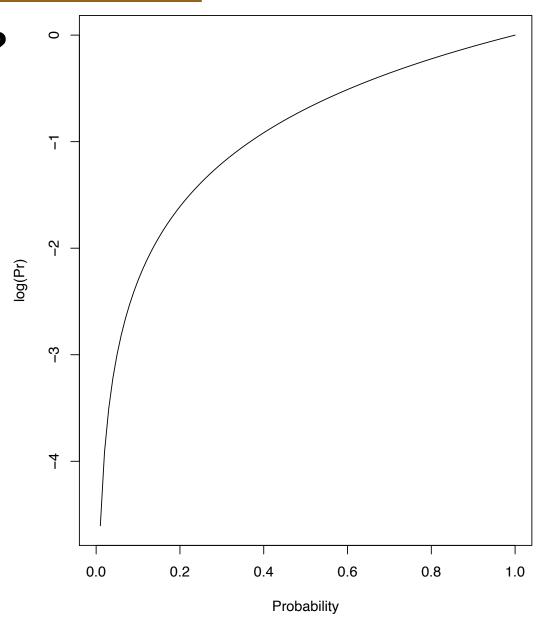
Likelihood(4-sided die) = $Pr\{2,1,1,4 \mid 4\text{-sided die}\} = \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4}$

Likelihood(4-sided die) = $f(X|4 - sided die) = \prod_{h=1}^{n} f(x_h|4 - sided die)$

Likelihood(θ) = $f(X|\theta) = \prod_{h=1}^{n} f(x_h|\theta)$

$$L(\theta) = f(X|\theta) = \prod_{h=1}^{n} f(x_h|\theta)$$

Likelihood – why do we always see *log*?

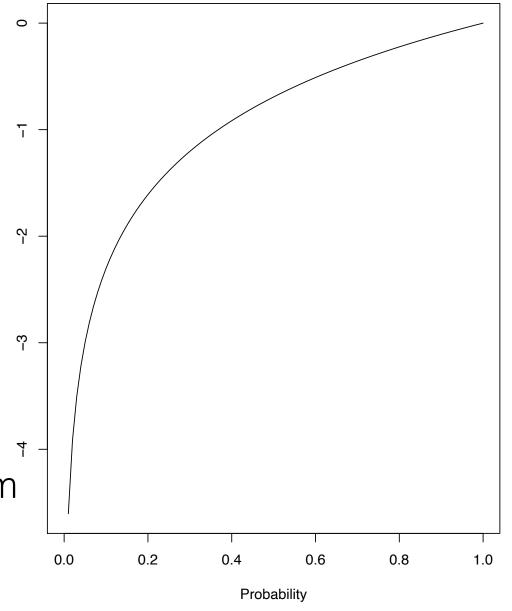


Likelihood – why do we always see *log*?

Many probabilities we calculate with phylogenies will be very small

Computers do not store small numbers accurately, usually to 16 decimal places

Thus we always see log(L) from a program where $log(x) = ln(x) = 1/e^x$



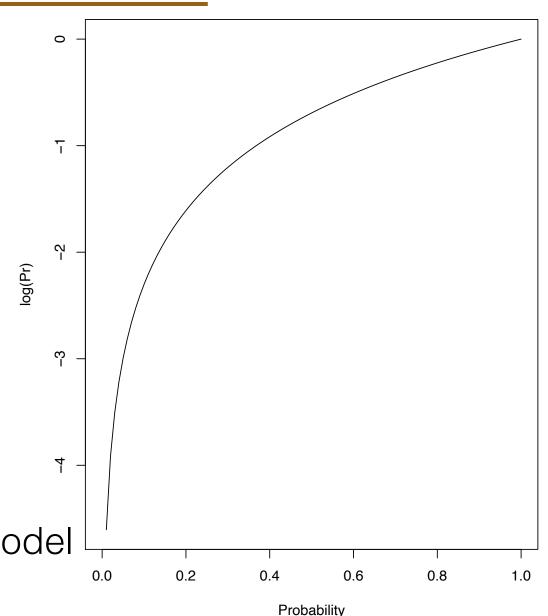
Likelihood – logs have rules

$$log(x \times y) = log(x) + \log(y)$$

 $Pr{2,1,1,4 \mid 10\text{-sided die}} = 1/10000$ $log(Pr{2,1,1,4 \mid 10\text{-sided die}}) = -9.21$

 $Pr{2,1,1,4 \mid 4\text{-sided die}} = 1/256$ $log(Pr{2,1,1,4 \mid 4\text{-sided die}}) = -5.55$

The *log-likelihood* is *maximized* for the model that surprises us the least



Likelihood – probability of observed data with respect to a particular model

Likelihood(4-sided die) = $Pr\{2,1,1,4 \mid 4\text{-sided die}\} = \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4}$

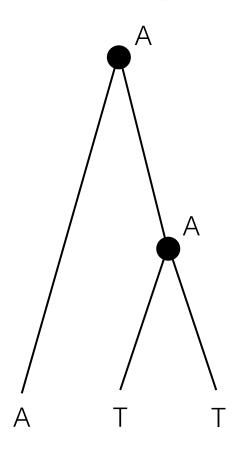
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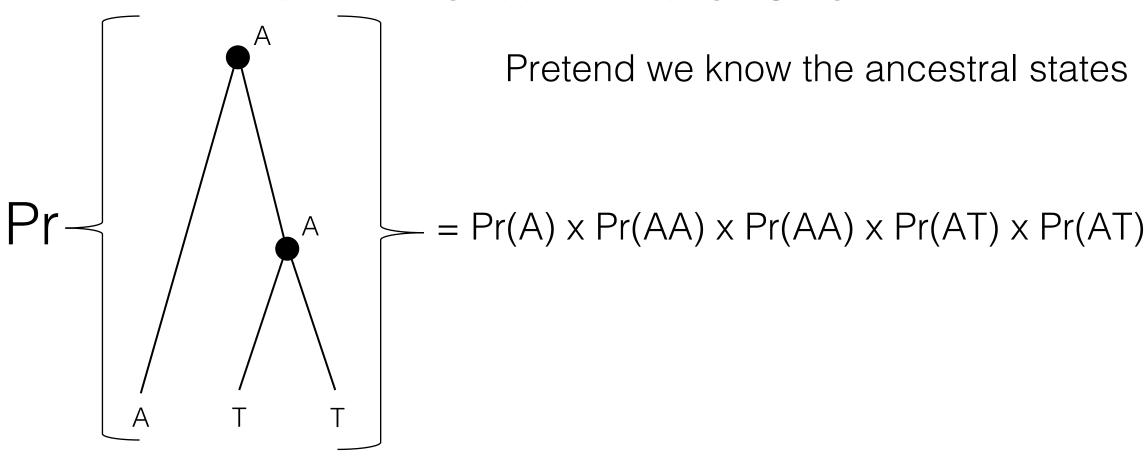
$$L(\theta) = f(X|\theta) = \prod_{h=1}^{n} f(x_h|\theta)$$

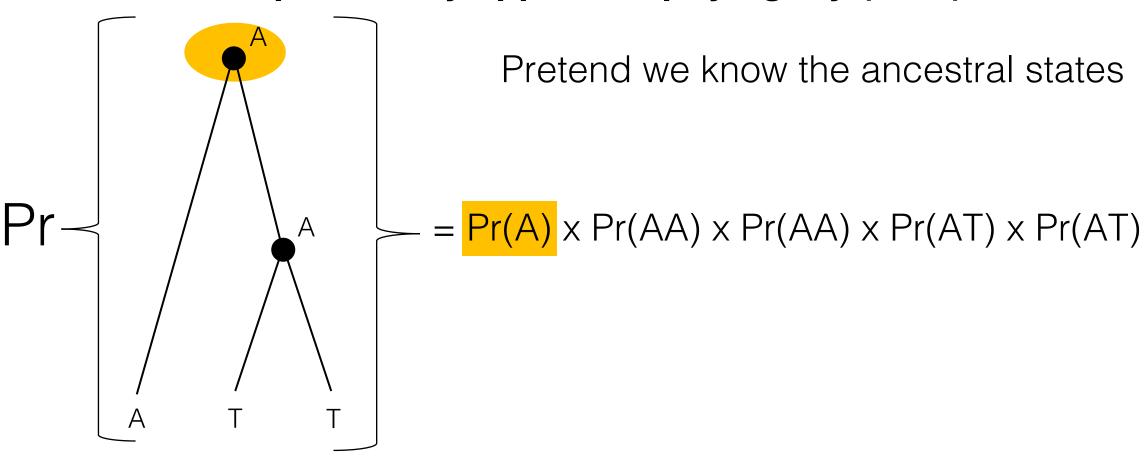
$$\ell = \log\{L(\theta)\} = \sum_{h=1}^{n} \log\{f(x_h|\theta)\}$$

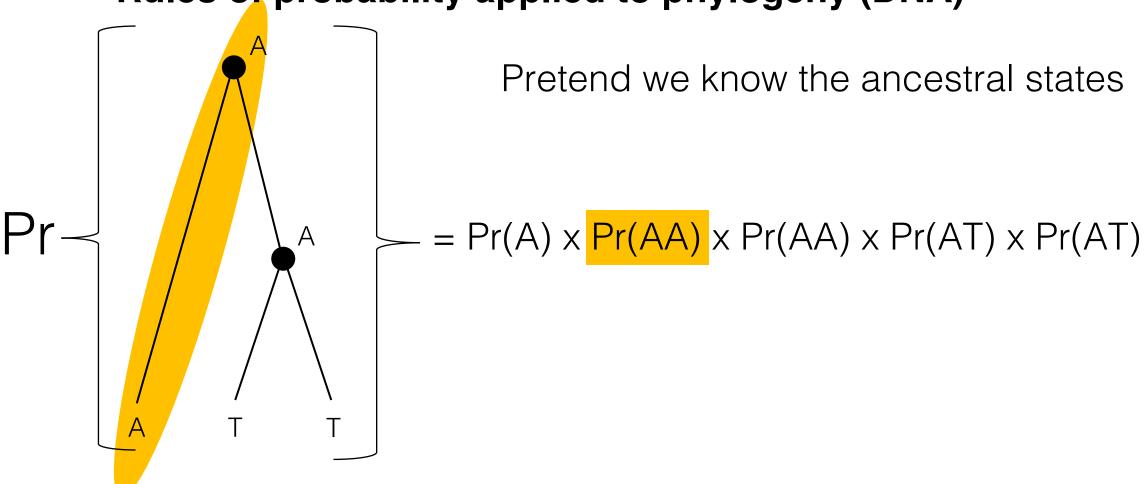
Rules of probability applied to phylogeny (DNA)

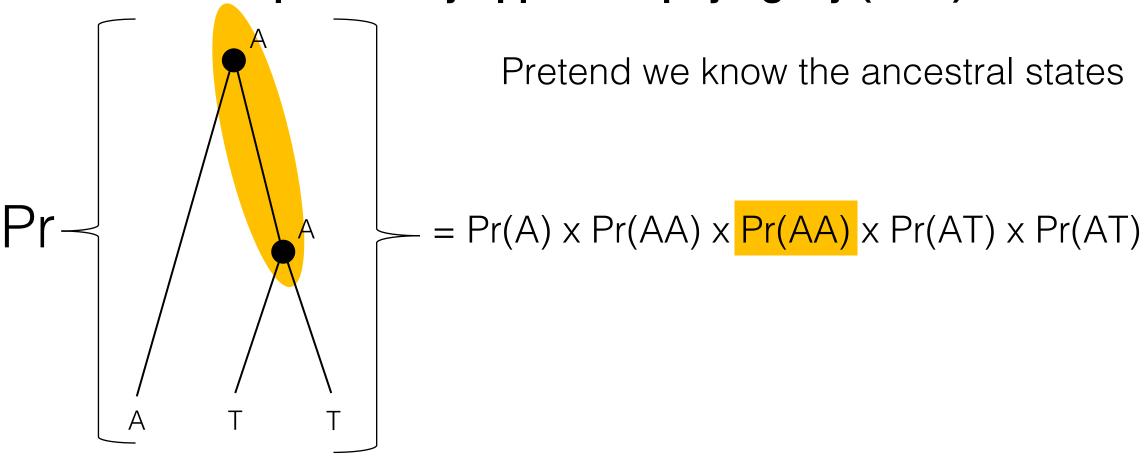


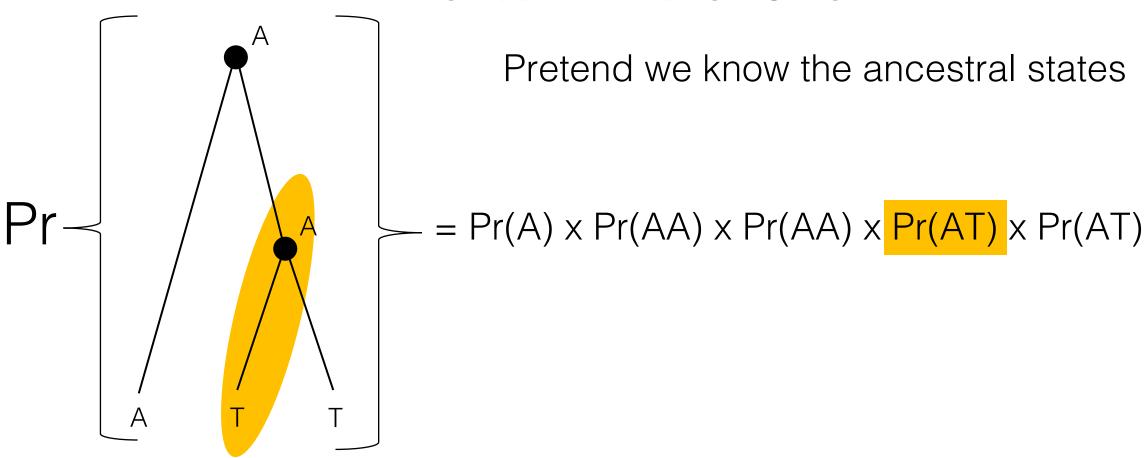
Pretend we know the ancestral states

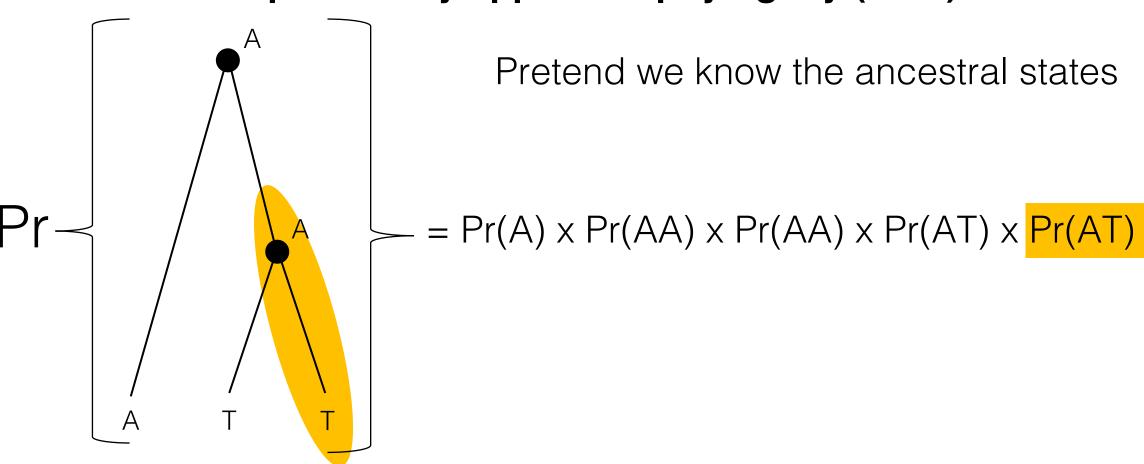


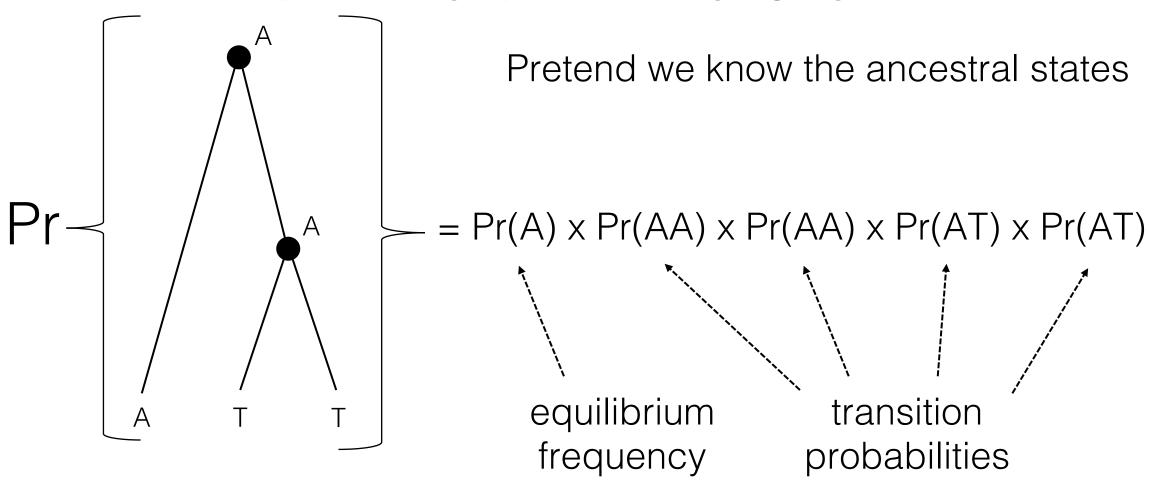


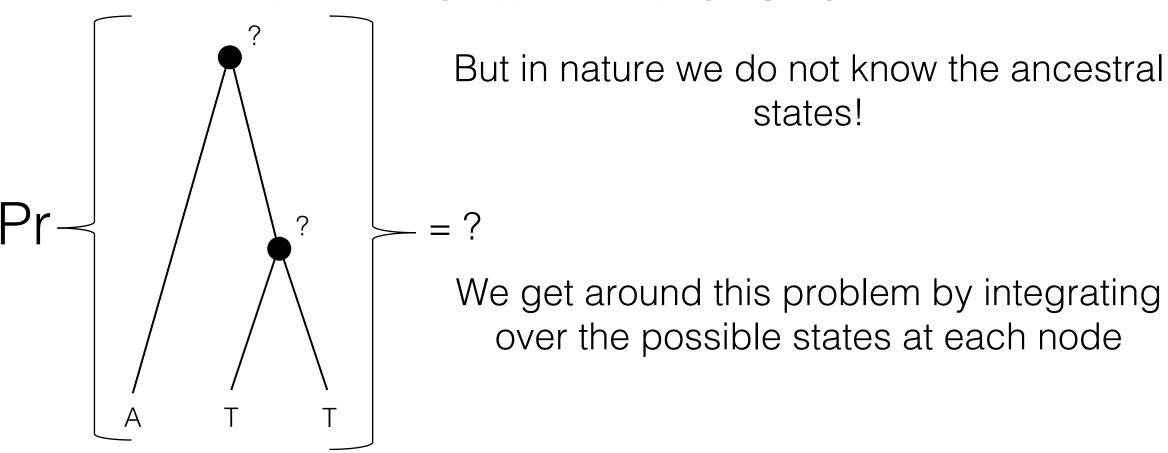


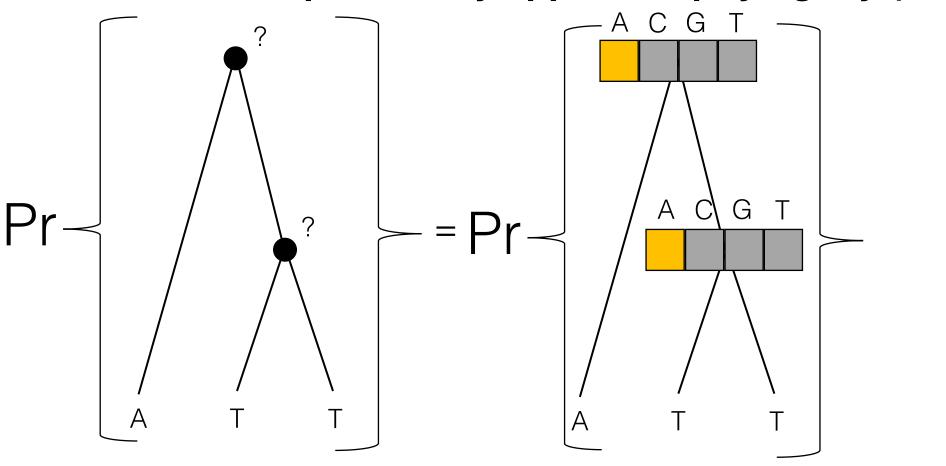


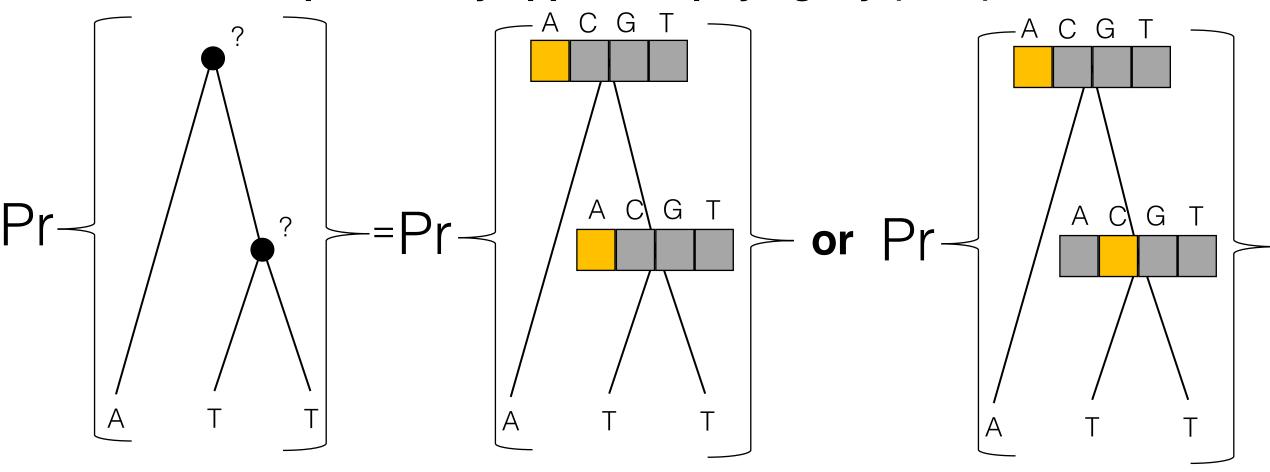


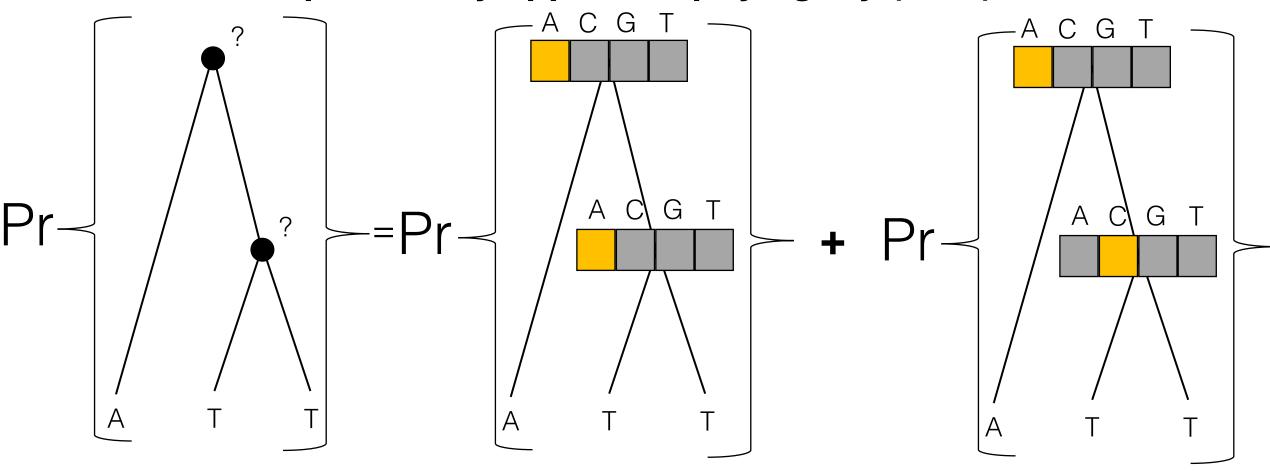


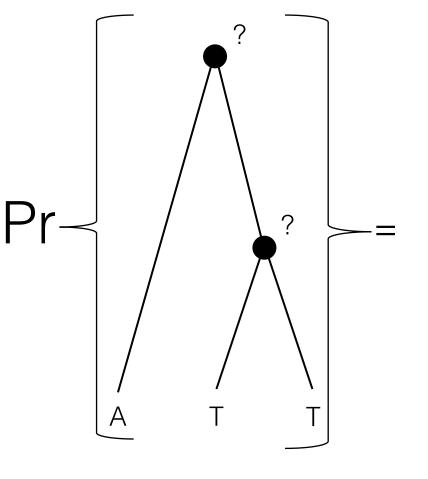


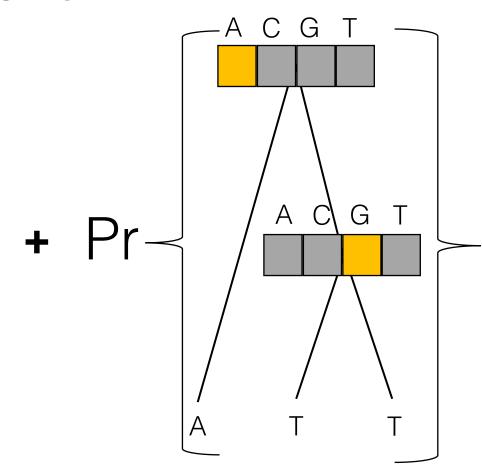


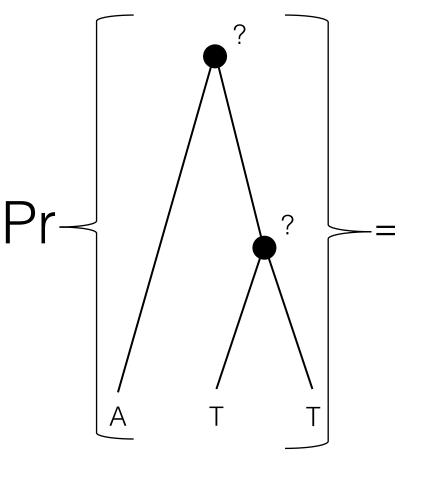


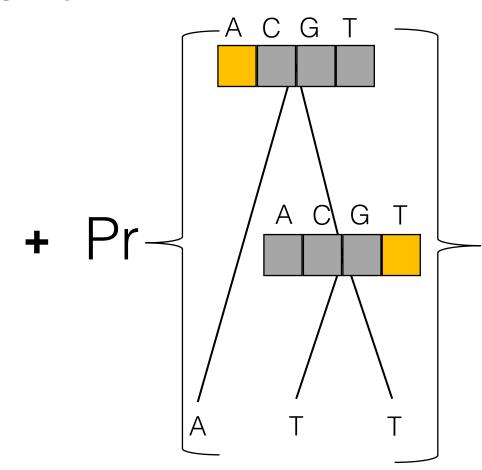


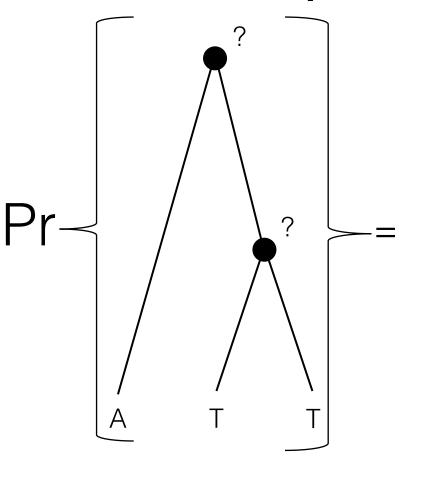


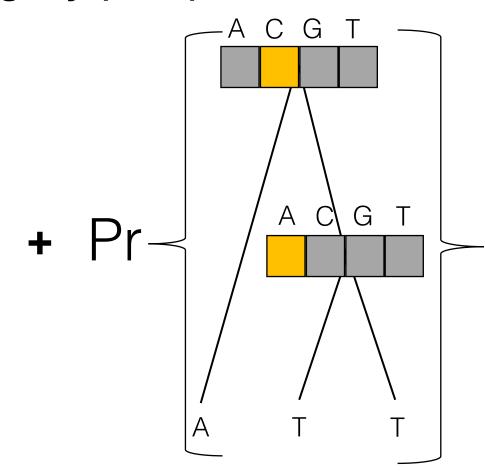


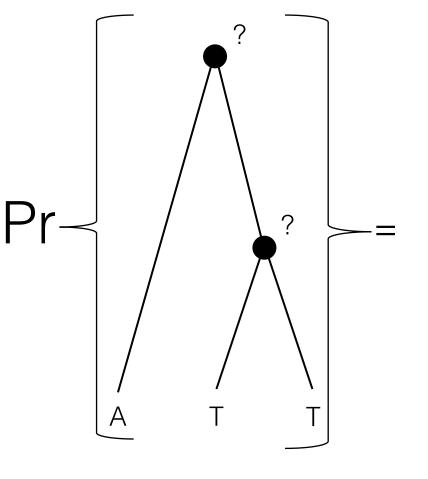


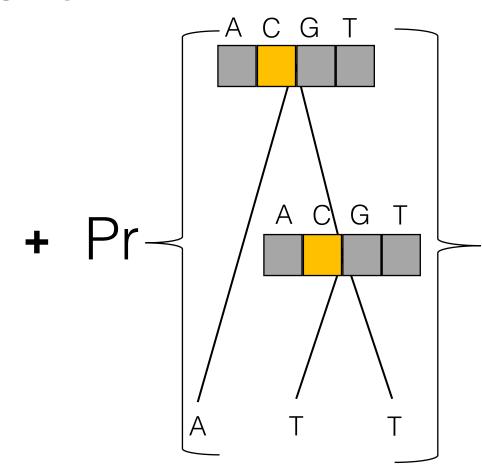


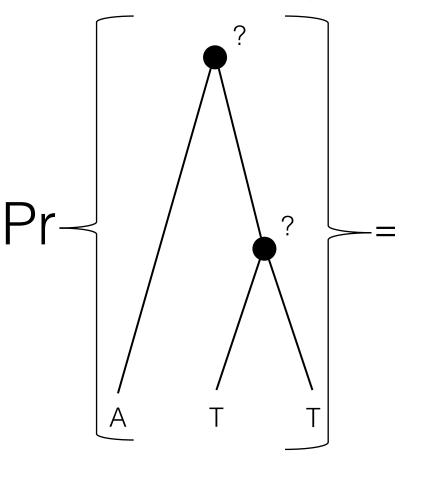


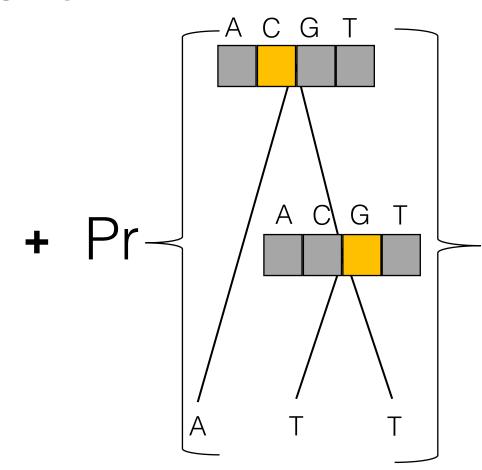


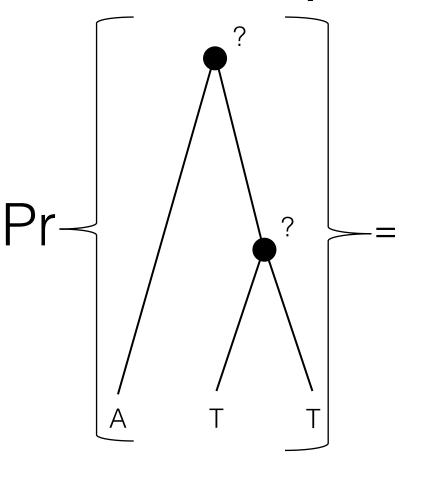


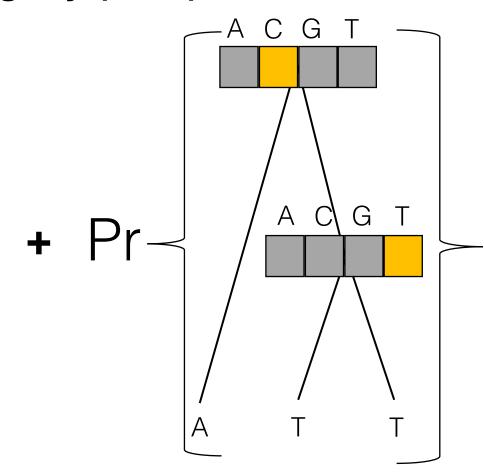


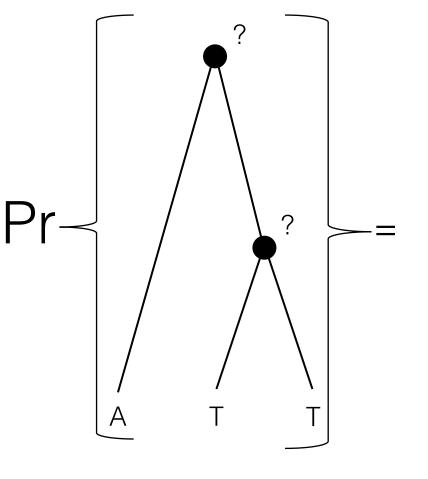


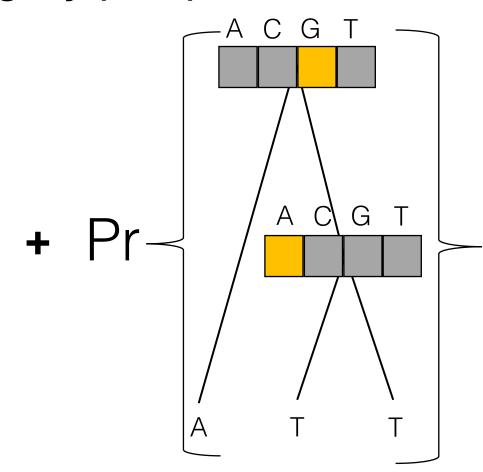


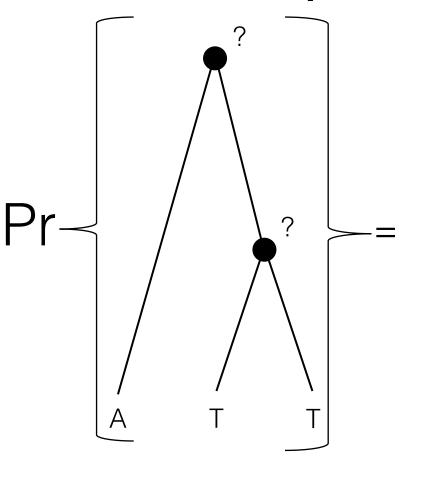


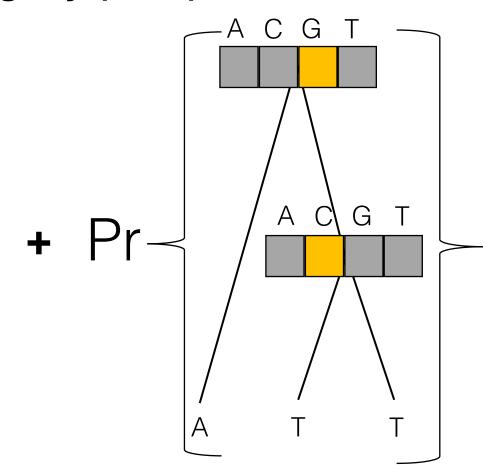


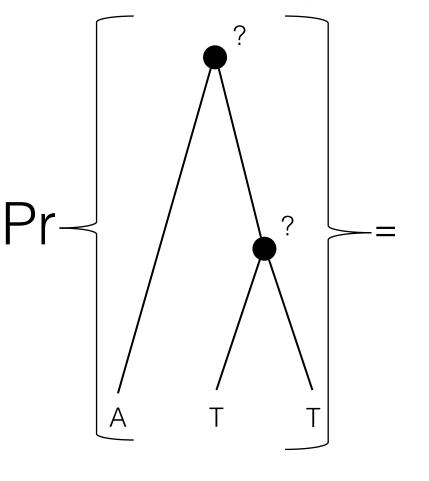


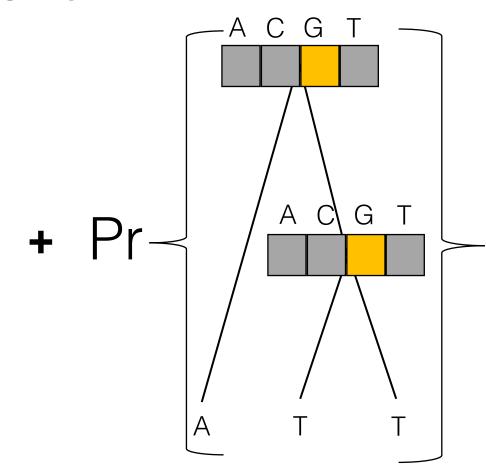


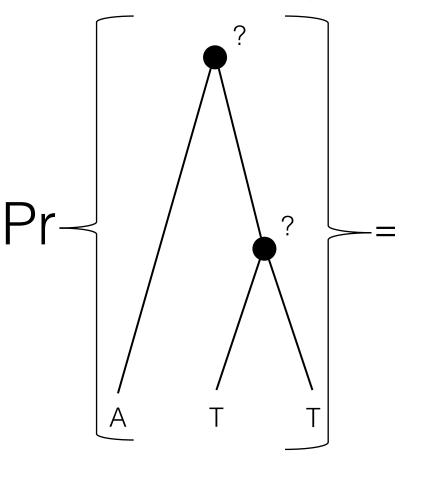


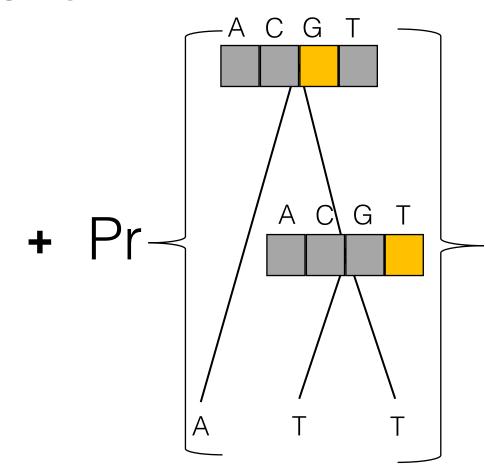


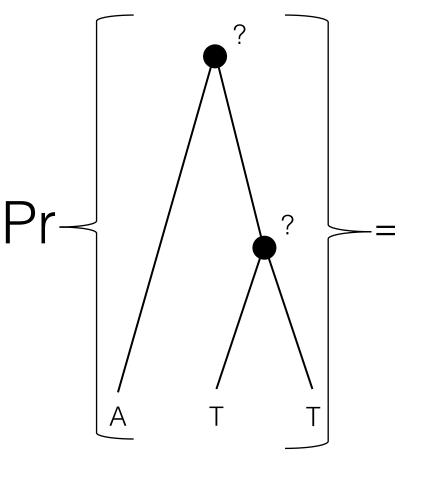


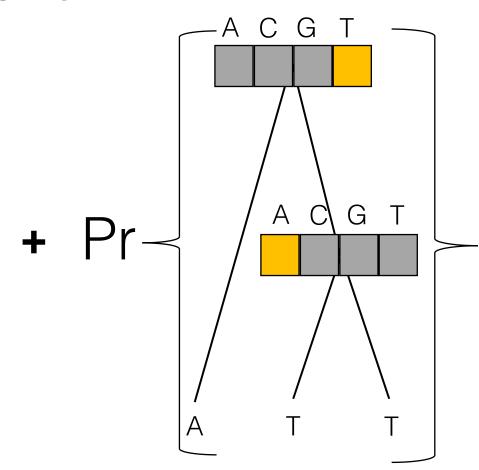


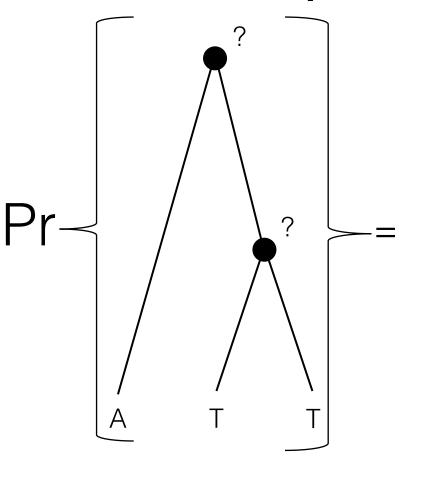


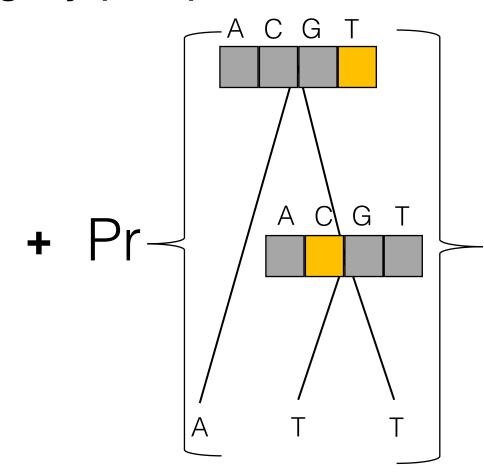


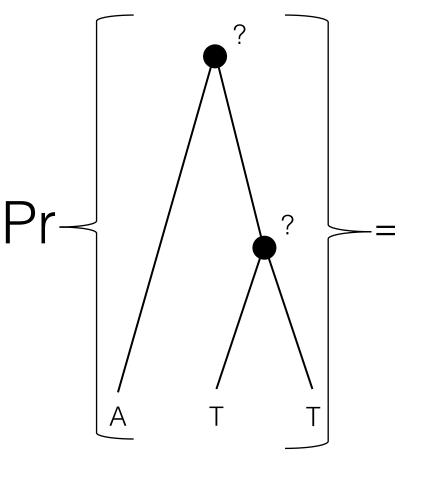


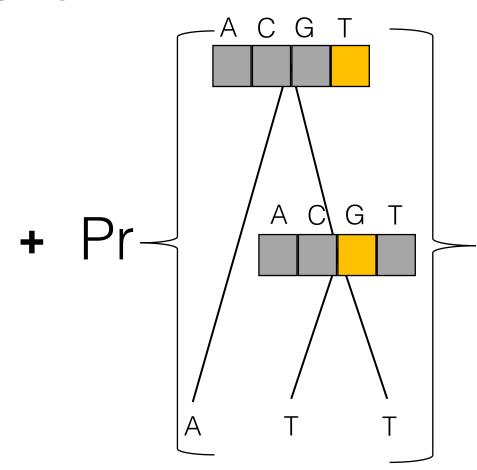


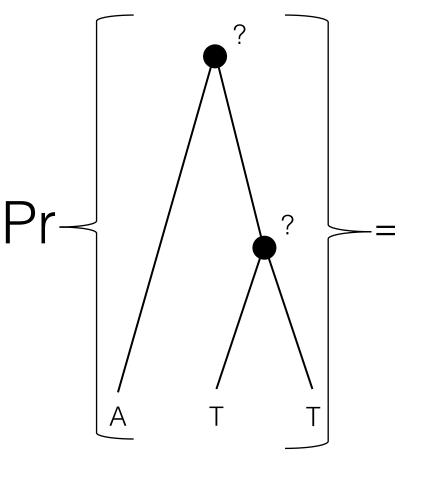


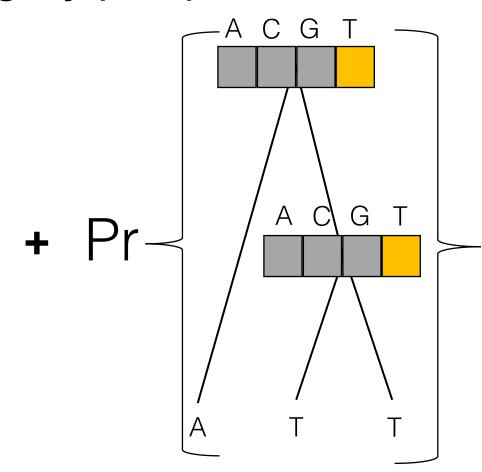


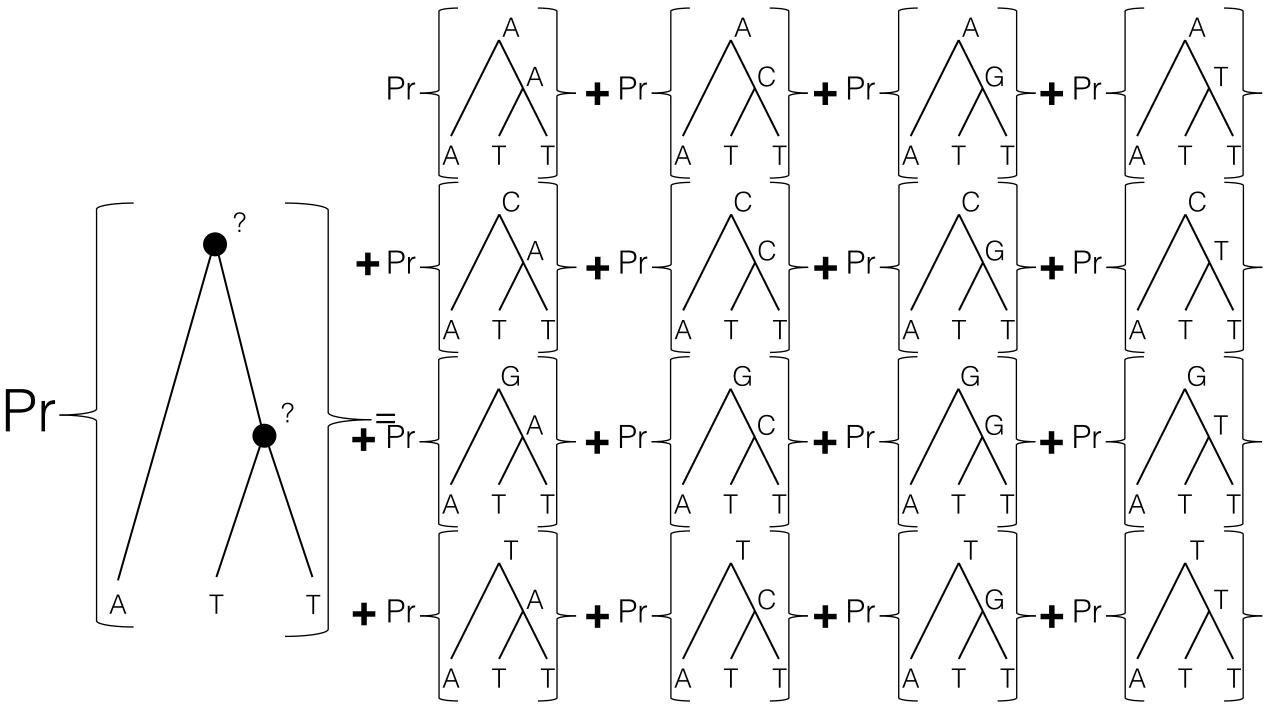


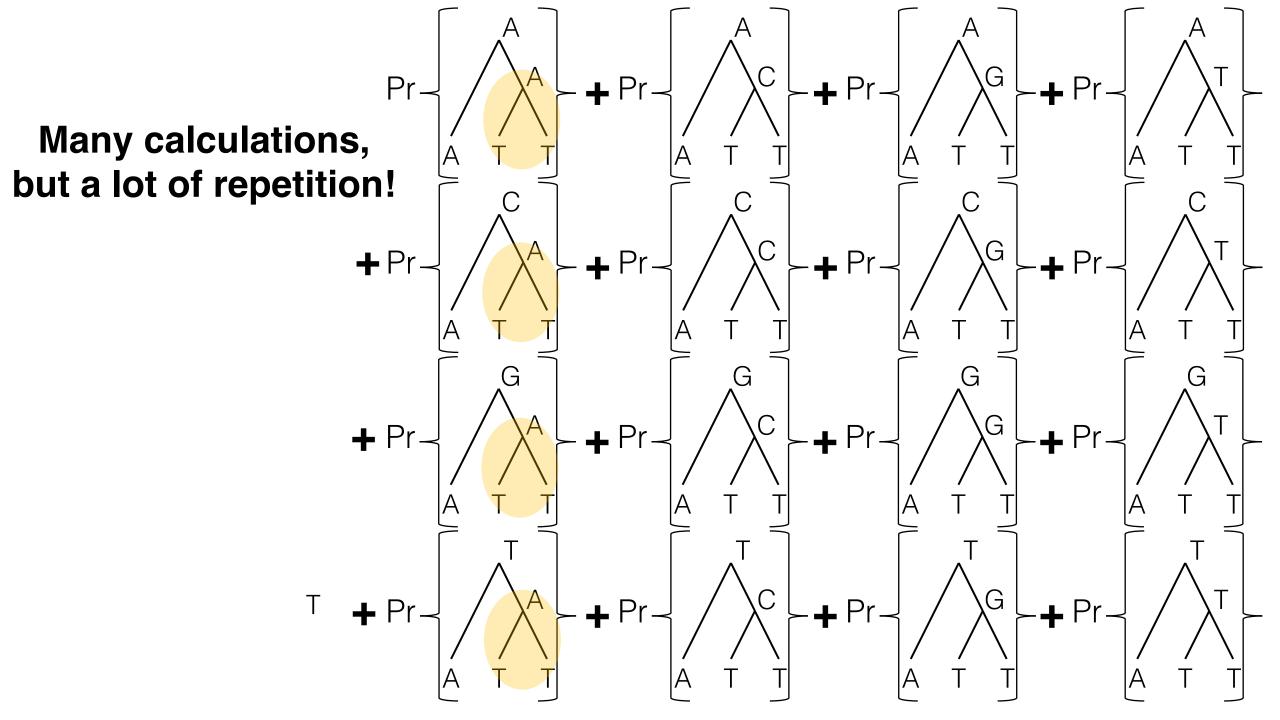


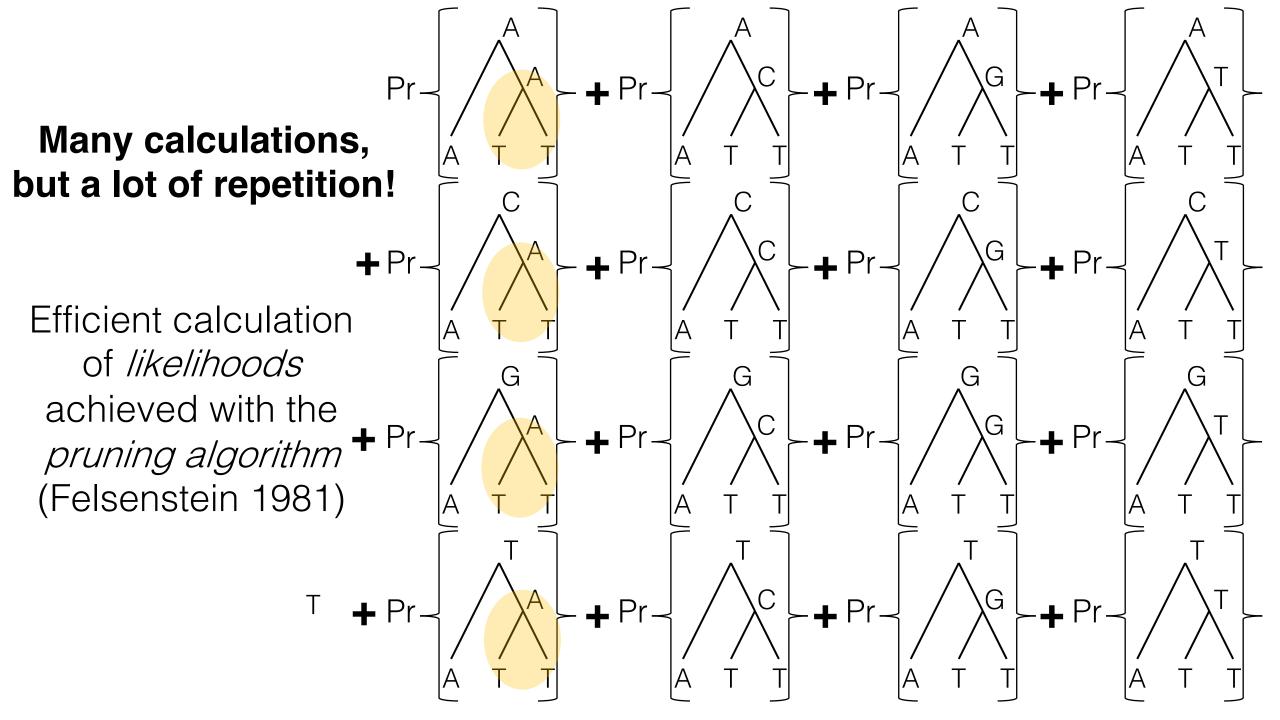


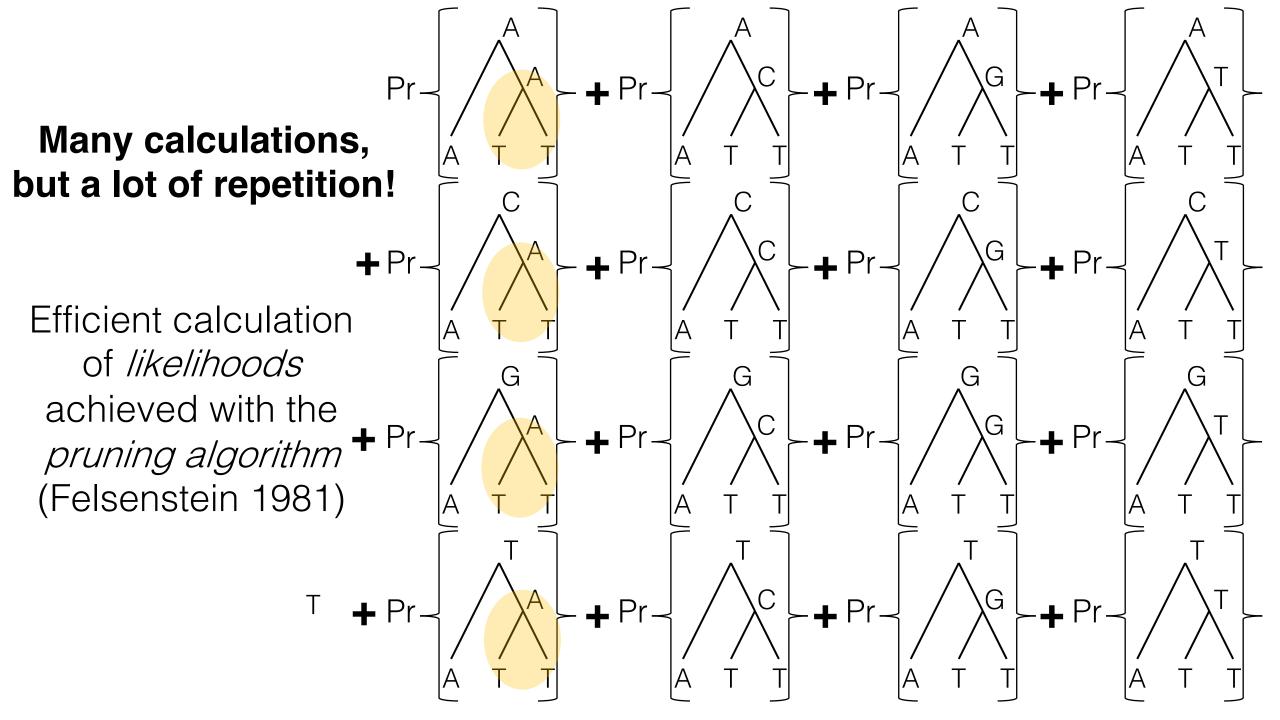


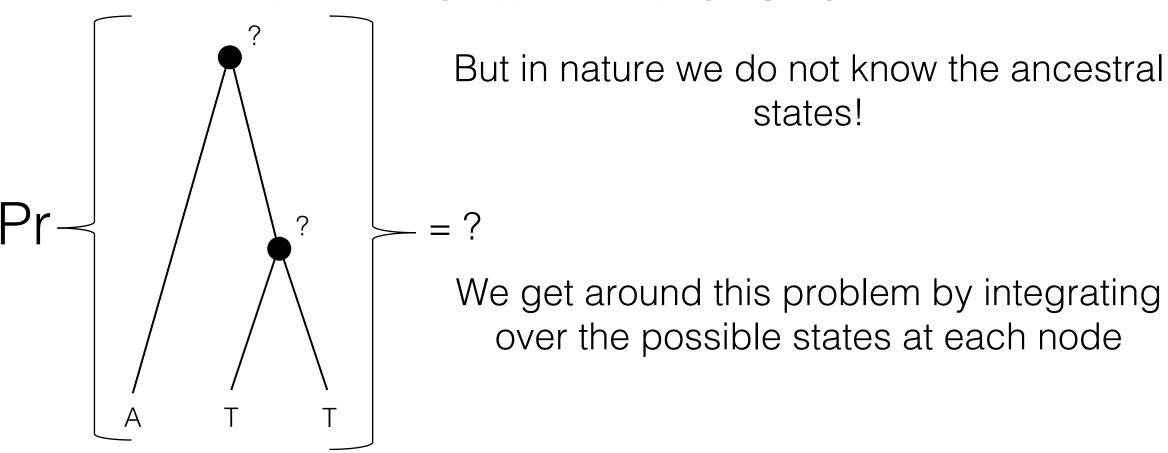












Learning Goals

Explain terminology

Primer on probability and likelihood

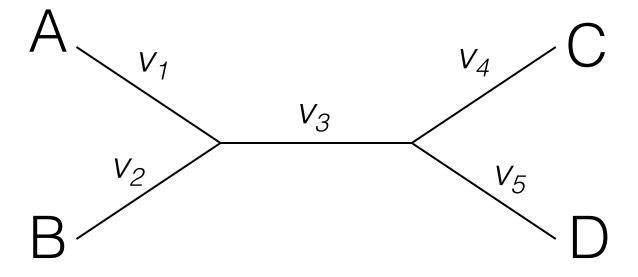
Models of molecular evolution

How to select a model

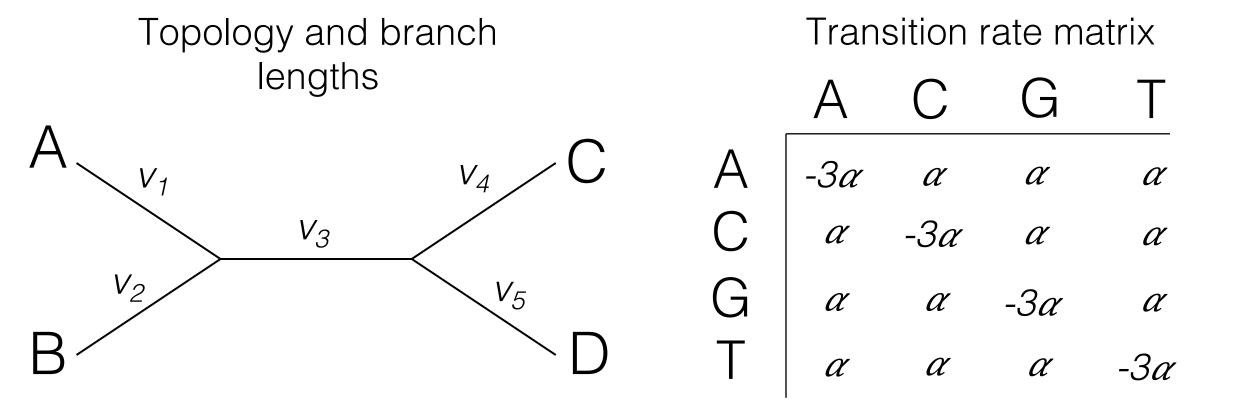
Application of models for phylogenetic estimation

We discussed the general likelihood function for a tree, but where do the conditional probabilities come from?

Topology and branch lengths

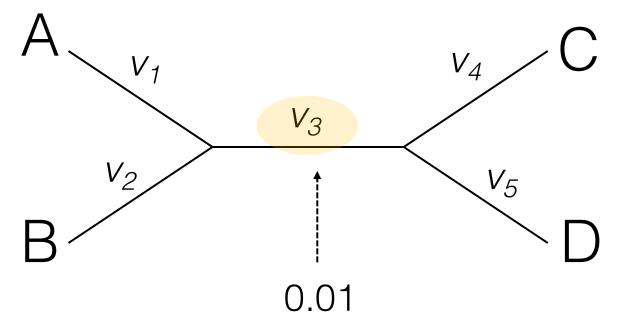


 v_i — length of branch *i* measured in **expected number of** substitutions per site



 v_i length of branch *i* measured in **expected number of** substitutions per site

Branch lengths (evolutionary distance) – the confounded measurement of rate and time



We expect 1 substitution for every 100 sites

Branch lengths = evolutionary distance

- These are the confounded measurement of rate and time

Consider a genome of 10,000bp

$$\frac{1 \, substitution}{1 \, million \, years} \times 100 \, million \, years = \frac{100 \, substitutions}{10,000 bp} = 0.01$$

$$\frac{10 \text{ substitutions}}{1 \text{ million years}} \times 10 \text{ million years} = \frac{100 \text{ substitutions}}{10,000bp} = 0.01$$

Understanding absolute time requires *clock models*

Transition rate matrix = describes how one base changes into another per unit of evolutionary distance *t*

corrects for *multiple hits*

Time-reversible $C \rightarrow G = G \rightarrow C$

Markov model

what happens in between the start and end state does not matter

	Α	C	G	Τ
A	-3α	α -3α	α	α
C	α	-3α	α	α
G	α	α	-3α	α
Τ	α	α	α	-3α

Let's derive the *transition*probabilities using the easiest rate
matrix, Juke and Cantor (1969)

	A	C	G	Т
Α	-3a	α	α	α
C	α	-3α	α	α
G	α	α	-3α	α
Т	α	α	α	-3α

Let's derive the *transition*probabilities using the easiest model, Juke and Cantor (1969) ------

The equilibrium frequencies (π_i) are probability of starting in state i $\pi_A = \pi_C = \pi_G = \pi_T = 0.25$

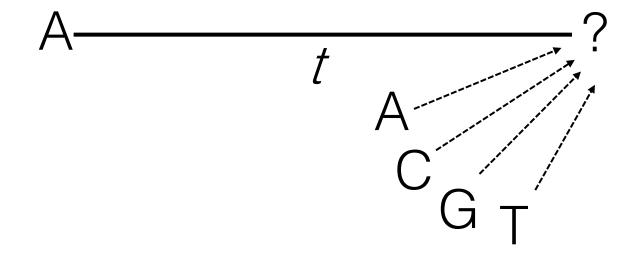
A is changing into C at rate α , G at rate α , and T at rate α . But remember, the process is reversible, so A stays an A with rate -3α

Α	C	G	Τ
-3α	α	α	α
α	-3α	α	α
α	α	-3α	α
α	α	α	-3α

Consider a single site that starts as an A

And some amount of time t passes

Site A can become anything at rate μ



Becuase rate μ represents *any* base change, a change from A to a specific base, α , is $\frac{1}{4}\mu$

We could rewrite as $\mu = 4\alpha$

The probability of something with rate μ over time t is given by the Poisson distribution!

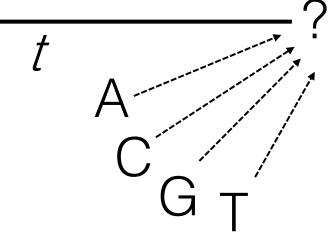
$$P(nothing\ happens) = e^{-\mu t}$$

$$P(at \ least \ one \ thing \ happens) = 1 - e^{-\mu t}$$

$$P(the\ thing\ that\ happened\ was\ a\ T) = \frac{1}{4}$$

$$P(end \ with \ T | \ start \ with \ A) = \frac{1}{4} \times (1 - e^{-\mu t})$$

$$= \frac{1}{4} \times (1 - e^{-4\alpha t})$$



remember! $\mu = 4\alpha$

What do all of the probability calculations look like?

$$P(end \ with \ T | \ start \ with \ A) = \frac{1}{4} \times (1 - e^{-4\alpha t})$$

$$P(end \ with \ G | \ start \ with \ A) = \frac{1}{4} \times (1 - e^{-4\alpha t})$$

$$P(end \ with \ C | \ start \ with \ A) = \frac{1}{4} \times (1 - e^{-4\alpha t})$$

$$P(end \ with \ A | \ start \ with \ A) = \frac{1}{4} \times (1 + 3e^{-4\alpha t})$$

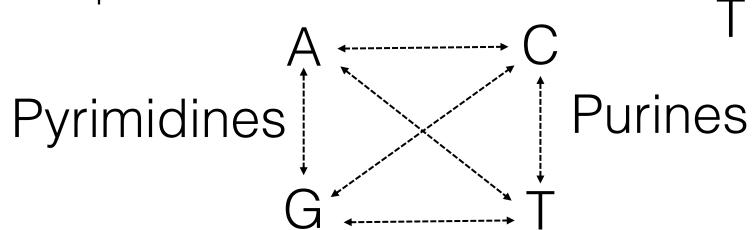
That -3α in our rate matrix ensures the probabilities sum to 1

 $4\alpha t$ is our branch length. We can now change the values of the model parameters to find the most likely tree!

Models can be more complex!

Some types of substitutions might occur at a different rate than others

Hasegawa, Kishino, and Yano (1985) assumed transitions would be more frequent than transversions



Α	С	G	Τ
_	$\pi_C eta$	$\pi_G eta \kappa$	$\pi_T eta$
$\pi_A eta$	_	$\pi_G eta$	$\pi_T eta \kappa$
$\pi_Aeta\kappa$	$\pi_C eta$	_	$\pi_T eta$
$\pi_A eta$	$\pi_C \beta \kappa$	$\pi_G eta$	_

Models can be more complex!

Maybe let all substitution types have their own rate and let the data decide?

GTR (Tavaré 1986)

A C G IA T_{C} π_{C} $\pi_{$

There are many more models in existence, but most are between JC69 and GTR.

People used to spend a lot of time trying to pick the best one for their data. Now, programs like RAxML always use GTR or IQTREE can a automate the model selection process for you.

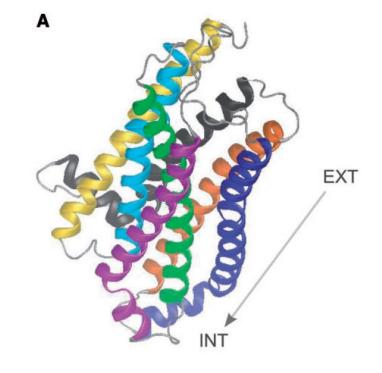
There are models for amino acids and codons too, which bring up additional theoretical complexities not covered here.

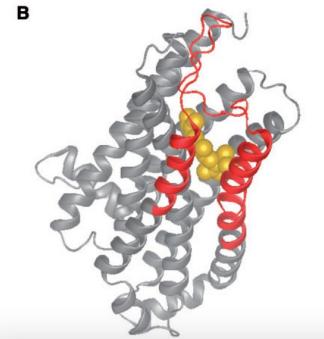
All models can incorporate rate variation though

Rate variation – some site evolve quickly and some evolve slowly

All models can incorporate rate variation though

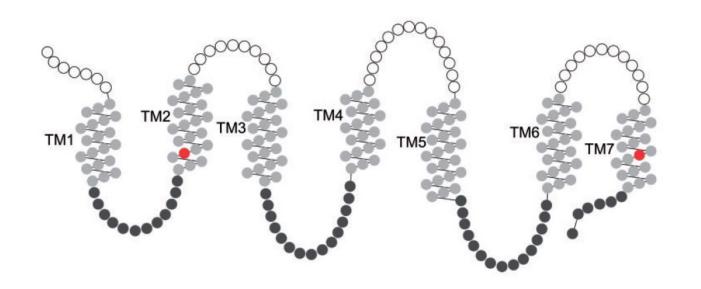
Rate variation – some site evolve quickly and some evolve slowly

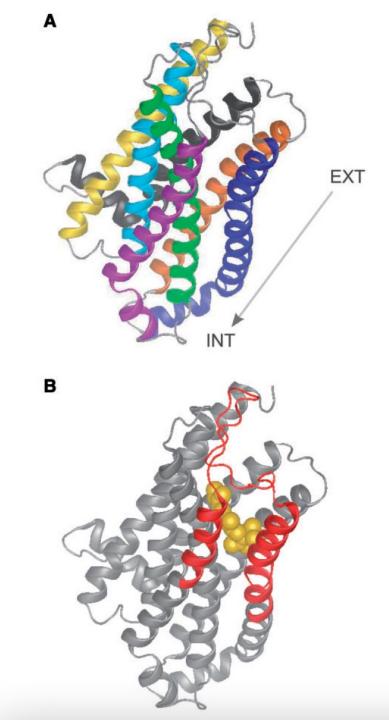




All models can incorporate rate variation though

Rate variation – some site evolve quickly and some evolve slowly





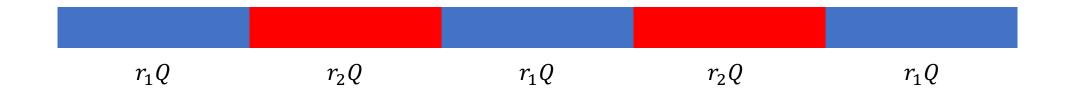
Yoder et al. 2015

All models can incorporate rate variation though

Rate variation – some site evolve quickly and some evolve slowly

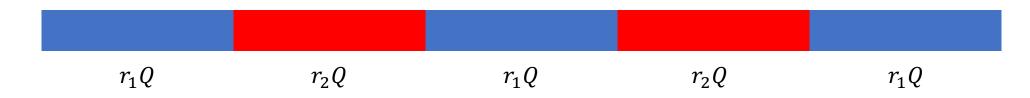
A site specific approach

Let different regions have different rates (r)



A site specific approach

Let different regions have different rates (r)



Our JC69 probability matrix will now look like:

For slow regions

$$P_{ij}(t) = \frac{1}{4} - \frac{1}{4}e^{-r_1 4\alpha t}$$

$$P_{ii}(t) = \frac{1}{4} + \frac{3}{4}e^{-r_1 4\alpha t}$$

For fast regions

$$P_{ij}(t) = \frac{1}{4} - \frac{1}{4}e^{-r_2 4\alpha t}$$

$$P_{ii}(t) = \frac{1}{4} + \frac{3}{4}e^{-r_2 4\alpha t}$$

A site specific approach

Pros

Convenient because we only have to estimate the likelihood of each site once

Cons

Requires a priori specification of regions (partitions)

Can still fit poorly for many sites

A mixture model approach

Integrate over multiple rates at each site

Consider we have 4 rates and *n* sites.

$$L(D|\theta) = \prod_{i=1}^{n} \left[\frac{1}{4} Pr(D_i|r_1) + \frac{1}{4} Pr(D_i|r_2) + \frac{1}{4} Pr(D_i|r_3) + \frac{1}{4} Pr(D_i|r_4) \right]$$

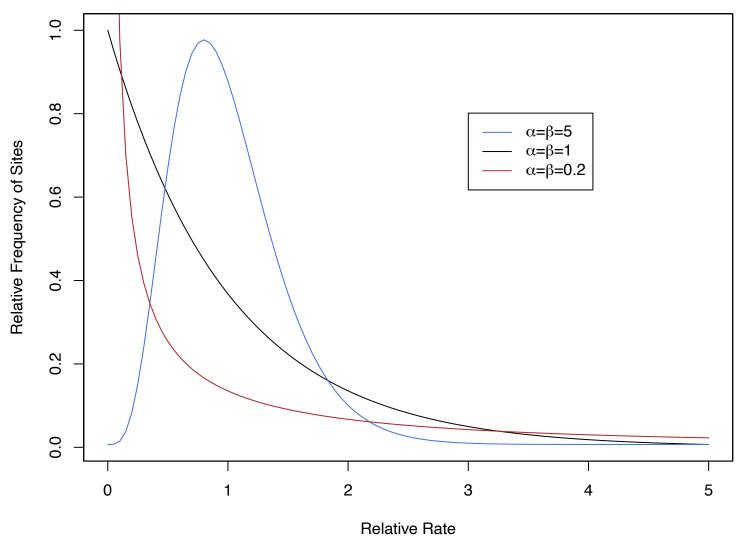
A mixture model approach

Where do those rates come from?

A mixture model approach

Assume rates follow a Gamma distribution

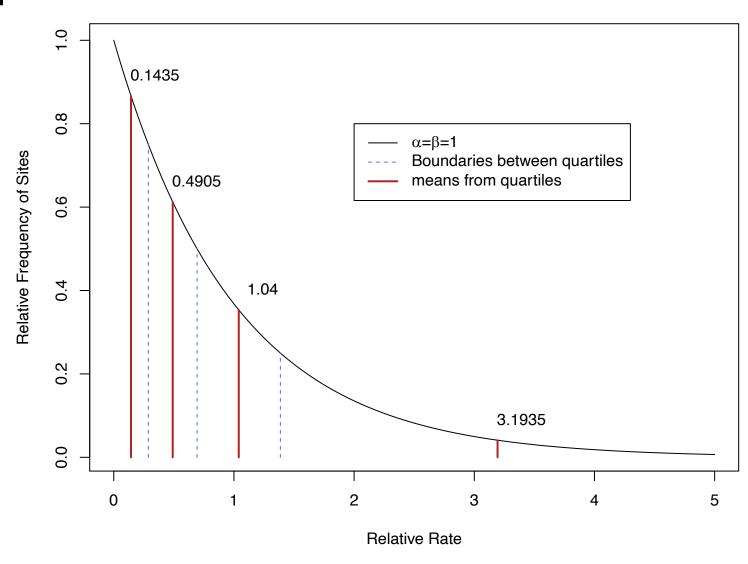
Higher values of alpha imply less rate variation



A mixture model approach

We get discrete rates by splitting the distribution into quantiles (often 4)

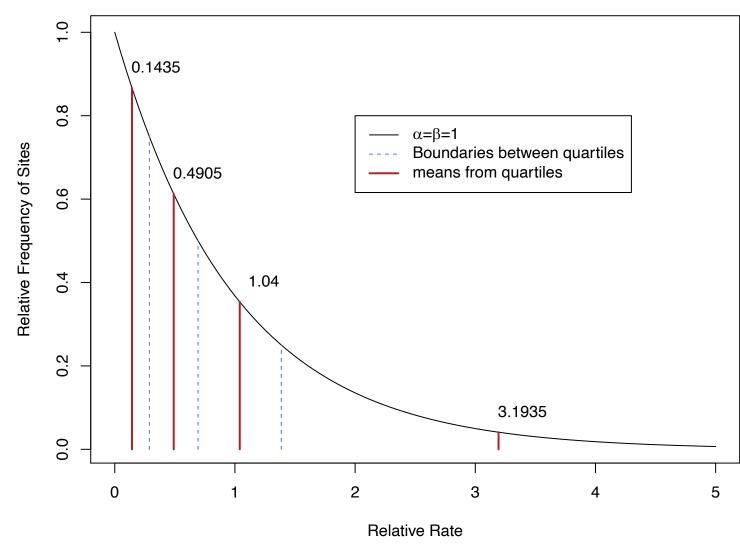
The rates are then the means of those quantiles



A mixture model approach

We get discrete rates by splitting the distribution into quantiles (often 4)

The rates are then the means of those quantiles



A mixture model approach

It is also possible to fit a model of rate heterogeneity where we do not assume the gamma distribution (free-rate model; Yang 1995)

The rate classes are approximated from the data directly

Estimating the rate classes requires having sufficient data and the gamma distribution is still widely used

Learning Goals

Explain terminology

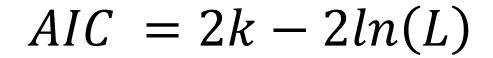
Primer on probability and likelihood

Models of molecular evolution

How to select a model

Application of models for phylogenetic estimation

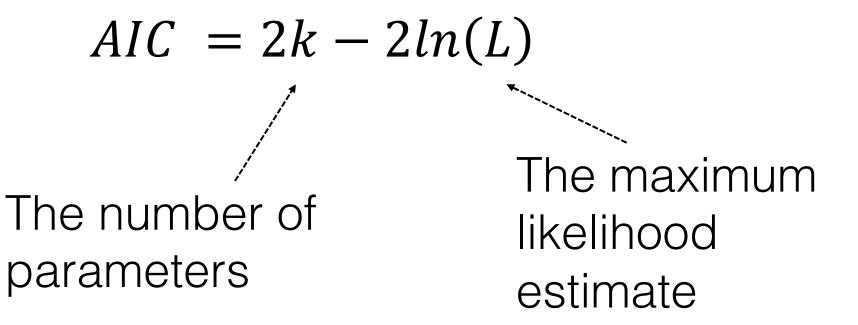
The Akaike Information Criterion (AIC)



The number of parameters

The maximum likelihood estimate

The Akaike Information Criterion (AIC)



Goal: Find the best (least worst) model among the set of possible models

$$AIC = 2k - 2ln(L)$$

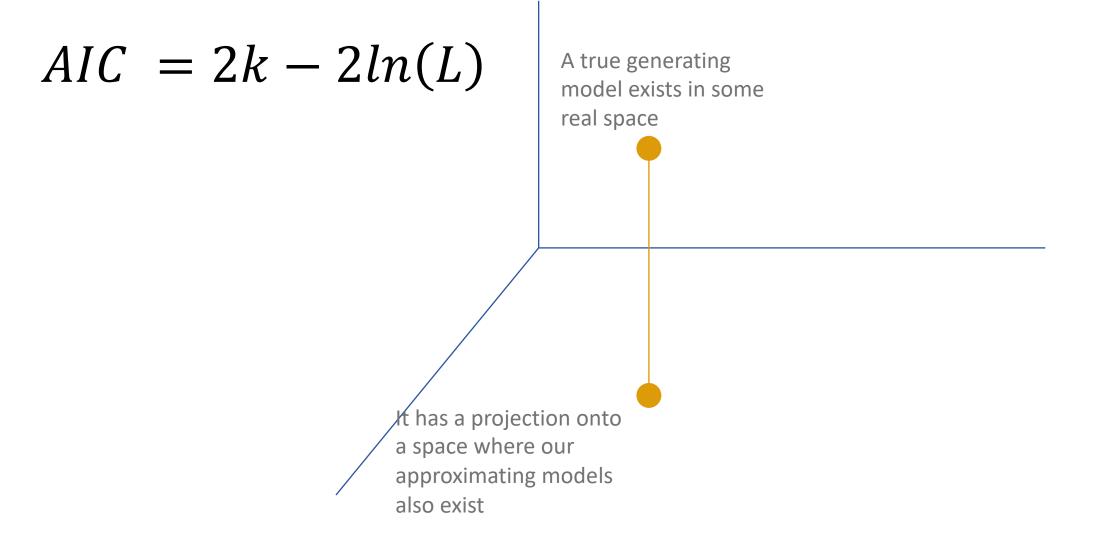
A true generating model exists in some real space

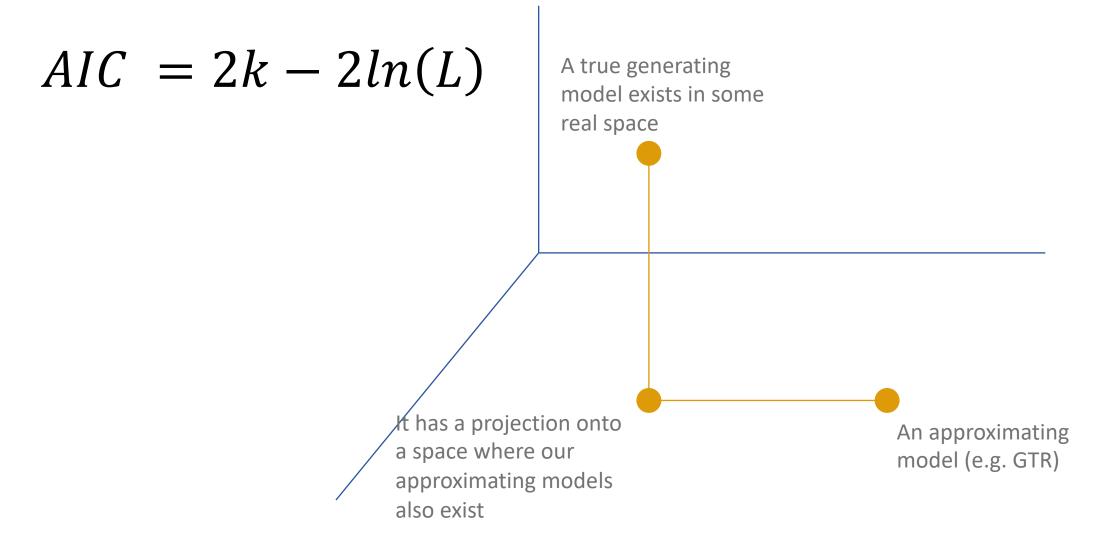


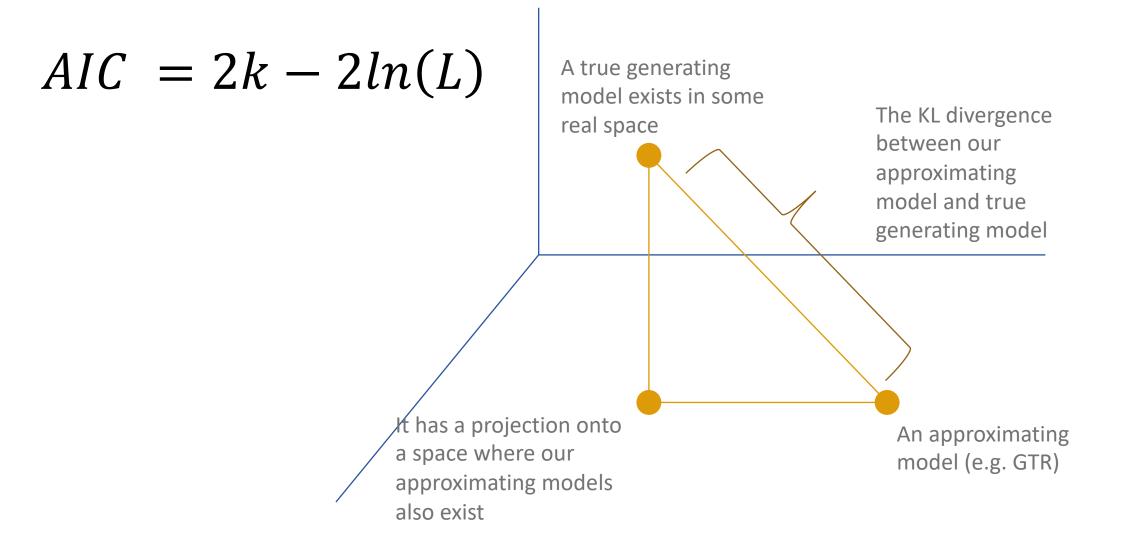
$$AIC = 2k - 2ln(L)$$

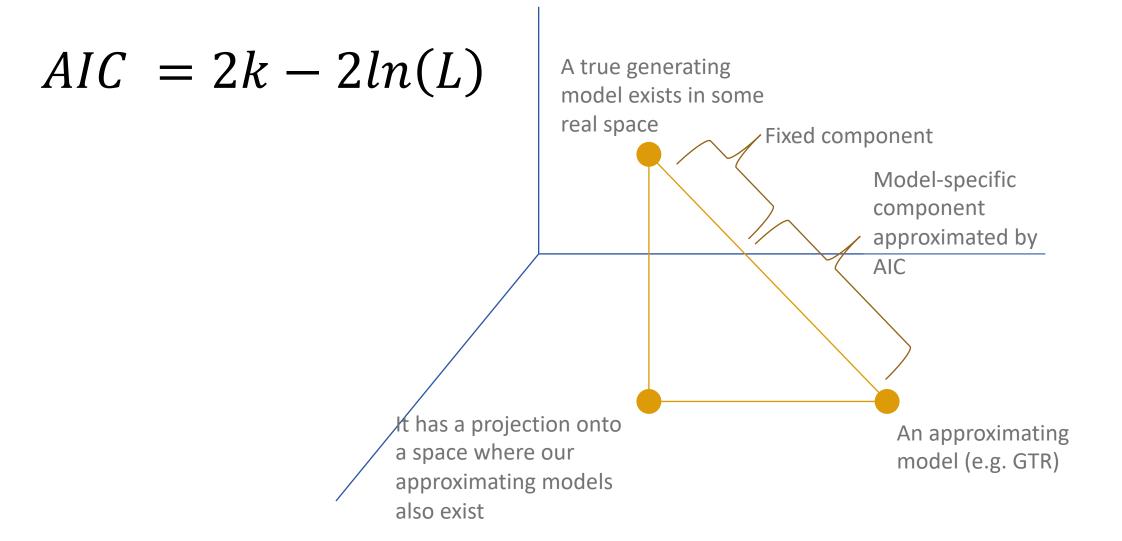
A true generating model exists in some real space











$$AIC = 2k - 2ln(L)$$

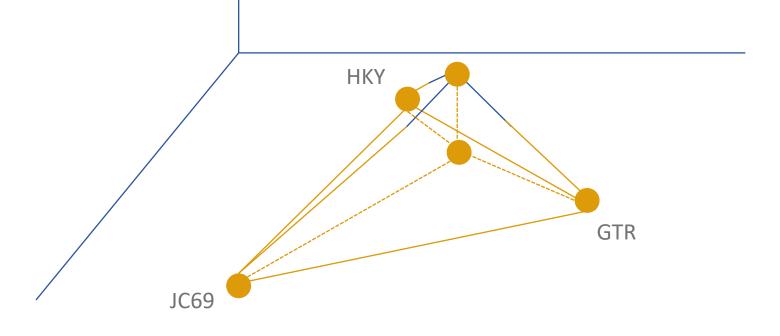
HKY

GTR

$$AIC = 2k - 2ln(L)$$

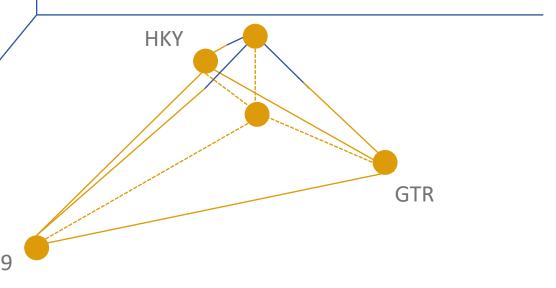
$$AIC = 2k - 2ln(L)$$

$$AIC = 2k - 2ln(L)$$



$$AIC = 2k - 2ln(L)$$

The best (least worst) model will have the lowest AIC score



$$AIC = 2k - 2ln(L)$$

Model	k	In(L)	AIC	ΔΑΙC
JC69	1	-533214	1066430	16825
GTR	8	-524800	1049608	3
HKY	5	-524800	1049605	0

The best model will have the lowest AIC score

Usually a difference of 2 AIC points is accepted

Alternatives to AIC

AIC Corrected for small sample size

ted for small sample size
$$AICc = 2k - 2ln(L) + \frac{2k^2 + 2k}{n - k - 1}$$

Bayesian information criterion

$$BIC = kln(n) - 2ln(L)$$

Not a lot of agreement about which is best. Phylogenetics often uses the AICc. Programs will often return all three.

Learning Goals

Explain terminology

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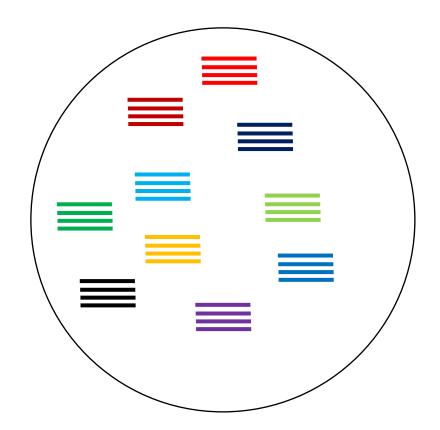
It is now possible to generate many loci for many species

We will cover different ways of generating sequence data on Wednesday

The good news is that if you can analyze 1 locus well, you can do it for 100 or many more

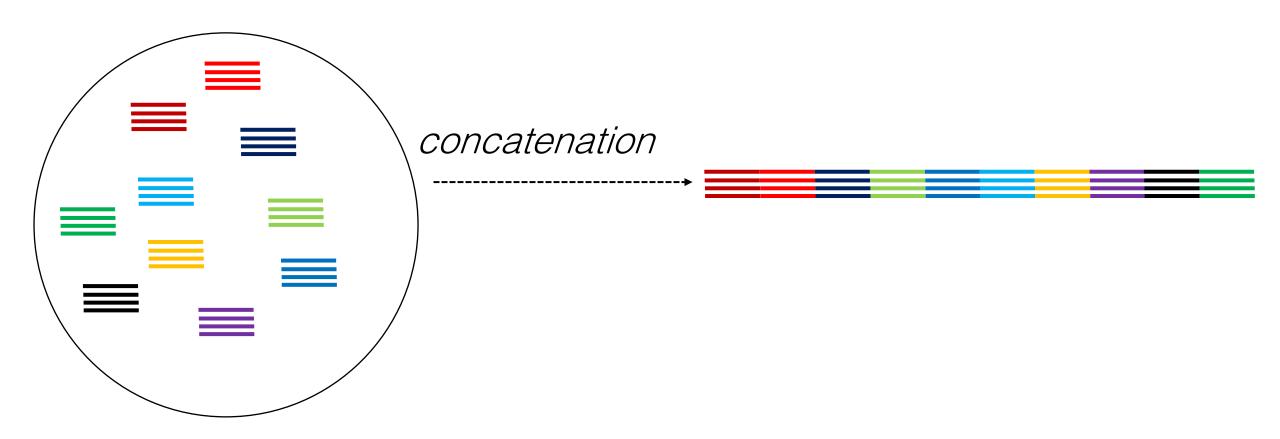
Concatenated maximum likelihood with partitioned mixture models

10 loci (genes) for 4 species

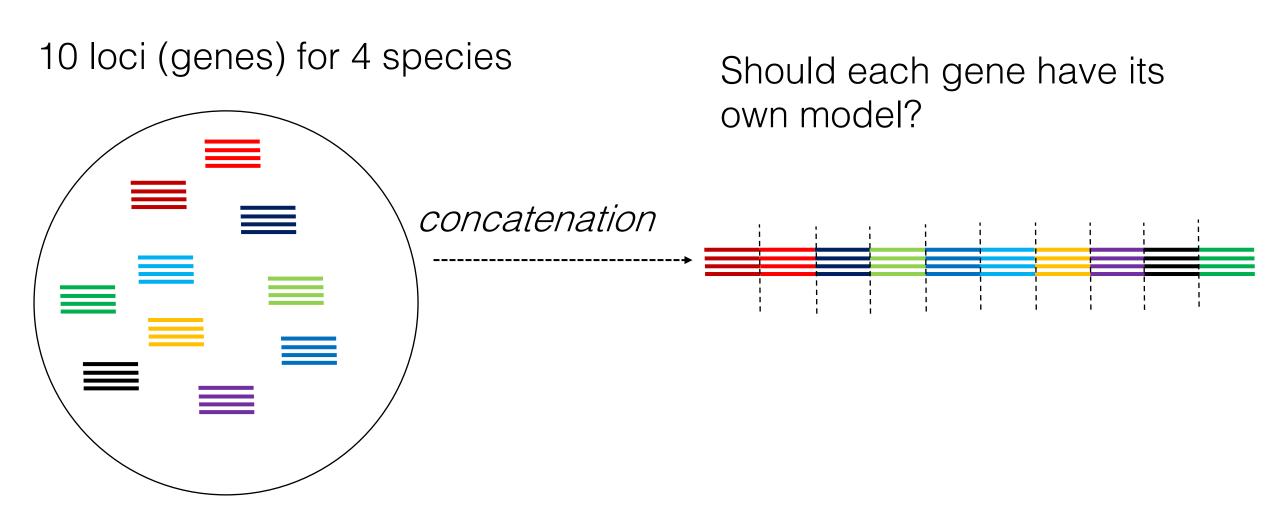


Concatenated maximum likelihood with partitioned mixture models

10 loci (genes) for 4 species



Concatenated maximum likelihood with partitioned mixture models



Concatenated maximum likelihood with partitioned mixture models

Used in almost every phylogenomics paper to some extent

Some notable controversies

Conflict between models based on the same data!

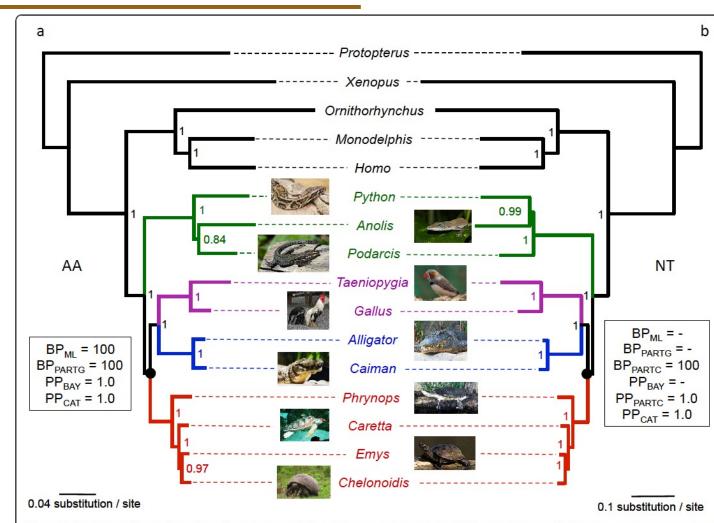


Figure 1 Phylogenetic relationships of amniotes as inferred from analyses of the 248-gene dataset. (a) Bayesian consensus topology obtained from analyses of the amino-acid dataset (62,342 sites) under the CAT-GTR + G4 mixture model. (b) Bayesian consensus topology obtained from analyses of the complete nucleotide dataset (187,026 sites) under the CAT-GTR + G4 mixture model. The nodal values indicate the clade Bayesian posterior probability (PP). Statistical support values obtained with different methods, models and data partitions detailed in Table 1 are reported in boxes for turtles plus archosaurs. Note the relative incongruence between the two trees concerning the position of *Python*. All pictures are from Wikimedia Commons, except for *Chelonoidis* from Y. Chiari. Please note also that the taxonomy of Galapagos turtles being currently revised, the appropriate species name for the *Chelonoidis* specimen included here might be *Chelonoidis* sp.

Table 1 Statistical support for the phylogenetic position of turtles based on the various reconstruction methods, substitution models, and data partitions.

	Amino acids	Nucleotides				
	All positions	All positions	Positions 1 + 2	Positions 3		
Total sites	62,342	187,026	124,684	62,342		
Constant sites	41,170 (66.0%)	99,638 (53.3%)	92,128 (73.9%)	7,510 (11.2%)		
Informative sites	8,749 (14.0%)	54,880 (29.3%)	14,009 (11.2%)	40,871 (65.6%)		
RaxML LG + G / GTR + G	Turtles + Archosaurs $BP_{ML} = 100$	Turtles + Crocodiles $BP_{ML} = 76$	Turtles + Archosaurs $BP_{ML} = 100$	Turtles + Crocodiles $BP_{ML} = 100$		
RaxML GTR + G partitioned by gene	Turtles + Archosaurs $BP_{PARTG} = 100$	Turtles + Crocodiles $BP_{PARTG} = 54$	-	-		
RaxML GTR + G partitioned by codon	_	Turtles + Archosaurs $BP_{PARTC} = 100$	-	-		
MrBayes WAG + G / GTR + G	Turtles + Archosaurs $PP_{BAY} = 1.0$	Turtles + Crocodiles $PP_{BAY} = 1.0$	Turtles + Archosaurs $PP_{BAY} = 1.0$	Turtles + Crocodiles $PP_{BAY} = 1.0$		
MrBayes GTR + G partitioned by codon	-	Turtles + Archosaurs $PP_{PARTC} = 1.0$	-	-		
PhyloBayes CAT-GTR + G	Turtles + Archosaurs $PP_{CAT} = 1.0$	Turtles + Archosaurs $PP_{CAT} = 1.0$	Turtles + Archosaurs $PP_{CAT} = 1.0$	Turtles + Archosaurs $PP_{CAT} = 1.0$		

The optimal partitioning for a phylogeny of Malpighiales is not obvious

Table 1. Characteristics of the four matrices and statistics of the best-scoring ML trees inferred from each of the four partitioning strategies

Matrix	Taxa/characters/ missing data %	Partitioning strategy	No. of partitions	Log- likelihood	AICc	ΔΑΙС	Coverage density	Fraction of triples	D	d	Terrace size
82-gene	58/72,828/17%	OnePart	1	-689042	1,378,328	166,322	1.00	1.00	1.00	1.00	1
		GenePart	82	-680357	1,362,435	150,429	0.88	1.00	1.00	1.00	1
		CodonPart	4	-680281	1,360,860	148,854	1.00	1.00	1.00	1.00	1
		MixtPart	13	-605772	1,212,006	0	1.00	1.00	1.00	1.00	1
Combined- 58 complete	58/81,117/12%	OnePart	1	-739270	1,478,784	193,023	1.00	1.00	1.00	1.00	1
		GenePart	91	-728235	1,458,355	172,594	0.88	1.00	1.00	1.00	1
		CodonPart	4	-730551	1,461,401	175,640	1.00	1.00	1.00	1.00	1
		MixtPart	15	-642632	1,285,761	0	1.00	1.00	1.00	1.00	1
Combined- incomplete	191/81,259/64%	OnePart	1	-892791	1,786,362	234,881	1.00	1.00	1.00	1.00	1
		GenePart	91	-879681	1,761,794	210,313	0.36	0.93	0.00	0.97	14,025
		CodonPart	4	-883407	1,767,647	216,166	1.00	1.00	1.00	1.00	1
		MixtPart	20	-775178	1,551,481	0	1.00	1.00	1.00	1.00	1
13-gene	186/15,574/15%	OnePart	1	-292212	585,198	47,256	1.00	1.00	1.00	1.00	1
		GenePart	13	-288145	577,294	39,352	0.93	1.00	1.00	1.00	1
		CodonPart	4	-289988	580,807	42,865	1.00	1.00	1.00	1.00	1
		MixtPart	14	-268460	537,942	0	1.00	1.00	1.00	1.00	1

Based on whole-chloroplast genomes

But it can affect the boostrap support of some major clades!

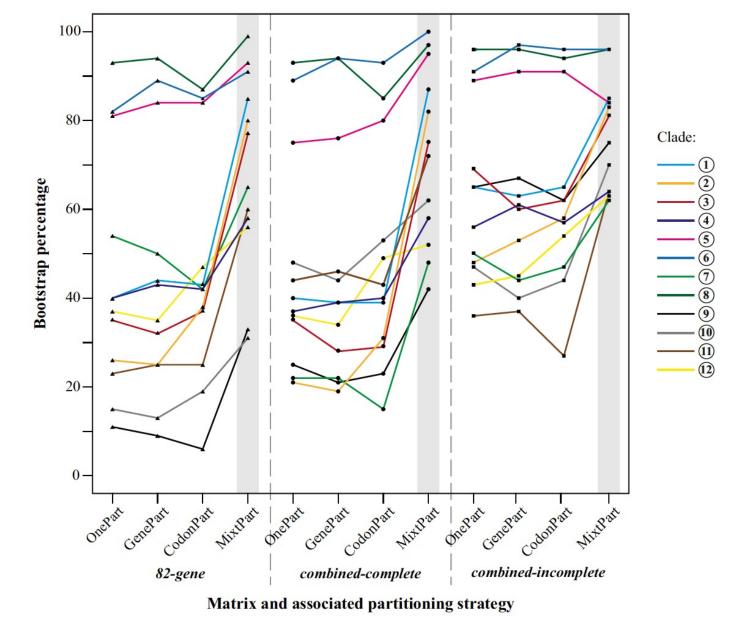


Fig. 3. ML BPs of the 12 additional clades we identified in Malpighiales (Fig. 1) inferred from three matrices and four partitioning strategies. The MixtPart partitioning strategy is highlighted in gray.

But it can affect the bootstrap support of some major clades!

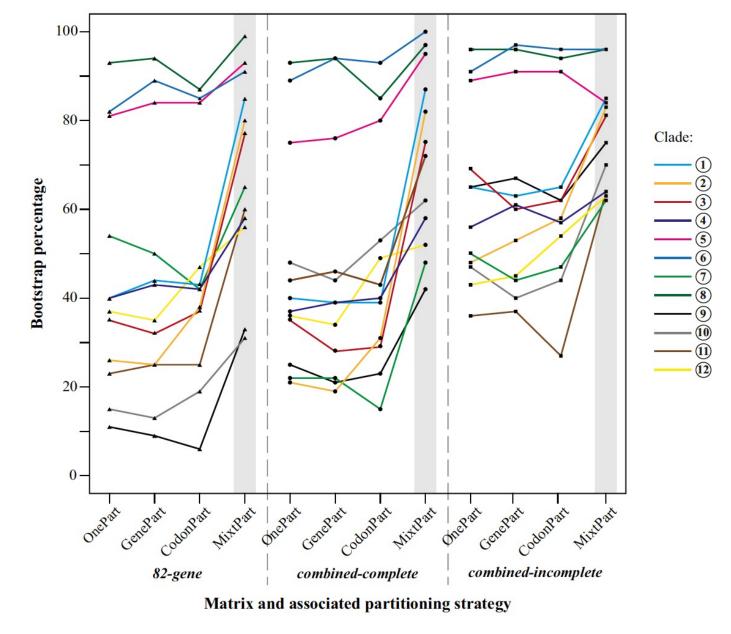
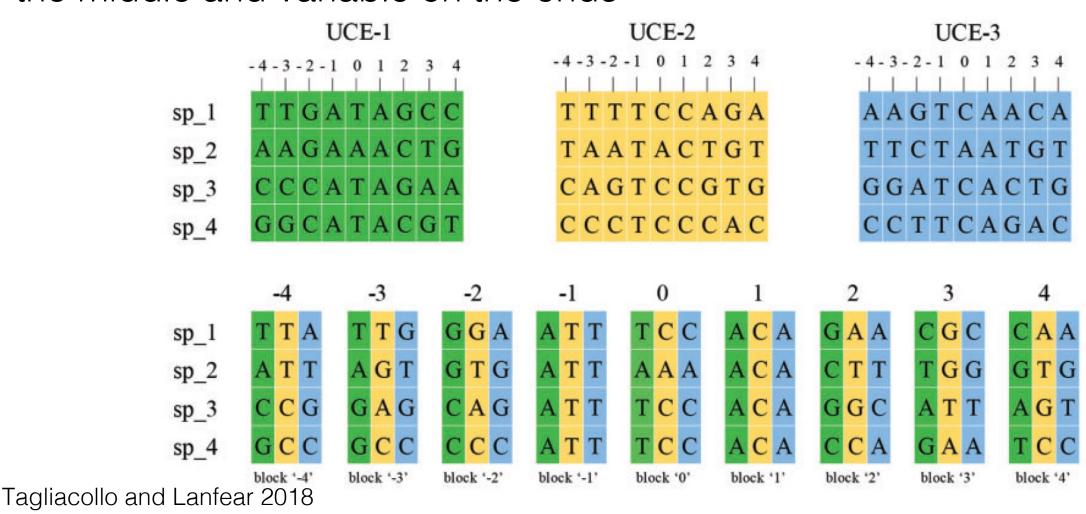


Fig. 3. ML BPs of the 12 additional clades we identified in Malpighiales (Fig. 1) inferred from three matrices and four partitioning strategies. The MixtPart partitioning strategy is highlighted in gray.

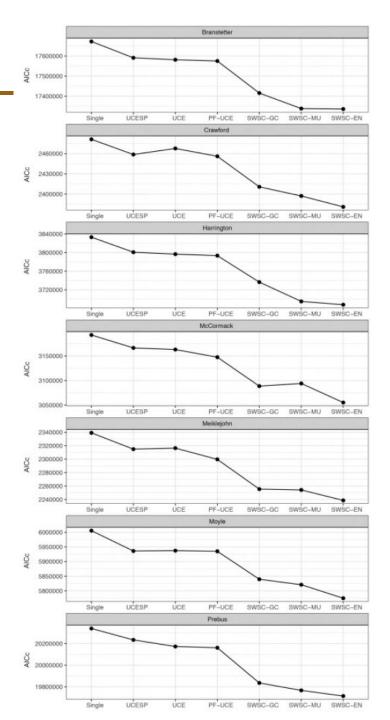
Target enrichment data is conserved in the middle and variable on the ends



Target enrichment data is conserved in the middle and variable on the ends

More sophisticated partitioning can lead to much better likelihood scores

Not clear if it will cause a different biological interpretation of the results though



Modern software now automate this model selection process for you!

This includes selecting among different types of substitution models and potentially different partitioning strategies.

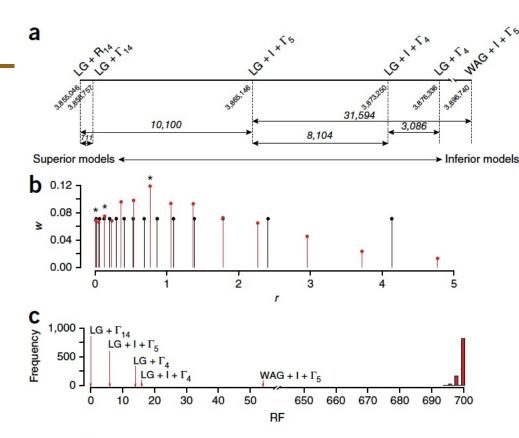


Figure 2 | Advantages provided by ModelFinder. (a) BIC scores of selected models of SE, given the alignment of bacterial and archaeal amino acids used by Wu $et~al.^{19}$. Models are listed above the thick horizontal line. Numbers along the line are BIC scores, and those in italics denote Δ BIC. (b) r_i and w_i values obtained under the R₁₄ model of RHAS (red lines and balls) and the Γ_{14} model of RHAS (black lines and balls) for the alignment analyzed by Wu $et~al.^{19}$. Stars indicate local peaks in the R₁₄ model. (c) RF distances between the most likely tree inferred under various models of SE. For comparison, a histogram with the distribution of 1,000 RF distances is included; each of these distances was obtained by comparing the most likely tree inferred under the LG + R₁₄ model of SE to a randomly generated tree with the same number of leaves.

IQTREE2 (Minh et al. 2020)

There are many software packages to choose from, why will we use IQTREE today?

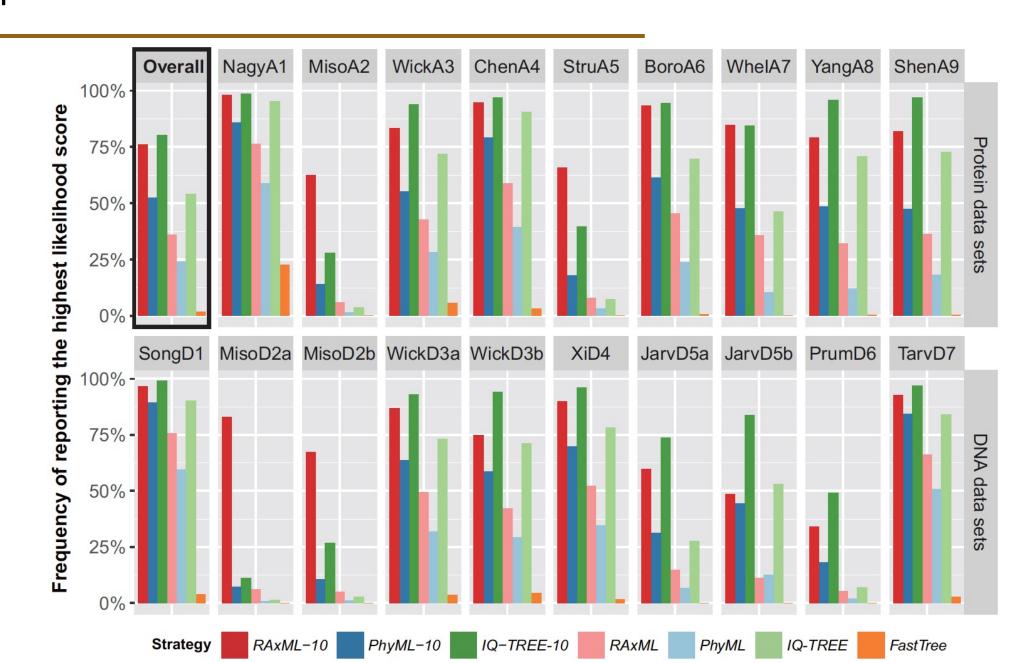
Fast

Automated model selection

Well-annotated log files

Many nice features when analyzing many loci

Returns best likelihoods most frequently



Zhou et al. 2018

End

When we come back we will analyze some data with IQTREE