Analytical Model to Infer Mask Peripheral Leakage Pattern in Large Population

Akshay Anand[1], Tso-Kang Wang[1], Tomas Solano[1], Rajat Mittal[2], Kenneth Breuer[3], Kourosh Shoele[1]

1)FAMU – FSU (Florida State University)

2) Johns Hopkins University

3) Brown University

74th APS DFD Meeting, Phoenix, Arizona

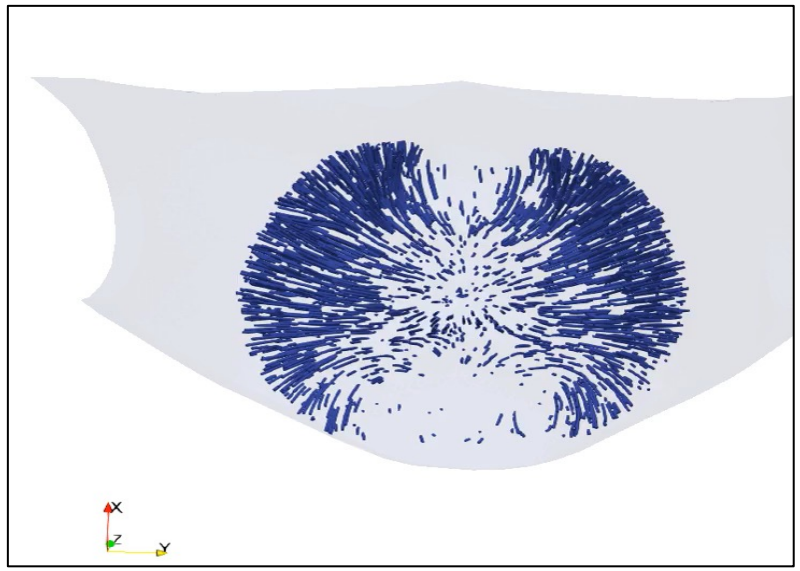
11.21.2021

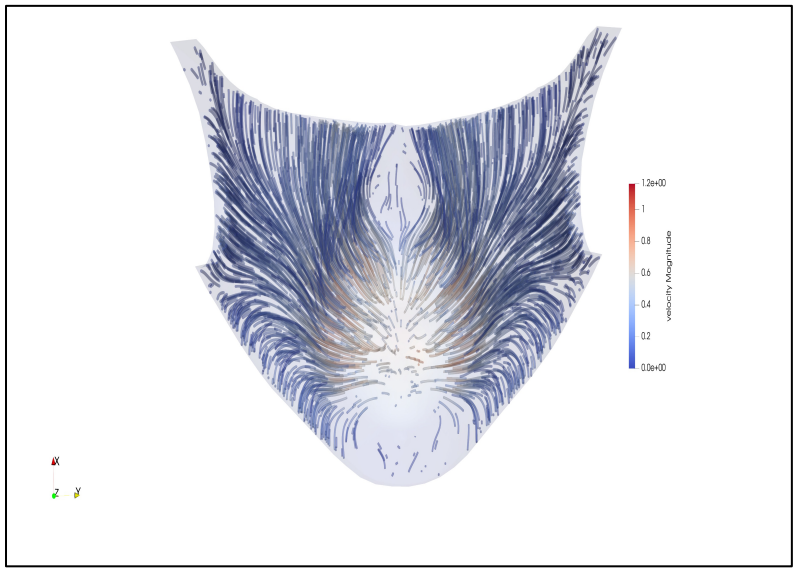
Outline

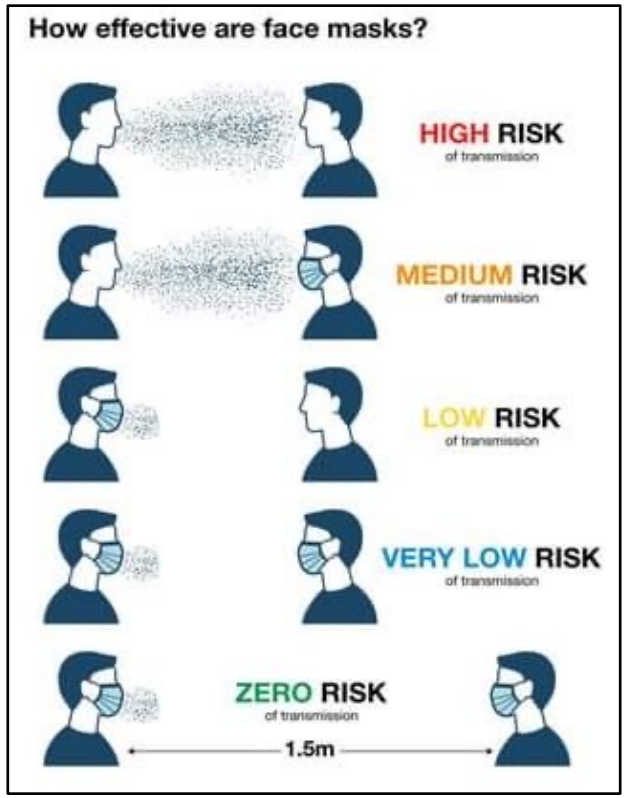
- Introduction
- Literature Review
- Methodology
- Results and Discussions
  - Effect of Number of Channels
  - Effect of Middle Area Periphery
  - Effect of Permeability Coefficient
  - Leakage Pattern for Facial Features
- Conclusions and Future Work

Introduction

- Face masks have been crucial in our defense against COVID-19 & other pathogenic agents
  - Does one size of mask fit all?
  - What are the main factors affecting the effectiveness of mask?



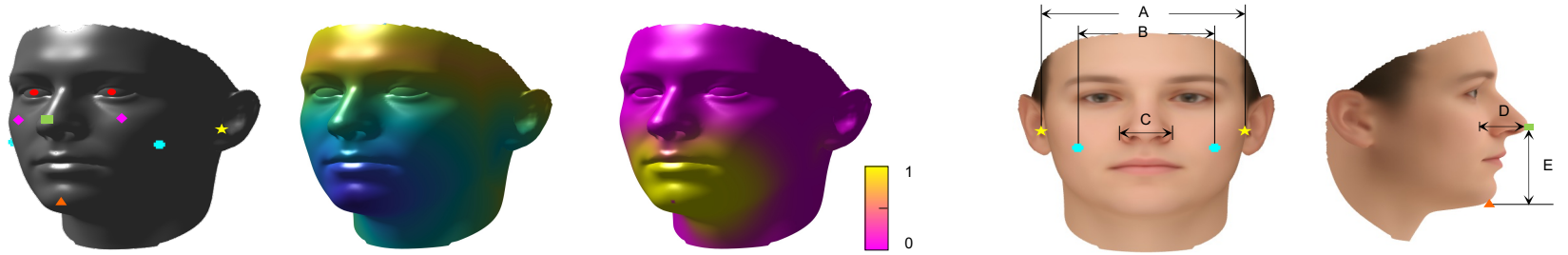




Flow physics in the integral region between face and mask in large population

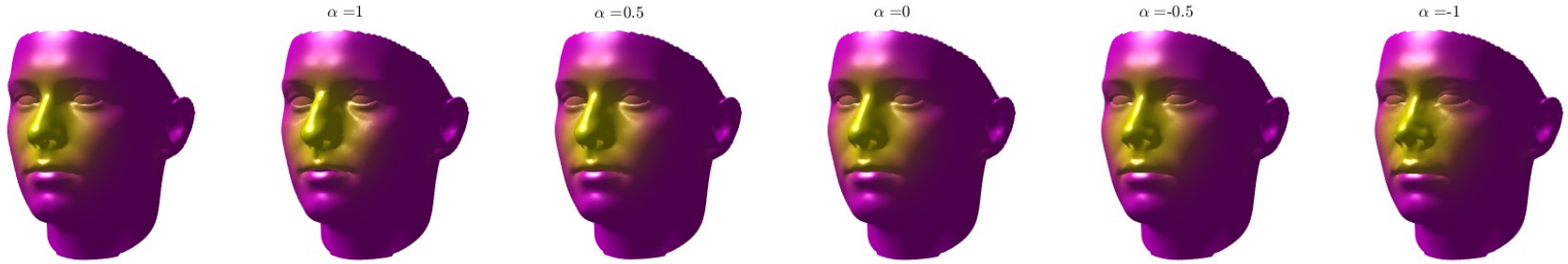
Mask Fitting Test Infrastructure

Face database[1] → Geodesic weighting → Weighted PCA w.r.t. features → Attach masks to[2,3] morphing faces → Correlation[2] relationship

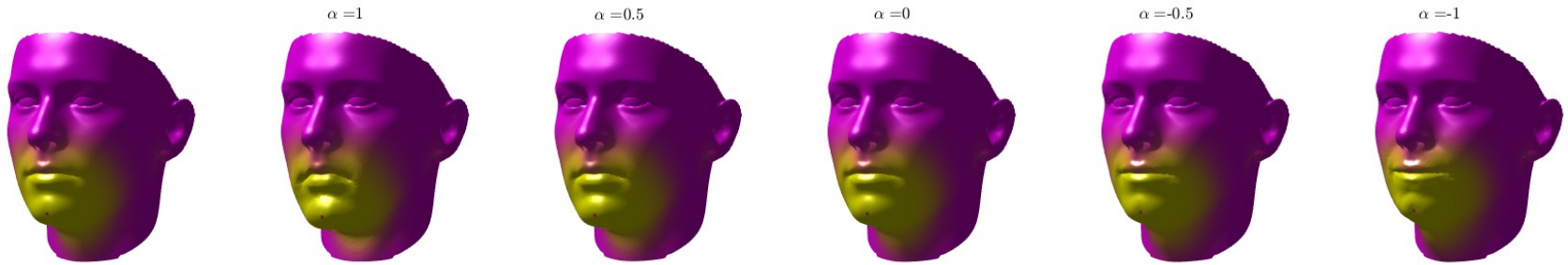


Examples

Nose



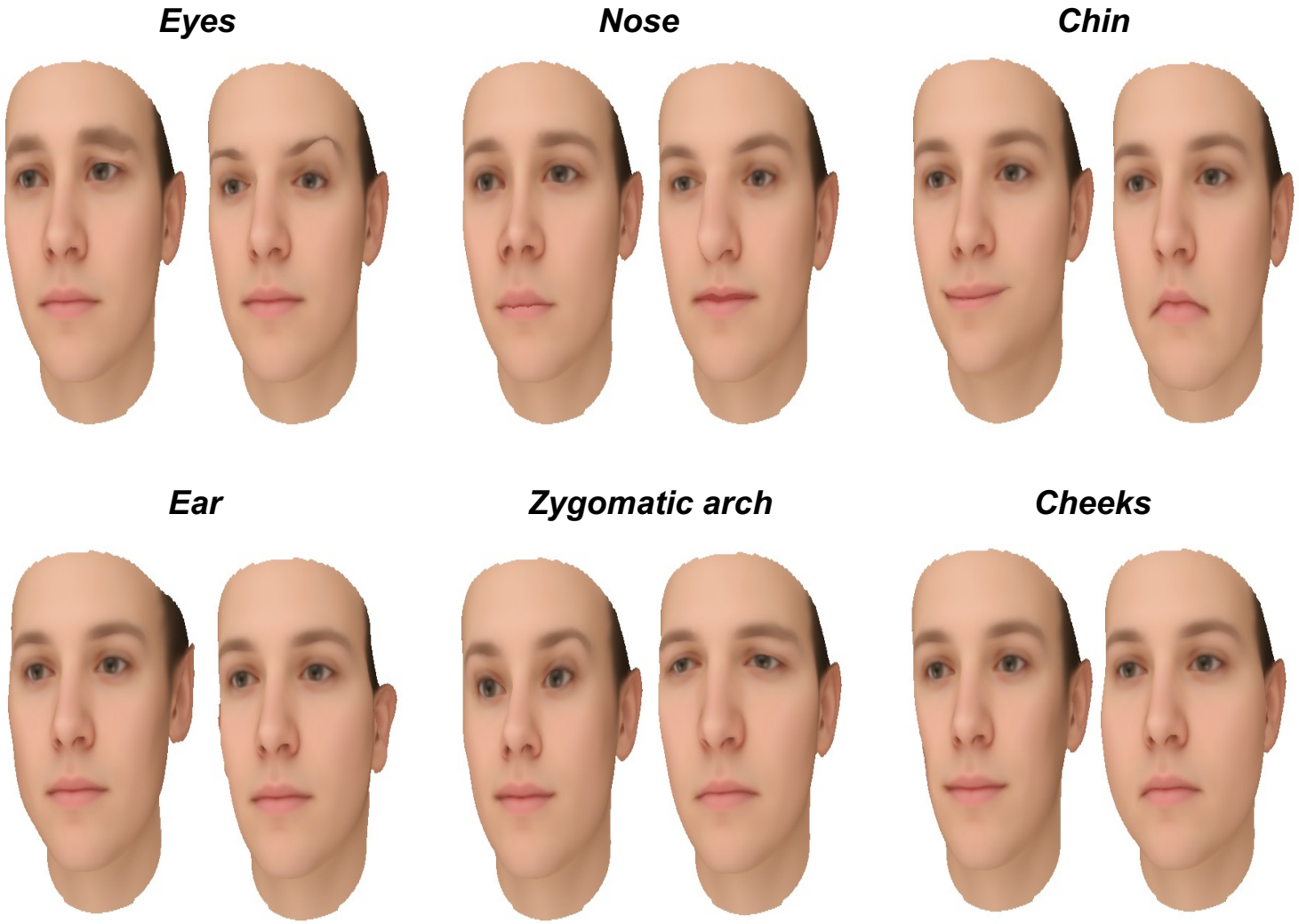
Chin

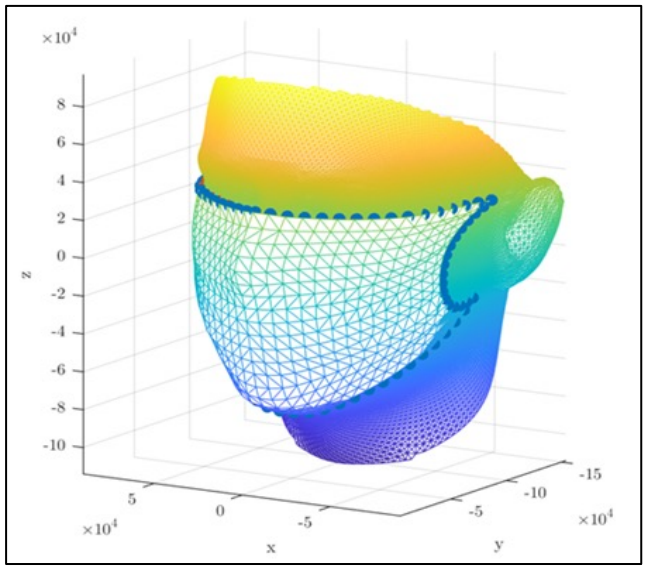


FAMU-FSU College of Engineering

[1.] Gilani et al. "Deep, dense and accurate 3D face correspondence for generating population specific deformable models. (2017)
[2.] Wang et al. "Bridge the Gap: Correlate Face Mask Leakage and Facial Features with 3D Morphable Face Models." (2021)
[3.] Solano et al. "One size fits all?: A simulation framework for face-mask fit on population-based faces." (2021)

Weighted PCA w.r.t Features

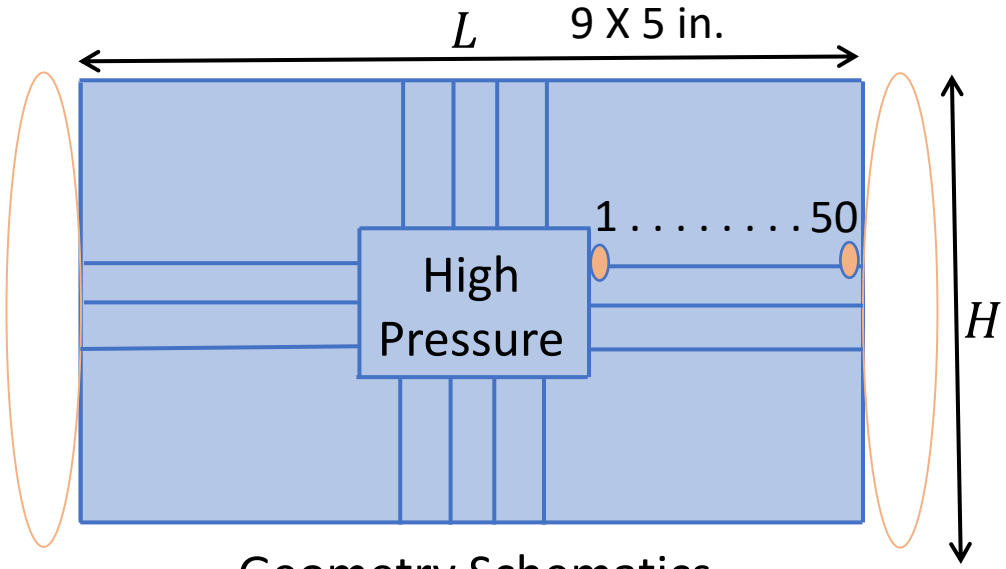




Mask is a part of PCA modes, the mean face and the mask are modified together by PCA modes

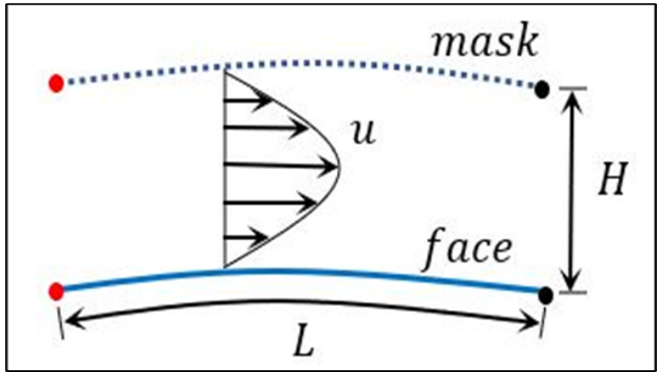
Wang et al. JESEE, 2021

Problem Set-Up

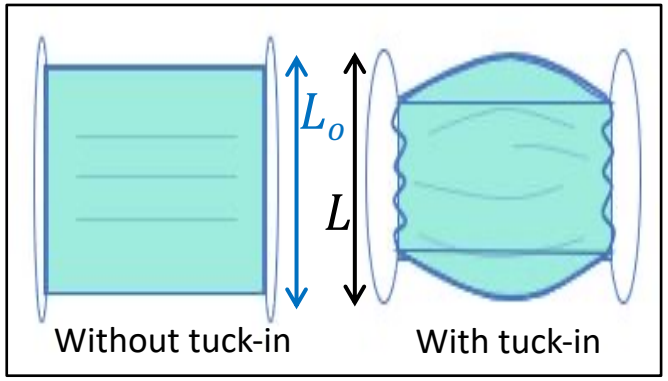


Geometry Schematics

Mask with multiple porous channel having longitudinally varying height



Velocity Profile



Tuck-in Ratio ($\frac{L_o}{L}$) 0.5

- If we know the solution for each mask; the velocity and pressure in channel are related to flux inside each channel
- With the iterative process we solve the flow that goes through inner domain and flow that goes to each channel (redistribution happens at entrance of each channel)
- This approach is used to compute the distribution of flow in each channel as well as the flow dynamics

[1.] Berman et al. "Laminar Flow In Channels with Porous Walls: J. Applied Physics (1953)

Governing Equations

- We assume laminar flow inside the channels

$$\frac{\partial v_x}{\partial x^*} + \frac{\partial v_y}{\partial y^*} = 0 \qquad x^* = \frac{x}{H}, y^* = \frac{y}{H}, L^* = \frac{L}{H}$$

$$\left(v_x \frac{\partial v_x}{\partial x^*} + v_y \frac{\partial v_x}{\partial y^*}\right) = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{\mu}{H}\left(\frac{\partial^2 v_x}{\partial x^{*2}} + \frac{\partial^2 v_x}{\partial y^{*2}}\right)$$

- Considering Kármán-Pohlhausen Momentum Integral[2] Technique

$$u = \frac{Q}{H(x)} f(x) g(y) \qquad v = g_v(x) n(y)$$

- $m(y)$ gives the shape of velocity profile in y direction

$$m(y) = 6\frac{y}{H}\left(1 - \frac{y}{H}\right) \qquad n(y) = 3\left(\frac{y}{H}\right)^2 - 2\left(\frac{y}{H}\right)^3$$

Simplest $m(y)$ profile is to assume is Poiseuille flow such that injection rate in channel does not affect y portion of v_x

- Channels has varying width and height; V_x takes this form

$$\int_0^H V_x \frac{\partial V_x}{\partial x} = \frac{6}{5}\frac{Q^2}{H}\left[f f' - \frac{H'}{H} f f\right]$$

- Flow through porous wall is governed by Darcy's law

$$\frac{dp}{dx} = -k^{-1}(u - V)$$

- 2nd order PDE into ODE; we solve for

$$\frac{df'}{dx} \text{ and } \frac{df}{dx} = f'$$

- Von – Kármán-Pohlhausen Approximations for the fluid part
- Iterative Approach for distribution of flux

The model was validated with the analytical channel flow data by Berman et al. (1953)

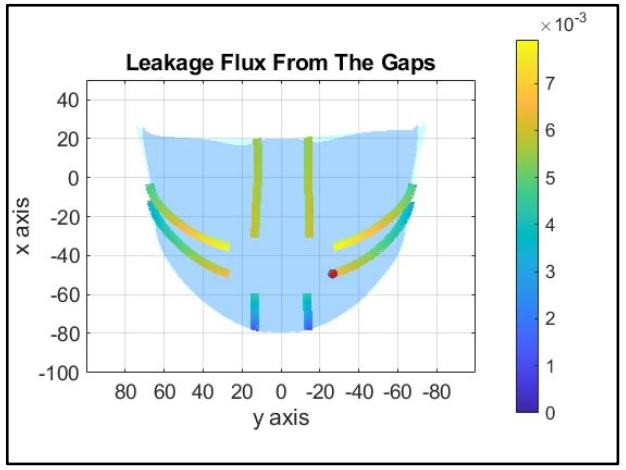
[1.] Berman et al. "Laminar Flow In Channels with Porous Walls" J. Applied Physics (1953)
[2.] Hartwig et al, "Analytical Model for Steady Flow Through a Finite Channel with one Porous Wall" Physics of Fluids (2014)

Optimal Number of Channels

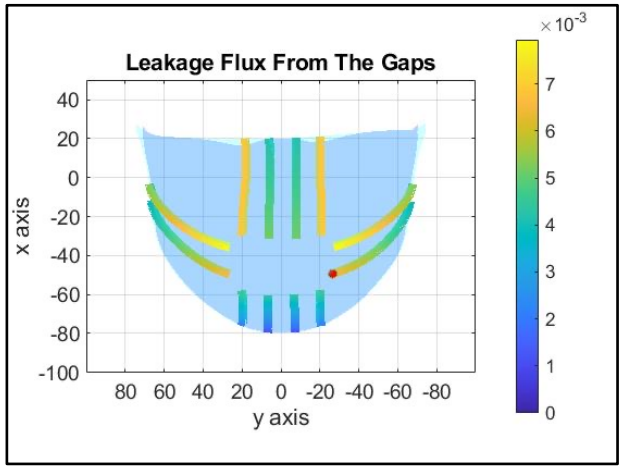
Nose, Mean face

$\propto = 0\ , \beta = 5$

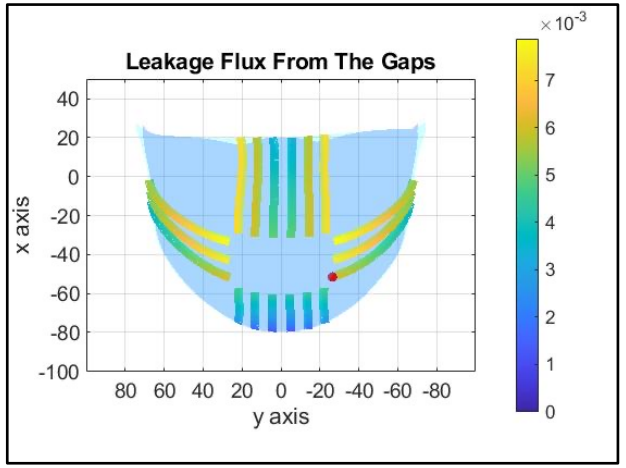
#Channels = 08



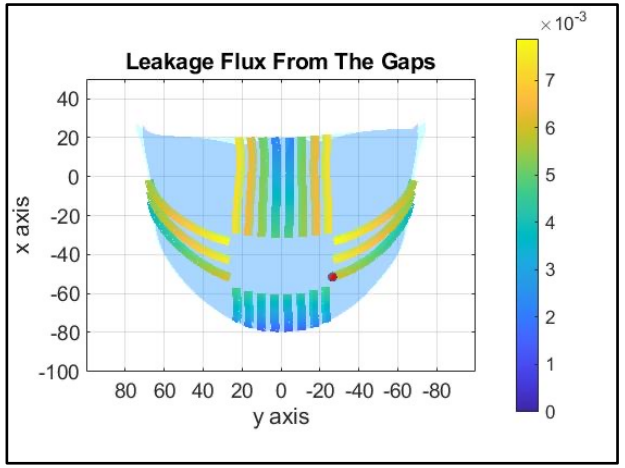
#Channels = 12

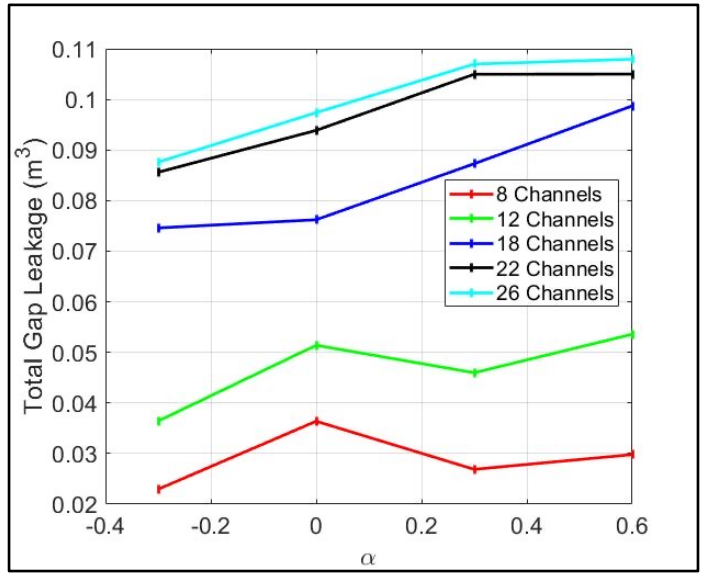


#Channels = 18



#Channels = 22

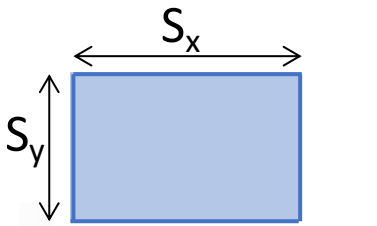




- Twenty-two is the sufficient number of channel

Effect of Inner Domain Periphery

$S_x = S_y = S$

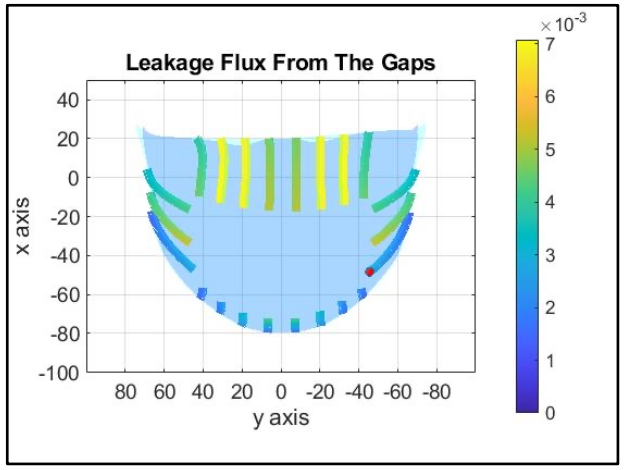


Nose, Mean face

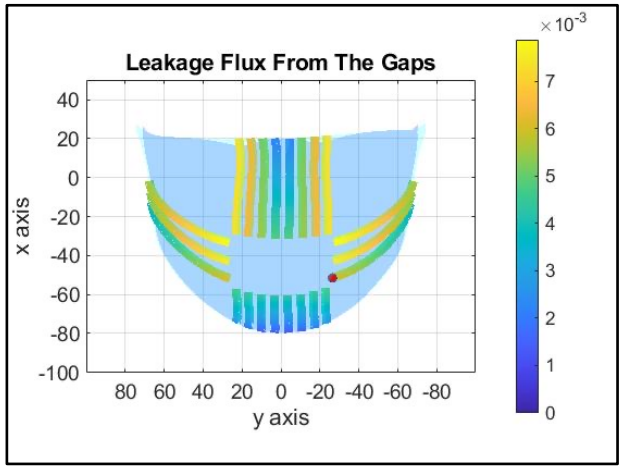
$\propto = 0\,, \beta = 5$

#Channel = 22

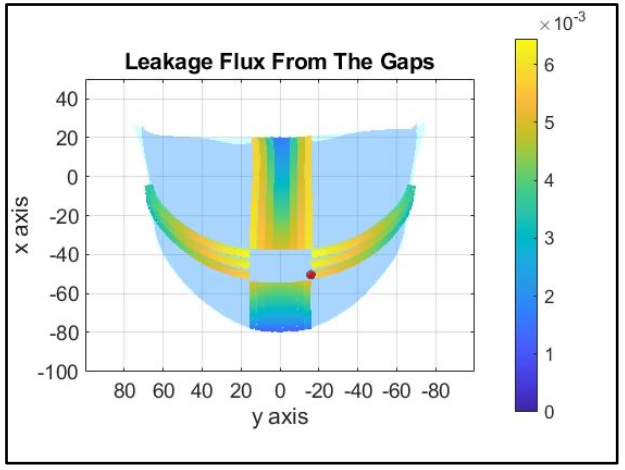
Sides = S/2



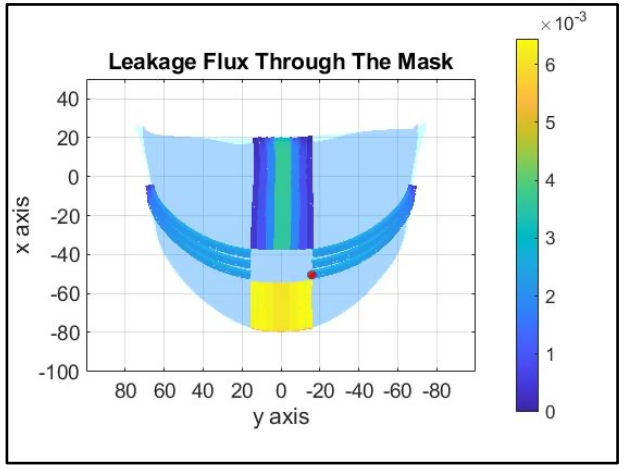
Sides = S/4



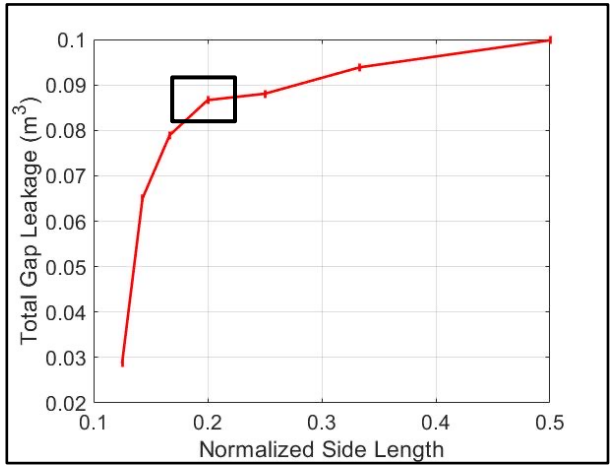
Sides = S/8

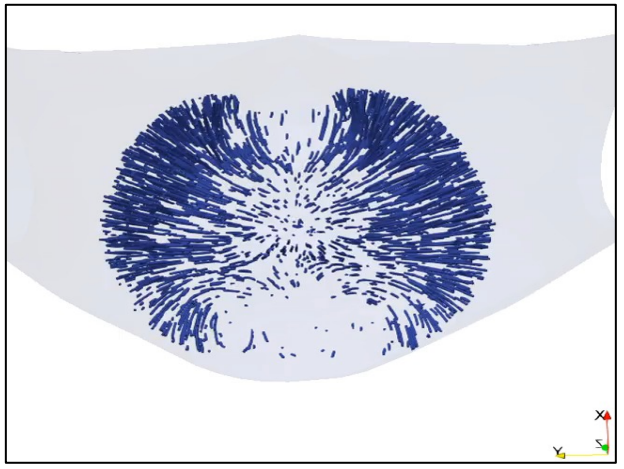


Sides = S/8



S/4 is a good choice

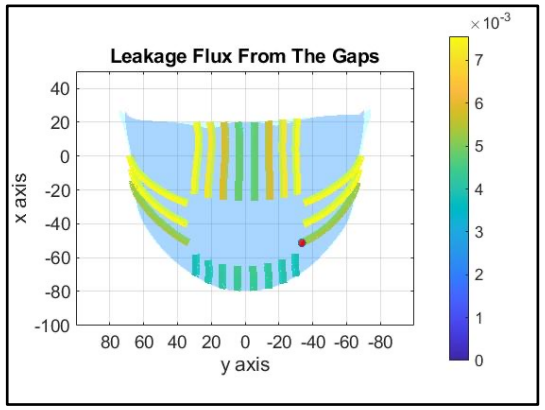


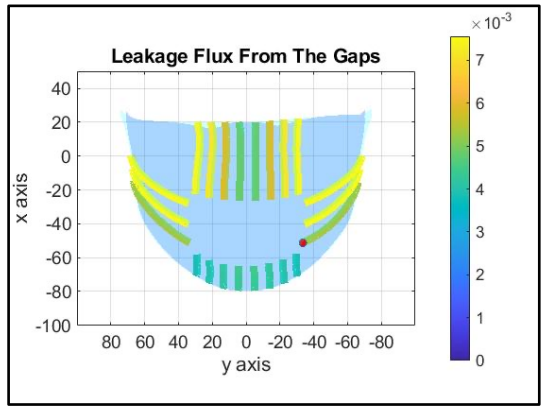


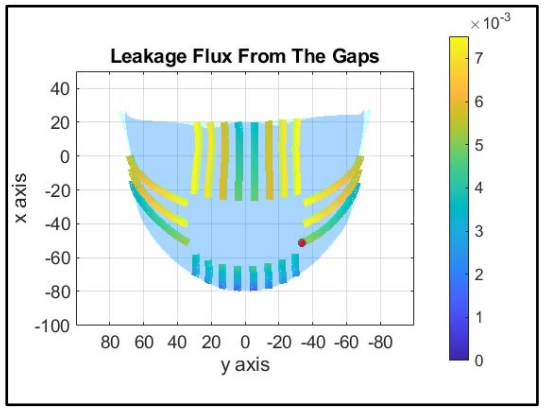
S ↙ Total Gap Leakage ↙

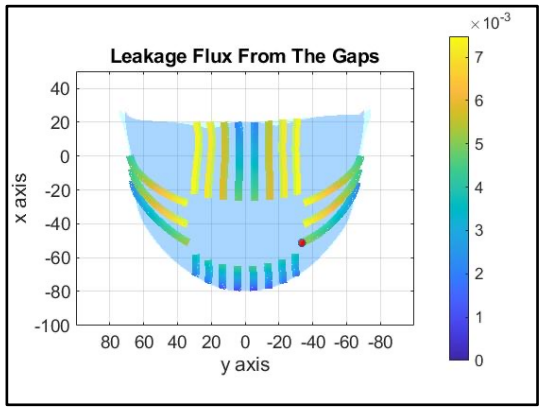
Effect of Permeability coefficient

$\beta = 0.1$ | $\beta = 1$ | $\beta = 5$ | $\beta = 10$







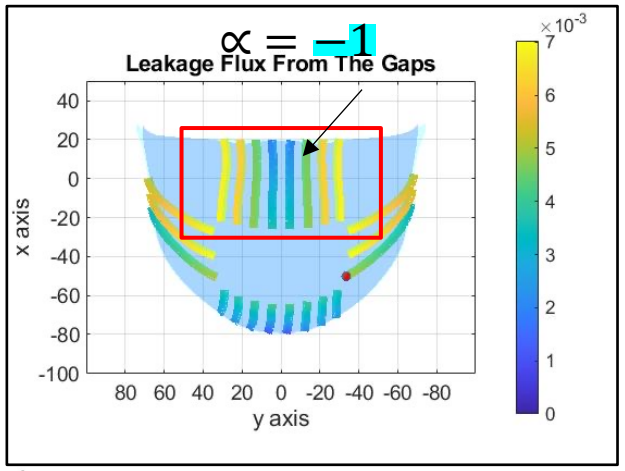


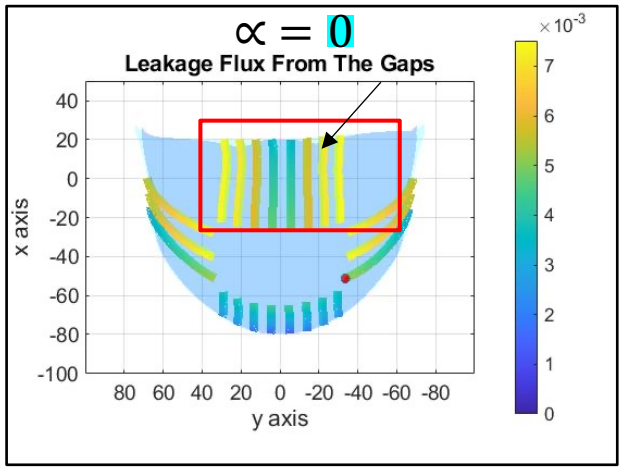
$$\beta = \frac{k}{\mu}$$

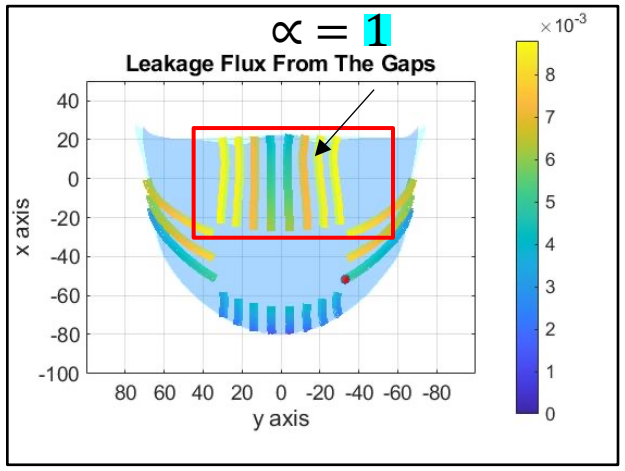
Nose, $\beta = 5$

$\propto = -1\ to + 1$

Channels = 22



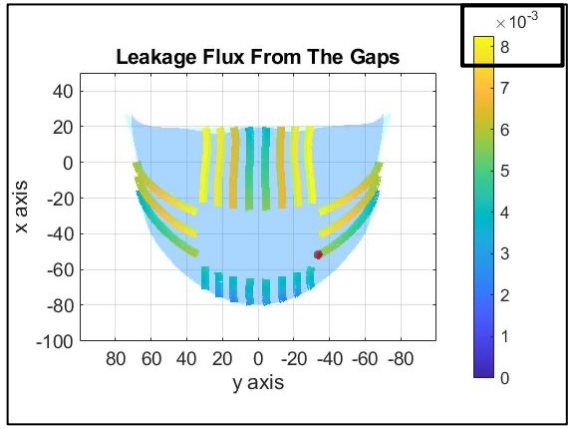


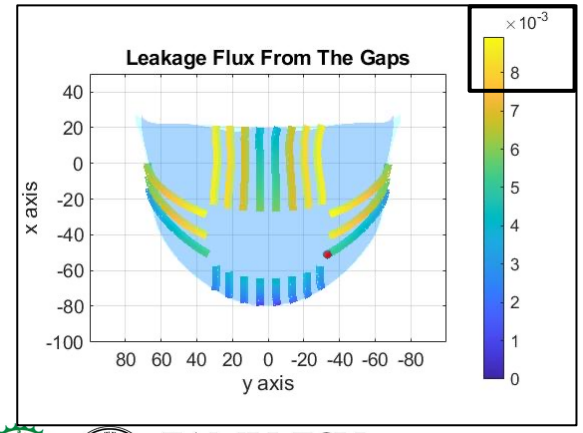


[1.] Aydin et al. "Performance of Fabrics for Home-Made Masks Against the Spread of Respiratory Infections Through Droplets: A Quantitative Study (2020)

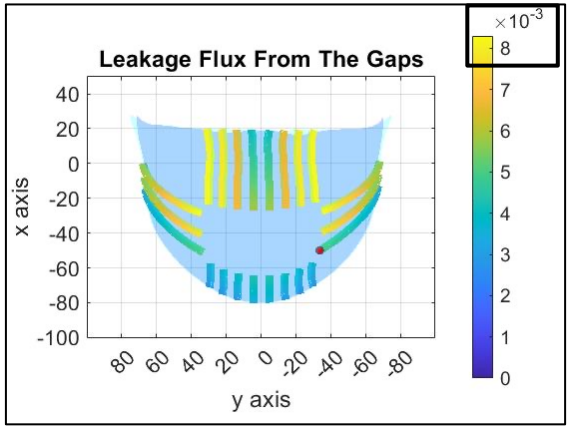
Leakage Flux Through Facial Features

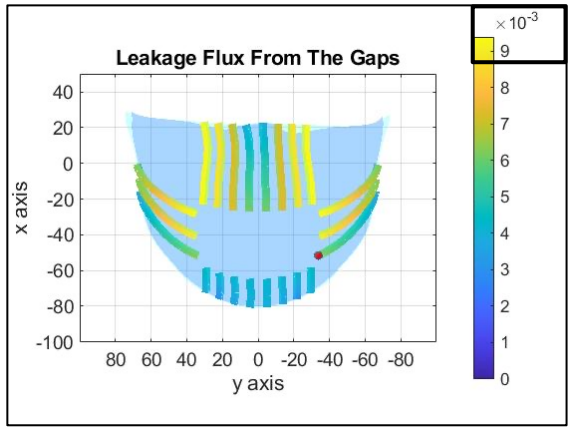
Forehead



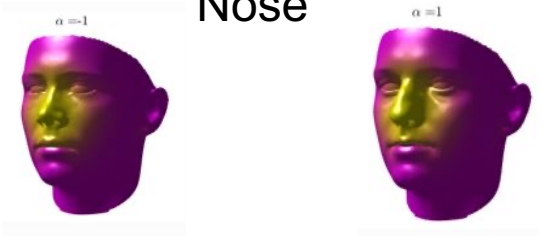


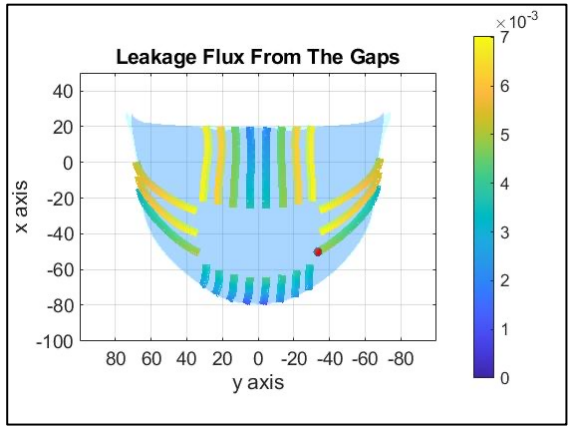
Eyes

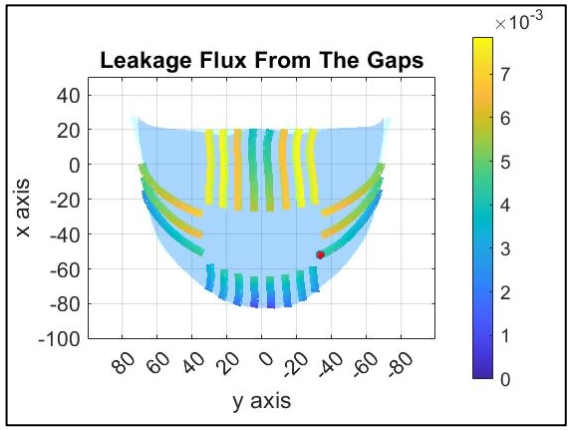




Nose



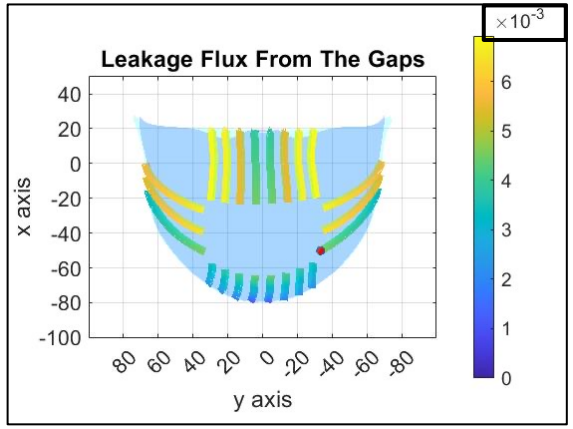


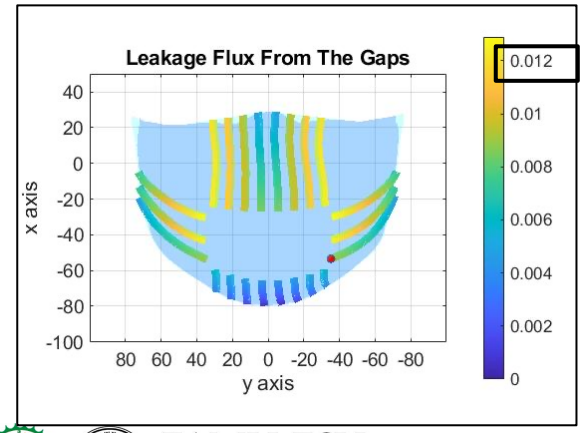


FAMU-FSU College of Engineering

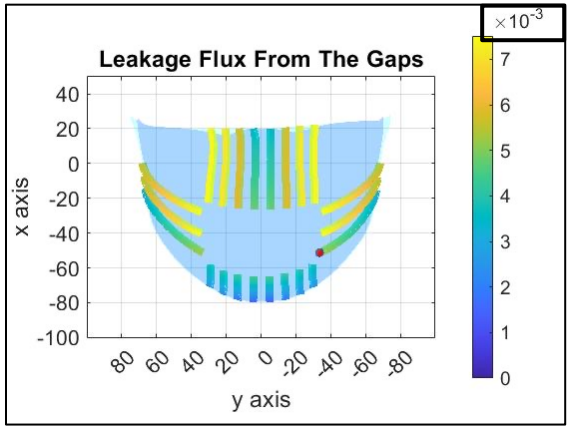
Leakage Flux Through Facial Features

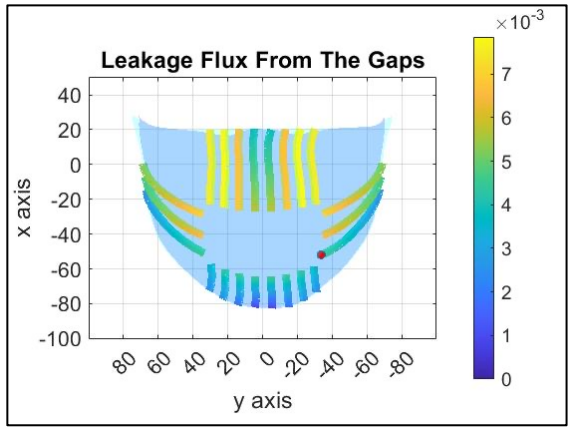
Cheek





Chin



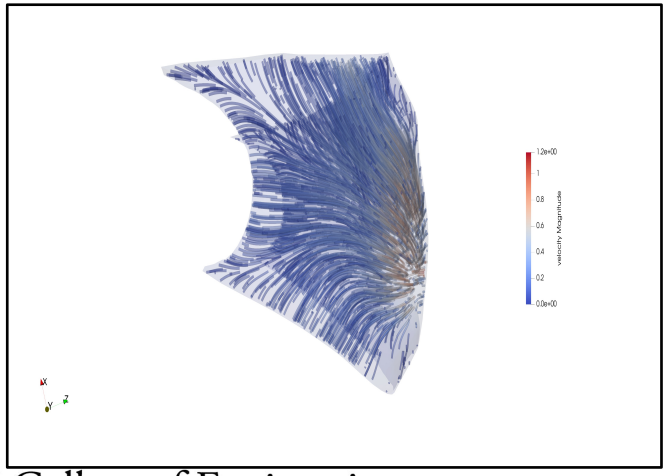


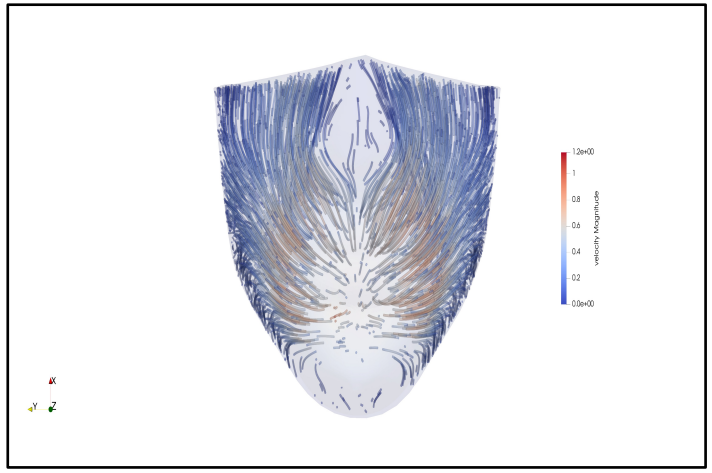
- Leakage flux from the facial feature forehead and eyelines does not have significant contribution
- Leakage flux from cheek and chin are more pronounced over other facial features

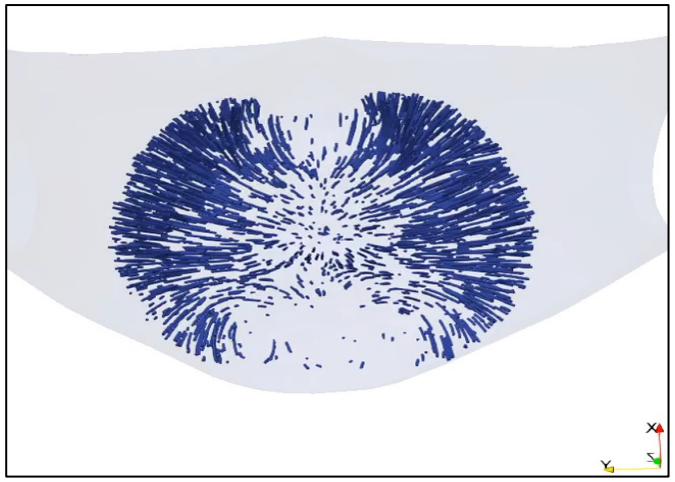
FAMU-FSU College of Engineering

Conclusion and Future Works

- Facial topology plays a significant role in the mask fitting on individuals face
- Majority of flow (air, aerosols) leaks from the gap if mask fit is poor
- Next, step is to connect these analysis with larger gaps and find correlation between different population
  - This approach can be used to quantify the flow
  - Developed model can be treated a multi-level component of computational framework to quantify the leakage pattern and mask efficacy







Acknowledgement

Funding Source:

- This material is based upon work supported by the National Science Foundation under Grant No. **CBET-2034992**.Any opinions, findings, and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the National Science Foundation

- Thanks to the computing resources provided by Florida State University's Research Computing Center (RCC)

Akshay Anand
aanand@fsu.edu