

# Stats\_project

November 26, 2024

Where to take driving exam? Candidate number: 50691

## 1 Abstract

My advice to XYZ (21 year old woman from Reading), supported by statistical evidence, is to take the driving exam in Reading. However the data is not perfect and she has to study hard for the test, as we do not know the skills/practice of people who take the exam in Reading and London.

```
[1]: ID <- 202215485
      source("XYZprofile.r")
      XYZprofile(ID)
      # With average driving skills.
```

The profile of XYZ:

- Age: 21
- Gender: Female
- Home address: Reading

```
[20]: library(dplyr)
      set.seed(100)
```

## 2 Exploratory data analysis:

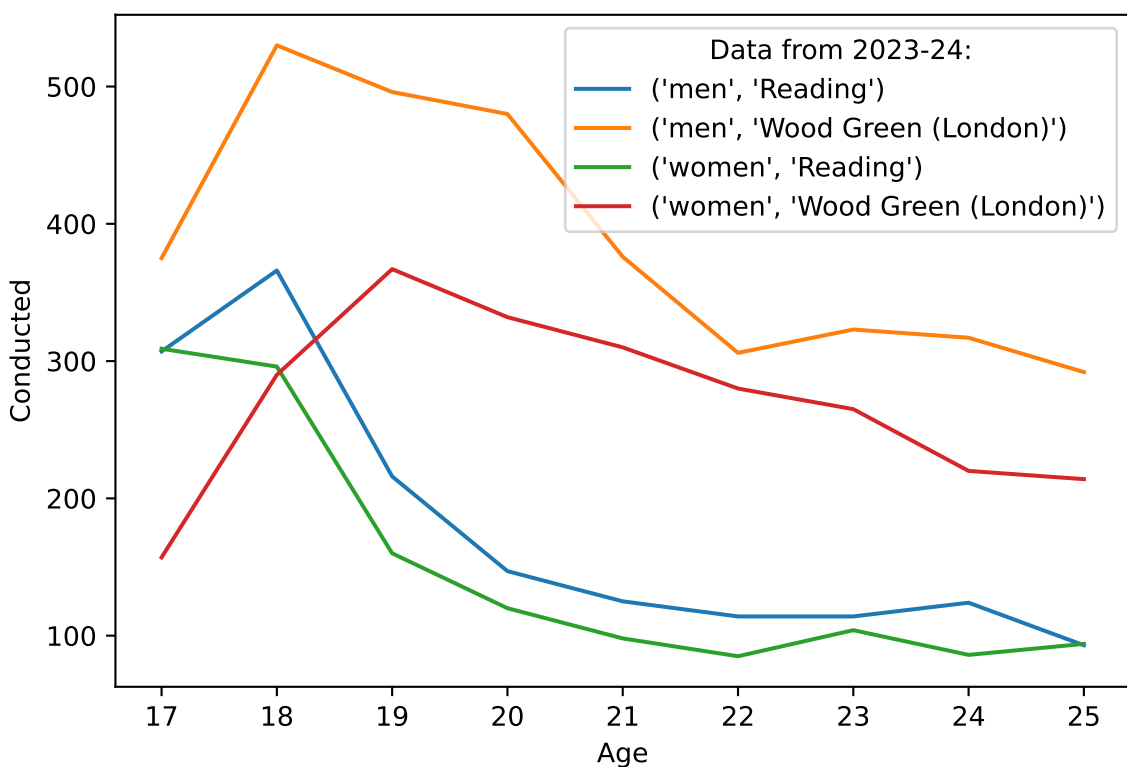
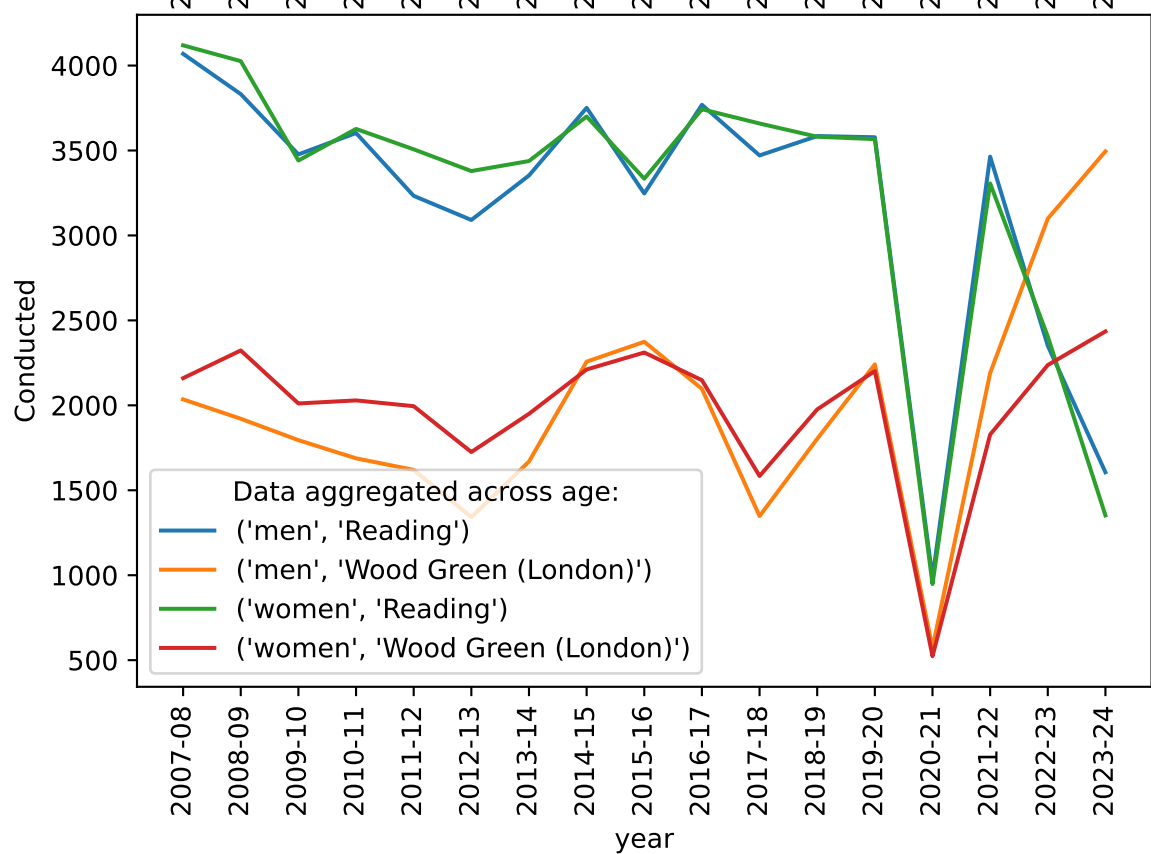
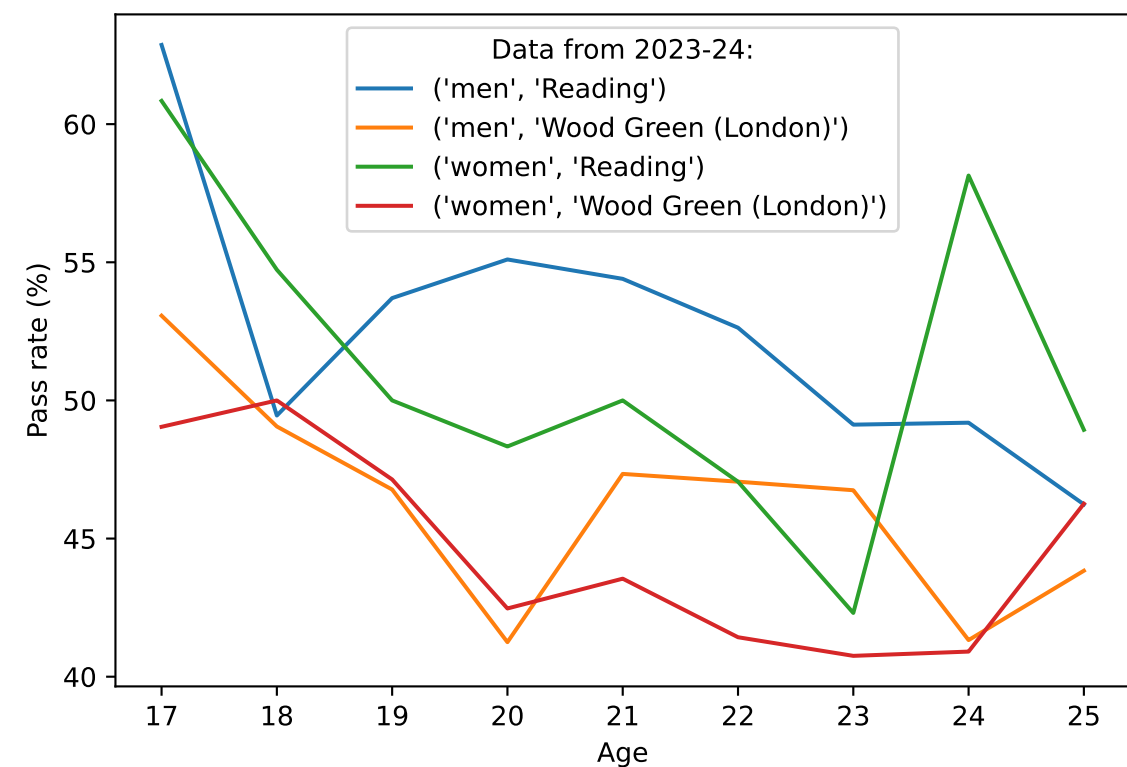
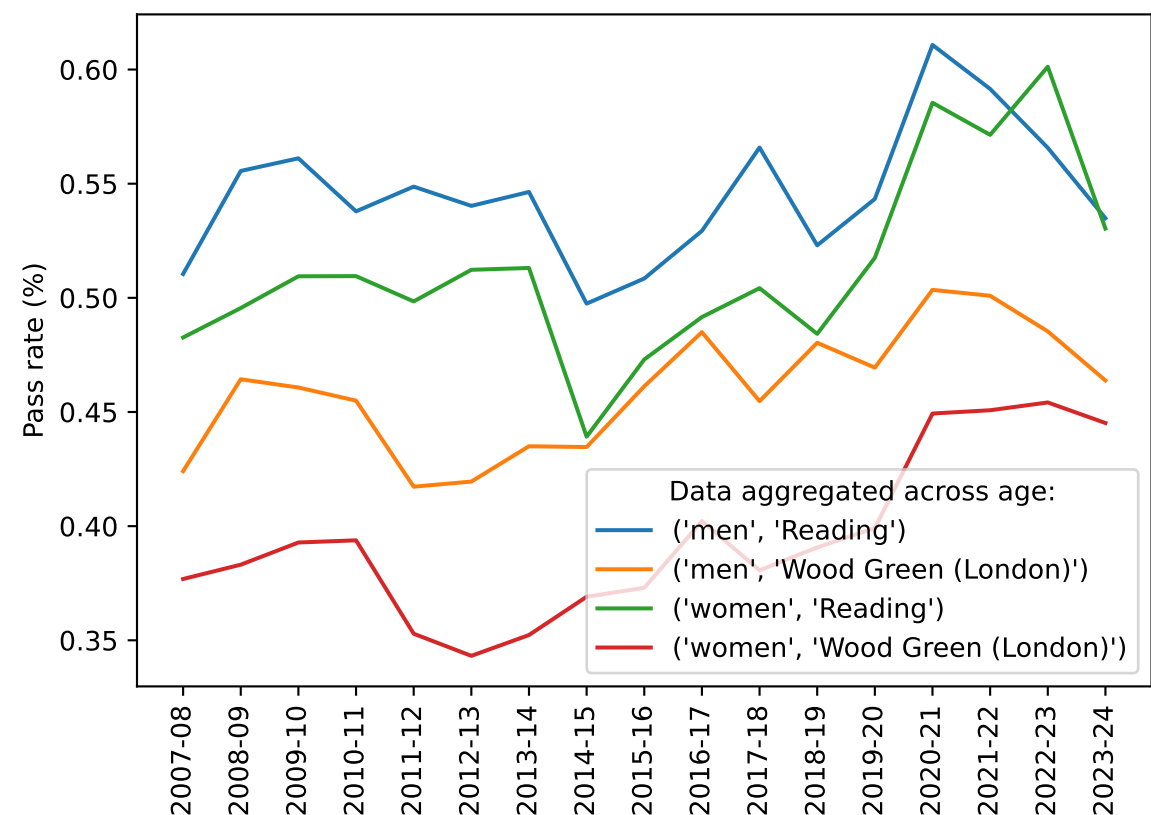
I downloaded and cleaned the data using Python, this code is omitted due to space limitations. I then created the plot below which serves as exploratory data analysis.

```
[3]: passing_rates_reading <- read.csv("passing_rates_reading.csv")
```

```
[4]: passing_rates_reading[15:20,]
```

		Location	Age	Conducted	Passes	Pass.rate....	gender	year
		<chr>	<dbl>	<dbl>	<dbl>	<dbl>	<chr>	<chr>
A data.frame: 6 × 7	15	Wood Green (London)	22	306	144	47.05882	men	2023-2
	16	Wood Green (London)	23	323	151	46.74923	men	2023-2
	17	Wood Green (London)	24	317	131	41.32492	men	2023-2
	18	Wood Green (London)	25	292	128	43.83562	men	2023-2
	19	Reading	17	309	188	60.84142	women	2023-2
	20	Reading	18	296	162	54.72973	women	2023-2

Evolution of passing rates and total exams conducted, across time (aggregated by age) and across age (for 2023-24)



```

p_rate_age_subset <- data.frame(subset(p_rate_age_subset, year=="2023-24" &
↳gender=="women"))

rownames(p_rate_age_subset) <- p_rate_age_subset$Location

#Put the elements in a nice vector for subsequent analysis.

pass_r <- p_rate_age_subset["Reading", "passes"]
tot_r <- p_rate_age_subset["Reading", "conducted"]
pass_l <- p_rate_age_subset["Wood Green (London)", "passes"]
tot_l <- p_rate_age_subset["Wood Green (London)", "conducted"]

data_for_test <- c(pass_r,tot_r,pass_l,tot_l)

return (data_for_test)
}

```

```

[6]: # This code was copy pasted from the latter permutation function, it is very
↳similar.
cond_mean_boot <- function(params, iter){
  # The pre processing function will give the parameters in the following
↳order:
  pass_r <-params[1]
  total_r <- params[2]
  pass_l <- params[3]
  total_l <-params[4]

  estimate_r <- round(pass_r/total_r, digits = 2)
  estimate_l <- round(pass_l/total_l, digits = 2)

  return(c(estimate_r,estimate_l))
}

```

```

[7]: param <- probs_for_test(passing_rates_reading,c(21))
cond_mean_boot(param,10000)

```

1. 0.5 2. 0.44

```

[8]: param <- probs_for_test(passing_rates_reading,c(20,21,22))
cond_mean_boot(param,10000)

```

1. 0.49 2. 0.43

```

[9]: param <- probs_for_test(passing_rates_reading,c(19,20,21,22,23))
cond_mean_boot(param,10000)

```

1. 0.48 2. 0.43

Predicted passing rate (women in 2023/24)	Women aged 21	Women aged 20-22	Women aged 19-23
Reading	0.5	0.49	0.48
London	0.44	0.43	0.43

To estimate XYZ's passing rate you can just divide the total number of people who pass over the total number of people who take the exam. The only difficult part is to determine who you should count. Following my exploratory analysis I only count women aged 21 for the first analysis and expand it to women aged 20-22 for the second one; arguing that women in this age range are very similar. In all cases I only use data from 2023-24.

In Reading, the expected probability of passing the exam seems to be roughly 50% for women. In London this expectation is a bit smaller, roughly 44% of women pass the test.

This analysis requires weak conditions (Law of Large Numbers) that are satisfied, however it is important to know that this expected passing rate is for a group of women aged xxx taking the exam in Reading or London (for 2023-24), if we knew the distribution of skill/practice time amongst these women, I would be able to give a more precise prediction for XYZ. I touch upon this in the Limitations section.

Technical note: I built a confidence interval for completeness using non-parametric bootstrap but I removed it as it was very related to the following testing part.

## 4 Where to take the test, are the passing rates for women higher in Reading?

Using the Permutation test I get to the conclusion that XYZ should take the exam in Reading, this test relies on few assumptions. I previously use the Wald test which gives inconclusive results under different stronger assumptions, so I prefer the results from the Permutation test.

### 4.1 Wald test:

This is a statistical test that can be derived under certain assumptions.

Issue 1: This is a test that requires you to have a very big sample so it might not work for my limited ones, especially in the case where I only include women aged 21.

Issue 2: This test requires an assumption on the distribution of the data that is not likely to hold. It basically requires that passing the exam is like flipping the same coin for everyone. But we know that people go in to the exam having practiced different amounts. Therefore a different coin is being flipped for each individual. For those that practice more, the coin that is flipped is more likely to result in a pass.

Technical note: This can be derived using MLE under the assumption that the data comes from two independent Binomial distributions.

```
[10]: wald_test <- function(params){
      pass_r <-params[1]
      size_r <- params[2]
```

```

pass_l <- params[3]
size_l <-params[4]

prob_r <-pass_r/size_r
prob_l <-pass_l/size_l

# Assuming a Binomial distribution this is the standard error of the
↪estimator.
se_mle <- sqrt( (prob_r)*(1-prob_r)/size_r + (prob_l)*(1-prob_l)/size_l )

WALD <- (prob_r-prob_l)/se_mle

return(WALD)
}

```

We are looking for values as close to 0 as possible. Those would allow us to say that it is better to take the exam in Reading.

```

[11]: p_val_wald <-function(wald) {
      p_val <-2*(1-pnorm(wald))
      return(round(p_val,digits=2))
    }

```

```

[12]: p_val_wald(wald_test(probs_for_test(passing_rates_reading,c(21))))

```

0.26

Under the most strict assumptions we can't recommend XYZ to go to Reading. We can try to add more similar people, women aged 20 and 22 to our sample.

```

[13]: p_val_wald(wald_test(probs_for_test(passing_rates_reading,c(20,21,22))))

```

0.07

This now shows moderately strong evidence in favour of taking the exam in Reading. However our assumptions may not be correct so this number could be meaningless.

```

[14]: p_val_wald(wald_test(probs_for_test(passing_rates_reading,c(19,20,21,22,23))))

```

0.07

## 4.2 Permutation test:

This is a very nice test that can compare two distributions or any statistic based on those distributions. It makes few assumptions, works for any distributions and does not require the sample to be very large. It is because of this lack of assumptions that I prefer to follow the recommendations from this test. The results from this test lead me to recommend taking the exam in Reading.

```

[21]:

```

```

permutation_test <- function(params, iter){
  # The pre processing function will give the parameters in the following
  # order:
  pass_r <- params[1]
  total_r <- params[2]
  pass_l <- params[3]
  total_l <- params[4]

  # We generate a fake sample, for London and Reading respectively
  # They will have the same number of passes and fails observed in the data.
  sample_r <- c(numeric(pass_r)+1, numeric(total_r-pass_r))
  sample_l <- c(numeric(pass_l)+1, numeric(total_l-pass_l))

  full_sample <- c(sample_r, sample_l)

  original_statistic <- mean(sample_r) - mean(sample_l)

  larger_than_og <- c()

  for (i in 1:iter){
    # Sampled without replacement. -> Permutation
    # There may be repeated permutations but the probability is small so I
    # will not correct for it as it would be very expensive to check.
    permutation <- sample(full_sample)
    # The first elements will be "reading people"
    perm_r <- permutation[1:total_r]

    # These are "london people"
    perm_l <- permutation[(total_r+1):length(permutation)]

    # Compute the statistic with the permuted sample.
    perm_statistic <- mean(perm_r) - mean(perm_l)
    # Keep TRUE=1 and FALSE=0
    larger_than_og <- c(larger_than_og, perm_statistic > original_statistic)
  }

  p_val_perm <- mean(larger_than_og)
  return(round(p_val_perm, digits=2))
}

```

```

[19]: params <- probs_for_test(passing_rates_reading, c(21))
      permutation_test(params, 10000)

```

0.11

We can see that even in the case where we only include women aged 21 we have moderately strong evidence in favour of going to Reading to take this exam.

```
[17]: params <- probs_for_test(passing_rates_reading,c(20,21,22))
      permutation_test(params,10000)
```

0.03

If we include women aged 20-22, we have very strong evidence in favour of going to Reading. So that is my final recommendation.

```
[18]: params <- probs_for_test(passing_rates_reading,c(19,20,21,22,23))
      permutation_test(params,10000)
```

0.03

We are looking for small numbers ( $z$ ), 0.1 or 0.05. These numbers mean that if the candidates repeated the test 100 times and the probabilities of passing were the same in both London and Reading, we would falsely recommend to go to Reading  $100 \cdot z$  times.

We can see in the table below that even for the most stringent assumptions - to only consider women aged 21 as a valid comparison group - the permutation test gives moderate evidence towards discarding the baseline hypothesis - passing rates being the same in both locations -. For the assumption that women aged 21 are identical in all important aspects to women aged 20 and 22, the permutation test gives strong evidence towards taking the exam in Reading.

Test p-values (women in 2023/24)	Women aged 21	Women aged 20-22	Women aged 19-23
Wald test	0.26	0.07	0.07
Permutation test	0.11	0.03	0.03

## 5 Limitations

- Time series: I largely avoid making predictions for this present year (2024-25) -which would be the most relevant quantity for XYZ-, because that is time series and a bit beyond the scope of this course. But it does look like passing rates in Reading are going down. And, since a post covid recovery, the number of exams taken in Reading have been going down quite a lot, so something strange could be going on there.
- Skill differences: My estimators aggregate/integrate over an unknown skill distribution that might vary across location. Practice/skill is a very important predictor on passing the exam or not. This might bias our results. e.g.

The exam is very difficult in London so people take more lessons to prepare than in Reading, therefore if we conditioned on practice time, the difference in passing rates between Reading and London would be greater.

Practice lessons are very expensive in London, that may drive people to practice less, if we conditioned on practice time, the difference in passing rates could be smaller or even negative. That might be because although driving in London is more difficult, you are going to be stuck in traffic during half of the exam, which minimizes opportunities to make mistakes.

Note that both these scenarios are in agreement with the fact that XYZ reports she has average skill/practice time.