Big Data Computing

Master's Degree in Computer Science

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SAPIENZA
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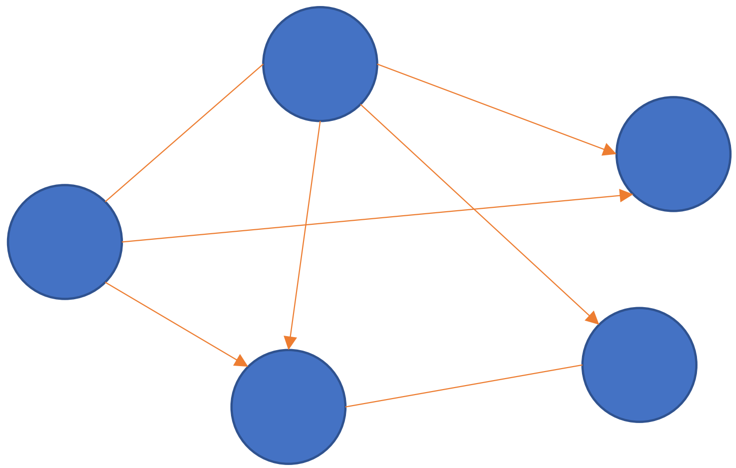
  - **Social Networks** (i.e., the set of social connections between people)
  - ...

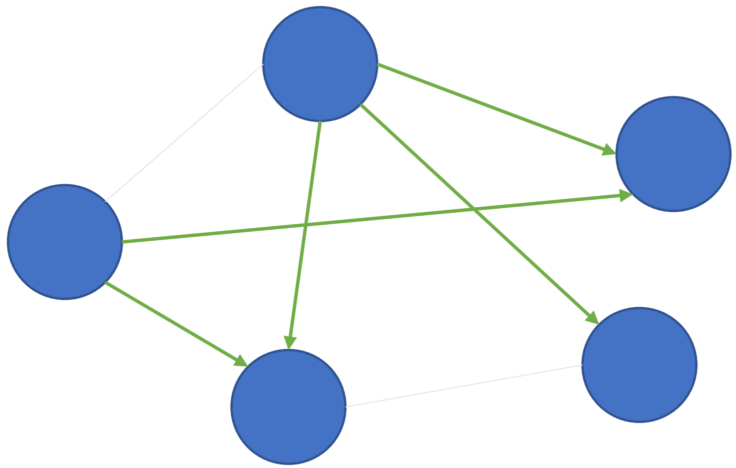
What is a Graph?

Informally, a set of vertices (nodes) connected by a set of edges (links)



What is a Graph?

edges may be directed



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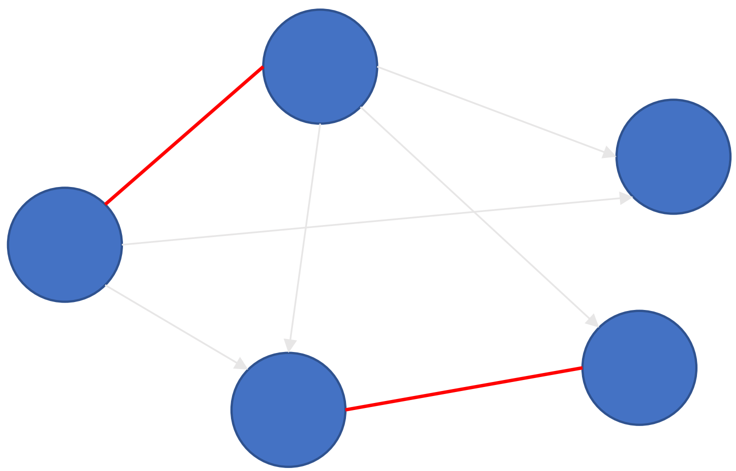

Directed vs. Undirected

Directed

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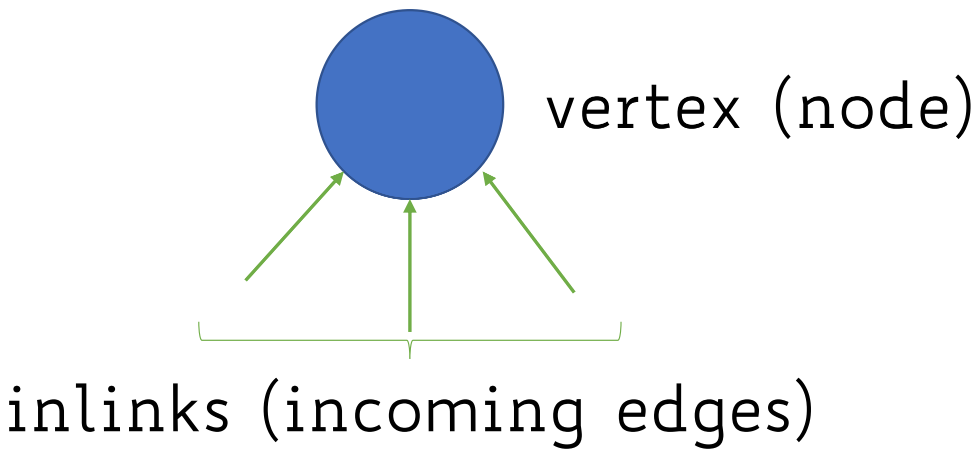

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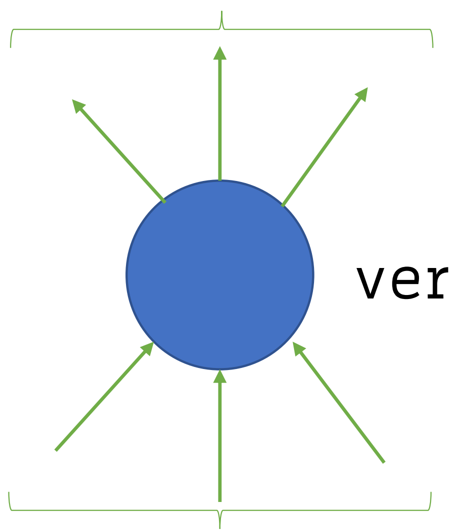

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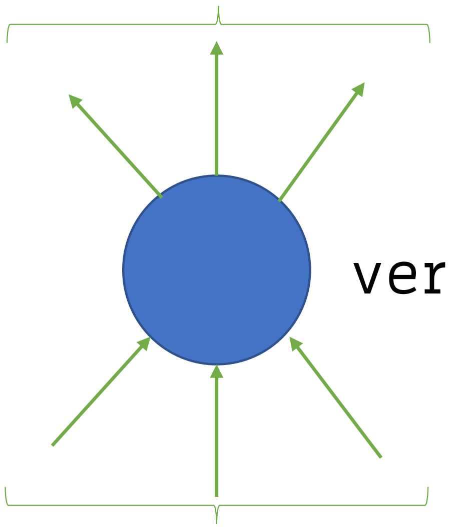

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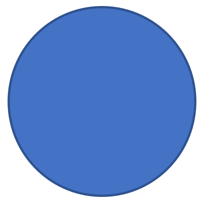

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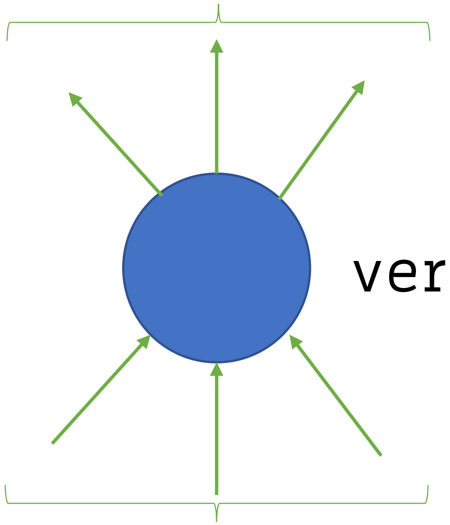

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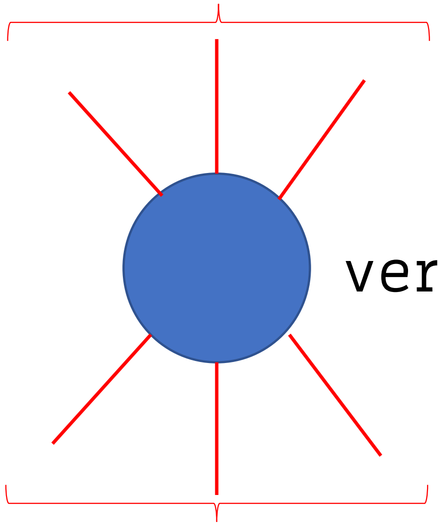

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Graph Formalism

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Note that an undirected graph is just a special case of a directed graph where the set of edges contain symmetric pairs of vertices

Node's Degree

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To be more explicit, in the case of a directed graph sometimes we distinguish between in-degree and out-degree

$$\text{in-deg}(v) = |\{u \in V | (u, v) \in E\}|$$

$$\text{out-deg}(v) = |\{u \in V | (v, u) \in E\}|$$

How Do We Represent Graphs?

3 main ways of representing graphs

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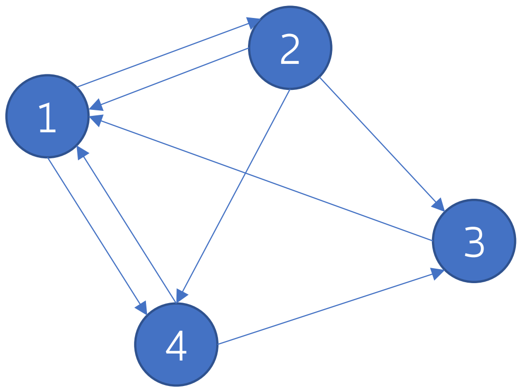

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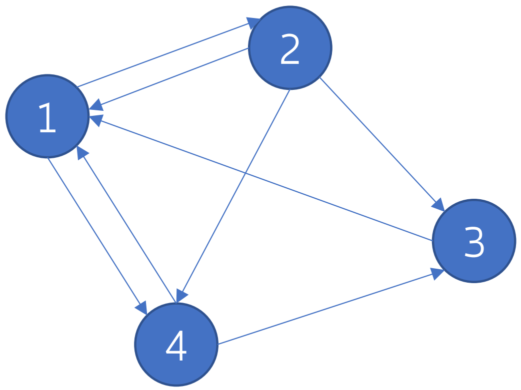

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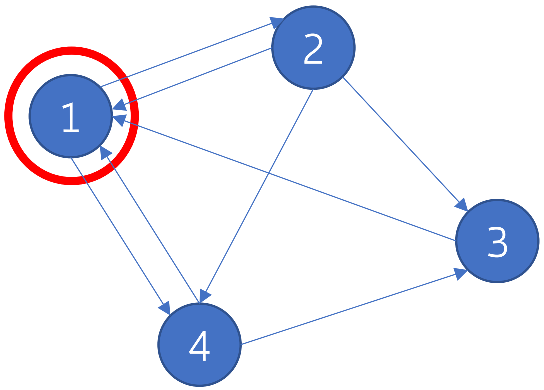

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- PROs:
  - Most intuitive representation
  - Ready-to-go for mathematical manipulation
- CONs:
  - Space inefficient (especially for loosely connected graphs, i.e., sparse matrices)
  - Easy to write yet hard to compute

Adjacency Lists

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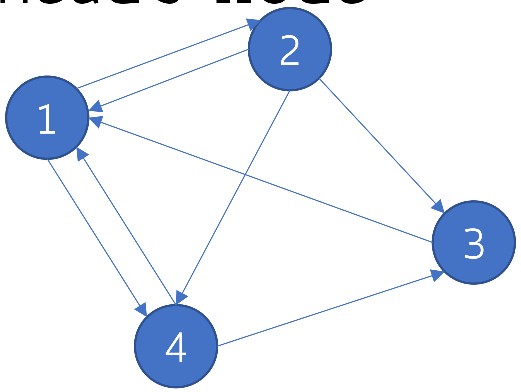

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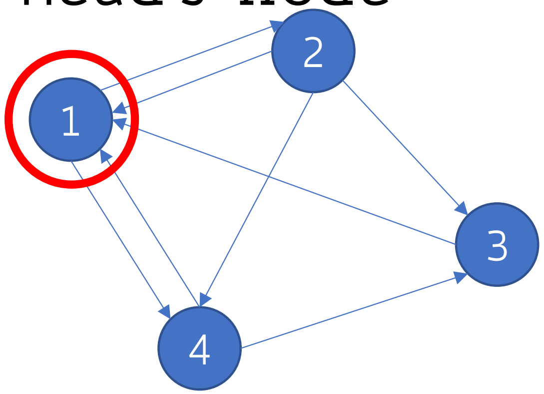

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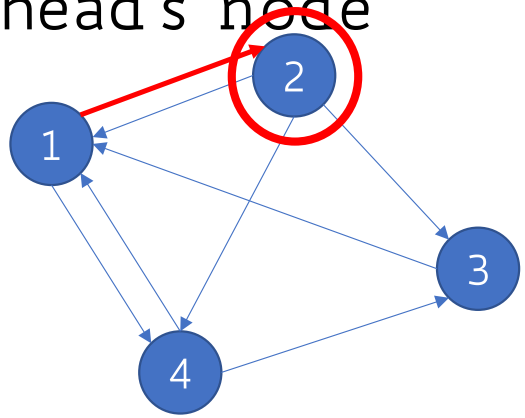

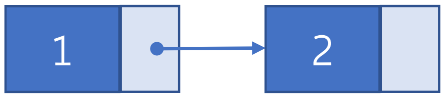

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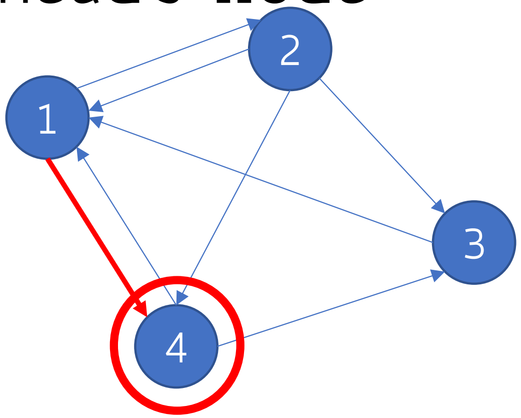

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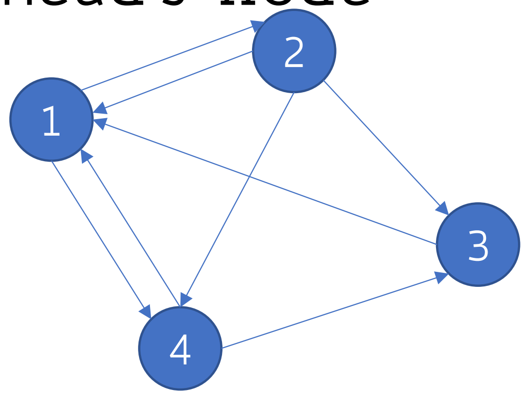

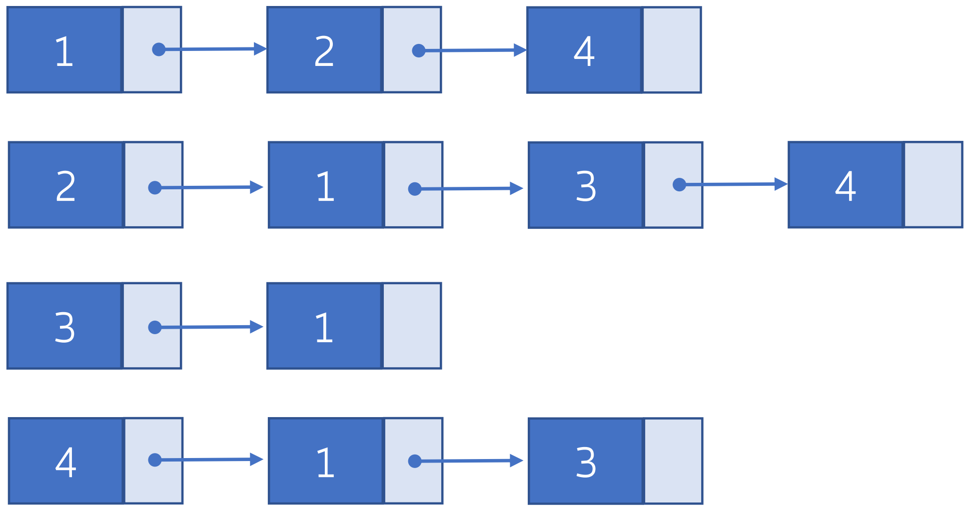

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Note that with adjacency matrix, any computation over incoming (outgoing) links reduces to a column (row) scan of the matrix

Edge Lists

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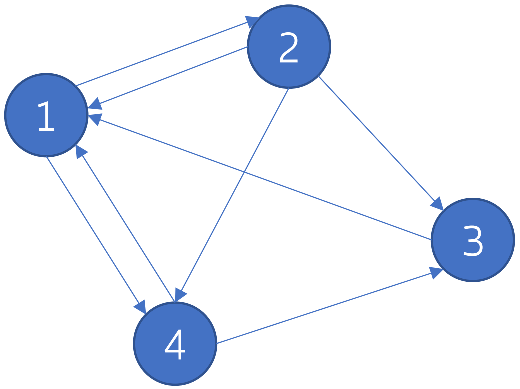

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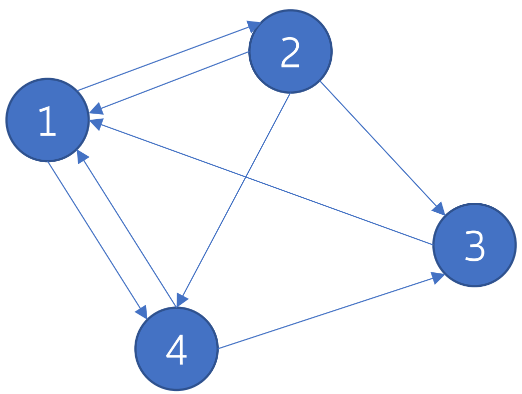

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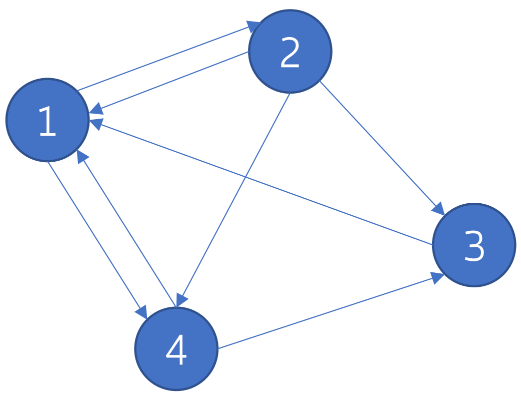

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Waste of space

Some Famous Graph Problems

Problems

Applications

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- The **Web graph** is a great test bed for link analysis

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- Other attempts: DMOZ, LookSmart

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- The list of top-k documents most similar to a query are returned (e.g., measuring **cosine similarity** between each query-document pair)

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The Web is huge and full of untrusted documents!

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Trustworthy web pages should point to
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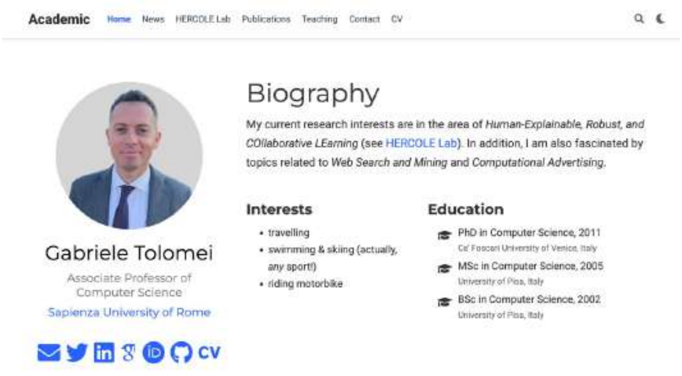

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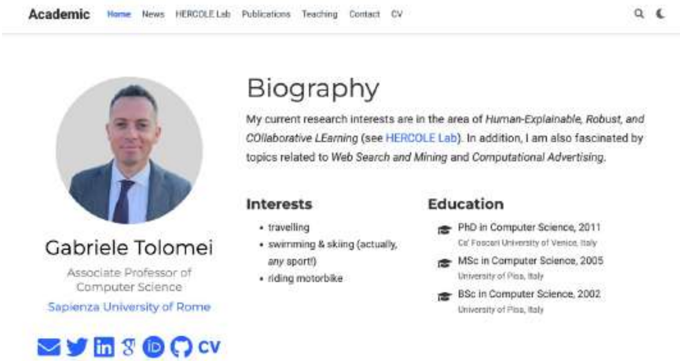

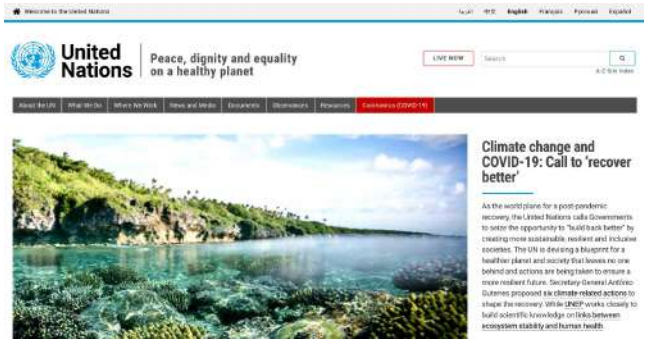

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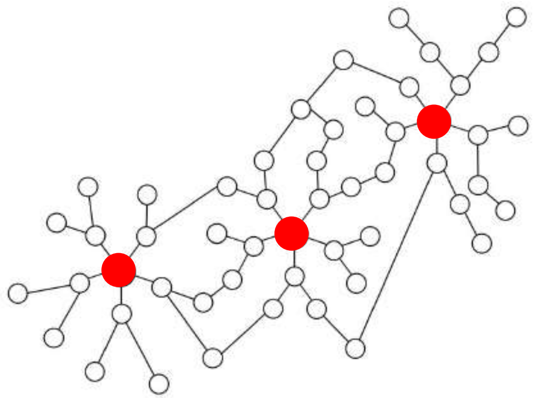

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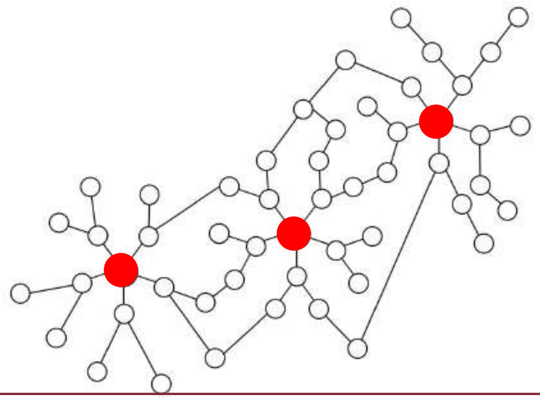

Rank nodes (i.e., assign them an importance score) on the basis of their connectivity

The Web as a "Scale-Free" Network

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They refer to graphs (i.e., networks) exhibiting such a behavior as scale-free networks

Barabási, A.L. and Albert,R. Emergence of scaling in random networks, Science, 286:509-512, October 15, 1999

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Generate a random graph where each pair of nodes is connected independently with the same probability p

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The degree of a node follows a **Binomial distribution**

$$deg(u) = Binomial(n-1, p)$$

The Erdős–Rényi Model: Approximations

Poisson (sparse graphs):

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Normal (dense graphs):

If n is large and p is not too small → $deg(u) \approx Normal(\mu, \sigma^2)$, where $\mu = p(n-1)$; $\sigma^2 = (n-1)p(1-p)$

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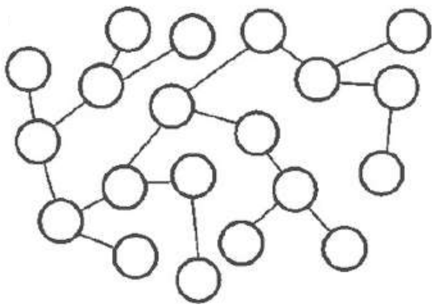
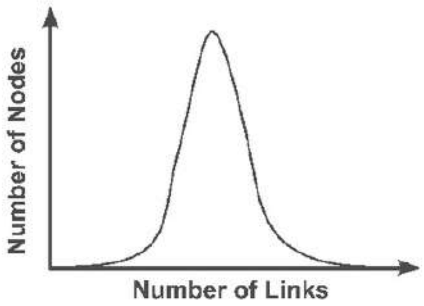
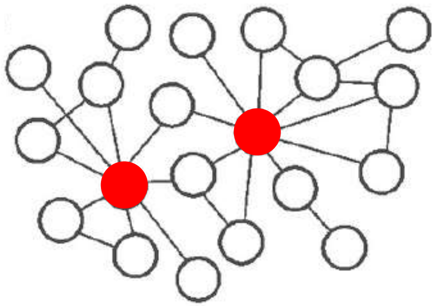


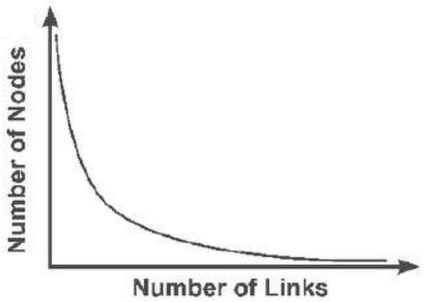


Random Graph

Most nodes have approximately the same number of links producing a bell-shaped curve of the degree distribution

The Web as a Scale-Free Network





Scale-Free Graph

Most nodes have few links, and few nodes (i.e., red ones) have a large number of links, resulting into a power law degree distribution

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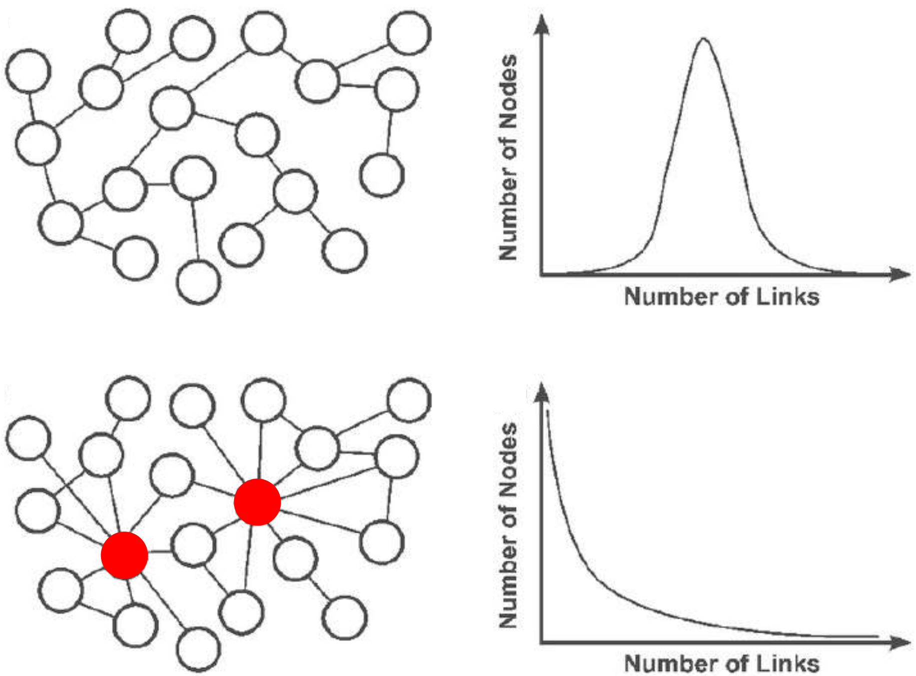

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Tëmkin, I. and Eldredge N. Networks and Hierarchies: Approaching Complexity in Evolutionary Theory. February 2015

Scale-Free Networks

The fraction of nodes in the network having k connections to other nodes follow a power law distribution

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The ratio of very connected nodes to the number of nodes in the rest of the network remains constant as the network changes in size

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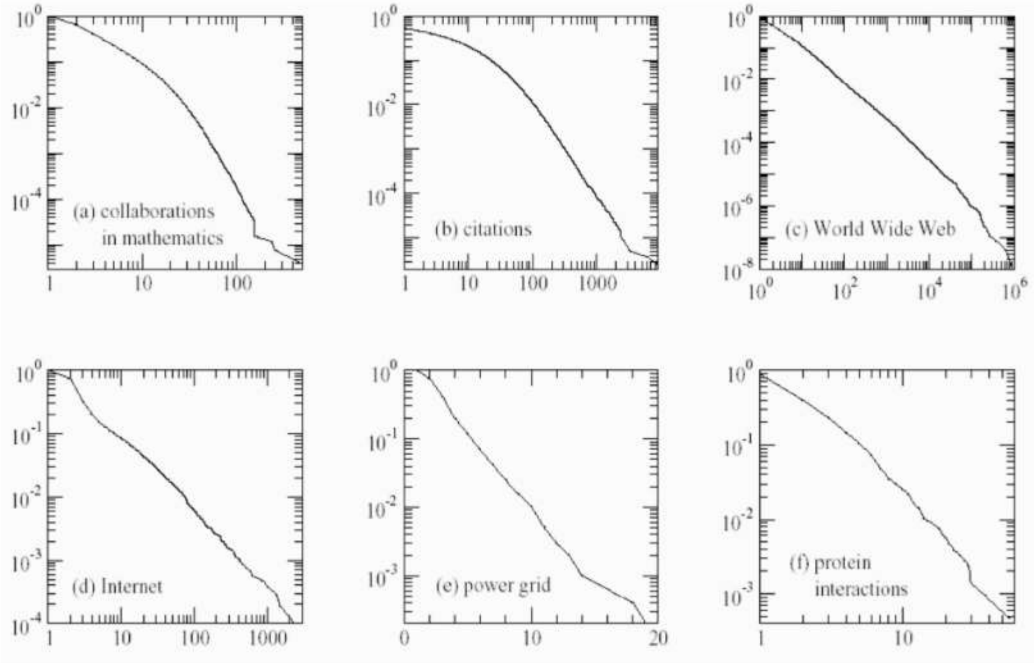
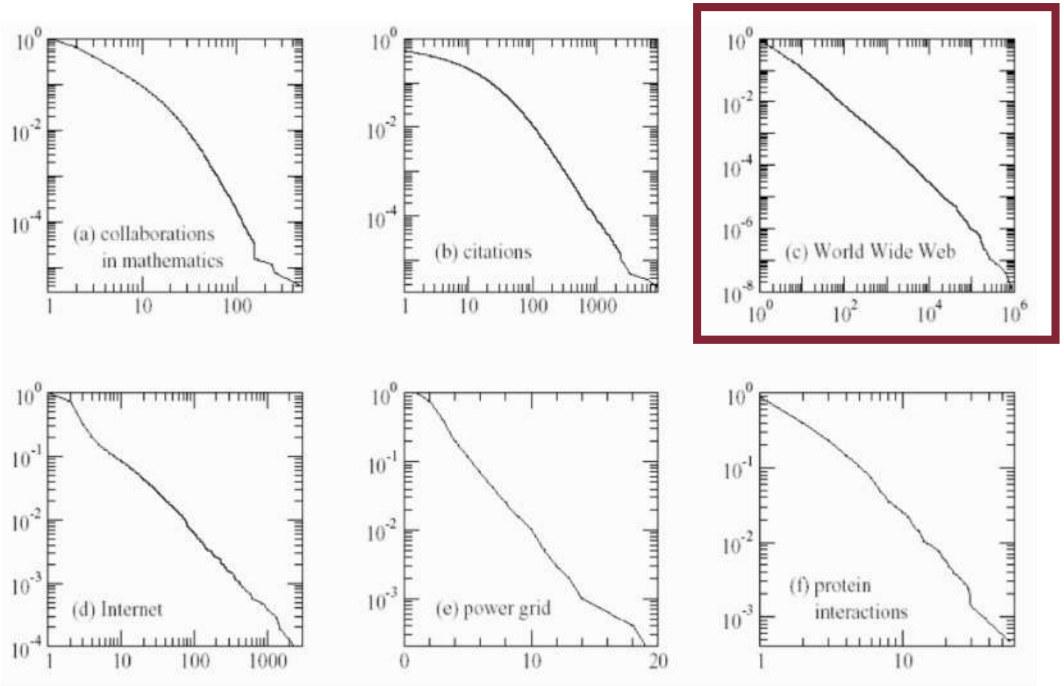
Scale-Free Networks: Examples



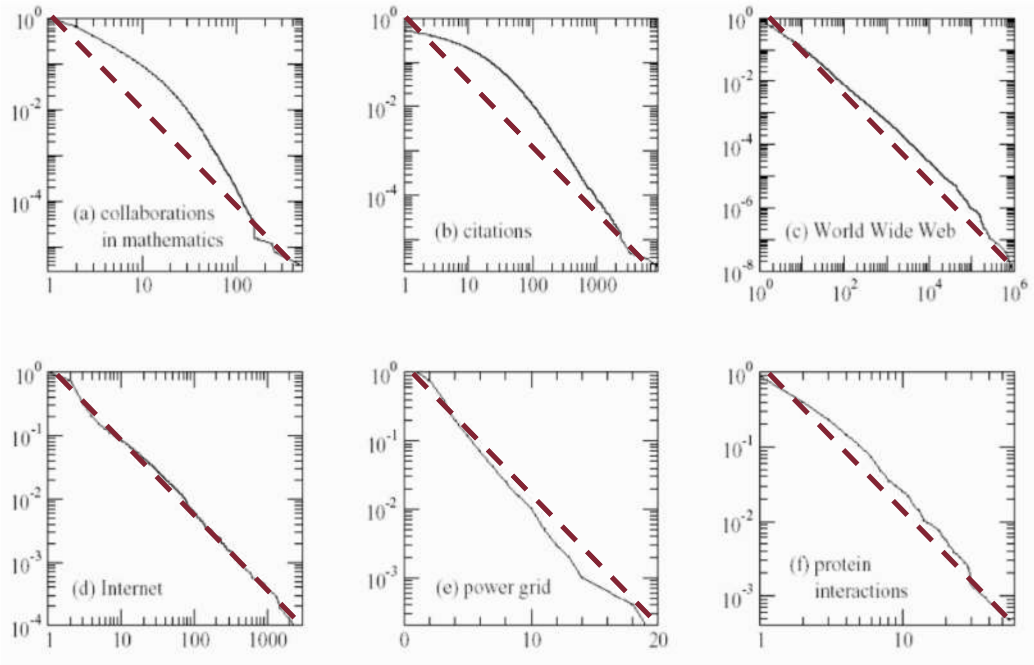
Many real-world networks are scale-free

Scale-Free Networks: Examples



The Web is one of those!

Scale-Free Networks: Examples



On log-log scale power law distributions look like straight lines

$$\log(p(k)) = \log(\alpha k^{-\gamma}) = \underbrace{\log(\alpha)}_{\text{constant } q} + \log(k^{-\gamma}) = q - \gamma \log(k)$$

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Several link analysis approaches to compute web page importance

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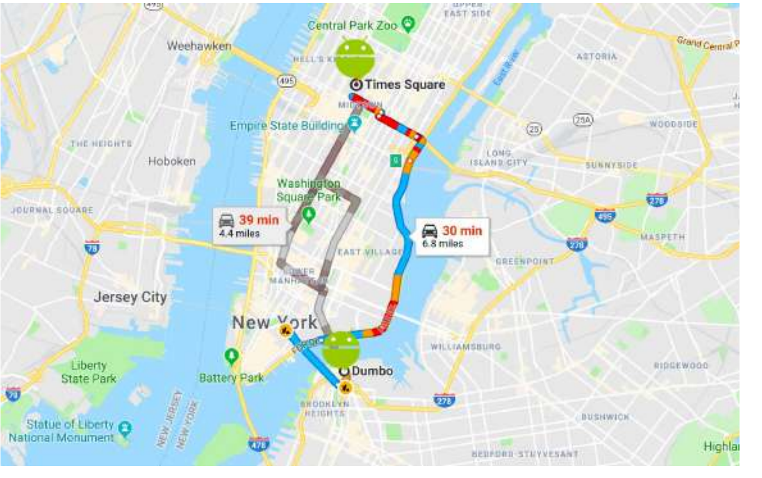

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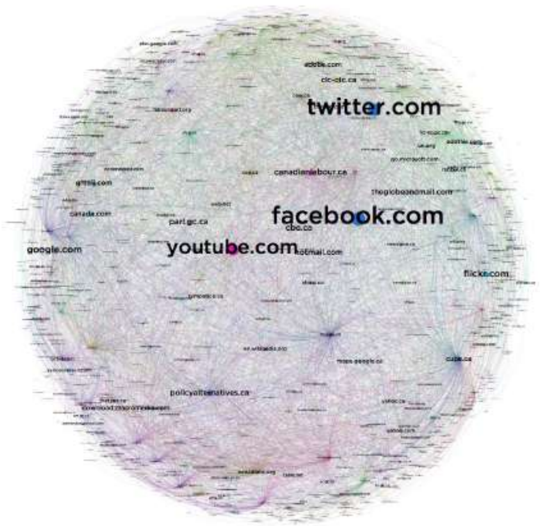

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- **Examples:**
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- **Idea:** Use node's connectivity to determine the **importance of a node**