Big Data Computing

Master's Degree in Computer Science

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SAPIENZA
UNIVERSITÀ DI ROMA

Recommendation Strategies

3 approaches to recommender systems

- Content-based filtering
- Collaborative filtering
  - Neighborhood-based (a.k.a. memory-based)
  - Latent-factor-based (a.k.a. model-based)
  - Hybrid (memory-model-based)
- Hybrid

COLLABORATIVE FILTERING

Collaborative Filtering (CF)

Idea

Recommend items to user u based on preferences of other users similar to u

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Core concept:

User-to-User or Item-to-Item similarity

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No need for explicit creation of user/item profiles

Collaborative Filtering: Approaches

3 main approaches to collaborative filtering

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Hybrid
(memory-model-based)

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Neighborhood-based (Memory-based) CF

Compute the relationship between users or items

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Evaluates a user's preference for an item based on ratings of "neighboring" users for that item

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Compute the relationship between users or items

User-based

Evaluates a user's preference for an item based on ratings of "neighboring" users for that item

Item-based

Evaluates a user's preference for an item based on ratings of "neighboring" items by the same user

USER-BASED COLLABORATIVE FILTERING

User-based Neighborhood

Given a user u and an item i not rated by u, we want to estimate r(u, i)

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From the set of users {u': u' != u} who have already rated i extract a subset of k neighbours of u

k-neighborhood of u is found on the basis of the similarity between user ratings without explicit user profiles

Estimate r(u, i) based on the ratings of users in the k-neighborhood of u

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In theory, rating prediction r(u,i) could be defined on any item i not rated by u

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In practice, we are interested only in estimating r(u,i) for those items i which have been rated by the u's k-neighborhood

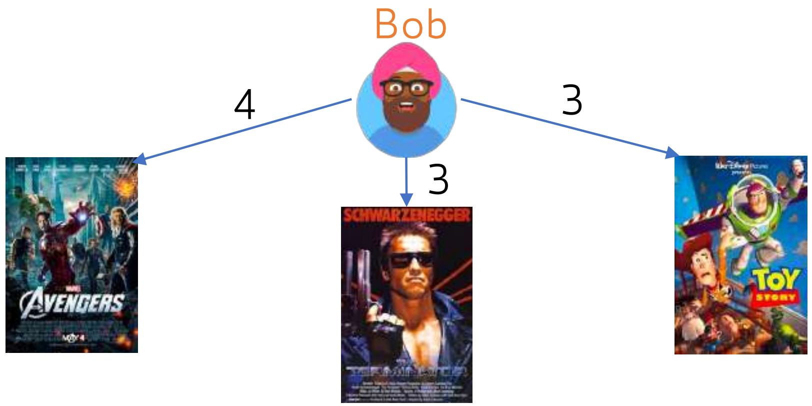
Intuitively, if a user v is not in the u's k-neighborhood then very likely u will not be interested in any item that only v has rated

In other words, the u's k-neighborhood must be computed first to narrow down the set of items which we must predict the rating of

User-based Neighborhood: Example

		MOVIES							
									
USERS	Alice	2		5	4	5	4		4
	Bob	4					3		3
	Carl	5	5	3	4	5	4		5
	...	...	...	...	...	...	...	...	...
	Zoe		1	3				5	4

User-based Neighborhood: Example



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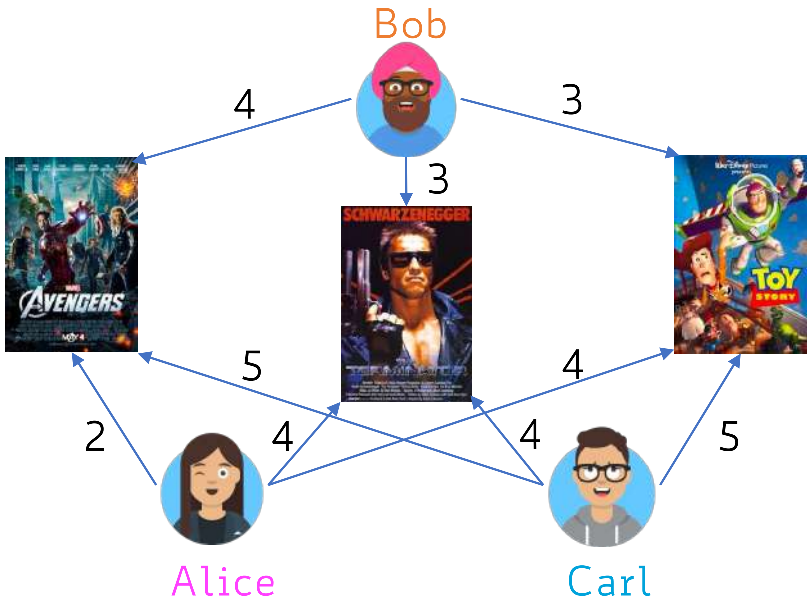

Alice and Carl are the 2-nearest neighbours of Bob
if we look at their rating behaviours

User-based Neighborhood: Example

Consider all the movies rated by Alice or Carl which Bob hasn't watched yet

User-based Neighborhood: Example

Predict the rating that Bob would give to each of those movies on the basis of Alice's and Carl's ratings

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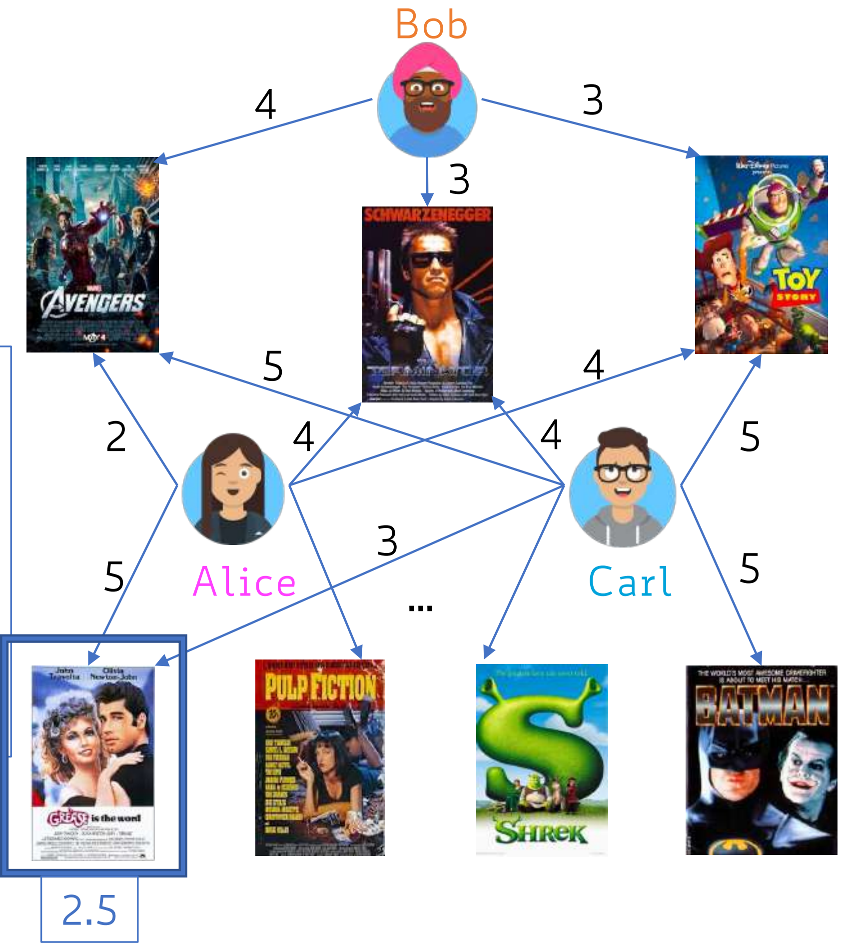

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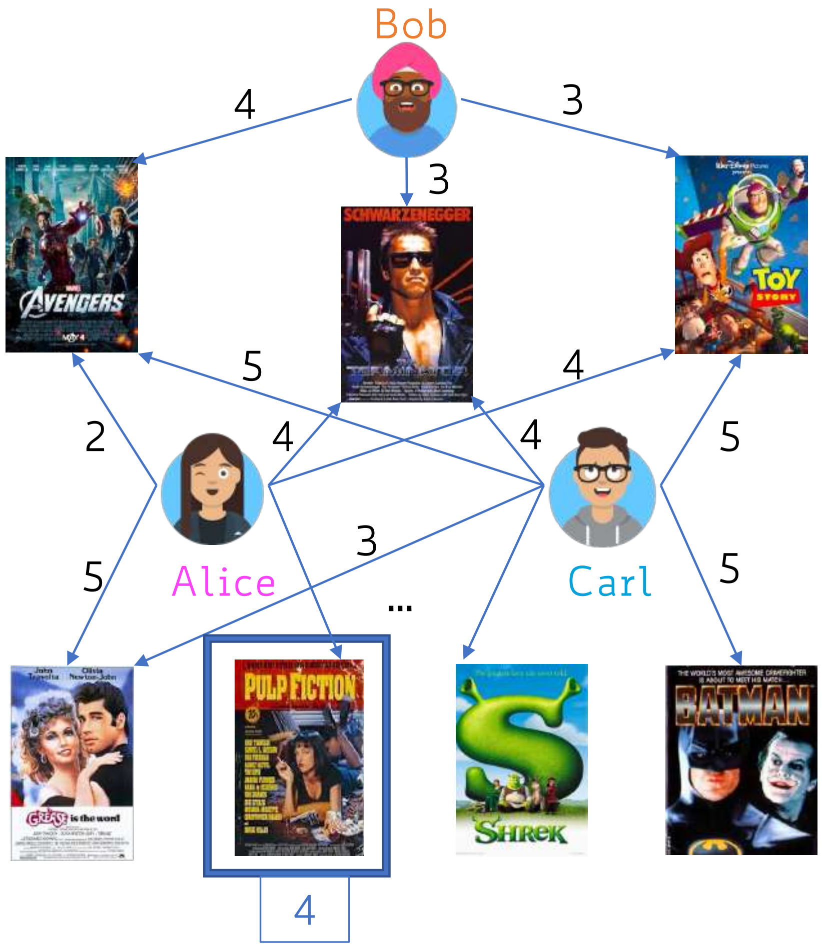

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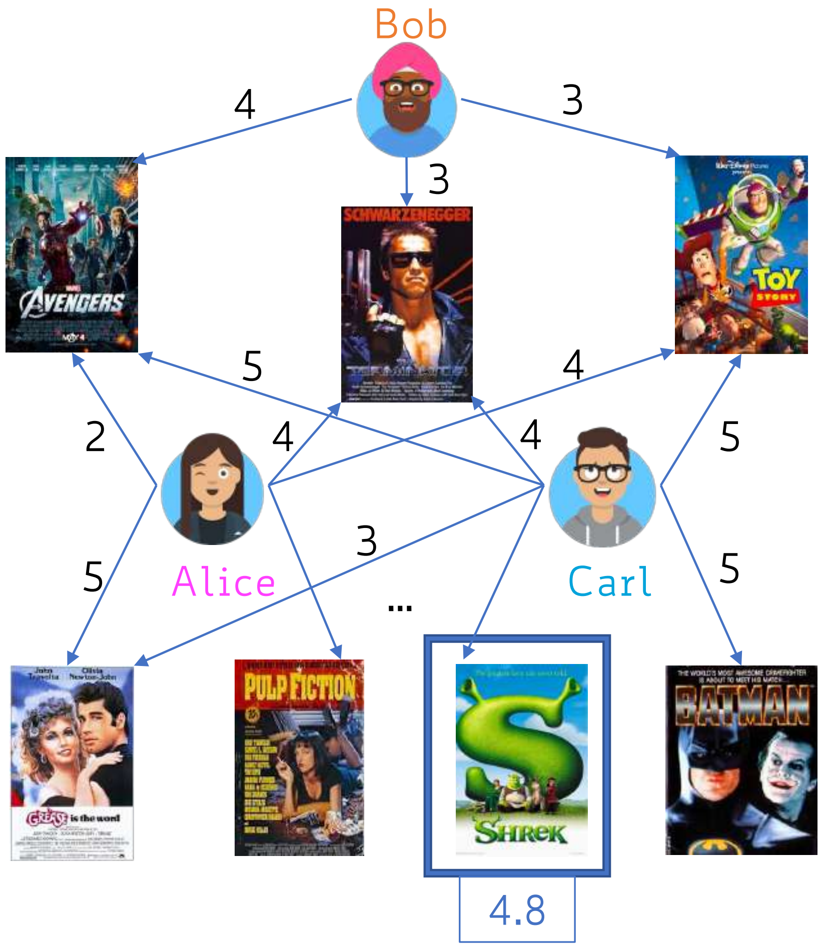

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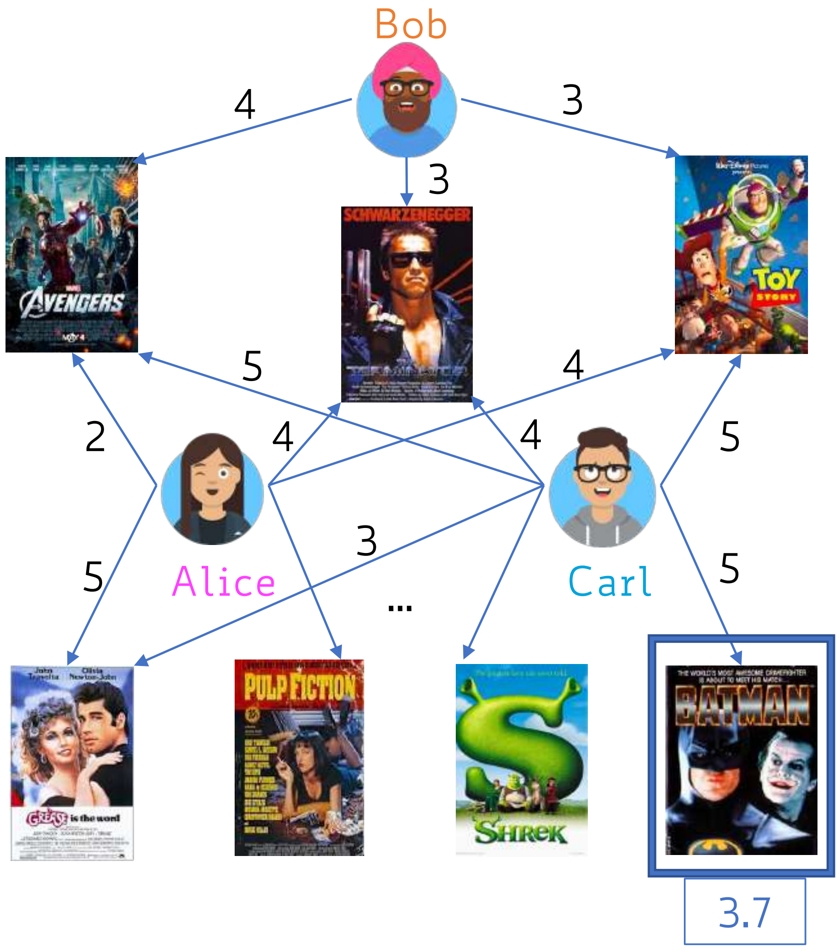
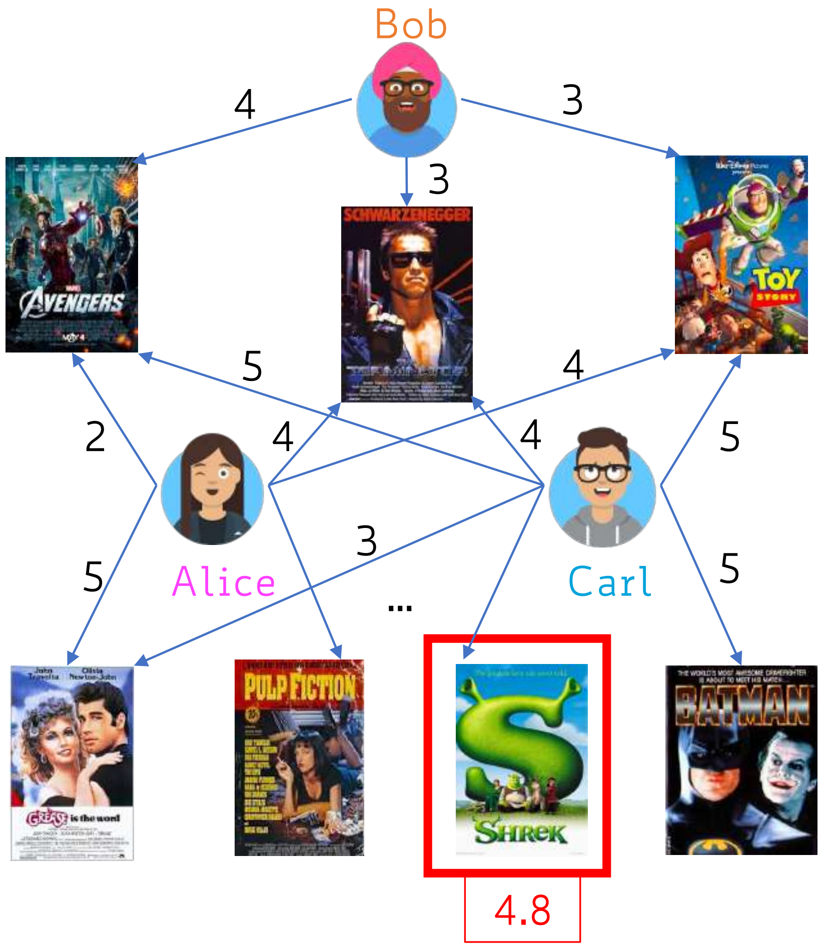


User-based Neighborhood: Example

Recommend the highest rated movie(s) to Bob!



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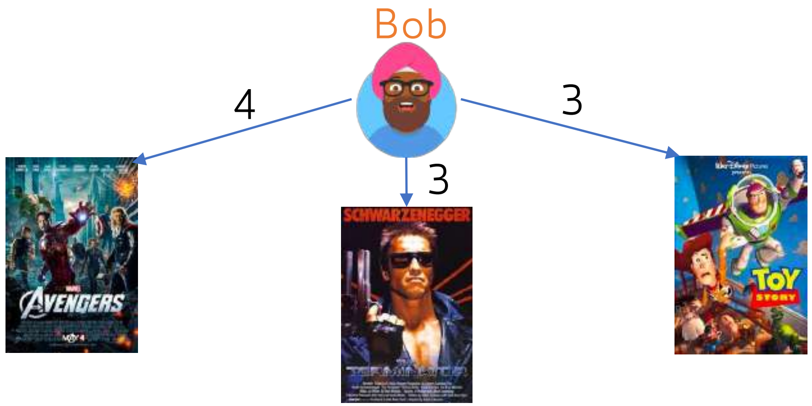
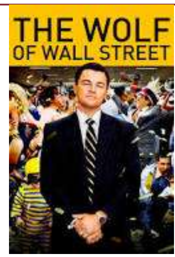


There is no point in predicting the rating of a movie which has only been rated by a user (Zoe) who is not in the Bob's neighborhood



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- Remember that we are **not** building any content-based user/item profile
- Intuitively, 2 users u_1 and u_2 are similar to each other if their ratings (of items) are similar
- Each user represented by her/his rating vector and similarity between them is measured in the item (rating) space

User-to-User Similarity

$\text{sim}(u, v)$ Similarity metric between any pair of users

		MOVIES							
		AVENGERS	BATMAN		PULP FICTION	SHREK	SCHWARZENEGGER	THE WOLF OF WALL STREET	TOY
USERS	Alice	2		5	4	5	4		4
	Bob	4					3		3
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Must capture the intuition: $\underbrace{\mathrm{sim}(\mathrm{Alice}, \mathrm{Carl})} > \underbrace{\mathrm{sim}(\mathrm{Alice}, \mathrm{Bob})}$

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$\mathbf{r}_{\text{Bob}}$

User-to-User Similarity: Jaccard Similarity

$$\mathrm{sim}(u,v) = J(\mathbf{r}_u, \mathbf{r}_v) = \frac{|\mathbf{r}_u \cap \mathbf{r}_v|}{|\mathbf{r}_u \cup \mathbf{r}_v|}$$

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$$\mathrm{sim}(\text{Alice}, \text{Bob}) = \frac{|\mathbf{r}_{\text{Alice}} \cap \mathbf{r}_{\text{Bob}}|}{|\mathbf{r}_{\text{Alice}} \cup \mathbf{r}_{\text{Bob}}|}$$

$$= \frac{3}{6} = 0.5$$

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$$= \frac{6}{7} \approx 0.86$$

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	...	...	...	...	...	...	...	...	...
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Problem!
Jaccard ignores rating values

User-to-User Similarity: Cosine Similarity

$$\mathrm{sim}(u, v) = \mathrm{cosine}(\mathbf{r}_u, \mathbf{r}_v) = \frac{\mathbf{r}_u \cdot \mathbf{r}_v}{||\mathbf{r}_u|| \, ||\mathbf{r}_v||}$$

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$$\mathrm{sim}(\mathrm{Alice}, \mathrm{Bob}) = \frac{\mathbf{r}_{\mathrm{Alice}} \cdot \mathbf{r}_{\mathrm{Bob}}}{||\mathbf{r}_{\mathrm{Alice}}|| \, ||\mathbf{r}_{\mathrm{Bob}}||}$$

$$= \frac{32}{\sqrt{102}\sqrt{44}} \approx 0.48$$

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$$\mathrm{sim}(\mathrm{Alice}, \mathrm{Carl}) = \frac{\mathbf{r}_{\mathrm{Alice}} \cdot \mathbf{r}_{\mathrm{Carl}}}{||\mathbf{r}_{\mathrm{Alice}}||\,||\mathbf{r}_{\mathrm{Carl}}||}$$

$$= \frac{102}{\sqrt{102}\sqrt{141}} \approx 0.85$$

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Problem!
Missing rating values are treated as 0s and have a negative effect

User-to-User Similarity: Pearson Correlation

$$\text{sim}(u, v) = \text{Pearson}(\mathbf{r}_u, \mathbf{r}_v) = \frac{(\mathbf{r}_u - \bar{\mathbf{r}}_u) \cdot (\mathbf{r}_v - \bar{\mathbf{r}}_v)}{\sqrt{(\mathbf{r}_u - \bar{\mathbf{r}}_u)^T \cdot (\mathbf{r}_u - \bar{\mathbf{r}}_u)} \times \sqrt{(\mathbf{r}_v - \bar{\mathbf{r}}_v)^T \cdot (\mathbf{r}_v - \bar{\mathbf{r}}_v)}}$$

		MOVIES							
USERS	Alice	-2		1	0	1	0		0
	Bob	2/3					-1/3		-1/3
	Carl	4/7	4/7	-10/7	-3/7	4/7	-3/7		4/7
	…	…	…	…	…	…	…	…	…
	Zoe		-9/4	-1/4				7/4	-1/4

Solution: Normalize ratings by subtracting the mean rating

Now 0 means neutral, and if we treat missing ratings as 0, it doesn't mean it's negative

User-to-User Similarity: Pearson Correlation

$\mathbf{r}'_u = \mathbf{r}_u - \bar{\mathbf{r}}_u$ mean-scaled rating vector of u

$\mathbf{r}'_v = \mathbf{r}_v - \bar{\mathbf{r}}_v$ mean-scaled rating vector of v

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$$= \frac{(\mathbf{r}_u - \bar{\mathbf{r}}_u) \cdot (\mathbf{r}_v - \bar{\mathbf{r}}_v)}{\sqrt{(\mathbf{r}_u - \bar{\mathbf{r}}_u)^T \cdot (\mathbf{r}_u - \bar{\mathbf{r}}_u)} \times \sqrt{(\mathbf{r}_v - \bar{\mathbf{r}}_v)^T \cdot (\mathbf{r}_v - \bar{\mathbf{r}}_v)}} = \text{Pearson}(\mathbf{r}_u, \mathbf{r}_v)$$

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Top-k most "similar" users to u

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Predicted rating given by user u to item i

$$\mathbf{r}_u[i] = r(u, i) = r_{u,i}$$

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$$r_{u,i} = \frac{1}{k} \sum_{u' \in \mathcal{U}^k} \mathrm{sim}(u, u') \cdot r_{u',i}$$

weighted average

User-based CF: Drawbacks

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Aging
user profiles changed quickly and the entire system model had to be recomputed

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- The model doesn't suffer from aging and therefore it does not need to be recomputed frequently

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$\mathbf{r}_{\text{Shrek}}$

Item-based Neighborhood: Example

Let's consider again Bob!

		MOVIES							
									
USERS	Alice	2		5	4	5	4		4
	Bob	4					3		3
	Carl	5	5	3	4	5	4		5
	...	...	...	...	...	...	...	...	...
	Zoe		1	3				5	4

Item-based Neighborhood: Example

Suppose we want to predict the rating Bob would give to Shrek

USERS	MOVIES							
Alice	2		5	4	5	4		4
Bob	4				?	3		3
Carl	5	5	3	4	5	4		5
...	...	...	...	...	...	...	...	...
Zoe		1	3				5	4

Item-based Neighborhood: Example

We first extract the subset of k most similar items to Shrek which have been rated by Bob

		MOVIES							
		AVENGERS	BATMAN	Grease	PULP FICTION	SHREK	SCHWARZENEGGER	THE WOLF OF WALL STREET	TOY
USERS	Alice	2		5	4	5	4		4
	Bob	4					3		3
	Carl	5	5	3	4	5	4		5
	...	...	...	...	...	...	...	...	...
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$\mathbf{r}_{\text{Shrek}}$

Item-based Neighborhood: Example

Suppose those are: The Avengers and The Terminator

		MOVIES							
USERS	Alice	2		5	4	5	4		4
	Bob	4					3		3
	Carl	5	5	3	4	5	4		5
	...	...	...	...	...	...	...	...	...
	Zoe		1	3				5	4

For example, item similarity is measured using Pearson's correlation

Item-based Neighborhood: Example

The predicted rating is computed as an aggregating function of the ratings that Bob gave to the k most similar movies to Shrek

		MOVIES							
									
USERS	Alice	2		5	4	5	4		4
	Bob	4				?	3		3
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Top-k most "similar" items to i among those rated by u

i's k-neighborhood

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Predicted rating given by user u to item i

$$\mathbf{r}_u[i] = r(u, i) = r_{u,i}$$

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2 possible ways of aggregating neighbors ratings

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$$\forall i' \in \mathcal{I}_u^k$$

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plain average

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$$r_{u,i} = \frac{1}{k} \sum_{i' \in \mathcal{I}_u^k} \mathrm{sim}(i, i') \cdot r_{u,i'}$$

weighted average

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- Item similarity can be computed from rating vectors (in user space)
- Analogous to user similarity of rating vectors (in item space):
  - Jaccard index
  - Cosine similarity (normalized = Pearson's correlation)
- Rating prediction using the same methods proposed for user-based CF
  - Plain average of ratings
  - Weighted average of ratings (taking item similarity into account)

Item-to-Item Collaborative Filtering

In general, item-based works better than user-based CF

Memory-based CF: Implementation

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- The most expensive step is finding the k most similar users (or items)
- This computation is too expensive to do online (for every user/item)
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- k-nearest neighbors search in high dimensions (i.e., quickly find the set of k nearest data points)

Memory-based CF: Implementation

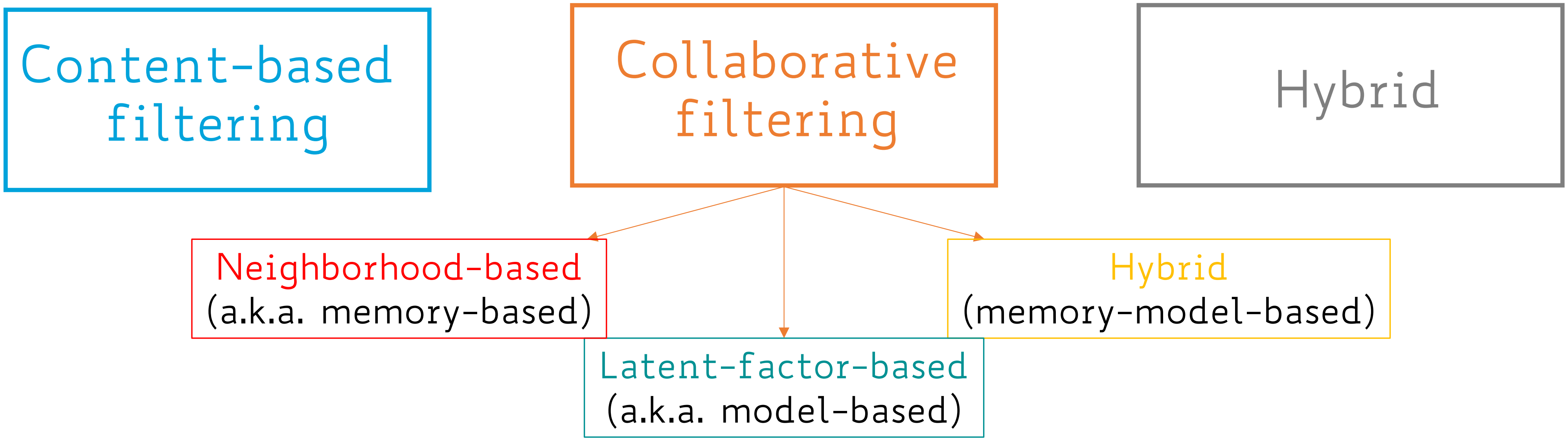
The curse of dimensionality (again!)

Memory-based CF: Implementation

Locality-Sensitive Hashing (LSH) approximation

Recommendation Strategies

3 approaches to recommender systems



LATENT FACTOR MODELS

Latent Factor (Model-based) CF

Tries to predict ratings by representing both items and users with a number of hidden factors inferred from observed ratings

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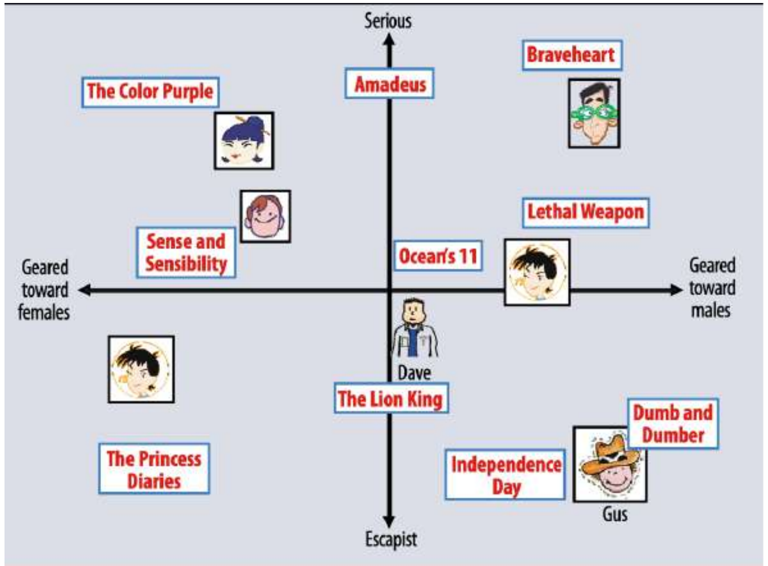

Example: 2 hidden factors

- Dim. 1: Male vs. Female
- Dim. 2: Serious vs. Escapist

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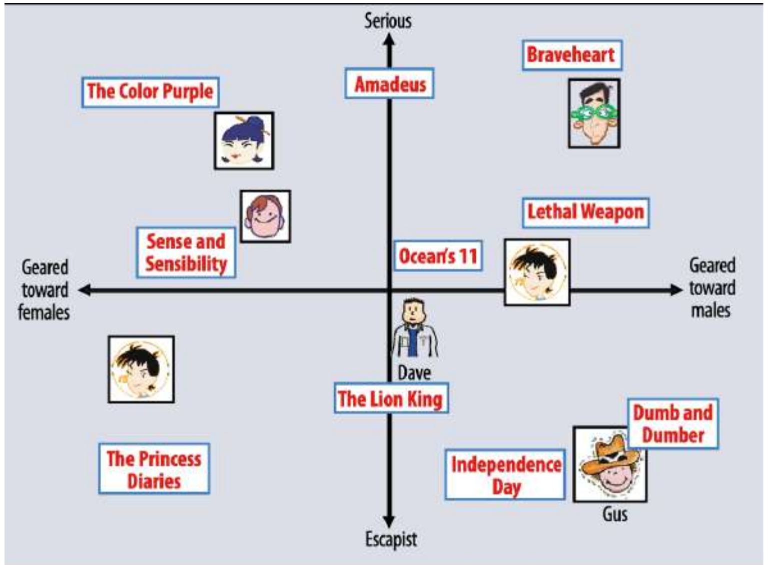

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A user's predicted rating for an item (movie) would equal the dot product of the movie and user vectors on the plot

Matrix Factorization

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- Such vectors are inferred (i.e., learned) from observed item ratings
- High correspondence between item and user factors leads to a recommendation

Matrix Factorization Framework

- Map both items and users to a **joint latent factor** d-dimensional space

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- That is why these features are often refer to as **latent features**

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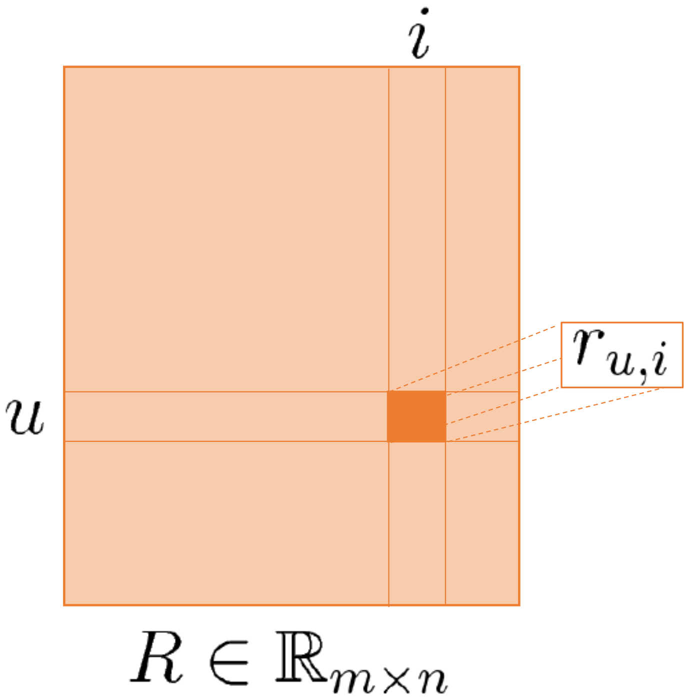
Recommendations for a user are generated by computing the estimated ratings for unseen items, and by taking the top-k highest rated ones

Matrix Factorization Framework



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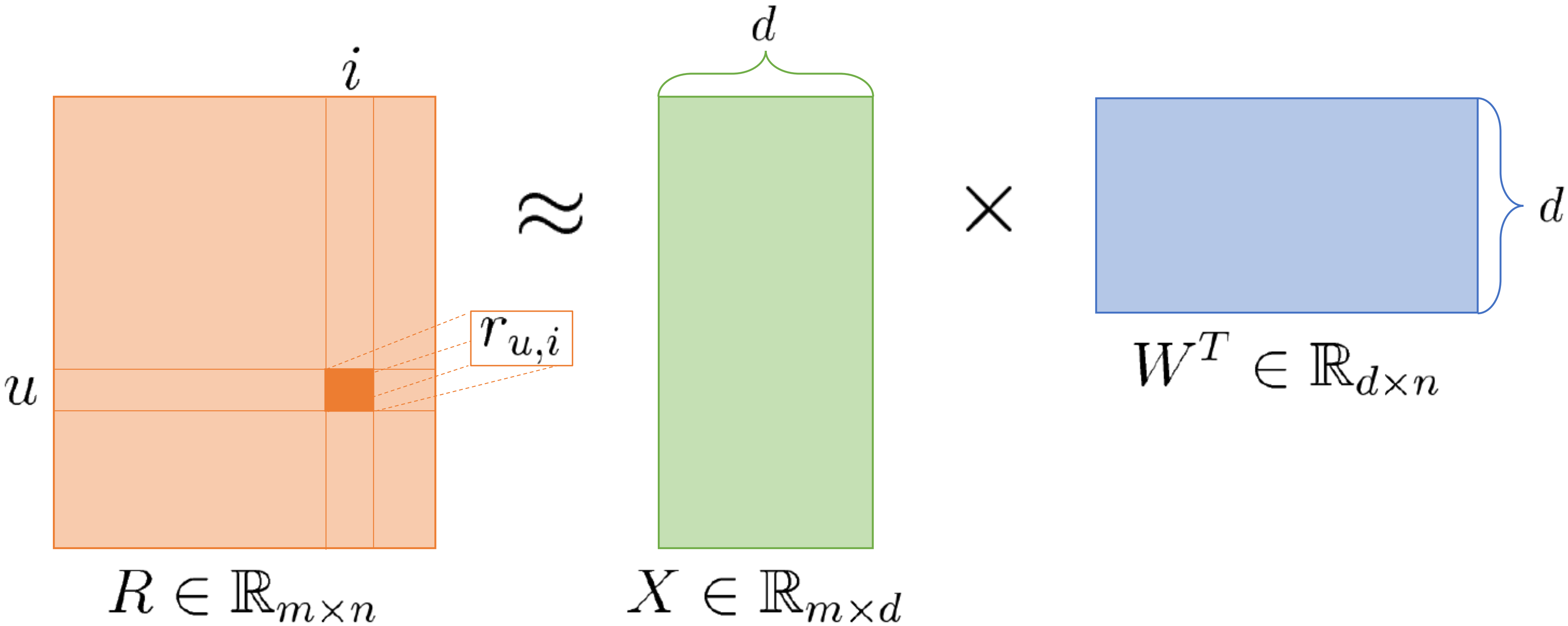

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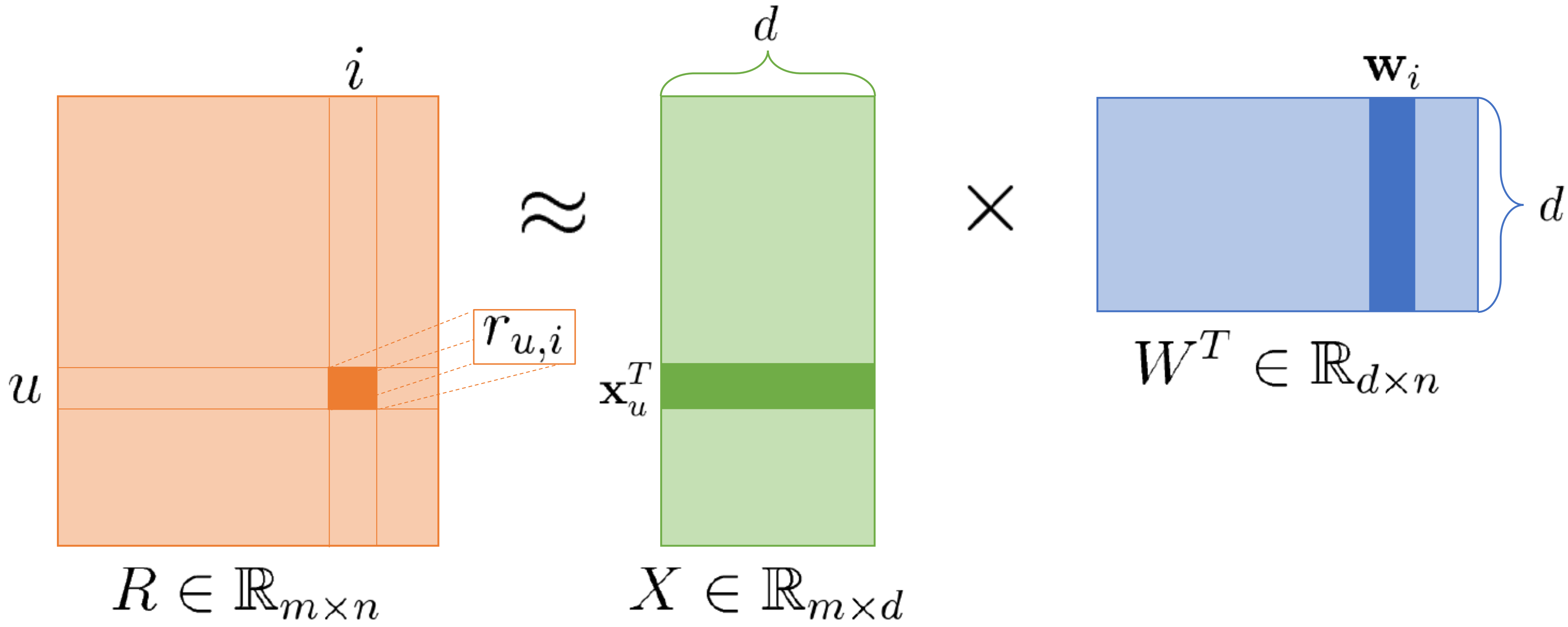

Approximate the user-item rating matrix R with the product of X x W^T

How Do We Learn X and W?

Assuming we have access to a dataset of observed ratings

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To actually learn the latent factor representations x_u and w_i we minimize the following loss function

$$L(X, W) = \underbrace{\sum_{(u,i) \in \mathcal{D}}}_{\text{Training set of observed ratings}} \left(r_{u,i} - \mathbf{x}_u^T \cdot \mathbf{w}_i \right)^2 + \lambda \left(\sum_{u \in \mathcal{D}} ||\mathbf{x}_u||^2 + \sum_{i \in \mathcal{D}} ||\mathbf{w}_i||^2 \right)$$

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Training set of observed ratings

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Mathematically convenient

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Still, how do we solve this?

Take-Home Message of Today

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- In large scale systems, this must be pre-computed offline
- At inference time, make use of ad hoc data structures (e.g., k-d trees) to efficiently compute the set of (approximated) nearest neighbors for a query user/item
- Latent Factor Models overcome this need