Big Data Computing

Master's Degree in Computer Science

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SAPIENZA
UNIVERSITÀ DI ROMA

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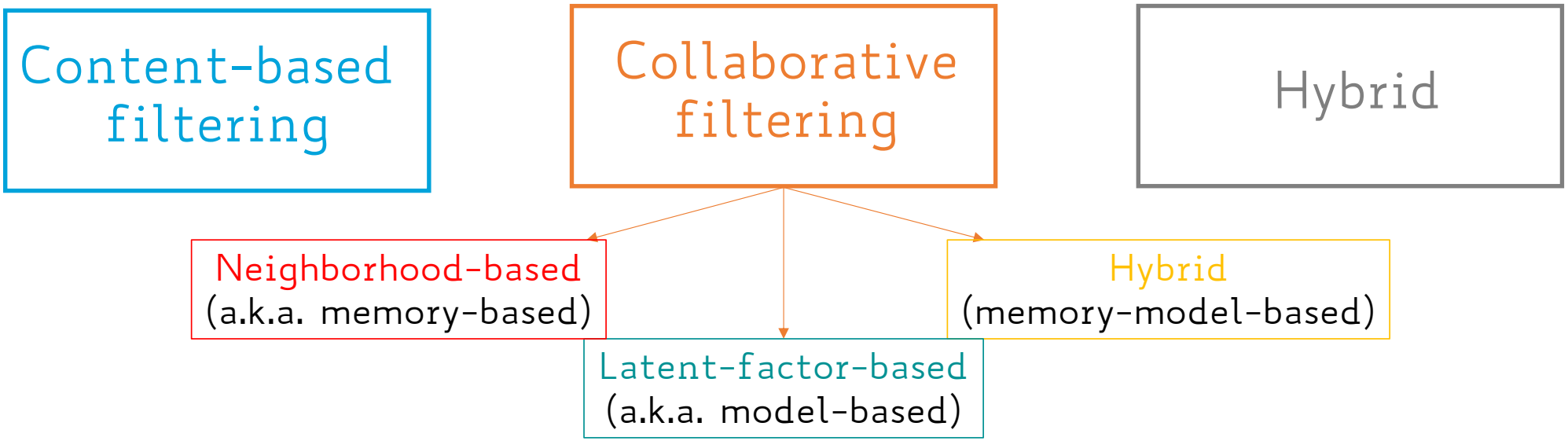
Recommendation Strategies

3 approaches to recommender systems



COLLABORATIVE FILTERING

Collaborative Filtering (CF)

Idea

Recommend items to user u based on preferences of other users similar to u

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Core concept:

User-to-User or Item-to-Item similarity

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Recommend items to user u based on preferences of other users similar to u

Core concept:
User-to-User or Item-to-Item similarity

No need for explicit creation of user/item profiles

Collaborative Filtering: Approaches

3 main approaches to collaborative filtering

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Neighborhood-based
(a.k.a. memory-based)

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3 main approaches to collaborative filtering

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Latent-factor-based
(a.k.a. model-based)

Collaborative Filtering: Approaches

3 main approaches to collaborative filtering

Neighborhood-based
(a.k.a. memory-based)

Hybrid
(memory-model-based)

Latent-factor-based
(a.k.a. model-based)

Neighborhood-based (Memory-based) CF

Compute the relationship between users or items

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Compute the relationship between users or items

User-based

Evaluates a user's preference for an item based on ratings of "neighboring" users for that item

Neighborhood-based (Memory-based) CF

Compute the relationship between users or items

User-based

Evaluates a user's preference for an item based on ratings of "neighboring" users for that item

Item-based

Evaluates a user's preference for an item based on ratings of "neighboring" items by the same user

USER-BASED COLLABORATIVE FILTERING

User-based Neighborhood

Given a user u and an item i not rated by u, we want to estimate r(u, i)

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From the set of users {u': u' != u} who have already rated i extract a subset of k neighbours of u

k-neighborhood of u is found on the basis of the similarity between user ratings without explicit user profiles

Estimate r(u, i) based on the ratings of users in the k-neighborhood of u

User-based Neighborhood

In theory, rating prediction r(u,i) could be defined on any item i not rated by u

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In practice, we are interested only in estimating r(u,i) for those items i which have been rated by the u's k-neighborhood

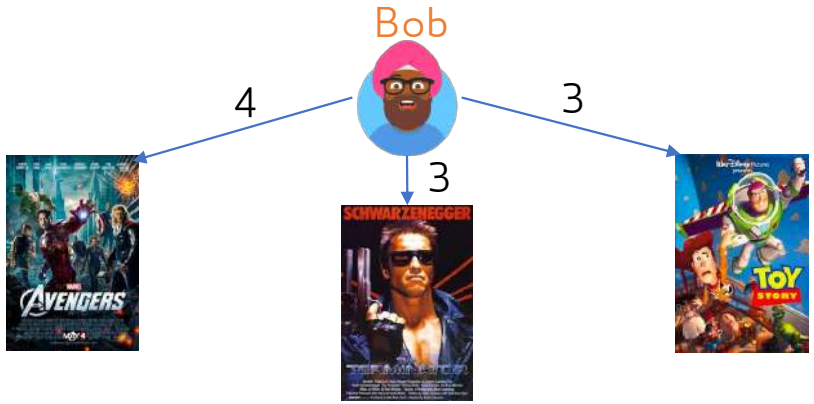
Intuitively, if a user v is not in the u's k-neighborhood then very likely u will not be interested in any item that only v has rated

In other words, the u's k-neighborhood must be computed first to narrow down the set of items which we must predict the rating of

User-based Neighborhood: Example

		MOVIES							
									
USERS	 Alice	2		5	4	5	4		4
	 Bob	4					3		3
	 Carl	5	5	3	4	5	4		5
	...	...	...	...	...	...	...	...	...
	 Zoe		I	3				5	4

User-based Neighborhood: Example



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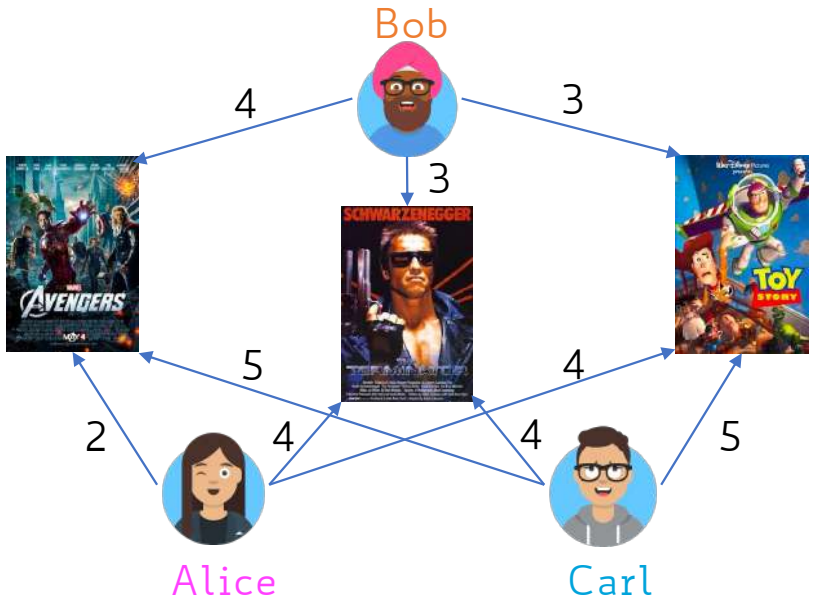

Alice and Carl are the 2-nearest neighbours of Bob if we look at their rating behaviours

User-based Neighborhood: Example

Consider all the movies rated by Alice or Carl which Bob hasn't watched yet

User-based Neighborhood: Example

Predict the rating that Bob would give to each of those movies on the basis of Alice's and Carl's ratings

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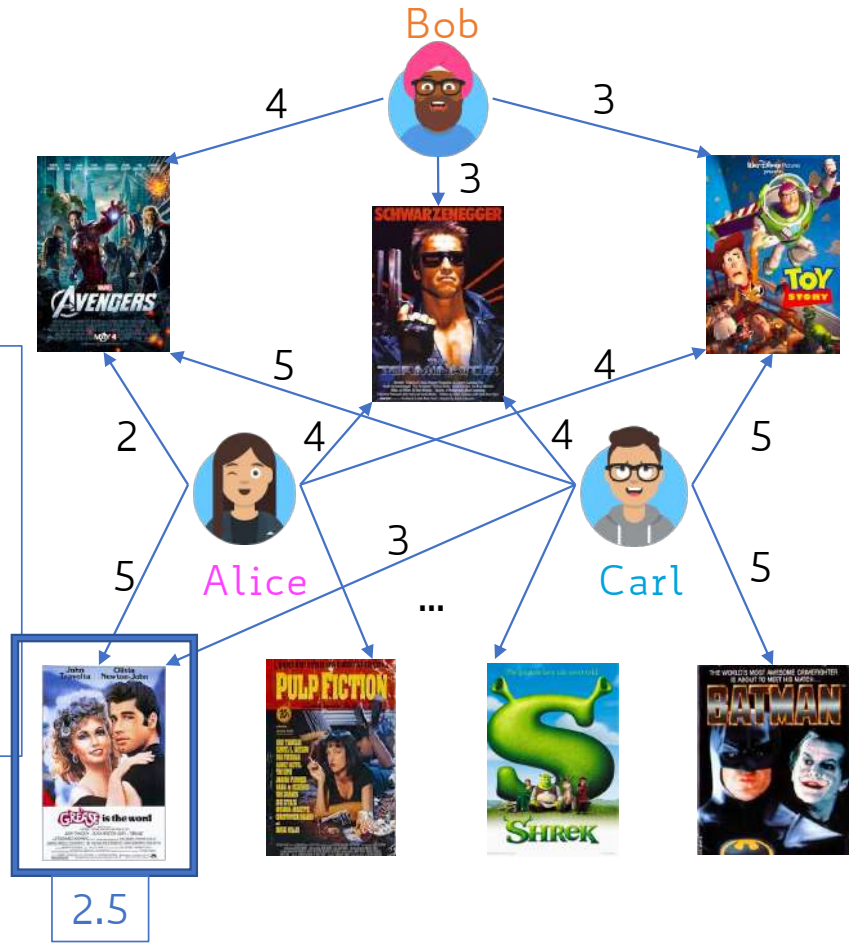

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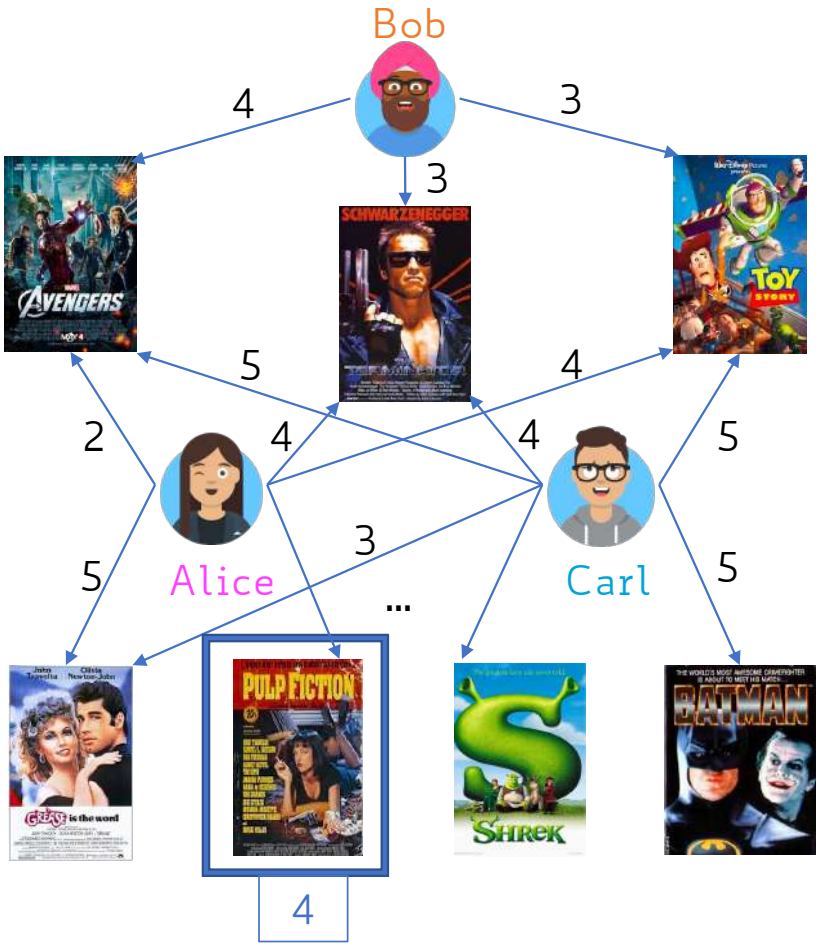

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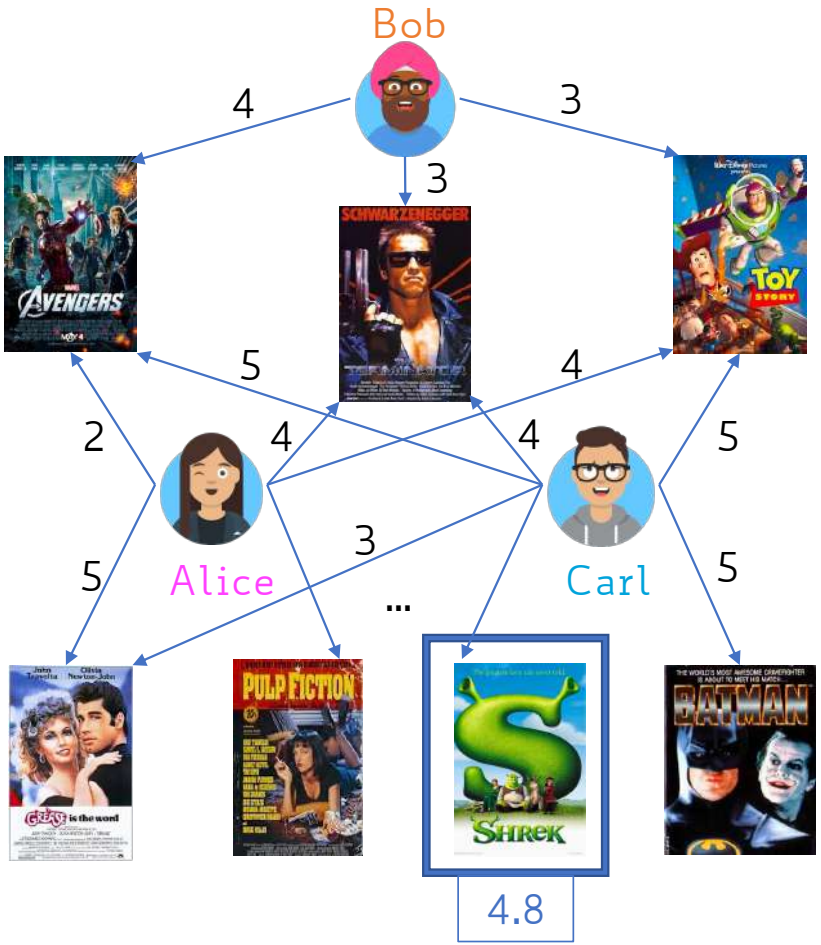
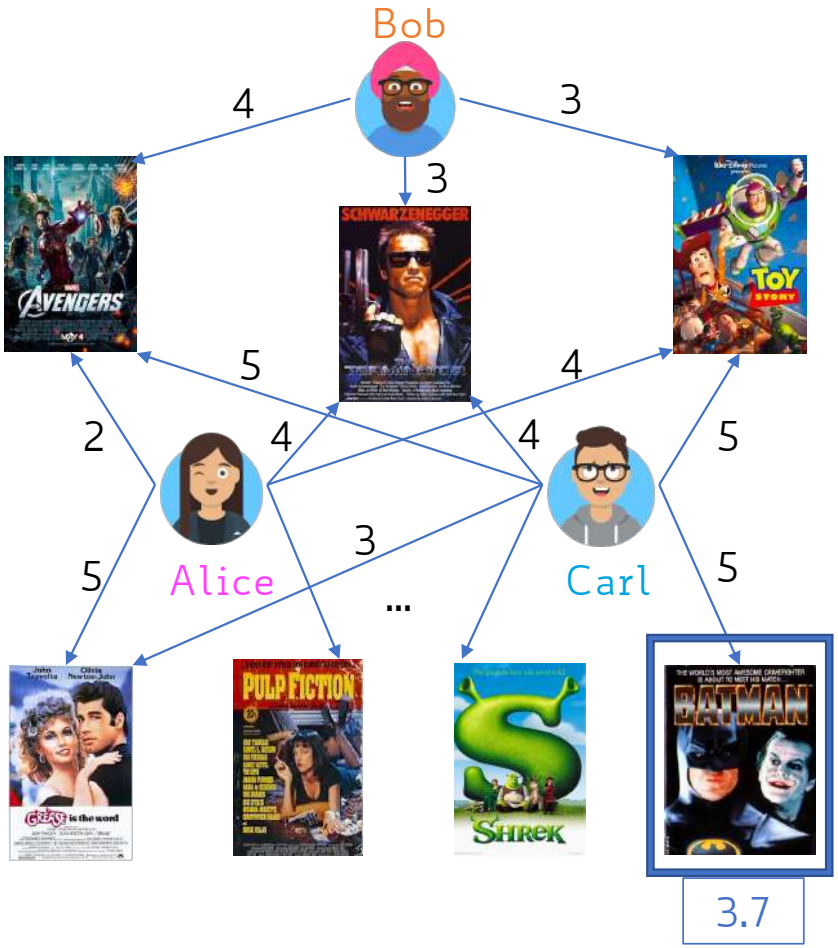


User-based Neighborhood: Example

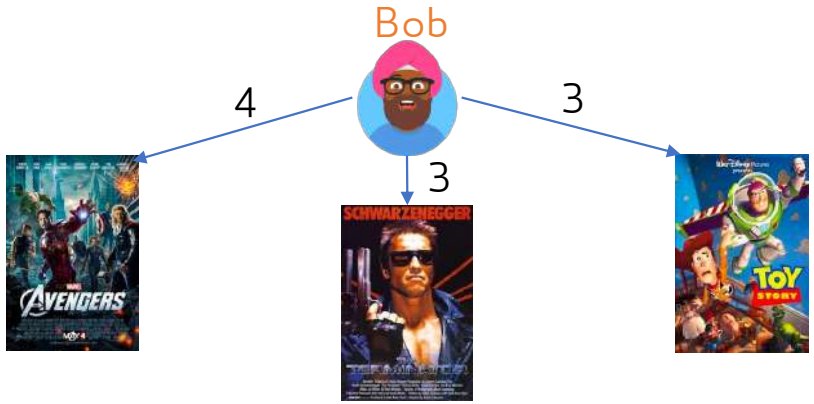
Predict the rating that Bob would give to each of those movies on the basis of Alice's and Carl's ratings



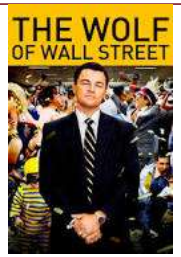
User-based Neighborhood: Example

Recommend the highest rated movie(s) to Bob!

User-based Neighborhood: Example



There is no point in predicting the rating of a movie which has only been rated by a user (Zoe) who is not in the Bob's neighborhood



User-to-User Similarity

- The key "trick" to discover the *k*-neighborhood of a given user *u* is the ability of finding users *u'* that are "similar" to *u*

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- Remember that we are **not** building any content-based user/item profile
- Intuitively, 2 users u_1 and u_2 are similar to each other if their ratings (of items) are similar
- Each user represented by her/his rating vector and similarity between them is measured in the item (rating) space

User-to-User Similarity

$\text{sim}(u, v)$ Similarity metric between any pair of users

		MOVIES							
									
USERS	 Alice	2		5	4	5	4		4
	 Bob	4					3		3
	 Carl	5	5	3	4	5	4		5
	...	...	...	...	...	...	...	...	...
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	...	...	...	...	...	...	...	...	...
	 Zoe		I	3				5	4

Must capture the intuition: $\underline{\text{sim}(\text{Alice}, \text{Carl})} > \underline{\text{sim}(\text{Alice}, \text{Bob})}$

User-to-User Similarity

$\mathbf{r}_u$ n-dimensional vector of ratings provided by user u (n = #movies)

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	Carl	5	5	3	4	5	4		5
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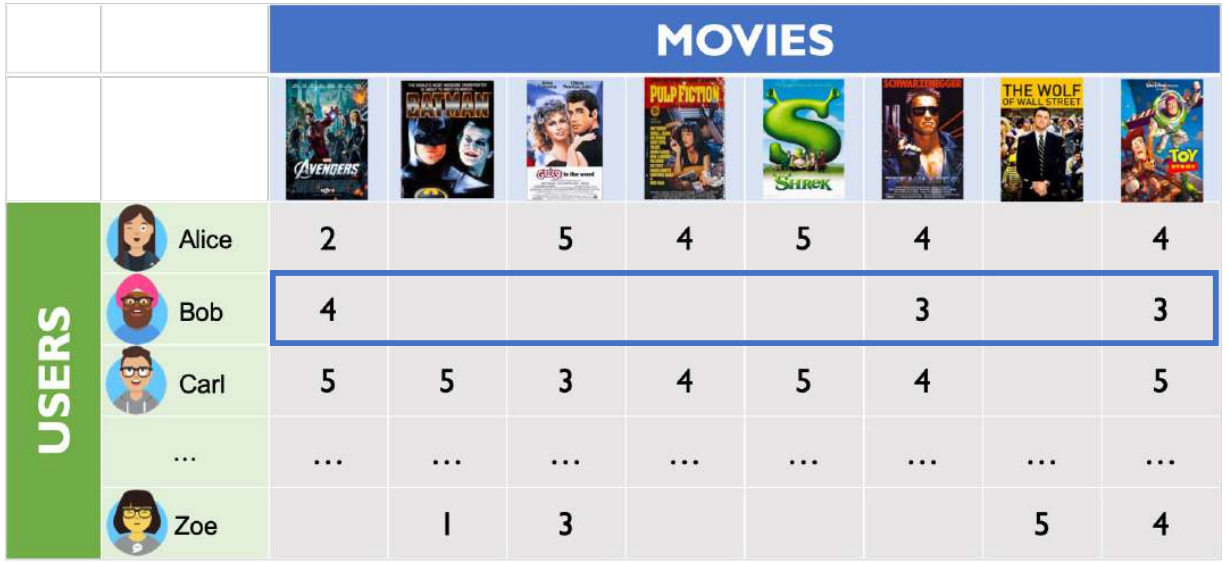

$\mathbf{r}_{\text{Bob}}$

User-to-User Similarity: Jaccard Similarity

$$\text{sim}(u, v) = J(\mathbf{r}_u, \mathbf{r}_v) = \frac{|\mathbf{r}_u \cap \mathbf{r}_v|}{|\mathbf{r}_u \cup \mathbf{r}_v|}$$

		MOVIES							
									
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		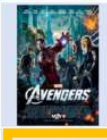							
USERS	Alice	2		5	4	5	4		4
	Bob	4					3		3
	Carl	5	5	3	4	5	4		5
	...	...	...	...	...	...	...	...	...
	Zoe		1	3				5	4

$$\text{sim}(\text{Alice}, \text{Bob}) = \frac{|\mathbf{r}_{\text{Alice}} \cap \mathbf{r}_{\text{Bob}}|}{|\mathbf{r}_{\text{Alice}} \cup \mathbf{r}_{\text{Bob}}|}$$

$$= \frac{3}{6} = 0.5$$

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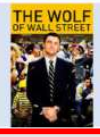
		MOVIES							
									
USERS	Alice	2		5	4	5	4		4
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	...	...	...	...	...	...	...	...	...
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$$\text{sim}(\text{Alice}, \text{Carl}) = \frac{|\mathbf{r}_{\text{Alice}} \cap \mathbf{r}_{\text{Carl}}|}{|\mathbf{r}_{\text{Alice}} \cup \mathbf{r}_{\text{Carl}}|}$$

$$= \frac{6}{7} \approx 0.86$$

User-to-User Similarity: Jaccard Similarity

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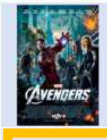
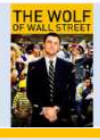
	...	...	...	...	...	...	...	...	...
	 Zoe		1	3				5	4

Problem!
Jaccard ignores rating values

User-to-User Similarity: Cosine Similarity

$$\text{sim}(u, v) = \text{cosine}(\mathbf{r}_u, \mathbf{r}_v) = \frac{\mathbf{r}_u \cdot \mathbf{r}_v}{||\mathbf{r}_u|| ||\mathbf{r}_v||}$$

		MOVIES							
									
USERS	Alice	2		5	4	5	4		4
	Bob	4					3		3
	Carl	5	5	3	4	5	4		5
	...	...	...	...	...	...	...	...	...
	Zoe		1	3				5	4

$$\text{sim}(\text{Alice}, \text{Bob}) = \frac{\mathbf{r}_{\text{Alice}} \cdot \mathbf{r}_{\text{Bob}}}{||\mathbf{r}_{\text{Alice}}|| ||\mathbf{r}_{\text{Bob}}||}$$

$$= \frac{32}{\sqrt{102}\sqrt{44}} \approx 0.48$$

User-to-User Similarity: Cosine Similarity

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								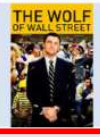	
USERS	Alice	2		5	4	5	4		4
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	...	...	...	...	...	...	...	...	...
	Zoe		1	3				5	4

$$\text{sim}(\text{Alice}, \text{Carl}) = \frac{\mathbf{r}_{\text{Alice}} \cdot \mathbf{r}_{\text{Carl}}}{||\mathbf{r}_{\text{Alice}}|| ||\mathbf{r}_{\text{Carl}}||}$$

$$= \frac{102}{\sqrt{102}\sqrt{141}} \approx 0.85$$

User-to-User Similarity: Cosine Similarity

$$\text{sim}(u, v) = \text{cosine}(\mathbf{r}_u, \mathbf{r}_v) = \frac{\mathbf{r}_u \cdot \mathbf{r}_v}{||\mathbf{r}_u|| ||\mathbf{r}_v||}$$

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	...	...	...	...	...	...	...	...	...
	Zoe		1	3				5	4

Problem!
Missing rating values are treated as Os and have a negative effect

User-to-User Similarity: Pearson Correlation

$$\text{sim}(u, v) = \text{Pearson}(\mathbf{r}_u, \mathbf{r}_v) = \frac{(\mathbf{r}_u - \bar{\mathbf{r}}_u) \cdot (\mathbf{r}_v - \bar{\mathbf{r}}_v)}{\sqrt{(\mathbf{r}_u - \bar{\mathbf{r}}_u)^T \cdot (\mathbf{r}_u - \bar{\mathbf{r}}_u)} \times \sqrt{(\mathbf{r}_v - \bar{\mathbf{r}}_v)^T \cdot (\mathbf{r}_v - \bar{\mathbf{r}}_v)}}$$

		MOVIES							
									
USERS	Alice	-2		1	0	1	0		0
	Bob	2/3					-1/3		-1/3
	Carl	4/7	4/7	-10/7	-3/7	4/7	-3/7		4/7
	...	...	...	...	...	...	...	...	...
	Zoe		-9/4	-1/4				7/4	-1/4

Solution:
Normalize ratings by subtracting the mean rating

Now 0 means neutral, and if we treat missing ratings as 0, it doesn't mean it's negative

User-to-User Similarity: Pearson Correlation

$\mathbf{r}'_u = \mathbf{r}_u - \bar{\mathbf{r}}_u$ mean-scaled rating vector of u

$\mathbf{r}'_v = \mathbf{r}_v - \bar{\mathbf{r}}_v$ mean-scaled rating vector of v

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$$= \frac{(\mathbf{r}_u - \bar{\mathbf{r}}_u) \cdot (\mathbf{r}_v - \bar{\mathbf{r}}_v)}{\sqrt{(\mathbf{r}_u - \bar{\mathbf{r}}_u)^T \cdot (\mathbf{r}_u - \bar{\mathbf{r}}_u)} \times \sqrt{(\mathbf{r}_v - \bar{\mathbf{r}}_v)^T \cdot (\mathbf{r}_v - \bar{\mathbf{r}}_v)}} = \text{Pearson}(\mathbf{r}_u, \mathbf{r}_v)$$

User-based Neighborhood: Predictions

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$$\mathcal{U}^k = \text{argmax}_{\mathcal{U}' \subseteq \mathcal{U} \setminus u, |\mathcal{U}'|=k} \sum_{u' \in \mathcal{U}'} \text{sim}(u, u')$$

Top-k most "similar" users to u

u's k-neighborhood

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Set of items rated by u's neighbors

$$\mathcal{I}^k = \{i \in \mathcal{I} : \mathbf{r}_{u',i} = \downarrow \wedge u' \in \mathcal{U}^k\}$$

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Predicted rating given by user u to item i

$$\mathbf{r}_u[i] = r(u, i) = r_{u,i}$$

User-based Neighborhood: Predictions

2 possible ways of aggregating neighbors ratings

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$$\forall i \in \mathcal{I}^k$$

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$$r_{u,i} = \frac{1}{k} \sum_{u' \in \mathcal{U}^k} r_{u',i}$$

plain average

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2 possible ways of aggregating neighbors ratings

$$\forall i \in \mathcal{I}^k$$

$$r_{u,i} = \frac{1}{k} \sum_{u' \in \mathcal{U}^k} r_{u',i}$$

plain average

$$r_{u,i} = \frac{\sum_{u' \in \mathcal{U}^k} \mathrm{sim}(u, u') * r_{u',i}}{\sum_{u' \in \mathcal{U}^k} \mathrm{sim}(u, u')}$$

weighted average

User-based CF: Drawbacks

3 main issues with user-based CF

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Efficiency
computing similarities between all pairs of users is expensive

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3 main issues with user-based CF

Sparsity
systems performed poorly when they had many items but comparatively few ratings

Efficiency
computing similarities between all pairs of users is expensive

Aging
user profiles changed quickly and the entire system model had to be recomputed

ITEM-BASED COLLABORATIVE FILTERING

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- The model doesn't suffer from aging and therefore it does not need to be recomputed frequently

Item-based Neighborhood

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Estimate r(u, i) based on the ratings of items in the k-neighborhood of i

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- Each item represented by the user ratings vector and similarity between them is measured in the user (rating) space

Item-to-Item Similarity

$\mathbf{r}_i$ m-dimensional vector of ratings provided for item i (m = #users)

		MOVIES							
									
USERS	 Alice	2		5	4	5	4		4
	 Bob	4					3		3
	 Carl	5	5	3	4	5	4		5
	...	...	...	...	...	...	...	...	...
	 Zoe		I	3				5	4

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$\mathbf{r}_{\text{Shrek}}$

Item-based Neighborhood: Example

Let's consider again Bob!

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USERS	 Alice	2		5	4	5	4		4
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Item-based Neighborhood: Example

Suppose we want to predict the rating Bob would give to Shrek

		MOVIES							
									
USERS	Alice	2		5	4	5	4		4
	Bob	4				?	3		3
	Carl	5	5	3	4	5	4		5
	...	...	...	...	...	...	...	...	...
	Zoe		1	3				5	4

Item-based Neighborhood: Example

We first extract the subset of k most similar items to Shrek which have been rated by Bob

		MOVIES							
									
USERS	 Alice	2		5	4	5	4		4
	 Bob	4					3		3
	 Carl	5	5	3	4	5	4		5
	...	...	...	...	...	...	...	...	...
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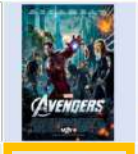
$\mathbf{r}_{\text{Shrek}}$

Item-based Neighborhood: Example

Suppose those are: The Avengers and The Terminator

		MOVIES							
									
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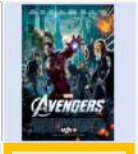
	...	...	...	...	...	...	...	...	...
	Zoe		1	3				5	4

For example, item similarity is measured using Pearson's correlation

Item-based Neighborhood: Example

The predicted rating is computed as an aggregating function of the ratings that Bob gave to the k most similar movies to Shrek

		MOVIES							
									
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$$\mathcal{I}_u^k = \text{argmax}_{\mathcal{I}'_u \subseteq \mathcal{I}_u, |\mathcal{I}'_u| = k} \sum_{i' \in \mathcal{I}'_u} \text{sim}(i, i')$$

Top-k most "similar" items to i among those rated by u

i's k-neighborhood

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Predicted rating given by user u to item i

$$\mathbf{r}_u[i] = r(u, i) = r_{u,i}$$

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$$\forall i' \in \mathcal{I}_u^k$$

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plain average

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$$r_{u,i} = \frac{\sum_{i' \in \mathcal{I}_u^k} \text{sim}(i, i') * r_{u,i'}}{\sum_{i' \in \mathcal{I}_u^k} \text{sim}(i, i')}$$

weighted average

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- Item similarity can be computed from rating vectors (in user space)
- Analogous to user similarity of rating vectors (in item space):
  - Jaccard index
  - Cosine similarity (normalized = Pearson's correlation)
- Rating prediction using the same methods proposed for user-based CF
  - Plain average of ratings
  - Weighted average of ratings (taking item similarity into account)

Item-to-Item Collaborative Filtering

In general, item-based works better than user-based CF

Memory-based CF: Implementation

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- *k*-nearest neighbors search in high dimensions (i.e., quickly find the set of k nearest data points)

Memory-based CF: Implementation

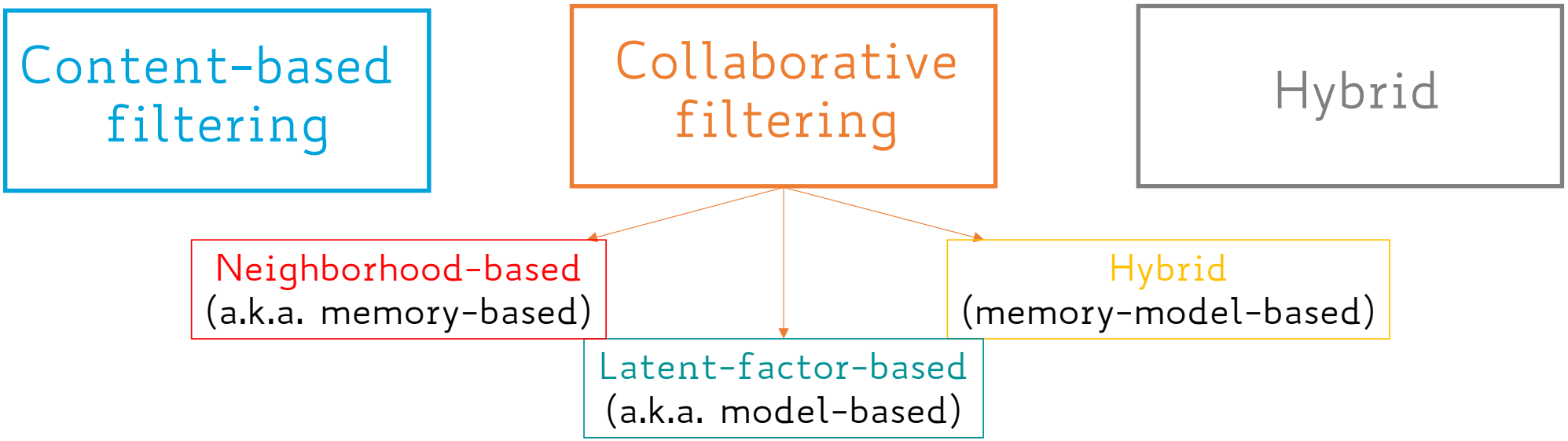
The curse of dimensionality (again!)

Memory-based CF: Implementation

Locality-Sensitive Hashing (LSH) approximation

Recommendation Strategies

3 approaches to recommender systems



LATENT FACTOR MODELS

Latent Factor (Model-based) CF

Tries to predict ratings by representing both items and users with a number of hidden factors inferred from observed ratings

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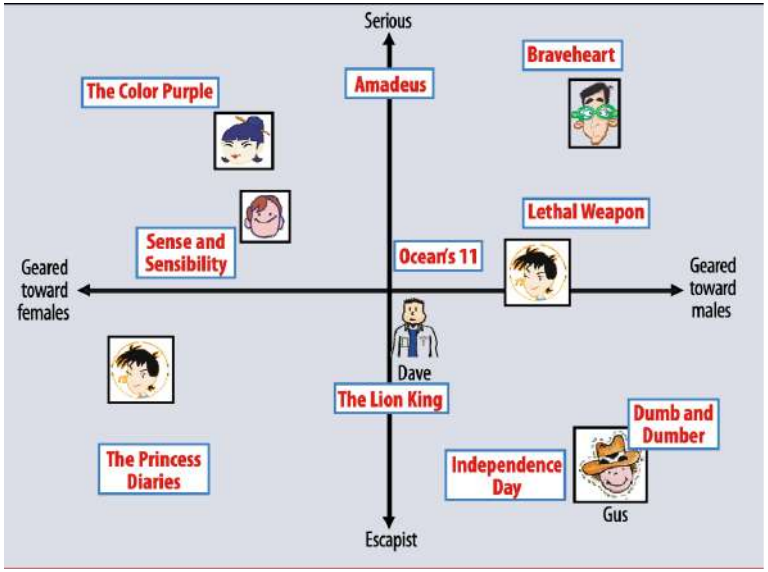

Example: 2 hidden factors

- Dim. 1: Male vs. Female
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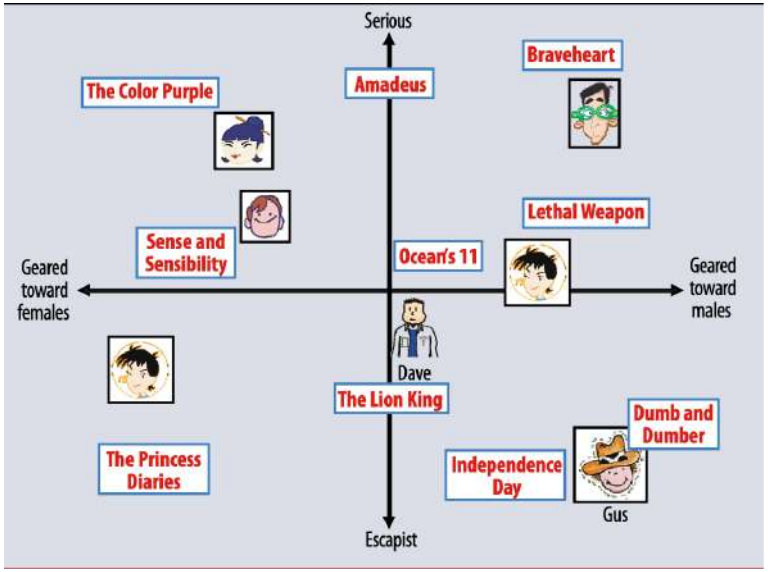

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A user's predicted rating for an item (movie) would equal the dot product of the movie and user vectors on the plot

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- High correspondence between item and user factors leads to a recommendation

Matrix Factorization Framework

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- That is why these features are often refer to as **latent features**

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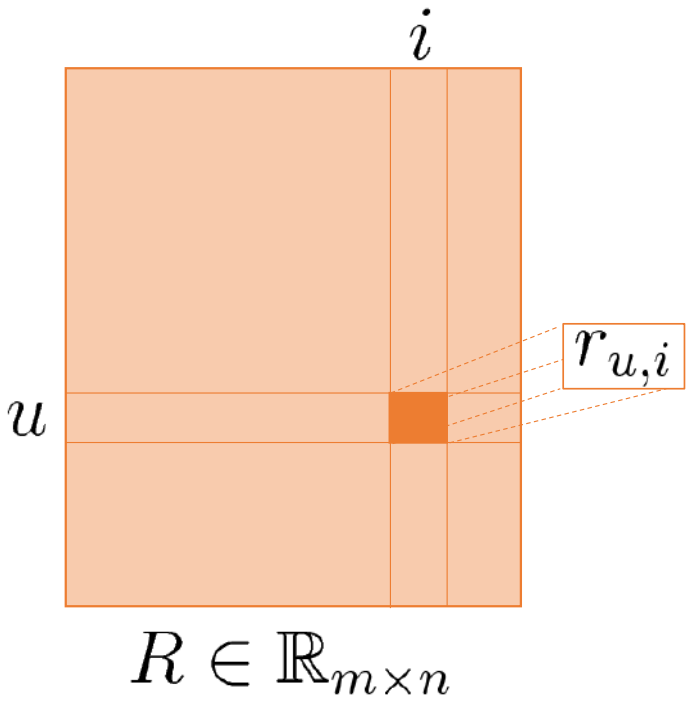
Recommendations for a user are generated by computing the estimated ratings for unseen items, and by taking the top-k highest rated ones

Matrix Factorization Framework



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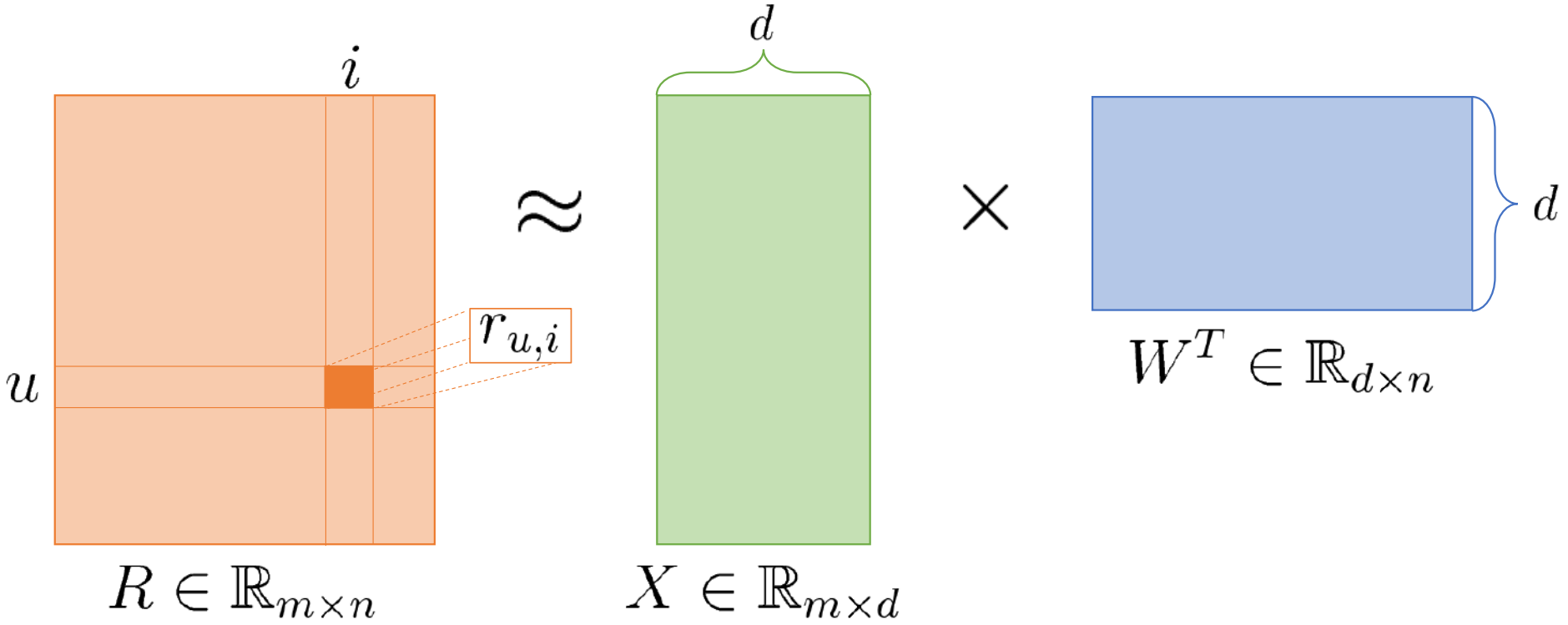

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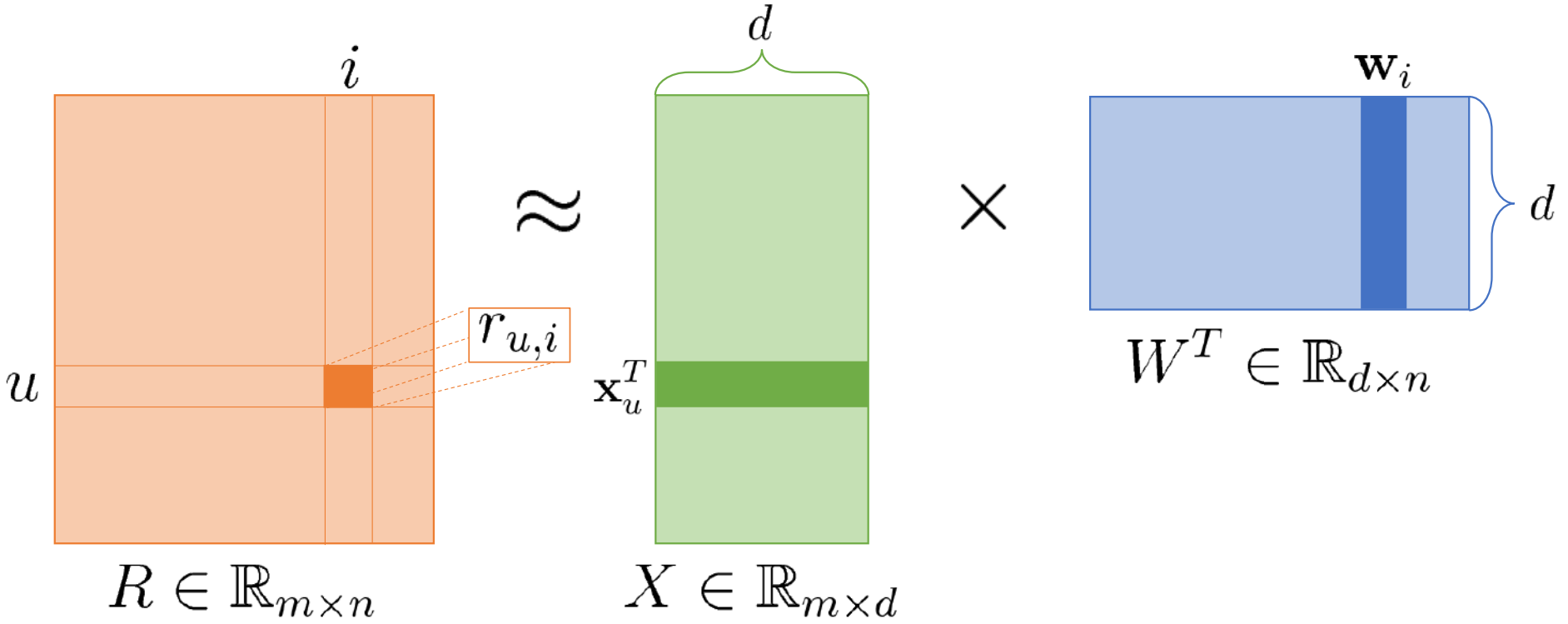

Approximate the user-item rating matrix R with the product of X x W^T

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$$L(X,W) = \sum_{(u,i)\in\mathcal{D}} \left(r_{u,i} - \mathbf{x}_u^T \cdot \mathbf{w}_i\right)^2 + \lambda\left(\sum_{u\in\mathcal{D}} ||\mathbf{x}_u||^2 + \sum_{i\in\mathcal{D}} ||\mathbf{w}_i||^2\right)$$

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regularization term

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Mathematically convenient

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Still, how do we solve this?

Take-Home Message of Today

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- In large scale systems, this must be pre-computed offline
- At inference time, make use of ad hoc data structures (e.g., k-d trees) to efficiently compute the set of (approximated) nearest neighbors for a query user/item
- Latent Factor Models overcome this need