

Sistemi Operativi I

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Recap of the Last Lecture

- Virtual Memory allows processes to extend their memory footprint beyond the limit of the physical RAM
- Combined with paging, it uses secondary storage (i.e., disks) as backup for unallocated frames
- Whenever a process requests a page, this could either be in main memory, never mapped, or on disk (**page fault**)
- Ideally, the OS should keep in main memory each process' **working set** to lower the chance of a page fault

Page Replacement: Motivation

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- If physical memory is full, a frame must be swapped out to make room for the swap-in page

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- If physical memory has still free frames, the page can be safely loaded into one of those
- If physical memory is full, a frame must be swapped out to make room for the swap-in page
- Several algorithms to select the page to evict from memory

Page Replacement Algorithms

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- FIFO (First-In-First-Out): throw out the page that has been in memory for longest time (i.e., the oldest)
 - Easy to implement but may remove frequently accessed pages
- MIN (OPT): remove the page that will not be accessed for the longest time (provably optimal [Belady 1966])
 - Needs to predict the future → very hard!

Page Replacement Algorithms

- Random: pick any page at random (works surprisingly well!)
- FIFO (First-In-First-Out): throw out the page that has been in memory for longest time (i.e., the oldest)
 - Easy to implement but may remove frequently accessed pages
- MIN (OPT): remove the page that will not be accessed for the longest time (provably optimal [Belady 1966])
 - Needs to predict the future → very hard!
- LRU (Least Recently Used): approximation of MIN, remove the page that has not been used in the longest time
 - Assumes the past is a good predictor of the future (not always true!)

FIFO Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1											
F_2											
F_3											

How many page faults (denoted by *)?

FIFO Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1											
F_2											
F_3											

Initially, no frame is loaded in memory at all
(pure demand paging)

FIFO Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1											
F_2											
F_3											

Virtual address within page A is referenced

FIFO Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1											
F_2											
F_3											

Virtual address within page A is referenced page fault

FIFO Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1	A*										
F_2											
F_3											

Virtual address within page A is referenced

page fault \Rightarrow

A loaded

FIFO = A

FIFO Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1	A*	A									
F_2											
F_3											

Virtual address within page B is referenced

FIFO Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1	A*	A									
F_2											
F_3											

Virtual address within page B is referenced page fault

FIFO Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1	A*	A									
F_2		B*									
F_3											

Virtual address within page B is referenced

page fault

B loaded

FIFO Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1	A*	A	A								
F_2		B*	B								
F_3											

Virtual address within page C is referenced

FIFO Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1	A*	A	A								
F_2		B*	B								
F_3											

Virtual address within page C is referenced page fault

FIFO Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1	A*	A	A								
F_2		B*	B								
F_3			C*								

Virtual address within page C is referenced → page fault → C loaded

FIFO Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1	A*	A	A	A							
F_2		B*	B	B							
F_3			C*	C							

Virtual address within page A is referenced

A is already loaded

FIFO = A → B → C

FIFO Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1	A*	A	A	A	A						
F_2		B*	B	B	B						
F_3			C*	C	C						

Virtual address within page B is referenced

B is already loaded

FIFO = A → B → C

FIFO Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1	A*	A	A	A	A	A					
F_2		B*	B	B	B	B					
F_3			C*	C	C	C					

Virtual address within page D is referenced

FIFO Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1	A*	A	A	A	A	A					
F_2		B*	B	B	B	B					
F_3			C*	C	C	C					

Virtual address within page D is referenced page fault

FIFO Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1	A*	A	A	A	A	D*					
F_2		B*	B	B	B	B					
F_3			C*	C	C	C					

Virtual address within page D is referenced

page fault

A replaced
D loaded

FIFO = B \rightarrow C \rightarrow D

FIFO Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1	A*	A	A	A	A	D*	D				
F_2		B*	B	B	B	B	B				
F_3			C*	C	C	C	C				

Virtual address within page A is referenced

FIFO Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1	A*	A	A	A	A	D*	D				
F_2		B*	B	B	B	B	B				
F_3			C*	C	C	C	C				

Virtual address within page A is referenced page fault

FIFO Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1	A*	A	A	A	A	D*	D				
F_2		B*	B	B	B	B	A*				
F_3			C*	C	C	C	C				

Virtual address within page A is referenced

page fault

B replaced
A loaded

FIFO = C → D → A

FIFO Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1	A*	A	A	A	A	D*	D	D			
F_2		B*	B	B	B	B	A*	A			
F_3			C*	C	C	C	C	C			

Virtual address within page D is referenced

D is already loaded

FIFO = C → D → A

FIFO Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1	A*	A	A	A	A	D*	D	D	D	C*	C
F_2		B*	B	B	B	B	A*	A	A	A	A
F_3			C*	C	C	C	C	B*	B	B	

Eventually, we get a total of 7 page faults

MIN Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1											
F_2											
F_3											

How many page faults (denoted by *)?

MIN Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1											
F_2											
F_3											

Initially, no frame is loaded in memory at all
(pure demand paging)

MIN Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1	A*	A	A	A	A						
F_2		B*	B	B	B						
F_3			C*	C	C						

Up to this point, the same as FIFO

MIN Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1	A*	A	A	A	A	A					
F_2		B*	B	B	B	B					
F_3			C*	C	C	C					

Virtual address within page D is referenced

MIN Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1	A*	A	A	A	A	A					
F_2		B*	B	B	B	B					
F_3			C*	C	C	C					

Virtual address within page D is referenced page fault

MIN Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1	A*	A	A	A	A	A					
F_2		B*	B	B	B	B					
F_3			C*	C	C	C					

Virtual address within page D is referenced

page fault

What's the page that will be requested the furthest away?

MIN Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1	A*	A	A	A	A	A					
F_2		B*	B	B	B	B					
F_3			C*	C	C	D*					

Virtual address within page D is referenced

page fault

C replaced
D loaded

MIN Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1	A*	A	A	A	A	A	A	A	A		
F_2		B*	B	B	B	B	B	B	B		
F_3			C*	C	C	D*	D	D	D		

Up to this point, no more page faults

MIN Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1	A*	A	A	A	A	A	A	A	A	A	
F_2		B*	B	B	B	B	B	B	B	B	
F_3			C*	C	C	D*	D	D	D	D	

Virtual address within page C is referenced

MIN Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1	A*	A	A	A	A	A	A	A	A	A	
F_2		B*	B	B	B	B	B	B	B	B	
F_3			C*	C	C	D*	D	D	D	D	

Virtual address within page C is referenced

page fault

What's the page that will be requested the furthest away?

MIN Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1	A*	A	A	A	A	A	A	A	A	A	
F_2		B*	B	B	B	B	B	B	B	B	C*
F_3			C*	C	C	D*	D	D	D	D	D

Virtual address within page C is referenced

page fault

B replaced
C loaded

B or D will be requested the furthest away (surely not A):
pick one (e.g., B)

MIN Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1	A*	A	A	A	A	A	A	A	A	A	A
F_2		B*	B	B	B	B	B	B	B	C*	C
F_3			C*	C	D*	D	D	D	D	D	D

Eventually, we get a total of 5 page faults

LRU Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1											
F_2											
F_3											

How many page faults (denoted by *)?

LRU Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1											
F_2											
F_3											

Initially, no frame is loaded in memory at all
(pure demand paging)

LRU Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1	A*	A	A	A	A						
F_2		B*	B	B	B						
F_3			C*	C	C						

Up to this point, the same as FIFO

LRU Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1	A*	A	A	A	A	A					
F_2		B*	B	B	B	B					
F_3			C*	C	C	C					

Virtual address within page D is referenced page fault

LRU Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1	A*	A	A	A	A	A					
F_2		B*	B	B	B	B					
F_3			C*	C	C	C					

Virtual address within page D is referenced

page fault

We can't look forward anymore!

LRU Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1	A*	A	A	A	A	A					
F_2		B*	B	B	B	B					
F_3			C*	C	C	D*					

Virtual address within page D is referenced

page fault

C replaced
D loaded

02/12/2025

C is the page that has not been used for the longest time in the past

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LRU Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1	A*	A	A	A	A	A	A	A	A		
F_2		B*	B	B	B	B	B	B	B		
F_3			C*	C	C	D*	D	D	D		

Up to this point, no more page faults

LRU Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1	A*	A	A	A	A	A	A	A	A	A	
F_2		B*	B	B	B	B	B	B	B	B	
F_3			C*	C	C	D*	D	D	D	D	

Virtual address within page C is referenced

LRU Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1	A*	A	A	A	A	A	A	A	A	A	
F_2		B*	B	B	B	B	B	B	B	B	
F_3			C*	C	C	D*	D	D	D	D	

Virtual address within page C is referenced

page fault

We can't look forward anymore!

LRU Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1	A*	A	A	A	A	A	A	A	A	C*	
F_2		B*	B	B	B	B	B	B	B	B	
F_3			C*	C	C	D*	D	D	D	D	

Virtual address within page C is referenced

page fault

A replaced
C loaded

02/12/2025

A is the page that has not been used for the longest time in the past

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LRU Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1	A*	A	A	A	A	A	A	A	A	C*	C
F_2		B*	B	B	B	B	B	B	B	B	B
F_3			C*	C	C	D*	D	D	D	D	D

Virtual address within page A is referenced

LRU Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1	A*	A	A	A	A	A	A	A	A	C*	C
F_2		B*	B	B	B	B	B	B	B	B	B
F_3			C*	C	C	D*	D	D	D	D	D

Virtual address within page A is referenced

page fault

We can't look forward anymore!

LRU Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1	A*	A	A	A	A	A	A	A	A	C*	C
F_2		B*	B	B	B	B	B	B	B	B	B
F_3			C*	C	C	D*	D	D	D	D	A*

Virtual address within page A is referenced

page fault

D replaced
A loaded

D is the page that has not been used for the longest time in the past

LRU Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1	A*	A	A	A	A	A	A	A	A	C*	C
F_2		B*	B	B	B	B	B	B	B	B	B
F_3			C*	C	C	D*	D	D	D	D	A*

Eventually, we get a total of 6 page faults

LRU Page Replacement: (Unlucky) Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, D, A, B, C, D, A, B, C

	A	B	C	D	A	B	C	D	A	B	C
F_1											
F_2											
F_3											

How many page faults (denoted by *)?

LRU Page Replacement: (Unlucky) Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, D, A, B, C, D, A, B, C

	A	B	C	D	A	B	C	D	A	B	C
F_1	A*	A	A								
F_2		B*	B								
F_3			C*								

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3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, D, A, B, C, D, A, B, C

	A	B	C	D	A	B	C	D	A	B	C
F_1	A*	A	A	D*							
F_2		B*	B	B							
F_3			C*	C							

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3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

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	A	B	C	D	A	B	C	D	A	B	C
F_1	A*	A	A	D*	D						
F_2		B*	B	B	A*						
F_3			C*	C	C						

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	A	B	C	D	A	B	C	D	A	B	C
F_1	A*	A	A	D*	D	D					
F_2		B*	B	B	A*	A					
F_3			C*	C	C	B*					

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4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, D, A, B, C, D, A, B, C

	A	B	C	D	A	B	C	D	A	B	C
F_1	A*	A	A	D*	D	D	C*				
F_2		B*	B	B	A*	A	A				
F_3			C*	C	C	B*	B				

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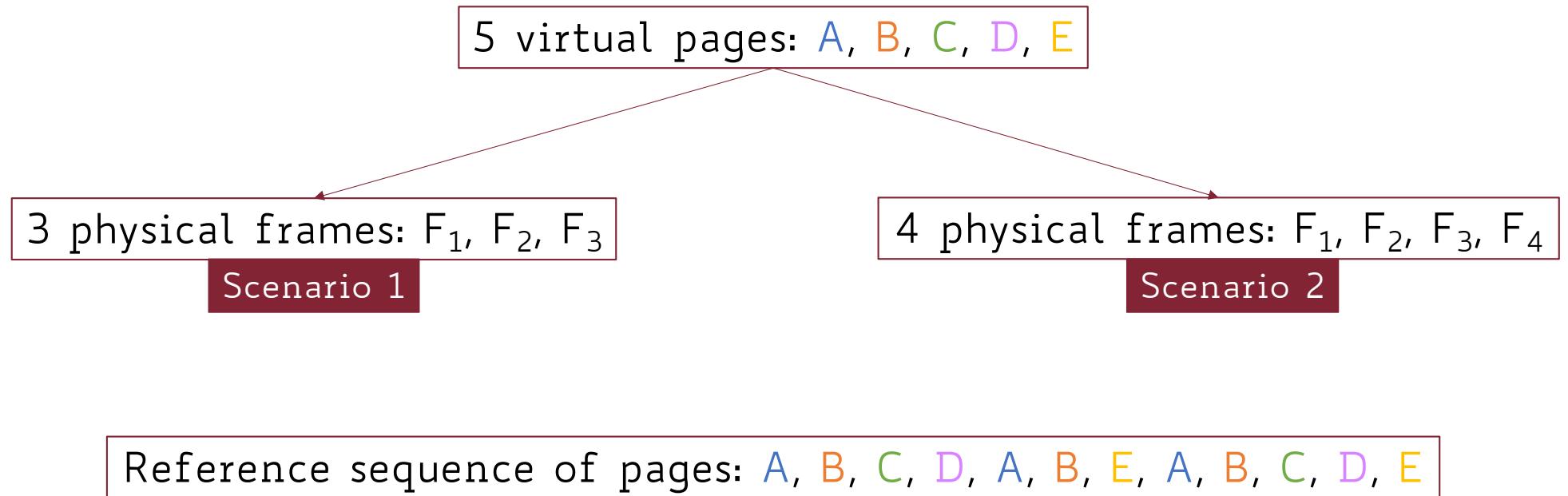
	A	B	C	D	A	B	C	D	A	B	C
F_1	A*	A	A	D*	D	D	C*	C	C	B*	B
F_2		B*	B	B	A*	A	A	D*	D	D	C*
F_3			C*	C	C	B*	B	B	A*	A	A

Eventually, we get a total of 11 page faults

Page Replacement: What If We Add Memory?

- Does adding memory always reduce the number of page faults?
- Intuitively, it would seem so...
- The answer, in fact, depends on the page replacement algorithm
- Let's see this with an example, using FIFO page replacement

FIFO Page Replacement: Example



FIFO Page Replacement: Example

	A	B	C	D	A	B	E	A	B	C	D	E
F_1	A*	A	A	D*	D	D	E*	E	E	E	E	E
F_2		B*	B	B	A*	A	A	A	A	C*	C	C
F_3			C*	C	C	B*	B	B	B	B	D*	D
F_1	A*	A	A	A	A	A	E*	E	E	E	D*	D
F_2		B*	B	B	B	B	B	A*	A	A	A	E*
F_3			C*	C	C	C	C	B*	B	B	B	B
F_4				D*	D	D	D	D	D	C*	C	C

FIFO Page Replacement: Example

	A	B	C	D	A	B	E	A	B	C	D	E
--	---	---	---	---	---	---	---	---	---	---	---	---

F_1	A*	A	A	D*	D	D	E*	E	E	E	E	E
F_2		B*	B	B	A*	A	A	A	A	C*	C	C
F_3			C*	C	C	B*	B	B	B	B	D*	D

9 page faults

F_1	A*	A	A	A	A	A	E*	E	E	E	D*	D
F_2		B*	B	B	B	B	B	A*	A	A	A	E*
F_3			C*	C	C	C	C	B*	B	B	B	B
F_4				D*	D	D	D	D	D	C*	C	C

10 page faults

Belady's Anomaly

Adding page frames may cause more page faults with some algorithms

LRU Page Replacement: Example

	A	B	C	D	A	B	E	A	B	C	D	E
--	---	---	---	---	---	---	---	---	---	---	---	---

F ₁	A*	A	A	D*	D	D	E*	E	E	C*	C	C
F ₂		B*	B	B	A*	A	A	A	A	A	D*	D
F ₃			C*	C	C	B*	B	B	B	B	B	B

9 page faults

F ₁	A*	A	A	A	A	A	A	A	A	A	A	E*
F ₂		B*	B	B	B	B	B	B	B	B	B	B
F ₃			C*	C	C	C	E*	E	E	E	D*	D
F ₄				D*	D	D	D	D	D	C*	C	C

8 page faults

With LRU, adding page frames always decreases the number of page faults

LRU Page Replacement: Example

	A	B	C	D	A	B	E	A	B	C	D	E
--	---	---	---	---	---	---	---	---	---	---	---	---

F ₁	A*	A	A	D*	D	D	E*	E	E	C*	C	C
F ₂		B*	B	B	A*	A	A	A	A	A	D*	D
F ₃			C*	C	C	B*	B	B	B	B	B	B

9 page faults

F ₁	A*	A	A	A	A	A	A	A	A	A	A	E*
F ₂		B*	B	B	B	B	B	B	B	B	B	B
F ₃			C*	C	C	C	E*	E	E	E	D*	D
F ₄				D*	D	D	D	D	D	C*	C	C

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	A	B	C	D	A	B	E	A	B	C	D	E
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F ₂		B*	B	B	A*	A	A	A	A	A	D*	D
F ₃			C*	C	C	B*	B	B	B	B	B	B
F ₁	A*	A	A	A	A	A	A	A	A	A	A	E*
F ₂		B*	B	B	B	B	B	B	B	B	B	B
F ₃			C*	C	C	C	E*	E	E	E	D*	D
F ₄				D*	D	D	D	D	D	C*	C	C

At each point in time 4-frame memory contains a subset of 3-frame

Page Replacement: Summary

- FIFO is easy to implement but may lead to too many page faults
- May suffer from Belady's Anomaly

Page Replacement: Summary

- MIN is the optimal choice but cannot be used in practice since future memory references are never known in advance

Page Replacement: Summary

- LRU is a fair approximation of MIN assuming the past is a good predictor of the future
 - Exploits the locality reference (small working set that fits in memory)
 - Works poorly when the locality reference doesn't hold (large working set)

LRU: Implementation Details

How could we implement LRU page replacement algorithm?

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First Idea

Keep a timestamp for each page with the time it has been last accessed
Remove the page with the highest difference w.r.t. current timestamp

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Keep a timestamp for each page with the time it has been last accessed
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Problems?

Every time a page is accessed its timestamp must be updated

Linear scan of all the pages to select the one to be removed

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How could we implement LRU page replacement algorithm?



Second Idea

Keep a list of pages with the most recently used in front and the least recently used at the end: every time a page is accessed move it to front

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Still too expensive as the OS must change multiple pointers on each memory access

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 - No total order of page access

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- The specific number of bits used and the frequency with which the reference byte is updated are adjustable

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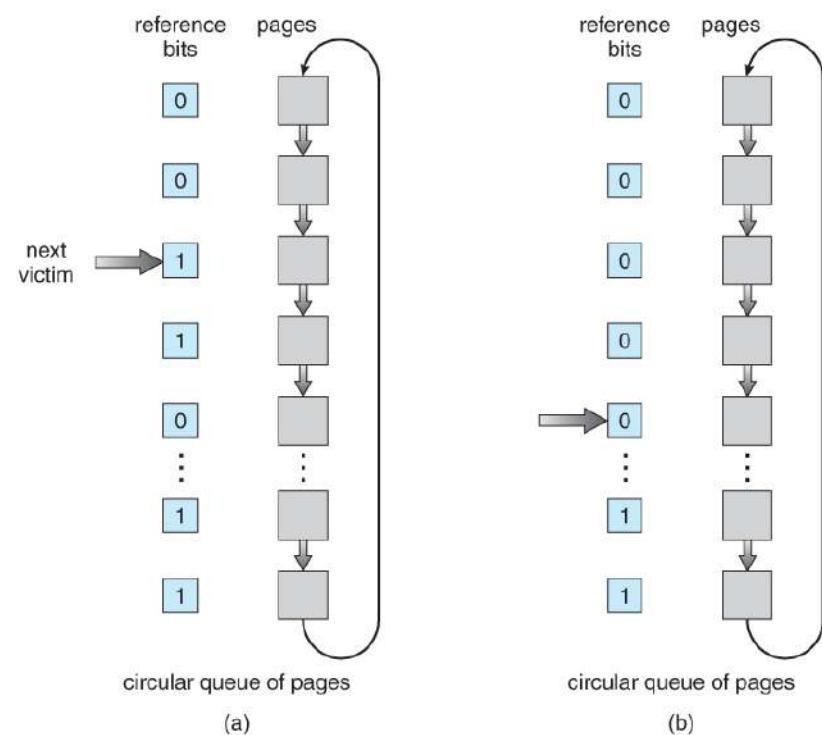
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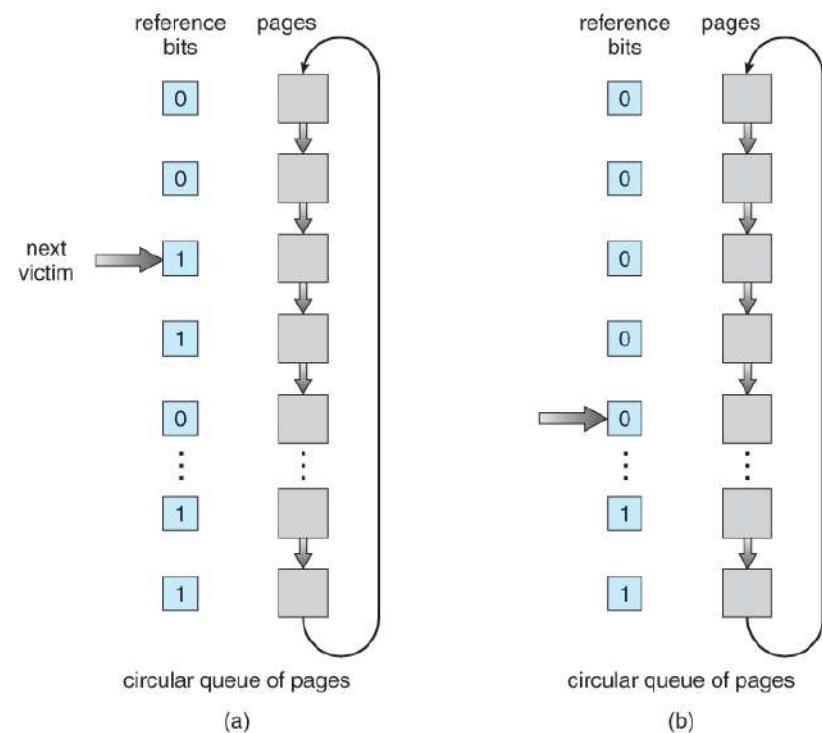
- Second Chance Algorithm → Single-Reference Bit + FIFO
- OS keeps frames in a FIFO circular list
- On every memory access, the reference bit is set to 1
- On a page fault, the OS scans the list of frames, checking the reference bit of the frame:
 - If this is 0, it replaces the page and sets it to 1
 - If this is 1, it sets it to 0 (second chance) and move to the next frame

Second Chance Algorithm (Clock)



A raw partitioning into: young vs. old frames

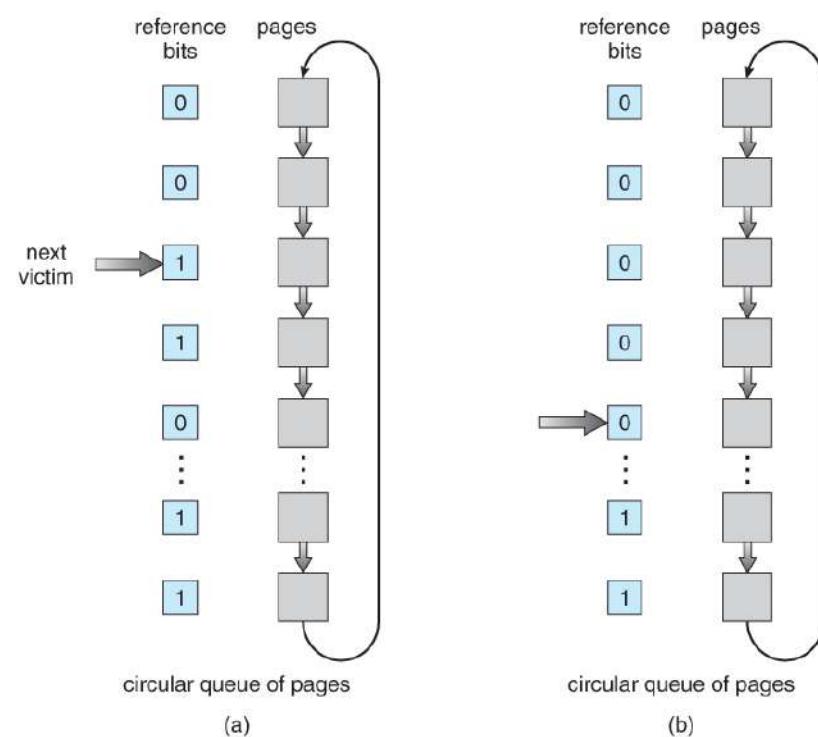
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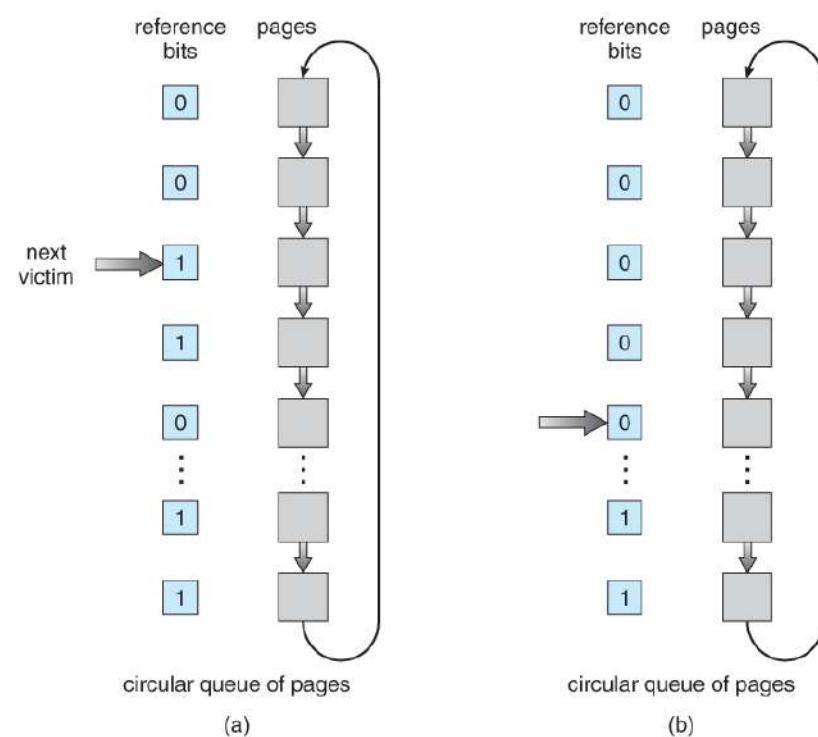


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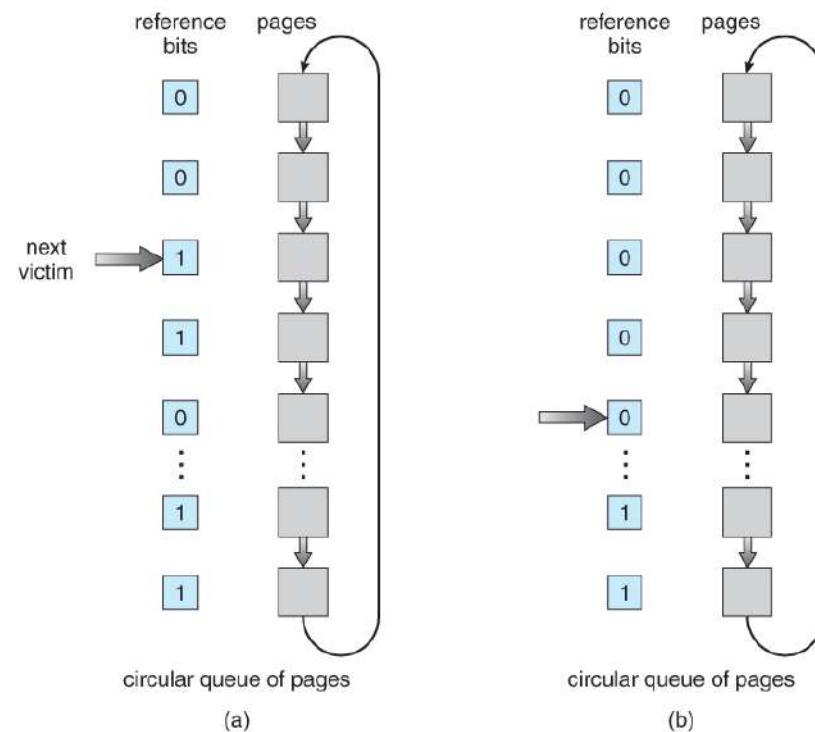
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This algorithm is also known as **clock** because it mimics the hands of a clock

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- Page replacement generally involves 2 I/O operations:
 - write the evicted page back to disk
 - read the newly referenced page from disk
- **Intuition:** It is cheaper to replace a page which has not been modified, since the OS does not need to write this back to disk

Enhanced Second Chance Algorithm

- OS should give preference to paging-out un-modified frames
- Yet, it can proactively write to disk modified frames for later

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- HW keeps a modify bit (in addition to the reference bit)
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- HW keeps a modify bit (in addition to the reference bit)
 - 1 means the page has been modified (different from the copy on disk)
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- Use both the reference and modify bits (r, m) to classify pages into:
 - (0, 0): neither recently used nor modified;
 - (0, 1): not recently used, but modified;
 - (1, 0): recently used, but clean
 - (1, 1): recently used and modified

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- Prioritize replacement of clean pages if possible

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- Multiple processes can however run concurrently on a single-CPU system
- The degree of multiprogramming is not fixed *a priori*, yet it is driven by the locality reference (a.k.a. 90÷10 rule)
- This allows a system to load the **working set** (i.e., few pages) of many processes, thereby increasing the degree of multiprogramming

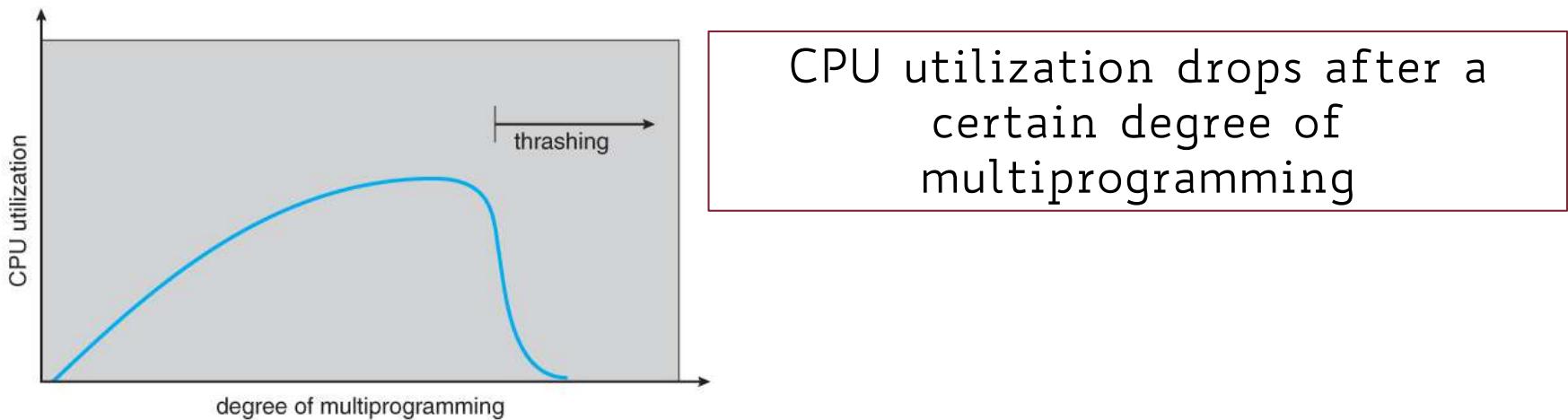
Multiprogramming and Thrashing

- When the degree of multiprogramming is too high, active working sets of running processes may saturate the whole memory capacity

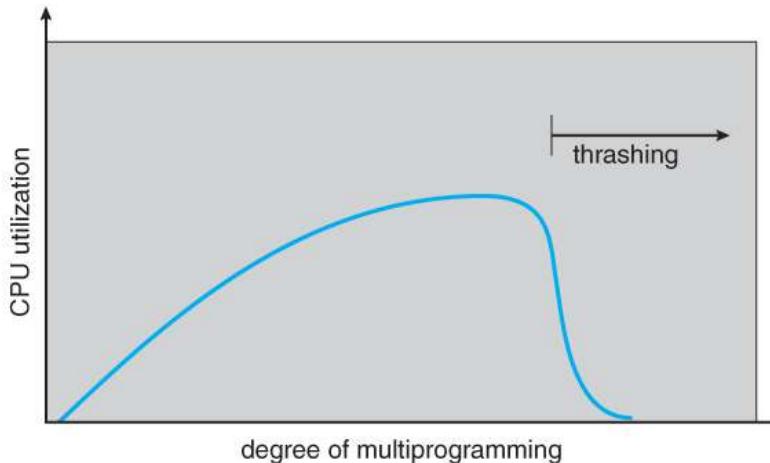
Multiprogramming and Thrashing

- When the degree of multiprogramming is too high, active working sets of running processes may saturate the whole memory capacity
- **Thrashing** → Memory is over-committed and pages are continuously tossed out while they are still in use
 - Memory access time approaches disk access time due to many page faults
 - Drastic degradation of performance

Multiprogramming and Thrashing



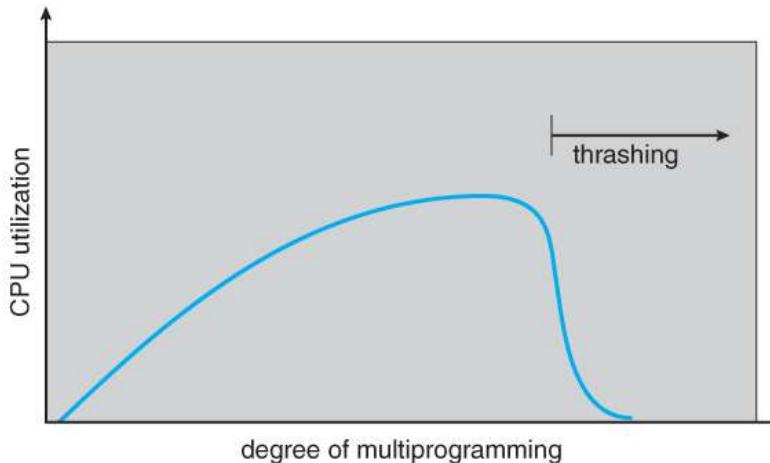
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CPU utilization drops after a certain degree of multiprogramming

Eventually, also CPU-bound processes turn into I/O-bound ones (due to page faults)

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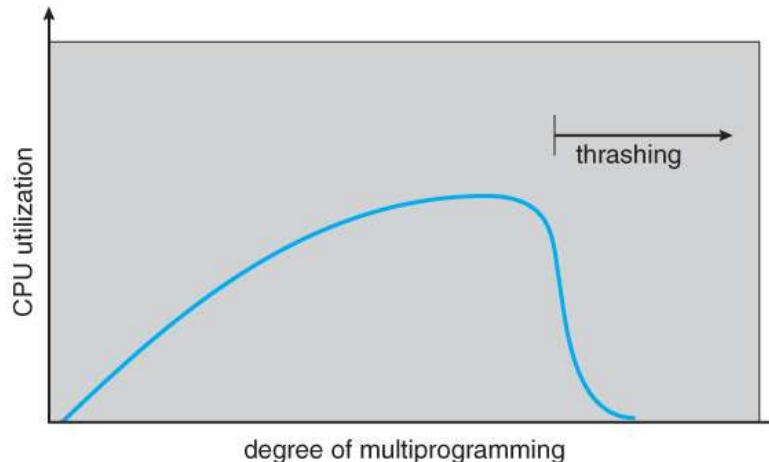


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Fixing the degree of multi-programming apriori may be a too inflexible option

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Ultimately, we want to give each process enough memory so as to avoid thrashing

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Global Allocation/Replacement

- All pages from all processes are in a single pool (single LRU queue)
- Upon page replacement, any page may be a potential victim, whether it currently belongs to the process seeking a free frame or not
- **PRO:** flexibility
- **CON:** thrashing more likely (no isolation)

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- Each process has its own fixed pool of frames
- LRU replacement affects only each process' frames
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m = number of available physical page frames

n = number of processes

S_i = size of the i -th process; $S = \sum_{i=1}^n S_i$ = total size of all processes

Equal Allocation/Replacement: $\frac{m}{n}$

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As allocations fluctuate over time, so does m
(processes must be swapped out or not started if not enough frames)

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- Intuitively this makes sense: the more memory a process needs the higher the chance it will refer a much larger memory range
- However, there might be cases where this is not true
 - e.g., a process allocates a 1GB array but only uses a small portion of it
- In other words, the working set of a process may not be correlated with its (theoretical) memory footprint

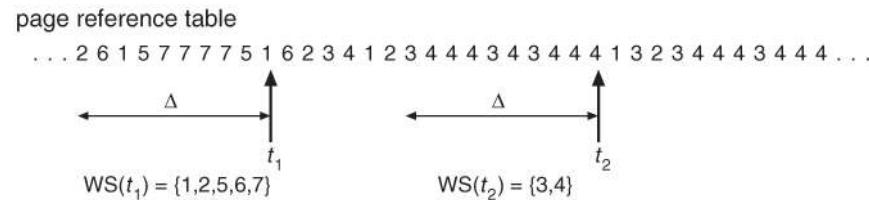
Matching the Working Set

- Goal → Give each process enough frames to contain its working set
 - Informally, the working set is the set of pages the process is using "right now"
 - More formally, the working set of a process at time t , $W(t)$, is the set of all pages referenced during $(t-\Delta, t)$

Matching the Working Set

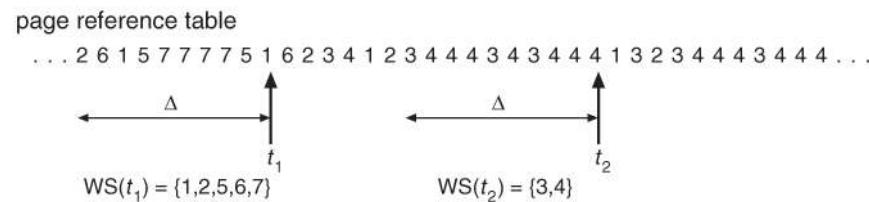
- Goal → Give each process enough frames to contain its working set
 - Informally, the working set is the set of pages the process is using "right now"
 - More formally, the working set of a process at time t , $W(t)$, is the set of all pages referenced during $(t-\Delta, t)$
 - Δ is often considered as a time window (e.g., last 100 ms.)

Determining the Working Set



The selection of Δ is critical to the success of the working set model

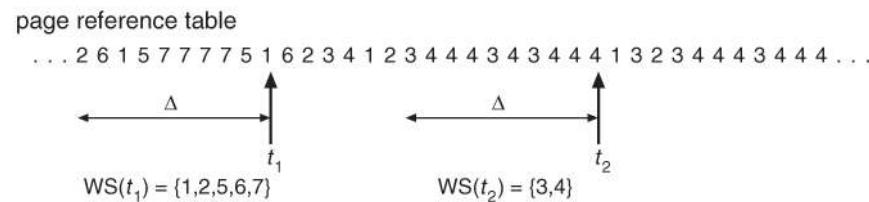
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Δ too small
it does not encompass all of the pages of the current locality

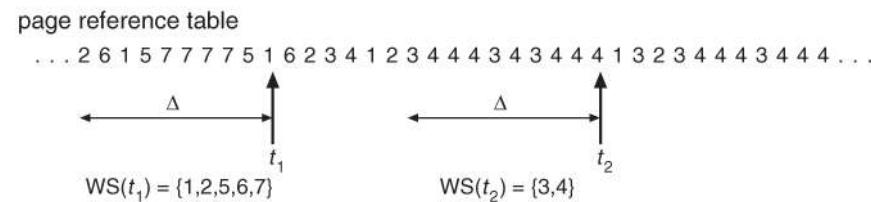
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Exact tracking is expensive: update the working set at each memory access

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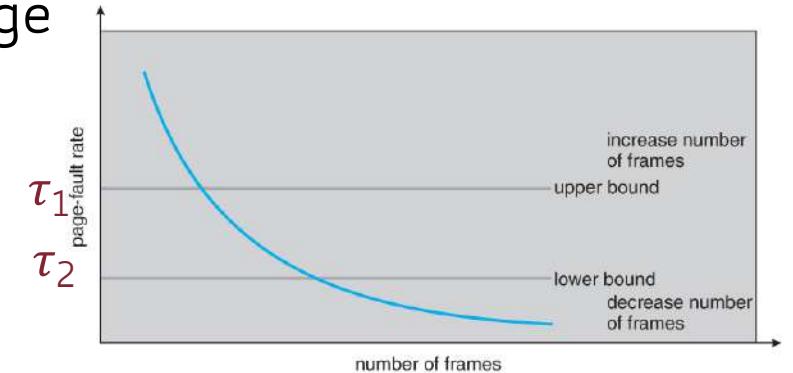
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- In both cases, keeping the exact sliding window is impractical, and the working set is often implemented with **sampling**
- Every k memory references (e.g., $k = 1,000$), consider the working set to be all pages referenced within *that* window

Tracking Page Fault Rate

- Ultimately, our goal is to minimize the **page fault rate**

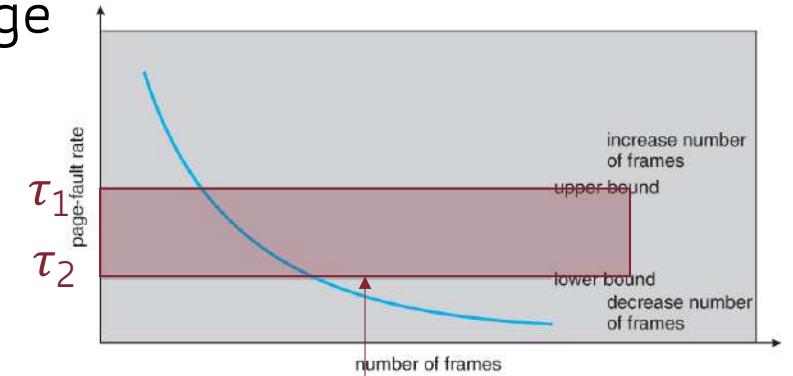
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Dynamically adjust allocated frames so as to keep processes in this area

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- The choice of page replacement algorithm is crucial when physical memory is limited
 - All algorithms approach to the optimum as the physical memory allocated to a process approaches to the virtual memory size
- The more processes running concurrently, the less physical memory each one can have
- The OS must dynamically adapt the number of frames per process based on its working set