

# Sistemi Operativi I

Corso di Laurea in Informatica

2025-2026



SAPIENZA  
UNIVERSITÀ DI ROMA

Gabriele Tolomei

Dipartimento di Informatica  
Sapienza Università di Roma

[tolomei@di.uniroma1.it](mailto:tolomei@di.uniroma1.it)

# Problems Seen So Far

- Contiguous allocation
  - Hard to grow or shrink process memory
- Fragmentation
  - Frequent compaction needed
- Process entirely loaded
  - Swapping helps but it may be too inefficient

# Paging

- A memory management scheme that addresses the problems above

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- External fragmentation is eliminated because pages have fixed size
  - Internal fragmentation may still occur though

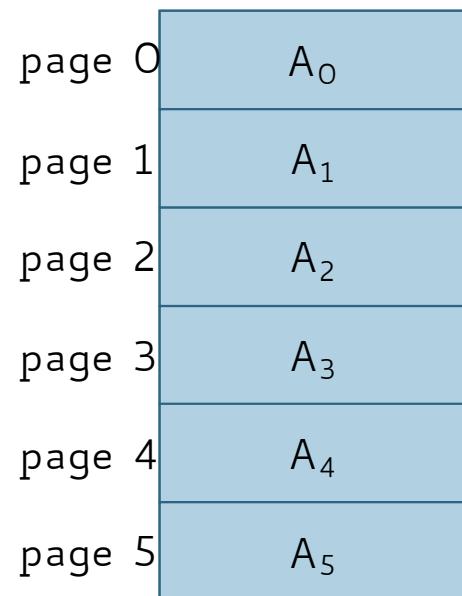
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## 90/10 Rule

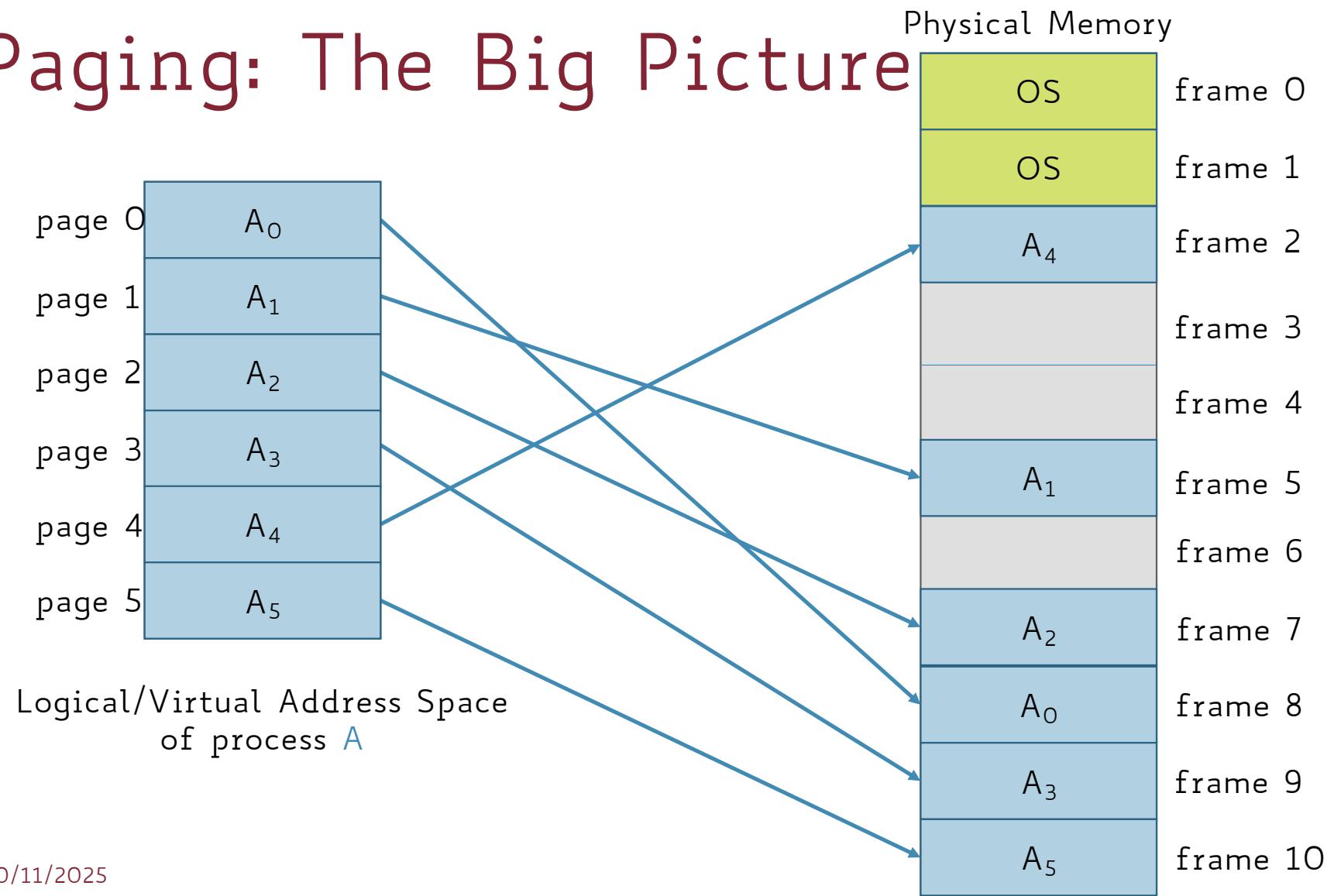
Processes spend **90%** of their time accessing only **10%** of their allocated memory space

# Paging: The Big Picture



Logical/Virtual Address Space  
of process A

# Paging: The Big Picture



# Basic OS Responsibilities for Paging

- The OS has **2 main responsibilities:**
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# Basic OS Responsibilities for Paging

- The OS has **2 main responsibilities**:
  - mapping between logical pages and physical frames
  - translating logical addresses to physical addresses
- All of this must be done efficiently!
  - Remember, memory addresses are referenced all the time
- OS needs dedicated support for doing it → **Page Table**

# Page Table: Mapping Pages to Frames

0	A <sub>0</sub>
1	A <sub>1</sub>
2	A <sub>2</sub>
3	A <sub>3</sub>
4	A <sub>4</sub>
5	A <sub>5</sub>

0	OS
1	OS
2	A <sub>4</sub>
3	
4	
5	A <sub>1</sub>
6	
7	A <sub>2</sub>
8	A <sub>0</sub>
9	A <sub>3</sub>
10	A <sub>5</sub>

# Page Table: Mapping Pages to Frames

Lookup table to retrieve what frame a page is stored in

0	A <sub>0</sub>
1	A <sub>1</sub>
2	A <sub>2</sub>
3	A <sub>3</sub>
4	A <sub>4</sub>
5	A <sub>5</sub>

Page	Frame
0	8
1	5
2	7
3	9
4	2
5	10

OS	0
OS	1
A <sub>4</sub>	2
	3
	4
A <sub>1</sub>	5
	6
A <sub>2</sub>	7
A <sub>0</sub>	8
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OS	0
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	3
	4
A <sub>1</sub>	5
	6
A <sub>2</sub>	7
A <sub>0</sub>	8
A <sub>3</sub>	9
A <sub>5</sub>	10

We have assumed **all** pages of a process are mapped to physical frames, but this is not always the case

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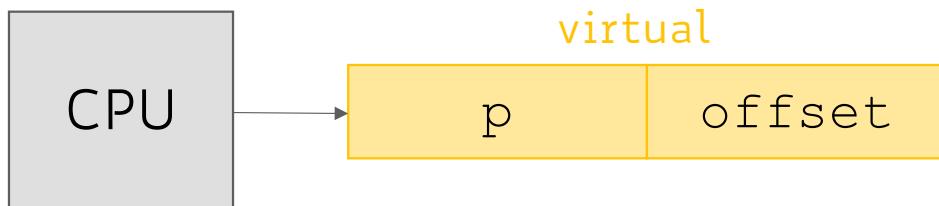
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- Processes use virtual (logical) addresses to refer to memory (not page number!)
- Virtual (logical) address space is still contiguous starting from 0
- Page table must ultimately translate virtual address to physical address

# Page Table: Virtual to Physical Address

virtual address consists of 2 parts:

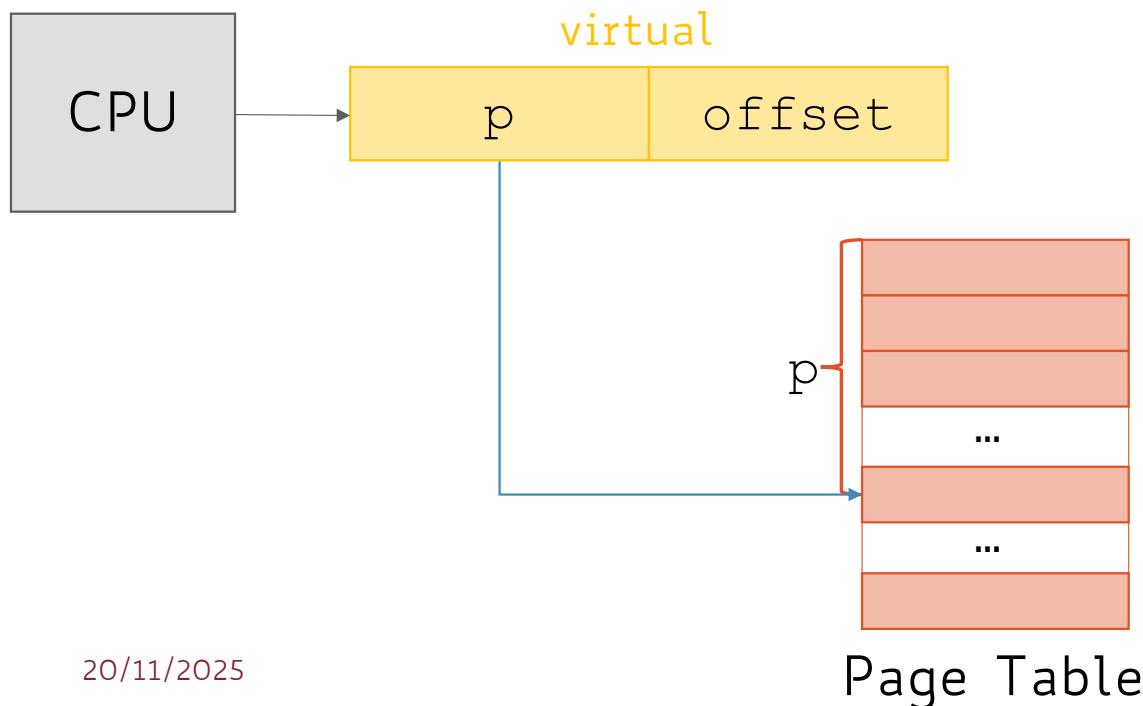
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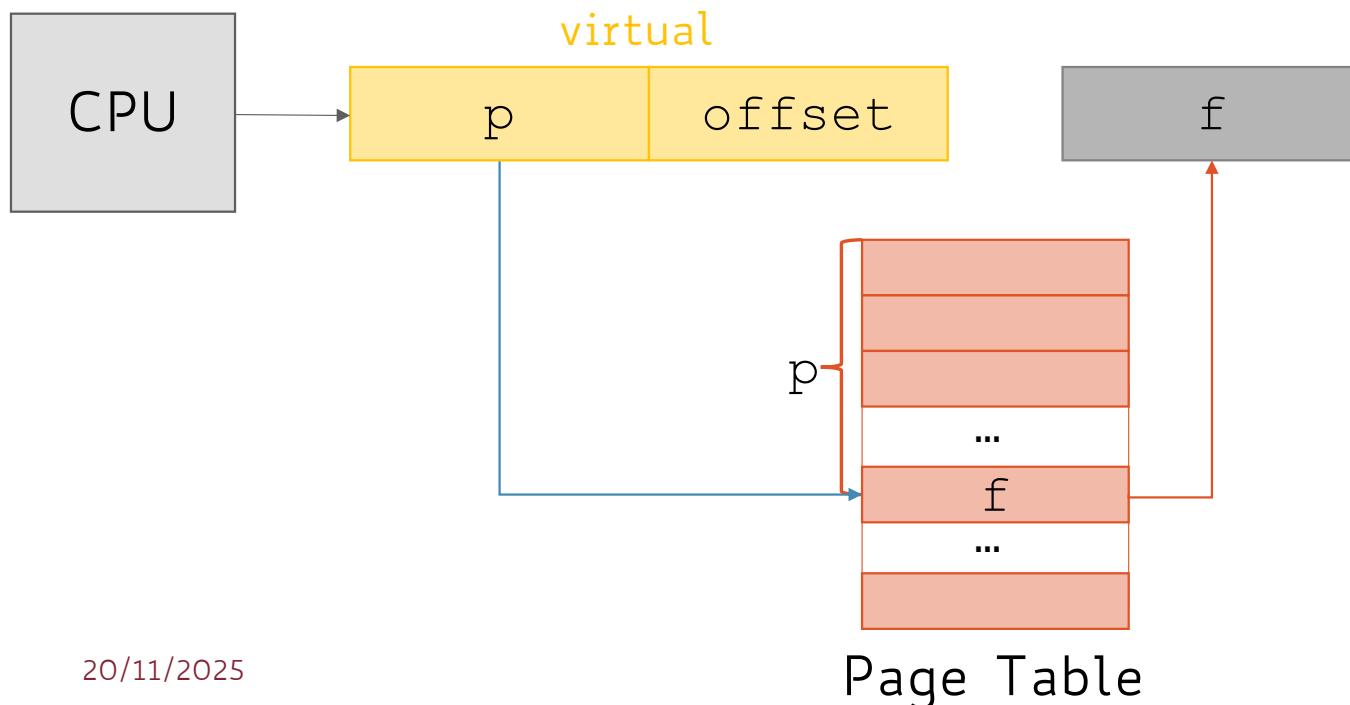
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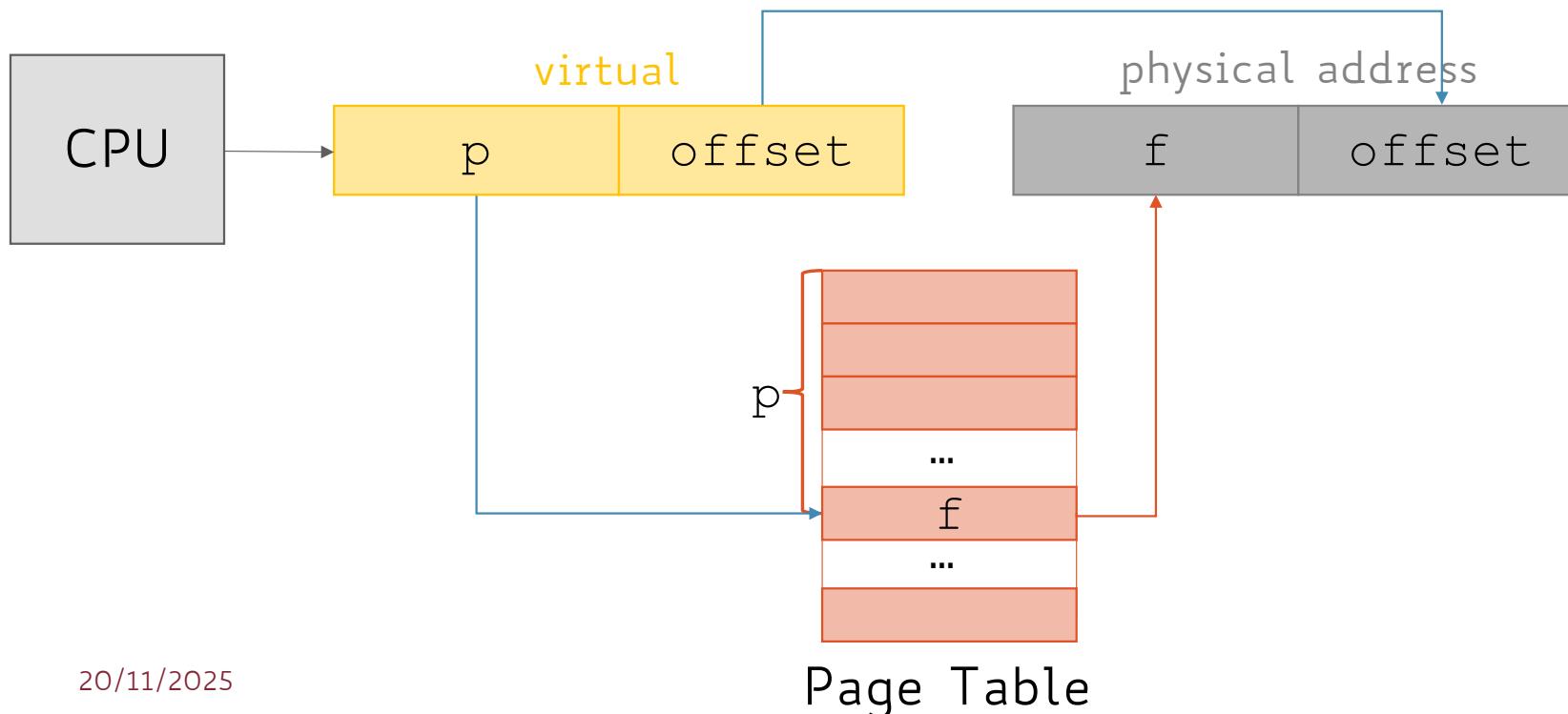
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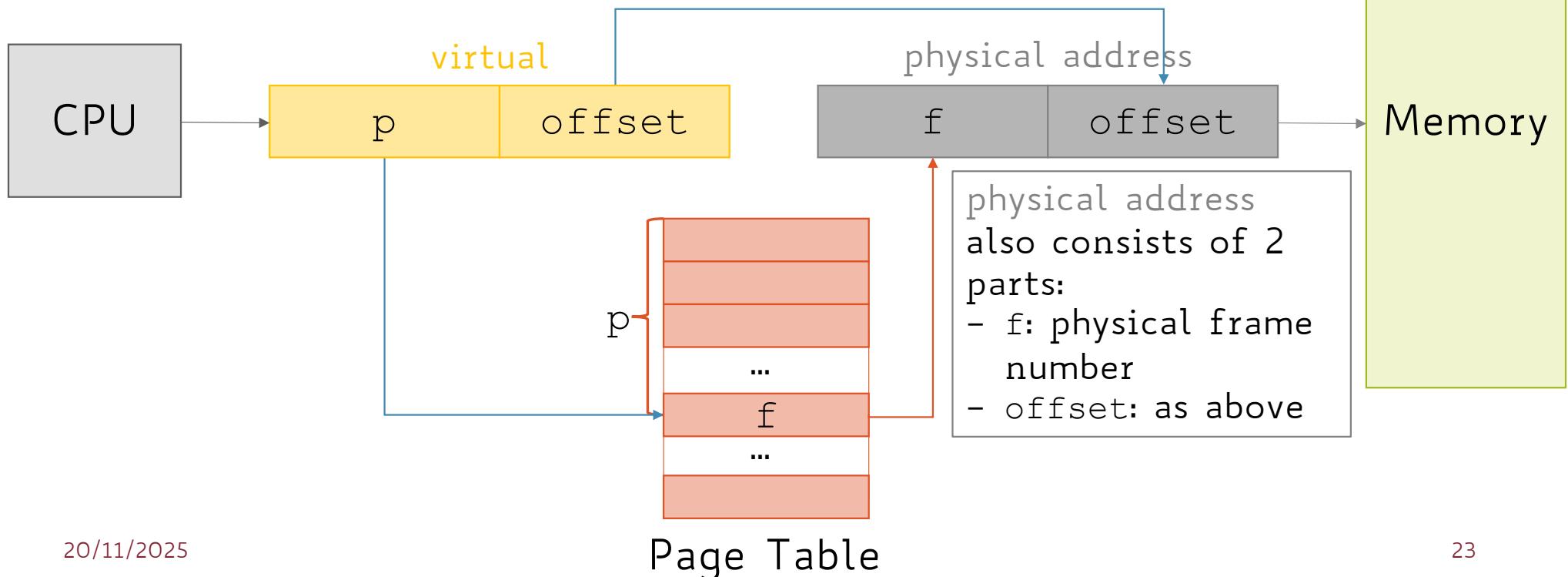
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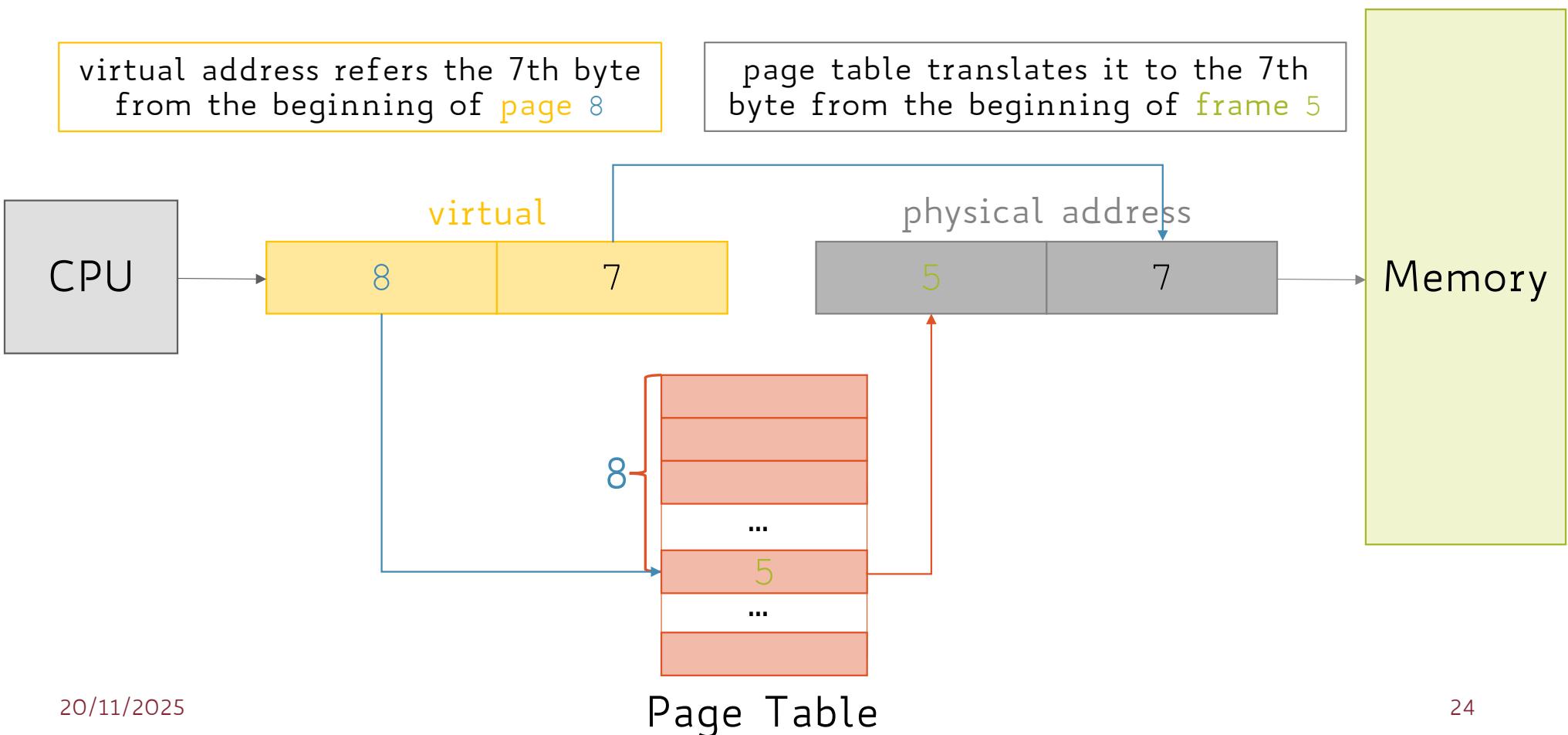
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# Page Table: Example of Address Translation



# Paging as Dynamic Relocation

- Paging is an advanced form of dynamic relocation
- Each virtual address is bound by the page table to a physical address
- Page table can be seen just as a set of base (relocation) registers, one for each frame
- Mapping is invisible to the user process: the OS maintains the page table and translation happens in hardware (via the MMU)
- Protection is provided similarly to dynamic relocation (limit register)

# Paging: Steps

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1. Get the page number (**p**) and the **offset** where the virtual address **x** resides
2. Use **p** to index into the page table to retrieve the frame number **f**
3. Combine **f** with **offset** to obtain the physical address **y**

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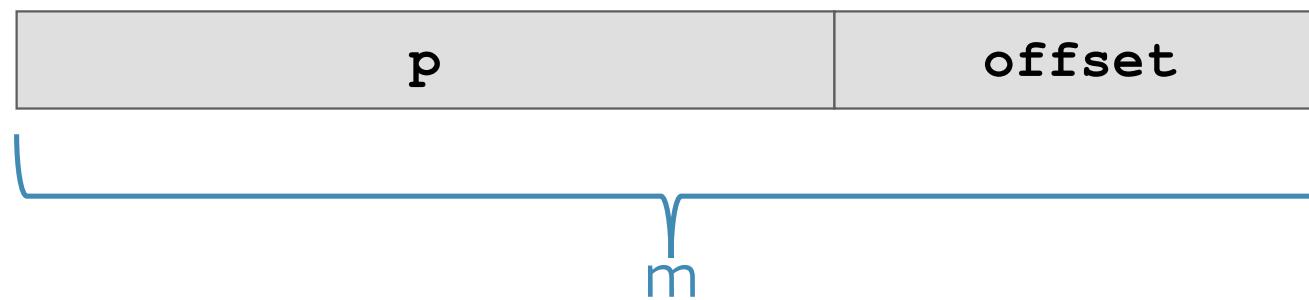
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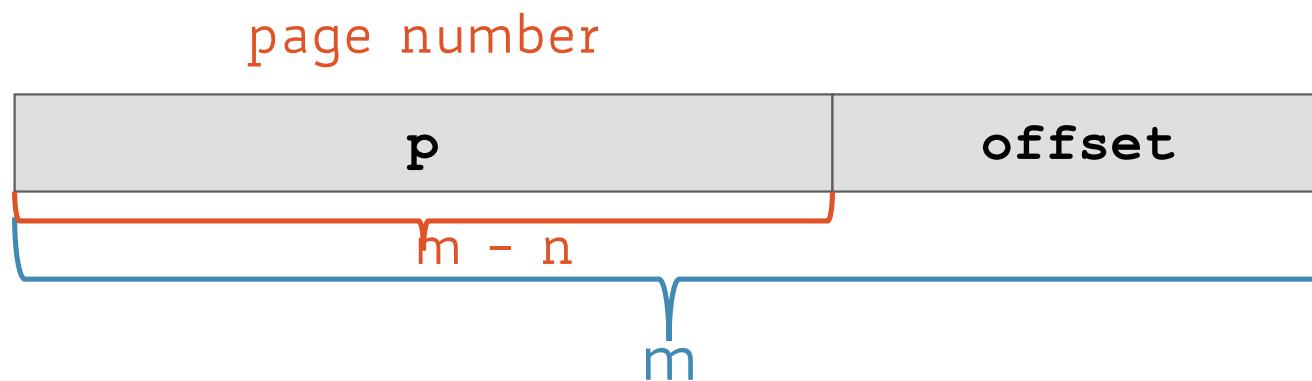
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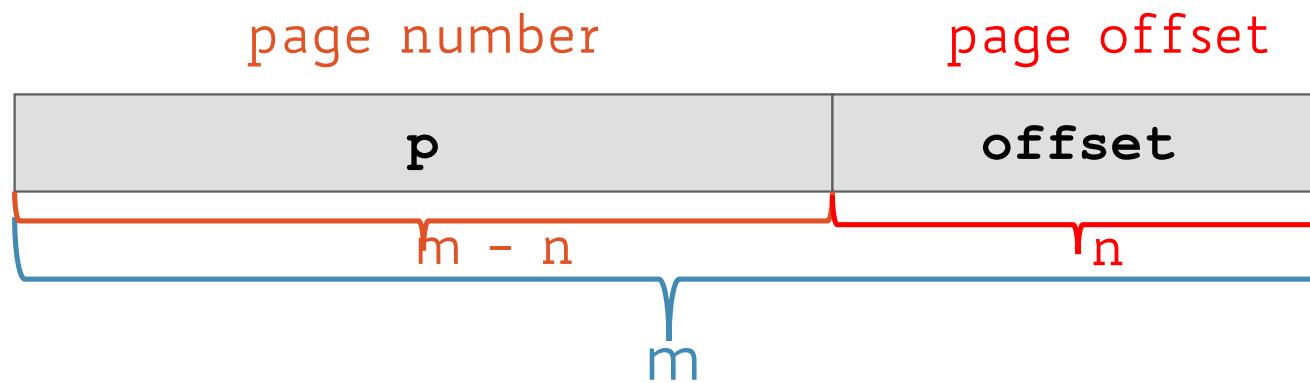
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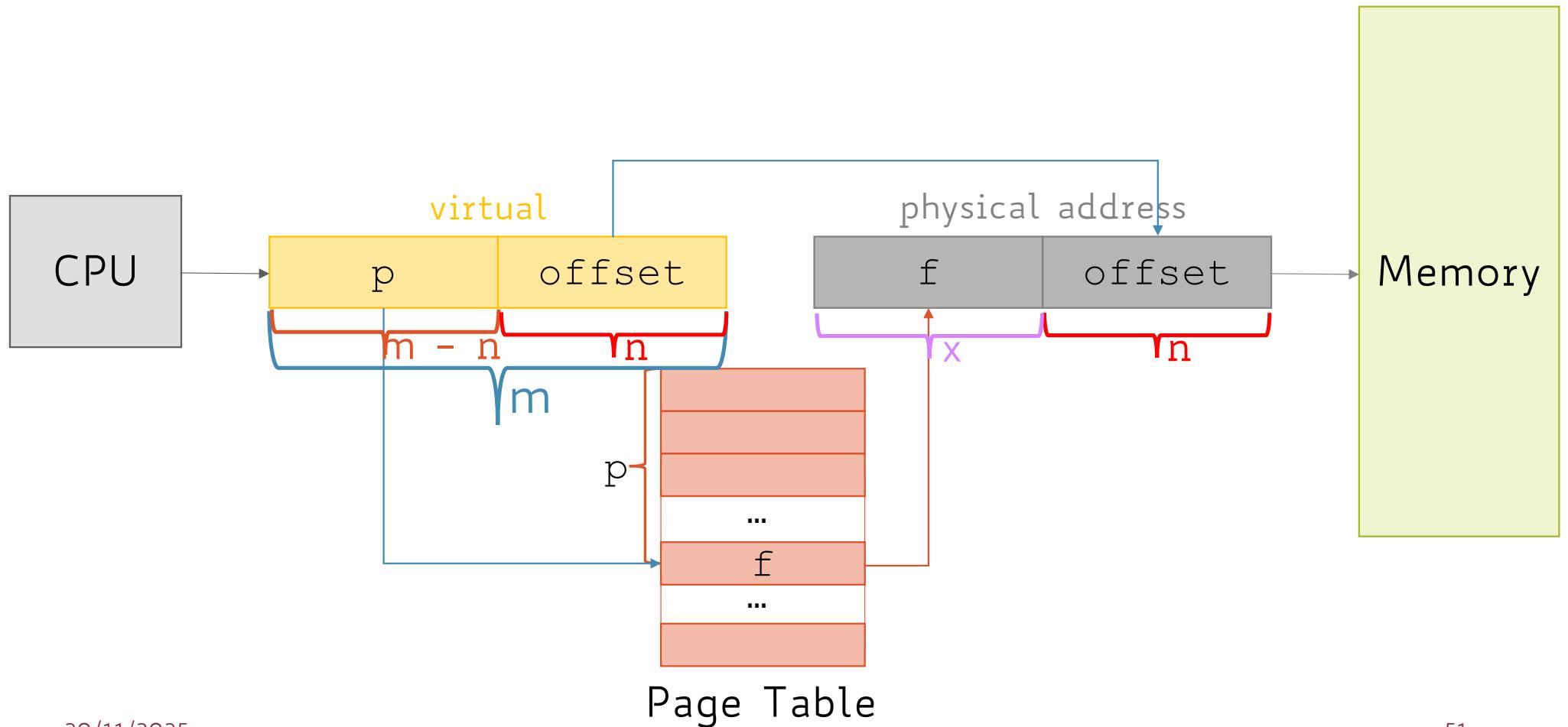
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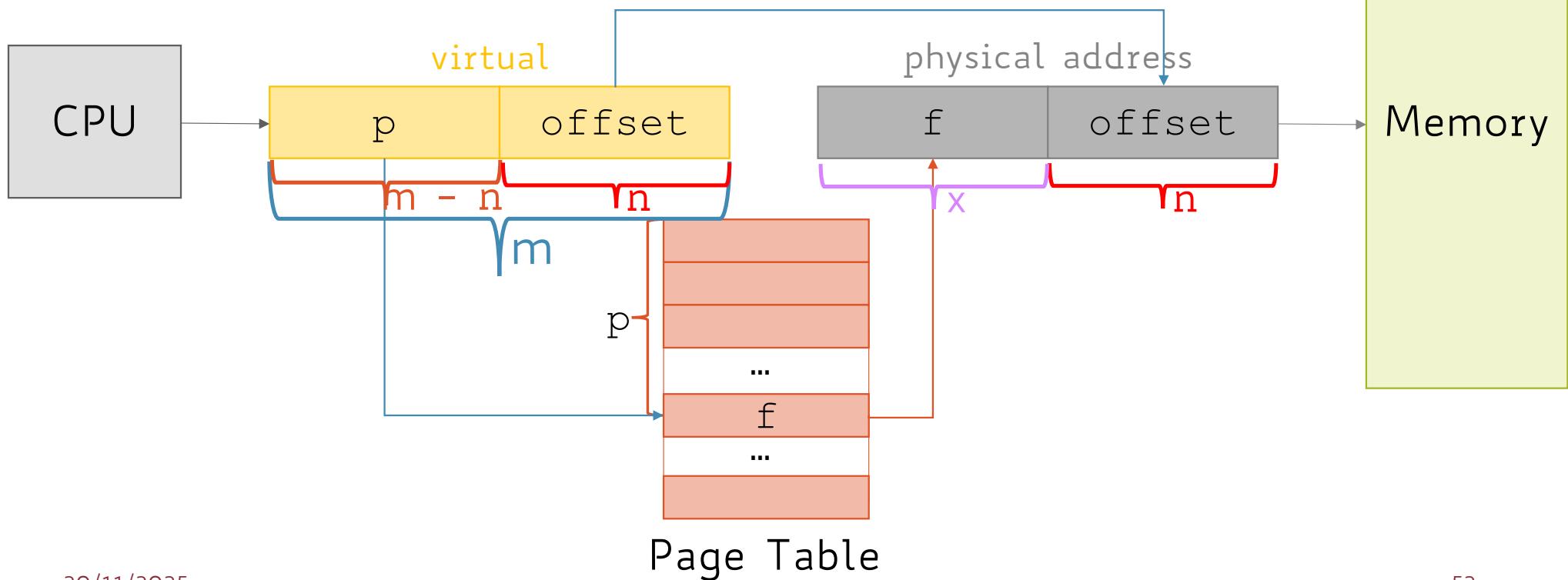
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NOTE

$m-n$  doesn't necessarily have to be equal to  $x$



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- Typical values of virtual address size is  $m = 32$  or  $64$  bits
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- Typical values of page/frame sizes is  $n = 12$  bits
  - Each page/frame is  $2^{12} = 4\text{KiB}$
- Assuming  $m = 32$  bits, there are  $2^{m-n} = 2^{20} = \sim 1\text{M}$  pages/frames
  - The page table has  $2^{20}$  entries (i.e., one for each page/frame)

# Paging: Practical Example

Suppose we have a virtual memory and a physical memory, both of size  $M = 1024B$  (1KiB)

Q1

How many bits are needed for a virtual/physical address (assuming single-byte addressing)

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10 bits to address  $M = 1024$  bytes (both for virtual and physical address)

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How big is the page table? (i.e., how many pages/entries does it have to index?)

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R2

$T = M / S = 1024 \text{ memory bytes} / 16 \text{ bytes per page} = 64 \text{ pages}$

# Paging: Practical Example

Q3

What is p and offset (i.e., how many bits for p and offset?)

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What is p and offset (i.e., how many bits for p and offset?)

R3

Our logical address is made of  $m = 10$  bits  
 $n = 4$  bits are used to represent the offset, as each page/frame is  $S = 16$  bytes  
 $m-n = 6$  bits are used to represent page number  $p$ , as there are  $T = 64$  pages

# Paging: Practical Example

Q4

Translate the virtual address  $x = 42$ , assuming the following page table

page	frame
0	12
1	5
2	37
3	0
...	..
63	29

# Paging: Practical Example

Q4

Translate the virtual address  $x = 42$ , assuming the following page table

page	frame
0	12
1	5
2	37
3	0
...	..
63	29

R4

$$p = x \text{ div } S = 42 \text{ div } 16 = 2$$

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Q4

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page	frame
0	12
1	5
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page	frame
0	12
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R4

$$p = x \text{ div } S = 42 \text{ div } 16 = 2$$

$$\text{offset} = x \text{ mod } S = 42 \text{ mod } 16 = 10$$

10th byte from the beginning of frame 37

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Modern computers however operate natively on multiple of bytes (i.e., words) rather than single-byte. Typical values of word length is: 16, 32 or 64 bits.

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How many bits are therefore needed to address the number of words available on  $M$ ?

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Q1

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R1

8 bits to address  $M = 1024/4 = 256$  4-byte words

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R3

Our logical address is now made of  $m = 8$  bits  
 $n = 2$  bits are used to represent the offset, as each page/frame is:  
 $S = 16$  bytes =  $4 * 4$ -byte words  
 $m-n = 6$  bits are used to represent page number p, as there are still  
 $T = 64$  pages

# Paging: Practical Example 2

Q4

Translate the virtual address  $x = 7$ , assuming the following page table

page	frame
0	12
1	5
2	37
3	0
...	..
63	29

# Paging: Practical Example 2

Q4

Translate the virtual address  $x = 7$ , assuming the following page table

page	frame
0	12
1	5
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...	..
63	29

Remember: now virtual address refers to a 4-byte word!

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Q4

Translate the virtual address  $x = 7$ , assuming the following page table

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0	12
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$S = 16 \text{ bytes} = 4 * 4\text{-byte words}$

Must be expressed in terms of number of words

R4

$$p = x \text{ div } S = 7 \text{ div } 4 = 1$$

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Q4

Translate the virtual address  $x = 7$ , assuming the following page table

page	frame
0	12
1	5
2	37
3	0
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R4

$$p = x \text{ div } S = 7 \text{ div } 4 = 1$$

$$\text{offset} = x \text{ mod } S = 7 \text{ mod } 4 = 3$$

3rd word from the beginning of frame 5

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- An advanced form of dynamic relocation that requires OS intervention (**Page Tables**)
- Standard paging alone can be inefficient...