

Sistemi Operativi

Corso di Laurea in Informatica

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SAPIENZA
UNIVERSITÀ DI ROMA

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Paging + Segmentation

- **Paging (OS' view of memory)**
 - Divide memory into fixed-size pages and map them to physical frames
- **Segmentation (compiler's view of memory)**
 - Divide process into logical segments (e.g., code, data, stack, heap)
- Combine paging with segmentation to get the best of both worlds
 - **Segmented Paging**

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Virtual Memory uses backing storage (i.e., disk) to store unused pages and give the illusion of infinite virtual address space

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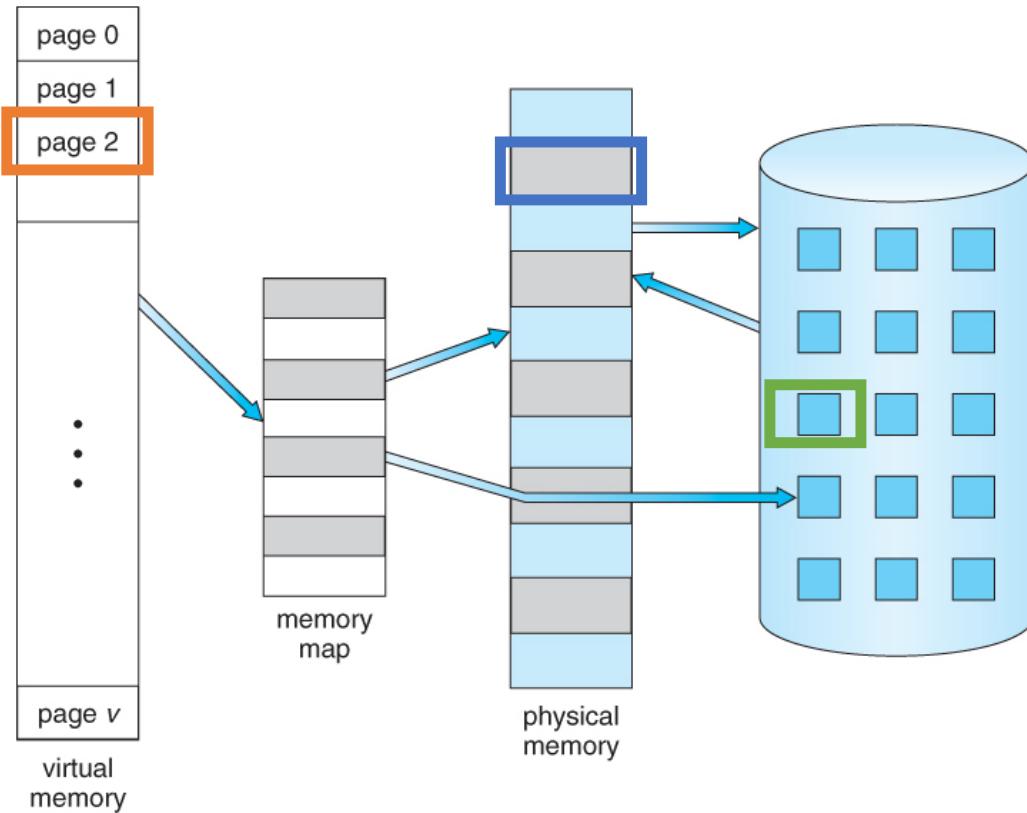
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 - Less I/O is needed for swapping processes in and out of memory, speeding things up

Virtual Memory: The Big Picture

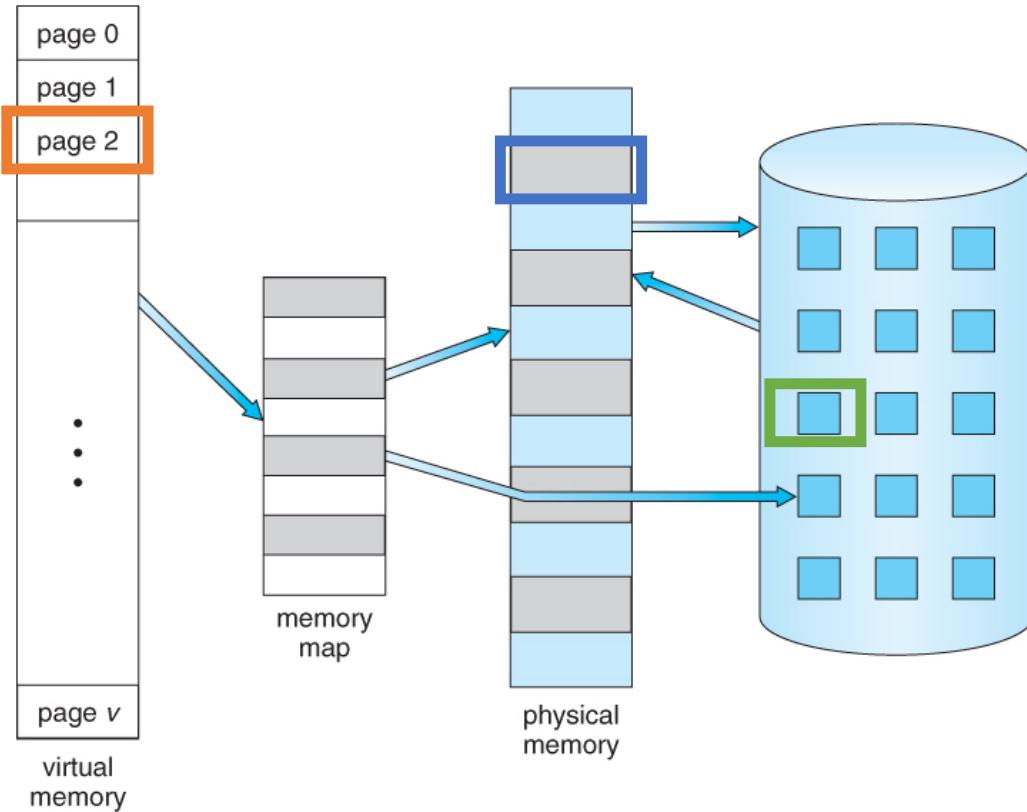


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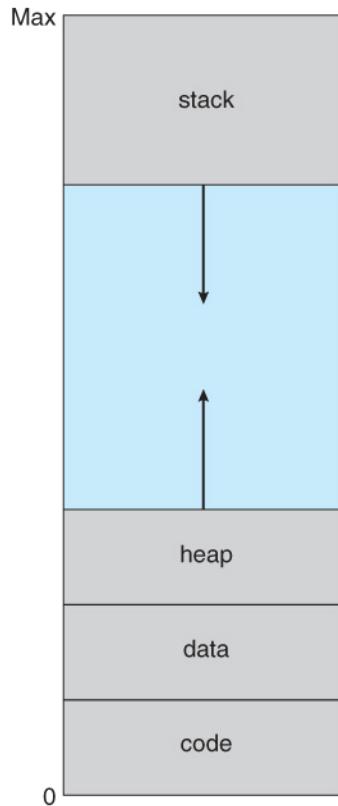
virtual memory can be much larger than physical memory



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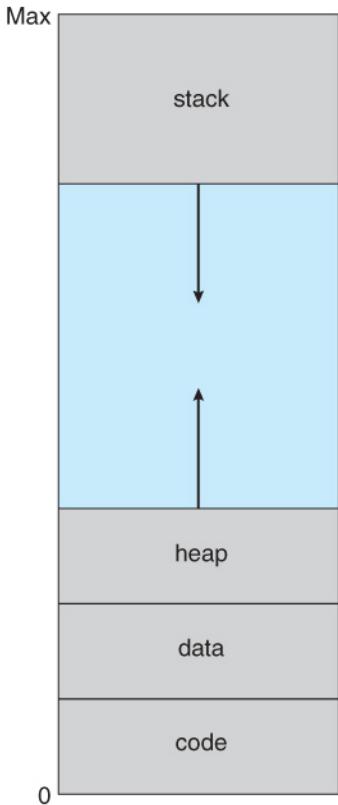
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The Sparseness of Virtual Address Space



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A lot of virtual memory addresses remain
unreferenced

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- Remember: access to disk is extremely slower than access to memory
- Therefore, memory accesses must reference pages that are in memory **with high probability**

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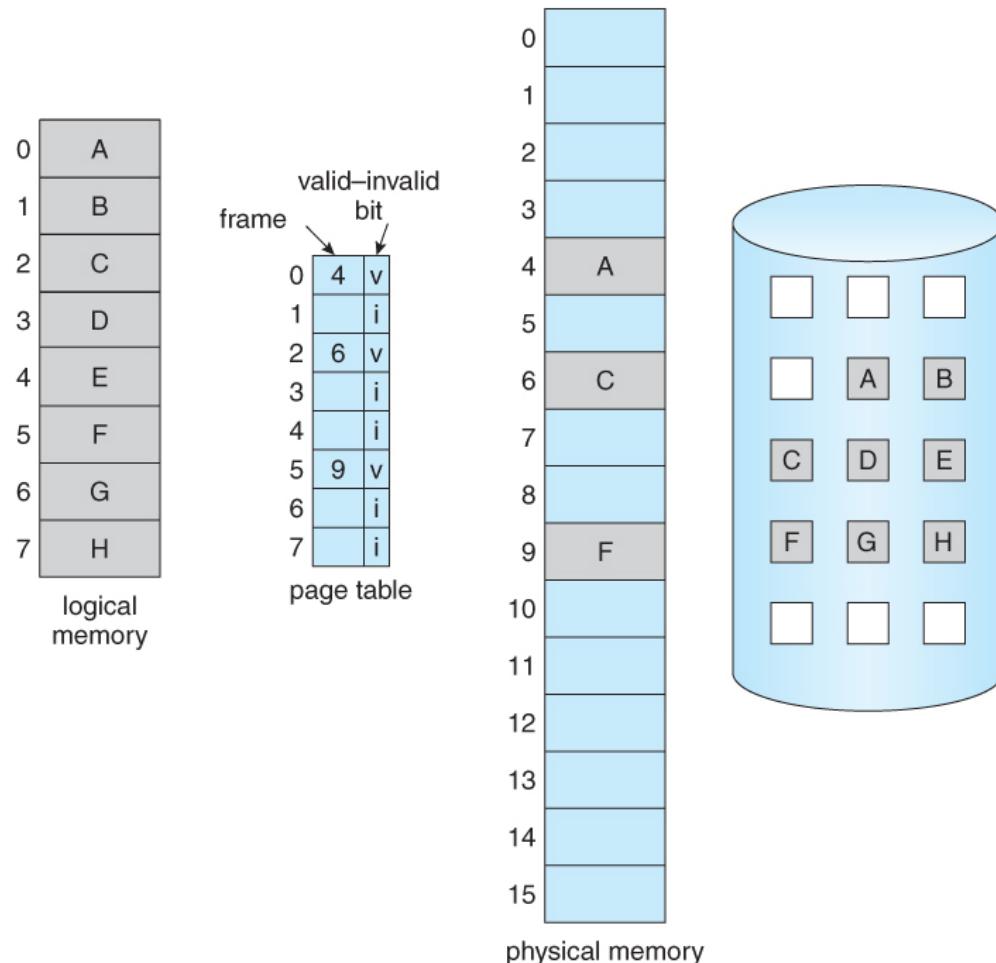
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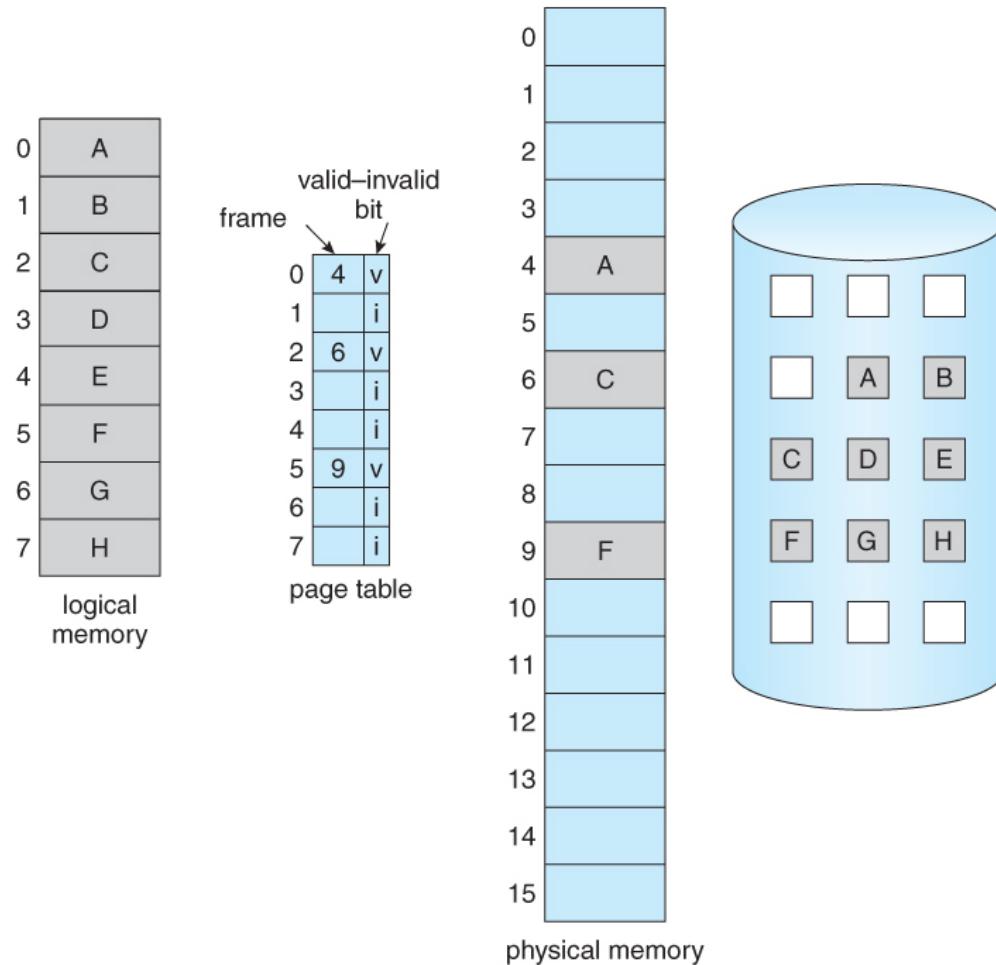
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- But in a reasonably small time frame, the working set stays the same

Virtual Memory: Basic Concepts



At each logical memory reference, a page table lookup is performed as usual

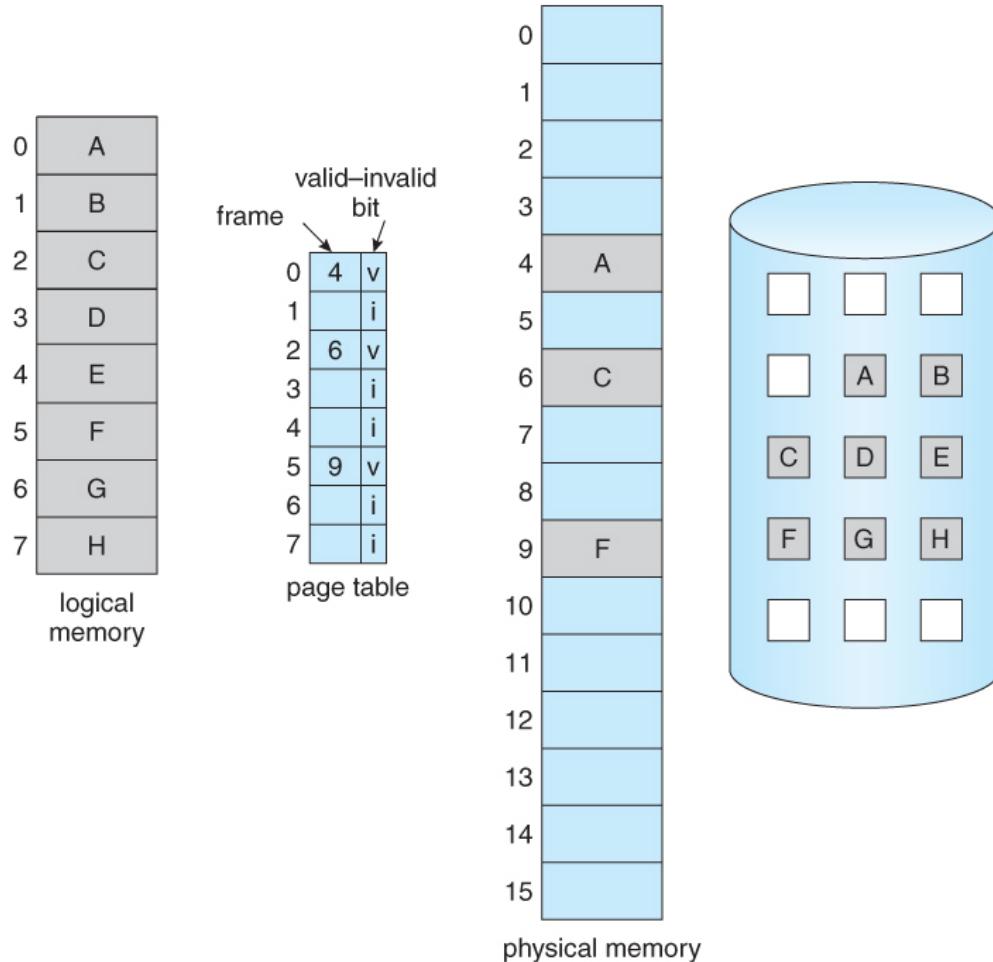
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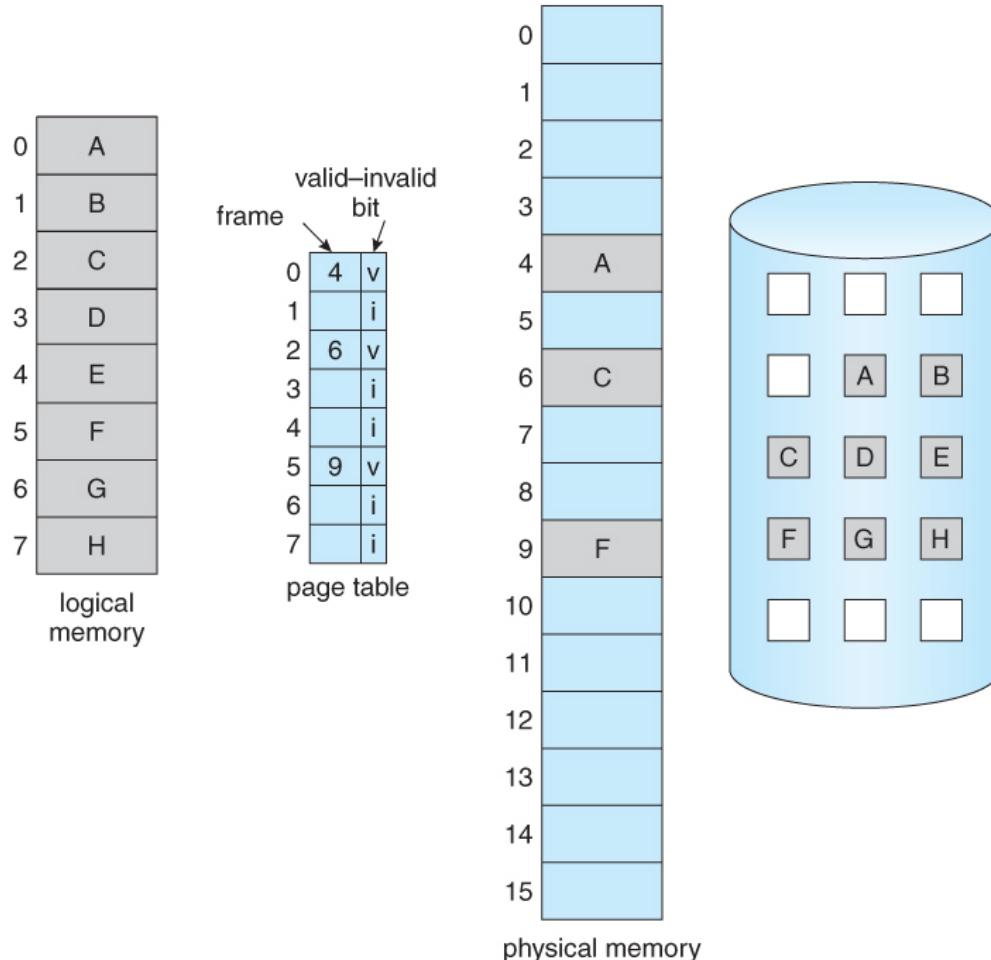


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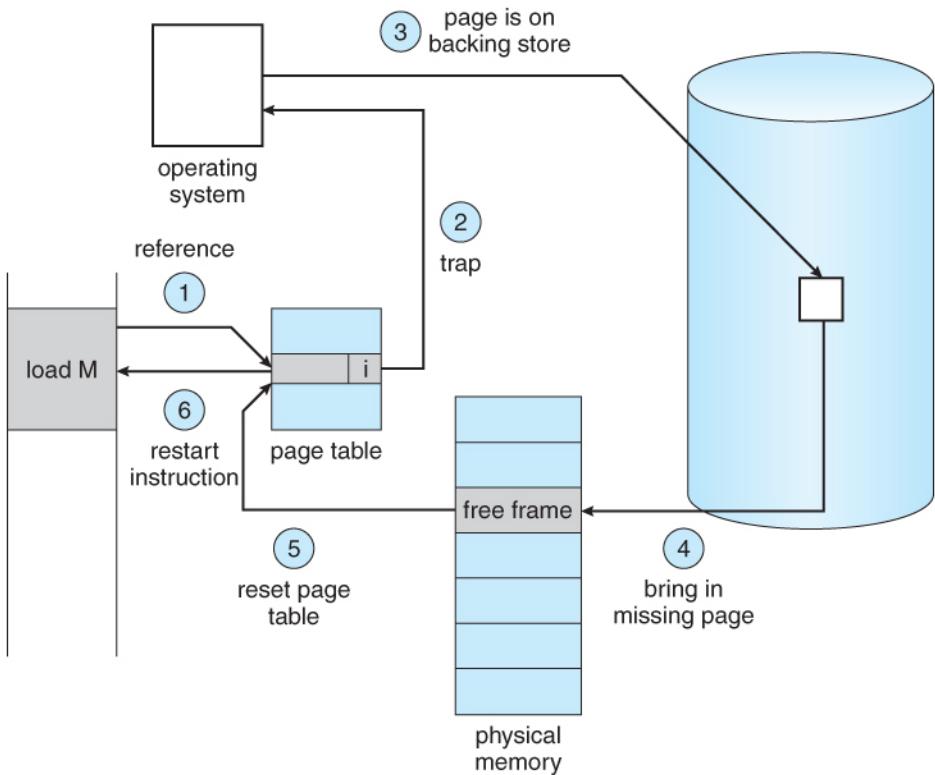
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Otherwise, a **page fault trap** occurs, and the page has to be loaded (i.e., fetched) from disk

Page Fault Handling

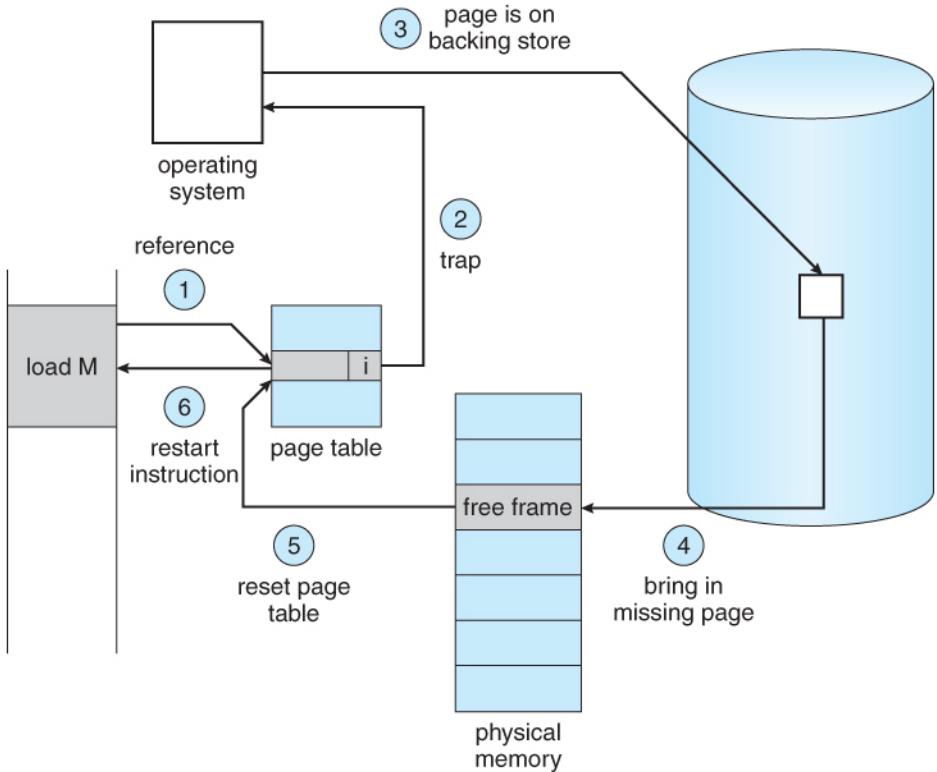
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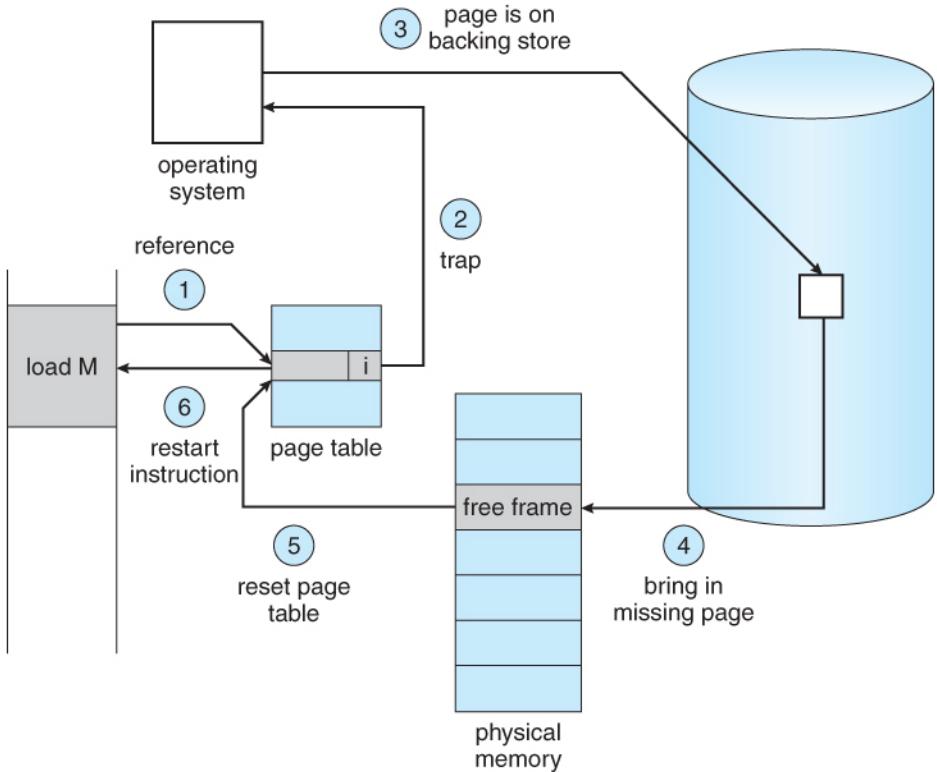
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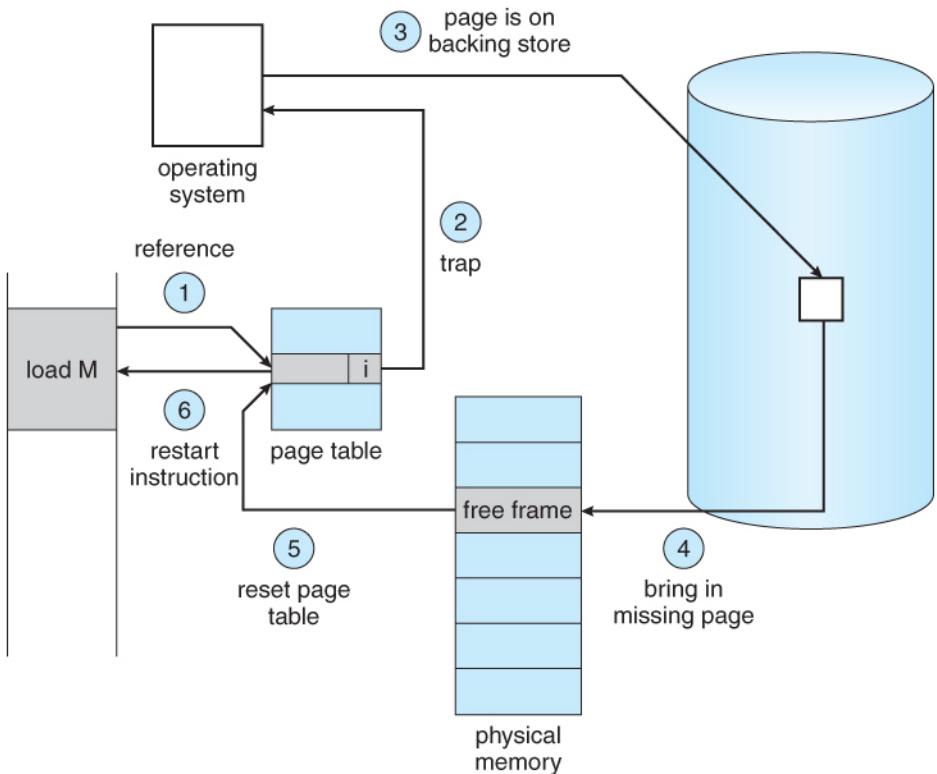


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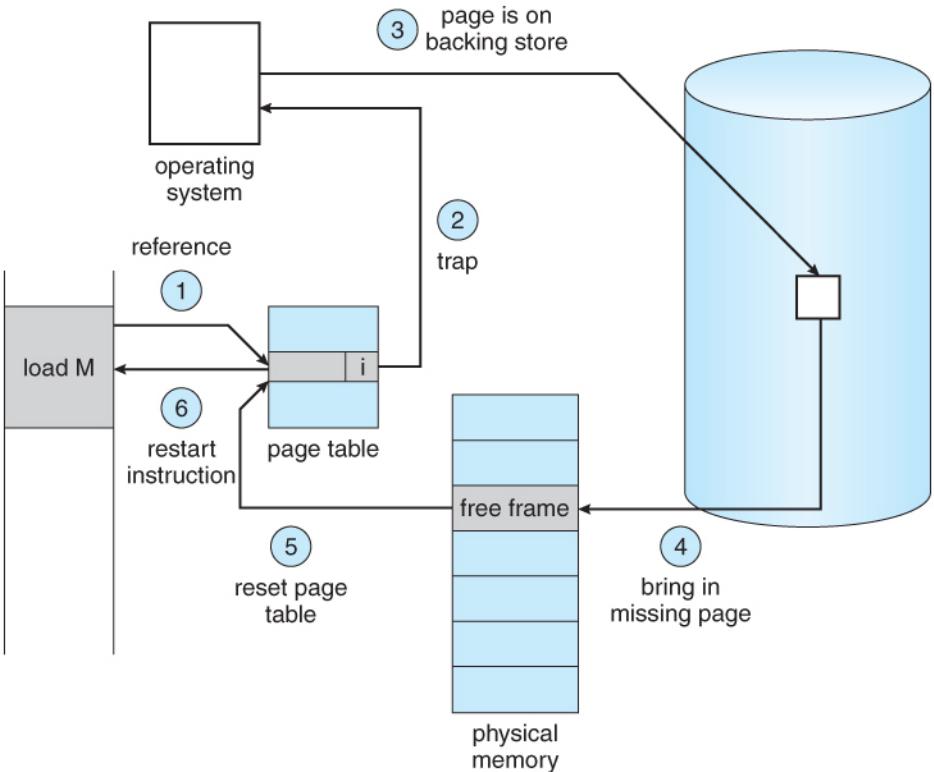
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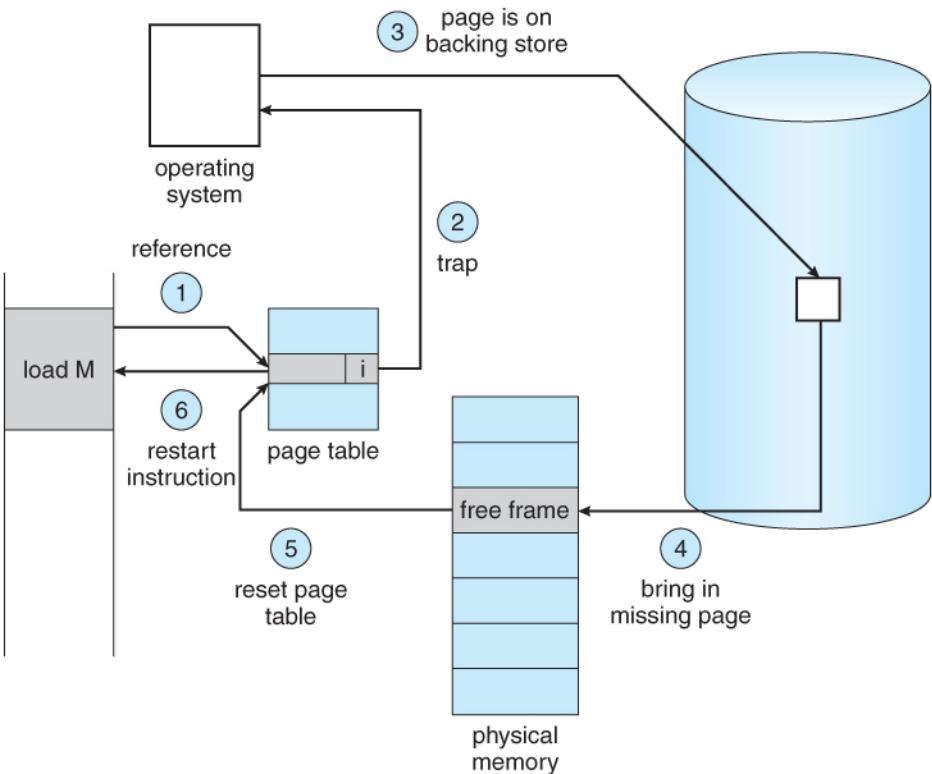
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5. When the I/O operation is complete, the process's page table is updated with the new frame number, and the bit is set to valid
6. The current process gets interrupted and the instruction that caused the page fault must be restarted from the beginning



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- If we get a TLB hit but the frame is not actually in main memory, we have to go fetch the page from disk anyway!
- TLB hit means the requested page entry is in the cache **and** the referenced frame is also in memory

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- If the requested page is not in the cache (TLB miss) and it is not even in memory (i.e., it is sitting on disk):
 - The OS picks a TLB entry to replace and fills it with the new entry as follows
 - invalidates the TLB entry
 - performs page fault trap operations
 - updates the TLB entry
 - restarts the faulting instruction

Page Fault Handling: Faulty Address

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- Architecture-dependant:
 - x86: hardware saves the virtual address that caused the fault (CR2 register)
 - On some platforms OS gets only address of faulting instruction, must simulate the instruction and try every address to find the one that generated the fault

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- To restart (from scratch) a faulty instruction the OS needs hardware support for saving:
 - The faulting instruction
 - The CPU state

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- **idempotent** vs. **non-idempotent** instructions
- **idempotent** → just restart the faulting instruction (hardware saves instruction address during page fault)
- **non-idempotent** → much more difficult to restart
 - `MOV [%R1], +(%R2)` → increment the value of R2 and store it to memory address in R1
 - What if memory address `[%R1]` causes the page fault?
 - Cannot naively redo the instruction from scratch otherwise R2 gets incremented twice

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- Even harder when using instructions that are not easily undoable
 - E.g., instructions that are used to move a block of memory at once
 - The block may span multiple pages: some of them can be in memory while some others not
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How to unwind those complicated side-effects?

Make sure all the addresses within the block to be transferred are in memory before starting executing the instruction

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- Still, an overhead must be paid every time a page fault occurs as the OS needs to interact with slower disk

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t_{MA} = physical memory access time

t_{FAULT} = time to handle a page fault

$p \in [0, 1]$ = probability of page fault

t_{ACCESS} = effective time for each memory reference

$$t_{ACCESS} = (1 - p) * t_{MA} + p * t_{FAULT}$$

Let's assume: $t_{MA} = 100$ nsec and $t_{FAULT} = 20$ msec = 20,000,000 nsec

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This heavily depends on p !

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The access time increases from just 100 nsec up to ~20.1 microsec

20 times slowdown factor

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$$\begin{aligned} 1.1 * 100 &= 100 - 100p + 20,000,000p = \\ 19,999,900p &= 110 - 100 = \end{aligned}$$

To achieve that goal, we can tolerate at most 1 page fault every about 2 million accesses!

$$p = \frac{10}{19,999,900} = \frac{1}{1,999,990} \approx 0,0000005 = 5 * 10^{-7}$$

Virtual Memory: Performance Example

More generally, given t_{MA} , t_{FAULT} , and a threshold $\varepsilon > 0$ if we want to find p s.t.:

$$t_{ACCESS} = (1 + \varepsilon) * t_{MA}$$

We substitute t_{ACCESS} and solve for p the resulting equation:

$$\begin{aligned}(1 - p) * t_{MA} + p * t_{FAULT} &= (1 + \varepsilon) * t_{MA} = \\ t_{MA} - p * t_{MA} + p * t_{FAULT} &= t_{MA} + \varepsilon * t_{MA} \\ p(t_{FAULT} - t_{MA}) &= \varepsilon * t_{MA} =\end{aligned}$$

$$p = \frac{\varepsilon * t_{MA}}{t_{FAULT} - t_{MA}}$$

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- So far, we have described how the OS (with the support of HW) manages page faults
- Still, the OS has to answer 2 fundamental questions:
 - When to load process' pages into main memory (**page fetching**)
 - Which page to remove from memory if this gets filled (**page replacement**)

Page Fetching Goals

- The overall goal is still to make physical memory look larger than it is
- Exploiting the locality reference of programs
- Keep in memory only those pages that is being used
- Keep on disk those pages that are unused
- Ideally, producing a memory system with the performance of main memory and the cost/capacity of disk!

Page Fetching Strategies

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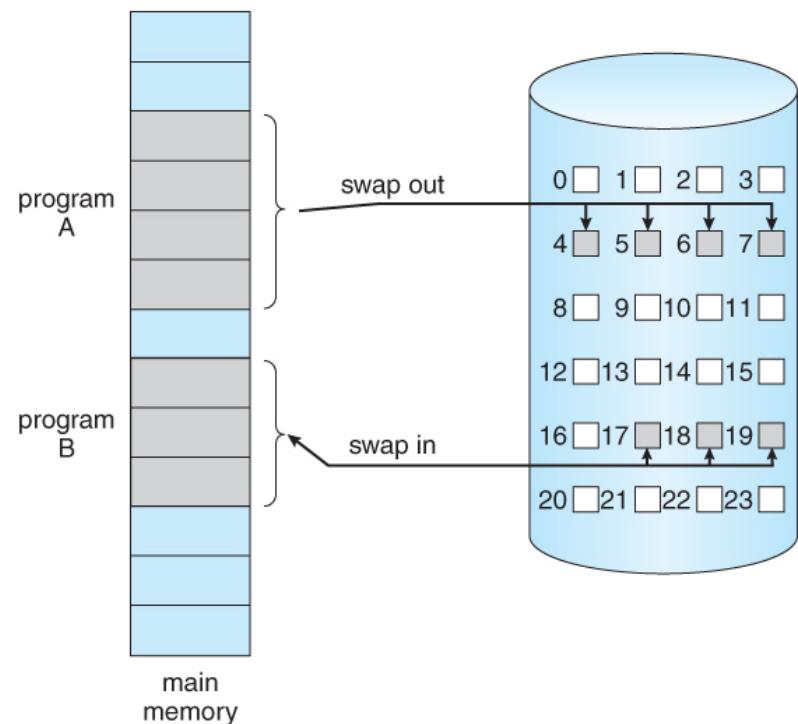
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Most modern OSs use **demand fetching**

(Pure) Demand Paging

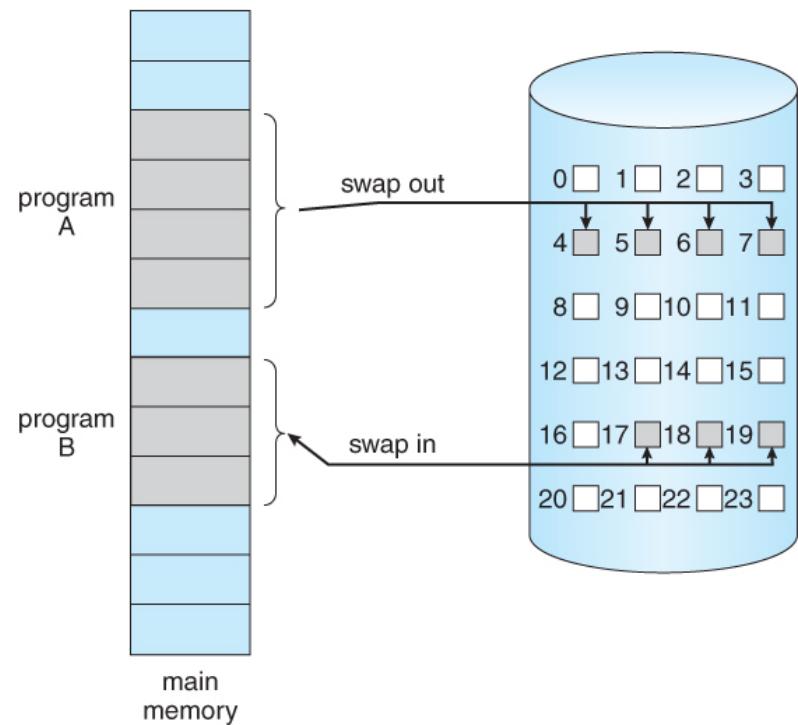
- When a process starts up, **none** of its pages are loaded
- Rather, a page is swapped in only when the process references it (upon a page fault)
- This is termed a **lazy swapper** or **pager**
- Opposite of loading all the pages at process startup!

Prefetching



The pager guesses when pages will be needed and load them ahead of time

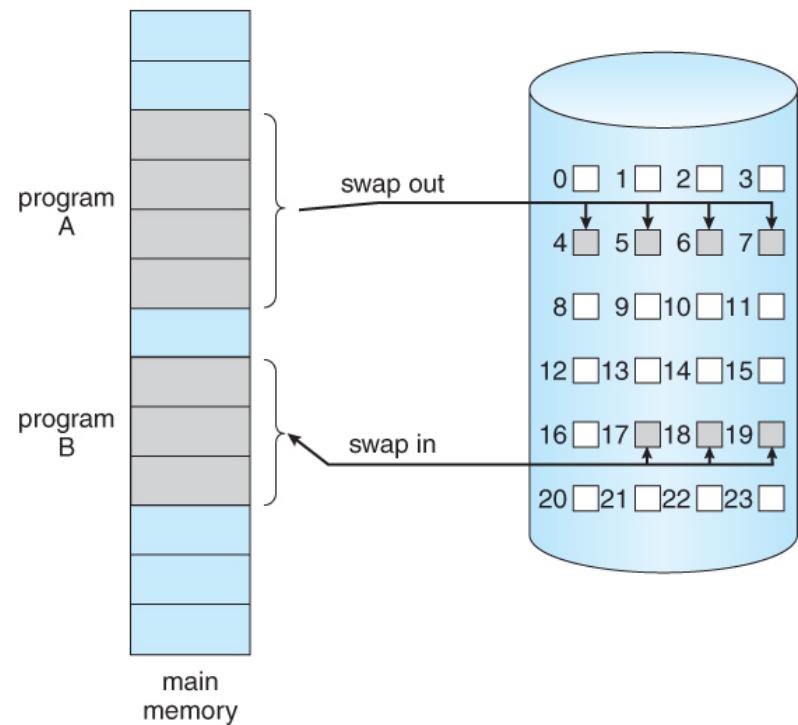
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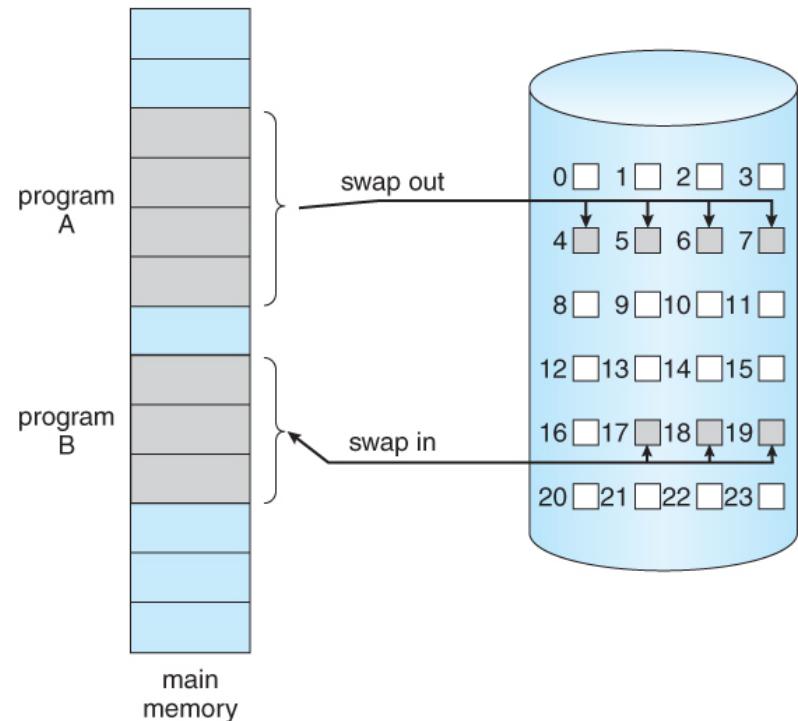


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Possible approach: upon page fault, load many pages instead of only the faulty one

works if program accesses memory sequentially

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- On Linux there exists a dedicated swap partition (on disk)
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- On Mac, instead, swap space is part of the file system
 - swapfiles

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- Depending on which kind of page is removed, different optimizations may apply upon page swap-out

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- **Data** page:
 - Data content does actually change!
 - Save it to a separate paging file, so that no changes are lost when it will be loaded in the future
 - Need to use the dedicated swap space

Page Replacement: Motivation

- On a page fault, we need to load a page from disk into memory
- If physical memory has still free frames, the page can be safely loaded into one of those
- However, if physical memory is full (i.e., all of its frames are loaded) a frame must be swapped out to make room for the swap-in page
- Several algorithms to select the page to evict from memory

Page Replacement Algorithms

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- **Random:** pick any page at random (works surprisingly well!)
- **FIFO (First-In-First-Out):** throw out the page that has been in memory for longest time (i.e., the oldest)
 - Easy to implement but may remove frequently accessed pages

Page Replacement Algorithms

- **Random:** pick any page at random (works surprisingly well!)
- **FIFO (First-In-First-Out):** throw out the page that has been in memory for longest time (i.e., the oldest)
 - Easy to implement but may remove frequently accessed pages
- **MIN (OPT):** remove the page that will not be accessed for the longest time (provably optimal [Belady 1966])
 - Needs to predict the future → very hard!

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 - Easy to implement but may remove frequently accessed pages
- **MIN (OPT):** remove the page that will not be accessed for the longest time (provably optimal [Belady 1966])
 - Needs to predict the future → very hard!
- **LRU (Least Recently Used):** approximation of MIN, remove the page that has not been used in the longest time
 - Assumes the past is a good predictor of the future (not always true!)

FIFO Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1											
F_2											
F_3											

How many page faults (denoted by *)?

FIFO Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1											
F_2											
F_3											

Initially, no frame is loaded in memory at all
(pure demand paging)

FIFO Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1											
F_2											
F_3											

Virtual address within page A is referenced

FIFO Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1											
F_2											
F_3											

Virtual address within page A is referenced



page fault

FIFO Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1	A*										
F_2											
F_3											

Virtual address within page A is referenced



page fault



A loaded

FIFO = A

FIFO Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1	A*	A									
F_2											
F_3											

Virtual address within page B is referenced

FIFO = A

FIFO Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1	A*	A									
F_2											
F_3											

Virtual address within page B is referenced → page fault

FIFO = A

FIFO Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1	A*	A									
F_2			B*								
F_3											

Virtual address within page B is referenced



page fault



B loaded

FIFO = A \rightarrow B

FIFO Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1	A*	A	A								
F_2		B*	B								
F_3											

Virtual address within page C is referenced

FIFO = A \rightarrow B

FIFO Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1	A*	A	A								
F_2		B*	B								
F_3											

Virtual address within page C is referenced



page fault

FIFO = A \rightarrow B

FIFO Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1	A*	A	A								
F_2		B*	B								
F_3			C*								

Virtual address within page C is referenced



page fault



C loaded

FIFO = A \rightarrow B \rightarrow C

FIFO Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1	A*	A	A	A							
F_2		B*	B	B							
F_3			C*	C							

Virtual address within page A is referenced



A is already loaded

FIFO = A \rightarrow B \rightarrow C

FIFO Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1	A*	A	A	A	A						
F_2		B*	B	B	B						
F_3			C*	C	C						

Virtual address within page B is referenced



B is already loaded

FIFO = A \rightarrow B \rightarrow C

FIFO Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1	A*	A	A	A	A	A					
F_2		B*	B	B	B	B					
F_3			C*	C	C	C					

Virtual address within page D is referenced

FIFO = A \rightarrow B \rightarrow C

FIFO Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1	A*	A	A	A	A	A					
F_2		B*	B	B	B	B					
F_3			C*	C	C	C					

Virtual address within page D is referenced



page fault

FIFO = A \rightarrow B \rightarrow C

FIFO Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1	A*	A	A	A	A	D*					
F_2		B*	B	B	B	B					
F_3			C*	C	C	C					

Virtual address within page D is referenced



page fault



A replaced
D loaded

FIFO = B \rightarrow C \rightarrow D

FIFO Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1	A*	A	A	A	A	D*	D				
F_2		B*	B	B	B	B	B				
F_3			C*	C	C	C	C				

Virtual address within page A is referenced

FIFO = B \rightarrow C \rightarrow D

FIFO Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1	A*	A	A	A	A	D*	D				
F_2		B*	B	B	B	B	B				
F_3			C*	C	C	C	C				

Virtual address within page A is referenced



page fault

FIFO = B \rightarrow C \rightarrow D

FIFO Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1	A*	A	A	A	A	D*	D				
F_2		B*	B	B	B	B	A*				
F_3			C*	C	C	C	C				

Virtual address within page A is referenced



page fault



B replaced
A loaded

FIFO = C → D → A

FIFO Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1	A*	A	A	A	A	D*	D	D			
F_2		B*	B	B	B	B	A*	A			
F_3			C*	C	C	C	C	C			

Virtual address within page D is referenced



D is already loaded

FIFO = C → D → A

FIFO Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1	A*	A	A	A	A	D*	D	D	D	C*	C
F_2		B*	B	B	B	B	A*	A	A	A	A
F_3			C*	C	C	C	C	B*	B	B	B

Eventually, we get a total of **7** page faults

MIN Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1											
F_2											
F_3											

How many page faults (denoted by *)?

MIN Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1											
F_2											
F_3											

Initially, no frame is loaded in memory at all
(pure demand paging)

MIN Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1	A*	A	A	A	A						
F_2		B*	B	B	B						
F_3			C*	C	C						

Up to this point, the same as FIFO

MIN Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1	A*	A	A	A	A	A					
F_2		B*	B	B	B	B					
F_3			C*	C	C	C					

Virtual address within page D is referenced

MIN Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1	A*	A	A	A	A	A					
F_2		B*	B	B	B	B					
F_3			C*	C	C	C					

Virtual address within page D is referenced



page fault

MIN Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1	A*	A	A	A	A	A					
F_2		B*	B	B	B	B					
F_3			C*	C	C	C					

Virtual address within page D is referenced



page fault

What's the page that will be requested the furthest away?

MIN Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1	A*	A	A	A	A	A					
F_2		B*	B	B	B	B					
F_3			C*	C	C	D*					

Virtual address within page D is referenced



page fault



C replaced
D loaded

C is the page that will be requested the furthest away

MIN Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1	A*	A	A	A	A	A	A	A	A		
F_2		B*	B	B	B	B	B	B	B		
F_3			C*	C	C	D*	D	D	D		

Up to this point, no more page faults

MIN Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1	A*	A	A	A	A	A	A	A	A	A	
F_2		B*	B	B	B	B	B	B	B	B	
F_3			C*	C	C	D*	D	D	D	D	

Virtual address within page C is referenced

MIN Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1	A*	A	A	A	A	A	A	A	A	A	
F_2		B*	B	B	B	B	B	B	B	B	
F_3			C*	C	C	D*	D	D	D	D	

Virtual address within page C is referenced



page fault

What's the page that will be requested the furthest away?

MIN Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1	A*	A	A	A	A	A	A	A	A	A	
F_2		B*	B	B	B	B	B	B	B	C*	
F_3			C*	C	C	D*	D	D	D	D	

Virtual address within page C is referenced



page fault



B replaced
C loaded

B or D will be requested the furthest away (surely not A): pick one (e.g., B)

MIN Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1	A*	A	A	A	A	A	A	A	A	A	A
F_2		B*	B	B	B	B	B	B	B	C*	C
F_3			C*	C	C	D*	D	D	D	D	D

Eventually, we get a total of **5** page faults

LRU Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1											
F_2											
F_3											

How many page faults (denoted by *)?

LRU Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1											
F_2											
F_3											

Initially, no frame is loaded in memory at all
(pure demand paging)

LRU Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1	A*	A	A	A	A						
F_2		B*	B	B	B						
F_3			C*	C	C						

Up to this point, the same as FIFO

LRU Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1	A*	A	A	A	A	A					
F_2		B*	B	B	B	B					
F_3			C*	C	C	C					

Virtual address within page D is referenced



page fault

LRU Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1	A*	A	A	A	A	A					
F_2		B*	B	B	B	B					
F_3			C*	C	C	C					

Virtual address within page D is referenced



page fault

We can't look forward anymore!

LRU Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1	A*	A	A	A	A	A					
F_2		B*	B	B	B	B					
F_3			C*	C	C	D*					

Virtual address within page D is referenced



page fault

C replaced
D loaded

C is the page that has not been used for the longest time in the past

LRU Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1	A*	A	A	A	A	A	A	A	A		
F_2		B*	B	B	B	B	B	B	B		
F_3			C*	C	C	D*	D	D	D		

Up to this point, no more page faults

LRU Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1	A*	A	A	A	A	A	A	A	A	A	
F_2		B*	B	B	B	B	B	B	B	B	
F_3			C*	C	C	D*	D	D	D	D	

Virtual address within page C is referenced

LRU Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1	A*	A	A	A	A	A	A	A	A	A	
F_2		B*	B	B	B	B	B	B	B	B	
F_3			C*	C	C	D*	D	D	D	D	

Virtual address within page C is referenced



page fault

We can't look forward anymore!

LRU Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1	A*	A	A	A	A	A	A	A	A	C*	
F_2		B*	B	B	B	B	B	B	B	B	
F_3			C*	C	C	D*	D	D	D	D	

Virtual address within page C is referenced



page fault

A replaced
C loaded

A is the page that has not been used for the longest time in the past

LRU Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1	A*	A	A	A	A	A	A	A	A	C*	C
F_2		B*	B	B	B	B	B	B	B	B	B
F_3			C*	C	C	D*	D	D	D	D	D

Virtual address within page A is referenced

LRU Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1	A*	A	A	A	A	A	A	A	A	C*	C
F_2		B*	B	B	B	B	B	B	B	B	B
F_3			C*	C	C	D*	D	D	D	D	D

Virtual address within page A is referenced



page fault

We can't look forward anymore!

LRU Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1	A*	A	A	A	A	A	A	A	A	C*	C
F_2		B*	B	B	B	B	B	B	B	B	B
F_3			C*	C	C	D*	D	D	D	D	A*

Virtual address within page A is referenced



page fault



D replaced
A loaded

D is the page that has not been used for the longest time in the past

LRU Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1	A*	A	A	A	A	A	A	A	A	C*	C
F_2		B*	B	B	B	B	B	B	B	B	B
F_3			C*	C	C	D*	D	D	D	D	A*

Eventually, we get a total of **6** page faults

LRU Page Replacement: (An Unlucky) Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, D, A, B, C, D, A, B, C

	A	B	C	D	A	B	C	D	A	B	C
F_1											
F_2											
F_3											

How many page faults (denoted by *)?

LRU Page Replacement: (An Unlucky) Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, D, A, B, C, D, A, B, C

	A	B	C	D	A	B	C	D	A	B	C
F_1	A*	A	A								
F_2		B*	B								
F_3			C*								

LRU Page Replacement: (An Unlucky) Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, D, A, B, C, D, A, B, C

	A	B	C	D	A	B	C	D	A	B	C
F_1	A*	A	A	D*							
F_2		B*	B	B							
F_3			C*	C							

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3 physical frames: F_1, F_2, F_3

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	A	B	C	D	A	B	C	D	A	B	C
F_1	A*	A	A	D*	D						
F_2		B*	B	B	A*						
F_3			C*	C	C						

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	A	B	C	D	A	B	C	D	A	B	C
F_1	A*	A	A	D*	D	D					
F_2		B*	B	B	A*	A					
F_3			C*	C	C	B*					

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F_1	A*	A	A	D*	D	D	C*				
F_2		B*	B	B	A*	A	A				
F_3			C*	C	C	B*	B				

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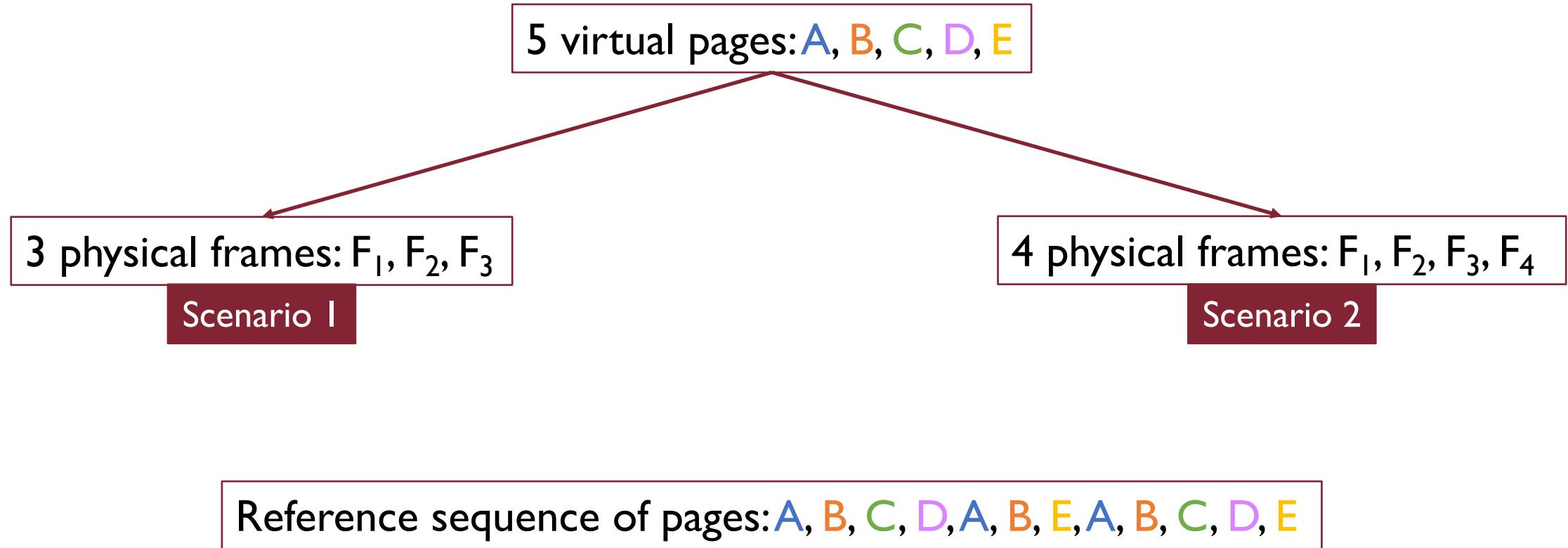
	A	B	C	D	A	B	C	D	A	B	C
F_1	A*	A	A	D*	D	D	C*	C	C	B*	B
F_2		B*	B	B	A*	A	A	D*	D	D	C*
F_3			C*	C	C	B*	B	B	A*	A	A

Eventually, we get a total of 11 page faults

Page Replacement: What If We Add Memory?

- Does adding memory always reduce the number of page faults?
- Intuitively, it would seem so...
- The answer, in fact, depends on the page replacement algorithm
- Let's see this with an example, using FIFO page replacement

FIFO Page Replacement: Example



FIFO Page Replacement: Example

	A	B	C	D	A	B	E	A	B	C	D	E
F_1	A*	A	A	D*	D	D	E*	E	E	E	E	E
F_2		B*	B	B	A*	A	A	A	A	C*	C	C
F_3			C*	C	C	B*	B	B	B	B	D*	D
F_4	A*	A	A	A	A	A	E*	E	E	E	D*	D
F_5	B*	B	B	B	B	B	A*	A	A	A	E*	
F_6		C*	C	C	C	C	B*	B	B	B	B	
F_7			D*	D	D	D	D	D	C*	C	C	

FIFO Page Replacement: Example

	A	B	C	D	A	B	E	A	B	C	D	E
--	---	---	---	---	---	---	---	---	---	---	---	---

F_1	A*	A	A	D*	D	D	E*	E	E	E	E	E
F_2		B*	B	B	A*	A	A	A	A	C*	C	C
F_3			C*	C	C	B*	B	B	B	B	D*	D

10 page faults

F_1	A*	A	A	A	A	A	E*	E	E	E	D*	D
F_2		B*	B	B	B	B	B	A*	A	A	A	E*
F_3			C*	C	C	C	C	B*	B	B	B	B
F_4				D*	D	D	D	D	D	C*	C	C

11 page faults

Belady's Anomaly

Adding page frames may cause more page faults with some algorithms

LRU Page Replacement: Example

	A	B	C	D	A	B	E	A	B	C	D	E
--	---	---	---	---	---	---	---	---	---	---	---	---

F ₁	A*	A	A	D*	D	D	E*	E	E	C*	C	C
F ₂		B*	B	B	A*	A	A	A	A	A	D*	D
F ₃			C*	C	C	B*	B	B	B	B	B	B

9 page faults

F ₁	A*	A	A	A	A	A	A	A	A	A	A	E*
F ₂		B*	B	B	B	B	B	B	B	B	B	B
F ₃			C*	C	C	C	E*	E	E	D*	D	
F ₄				D*	D	D	D	D	D	C*	C	C

8 page faults

With LRU, adding page frames **always** decreases the number of page faults

LRU Page Replacement: Example

	A	B	C	D	A	B	E	A	B	C	D	E
--	---	---	---	---	---	---	---	---	---	---	---	---

F ₁	A*	A	A	D*	D	D	E*	E	E	C*	C	C
F ₂		B*	B	B	A*	A	A	A	A	A	D*	D
F ₃			C*	C	C	B*	B	B	B	B	B	B

9 page faults

F ₁	A*	A	A	A	A	A	A	A	A	A	A	E*
F ₂		B*	B	B	B	B	B	B	B	B	B	B
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With LRU, adding page frames **always** decreases the number of page faults

Why?

LRU Page Replacement: Example

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F ₃			C*	C	C	B*	B	B	B	B	B	B
F ₄	A*	A	A	A	A	A	A	A	A	A	E*	
F ₅	B*	B	B	B	B	B	B	B	B	B	B	B
F ₆		C*	C	C	C	C	E*	E	E	E	D*	D
F ₇			D*	D	D	D	D	D	D	C*	C	C

At each point in time 4-frame memory contains a subset of 3-frame

Can't do any worse!

Page Replacement: Summary

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 - May suffer from Belady's Anomaly

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- **FIFO** is easy to implement but may lead to too many page faults
 - May suffer from Belady's Anomaly
- **MIN** is the optimal choice but cannot be used in practice since future memory references are never known in advance
- **LRU** is a fair approximation of MIN assuming the past is a good predictor of the future
 - Exploits the locality reference (small working set that fits in memory)
 - Works poorly when the locality reference doesn't hold (large working set)

LRU: Implementation Details

How could we implement LRU page replacement algorithm?

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First Idea

Keep a timestamp for each page with the time it has been last accessed
Remove the page with the highest difference w.r.t. current timestamp

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Linear scan of all the pages to
select the one to be removed

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Second Idea

Keep a list of pages with the most recently used in front and the least recently used at the end: every time a page is accessed move it to front

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Still too expensive as the OS must change multiple pointers on each memory access

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- **Single-Reference Bit →** Maintain 1 bit for each page table entry
 - Initially, all bits for all pages are set to 0
 - On each access to a page, the HW sets the reference bit to 1
 - Enough to distinguish pages that have been accessed since the last clear
 - No total order of page access

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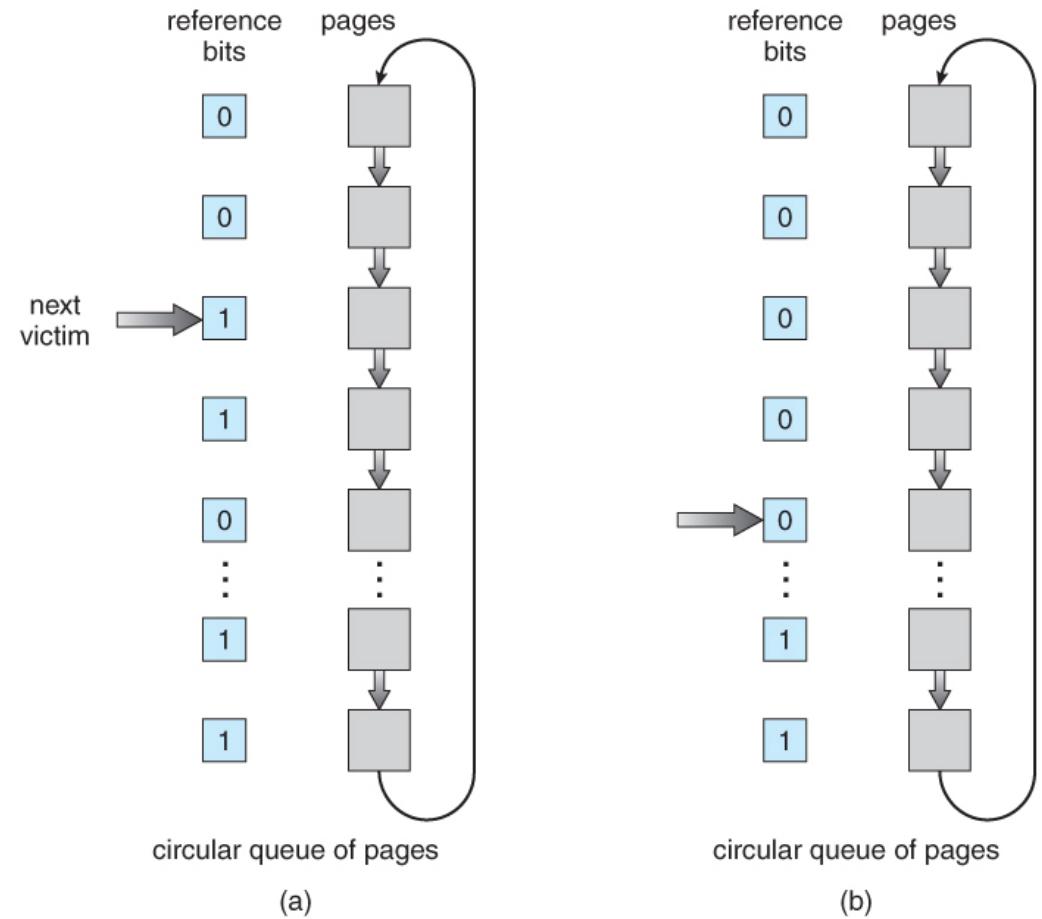
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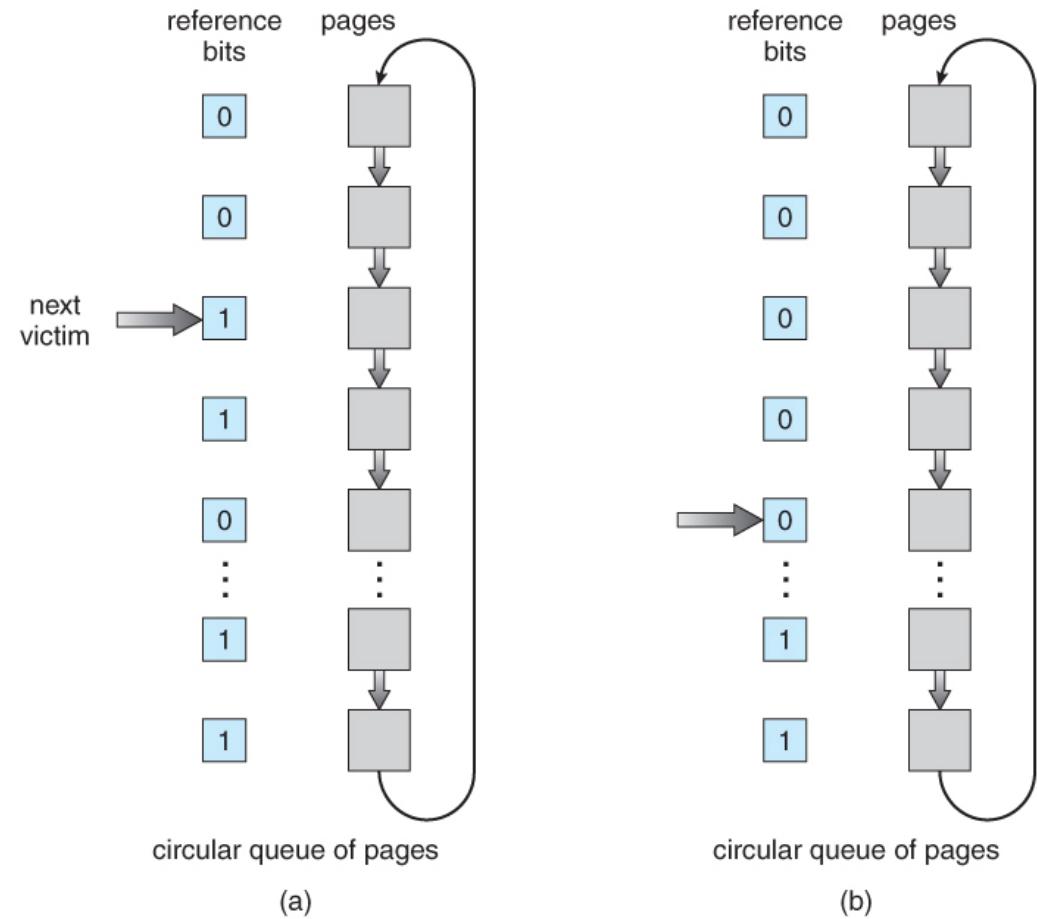
- Second Chance Algorithm → Single-Reference Bit + FIFO
- OS keeps frames in a FIFO circular list
- On every memory access, the reference bit is set to 1
- On a page fault, the OS scans the list of page table, checking the reference bit of the frame:
 - If this is 0, it replaces the page and sets it to 1
 - If this is 1, it sets it to 0 (second chance) and move to the next frame

Second Chance Algorithm (Clock)



A raw partitioning into: young vs. old frames

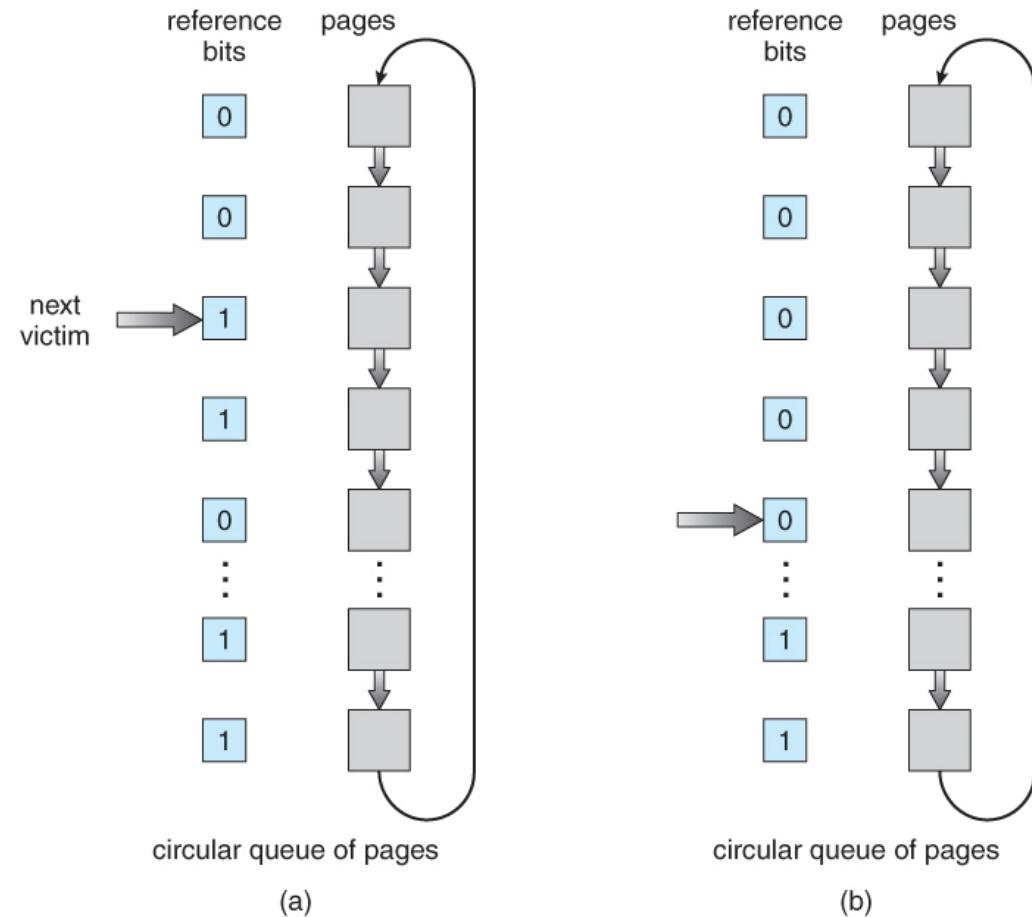
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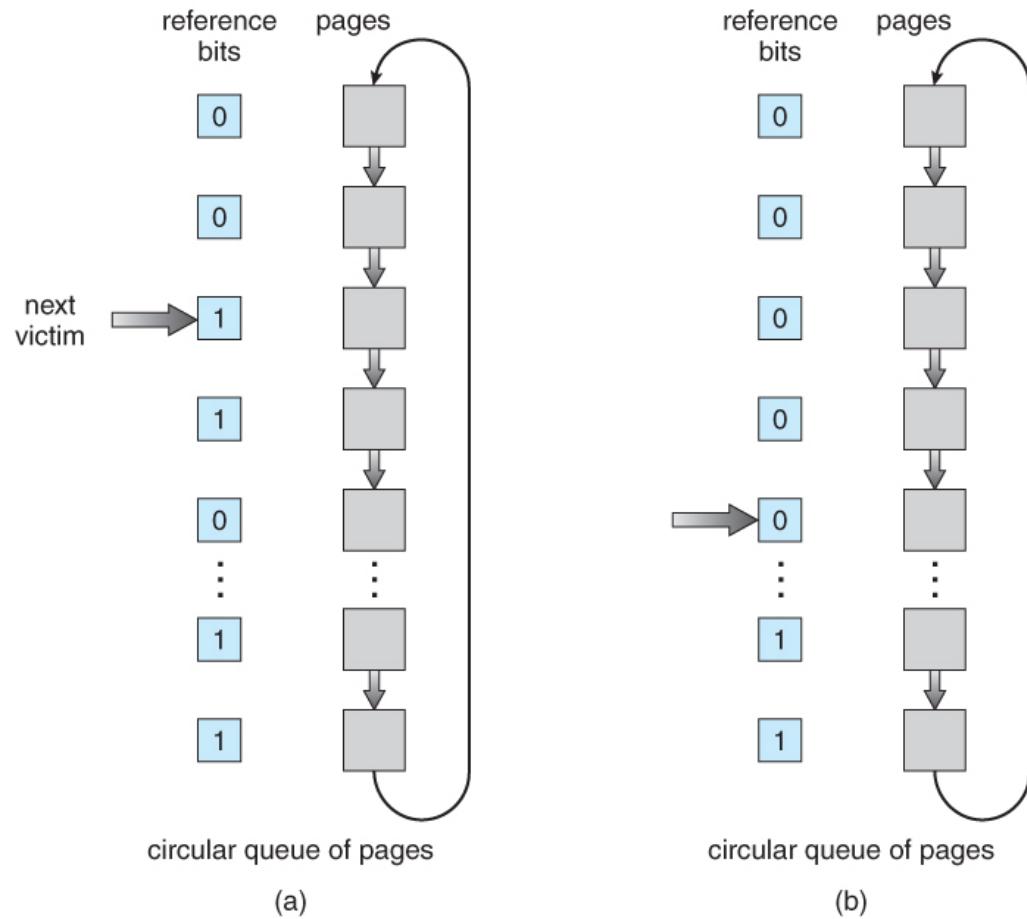


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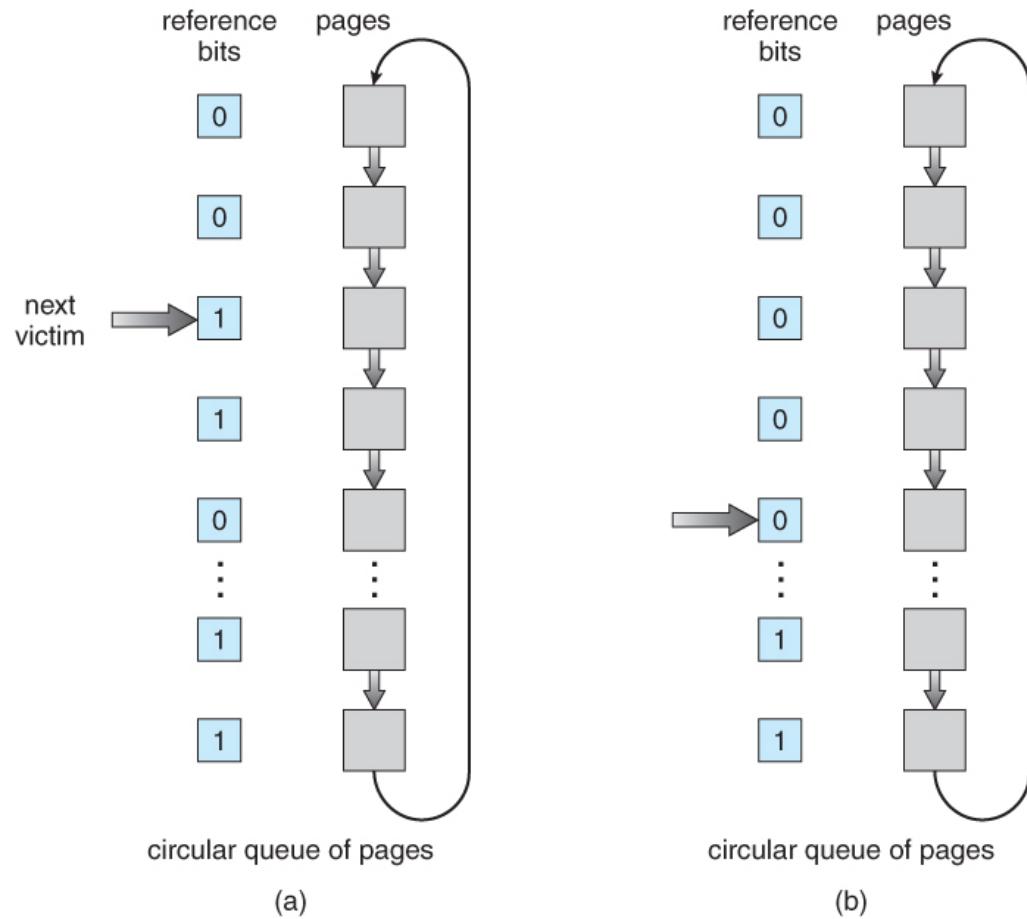
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This algorithm is also known as **clock** because it mimics the hands of a clock

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- Page replacement generally involves 2 I/O operations:
 - write the evicted page back to disk
 - read the newly referenced page from disk
- **Intuition:** It is cheaper to replace a page which has not been modified, since the OS does not need to write this back to disk
- OS should give preference to paging-out un-modified frames
 - Yet, it can proactively write to disk modified frames for later

Enhanced Second Chance Algorithm

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- Use both the reference and modify bits (r, m) to classify pages into:
 - (0, 0): neither recently used nor modified;
 - (0, 1): not recently used, but modified;
 - (1, 0): recently used, but clean
 - (1, 1): recently used and modified

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- The main difference between this algorithm and the standard clock is the preference for replacing clean pages if possible

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- Multiple processes can however run concurrently on a single-CPU system
- The degree of multiprogramming is not fixed apriori, yet it is driven by the locality reference (a.k.a. 90÷10 rule)
- This allows a system to load the **working set** (i.e., few pages) of many processes, thereby increasing the degree of multiprogramming

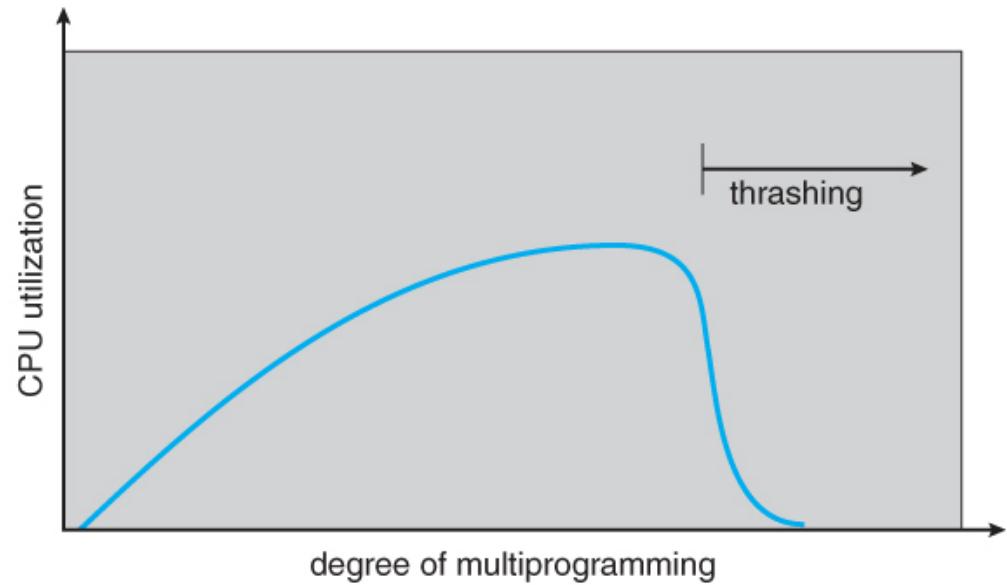
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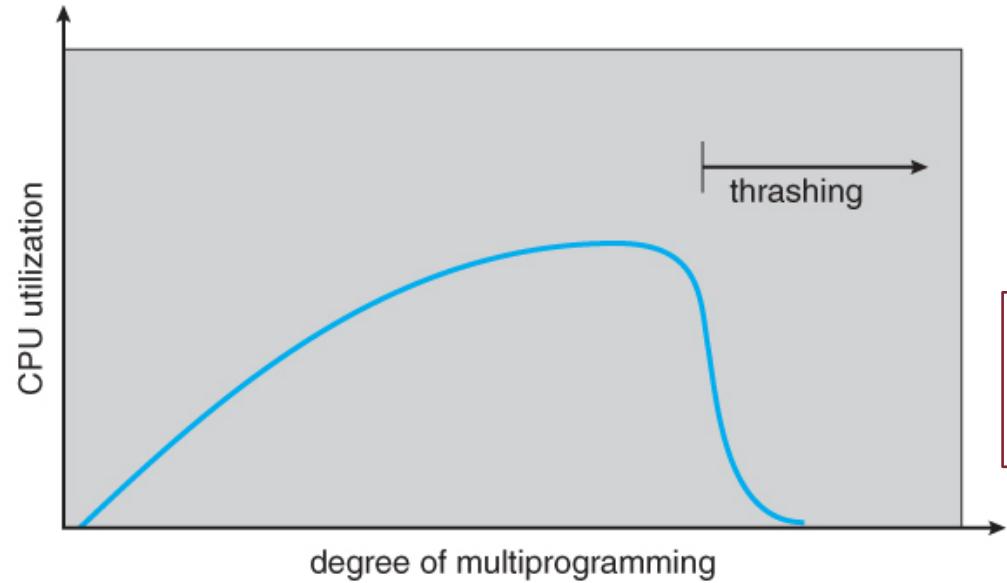
- When the degree of multiprogramming is too high, active working sets of running processes may saturate the whole memory capacity
- **Thrashing** → Memory is over-committed and pages are continuously tossed out while they are still in use
 - Memory access time approaches disk access time due to many page faults
 - Drastic degradation of performance

Multiprogramming and Thrashing



CPU utilization drops after a certain degree of multiprogramming

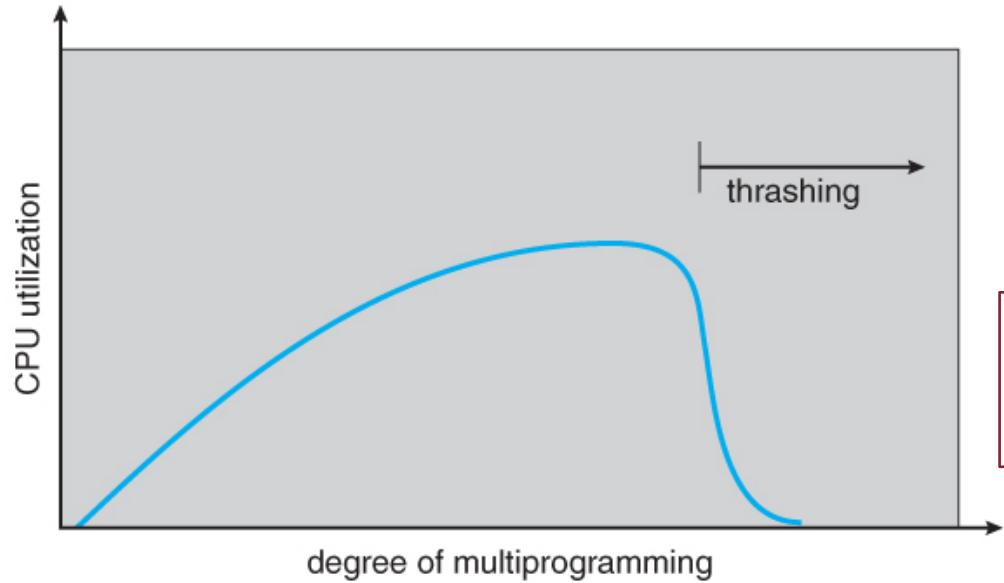
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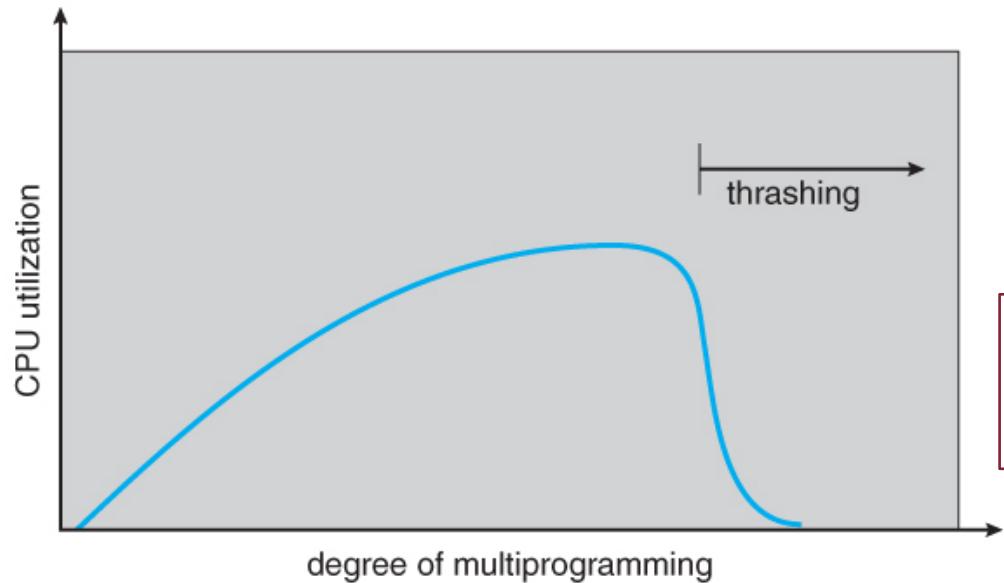


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Fixing the degree of multi-programming apriori may be a too inflexible option

Allocation/Replacement Policies

Ultimately, we want to give each process enough memory so as to avoid thrashing

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Global Allocation/Replacement

- All pages from all processes are in a single pool (single LRU queue)
- Upon page replacement, any page may be a potential victim, whether it currently belongs to the process seeking a free frame or not
- **PRO:** flexibility
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Local Allocation/Replacement

- Each process has its own fixed pool of frames
- Run only group of processes that fits in memory
- LRU replacement affects only each process' frames
- **PRO:** isolation
- **CON:** performance (a process may not be given enough memory)

Local Allocation/Replacement

m = number of available physical page frames

n = number of processes

S_i = size of the i -th process; $S = \sum_{i=1}^n S_i$ = total size of all processes

Equal Allocation/Replacement: $\frac{m}{n}$

Proportional Allocation/Replacement: $\frac{m * S_i}{S}$

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As allocations fluctuate over time, do does m
(processes must either be swapped out or not allowed to start if not enough frames)

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- However, there might be cases where this is not true
 - e.g., a process allocates a 1GB array but then only uses a small portion of it
 - In other words, the working set of a process may not be correlated with its memory footprint

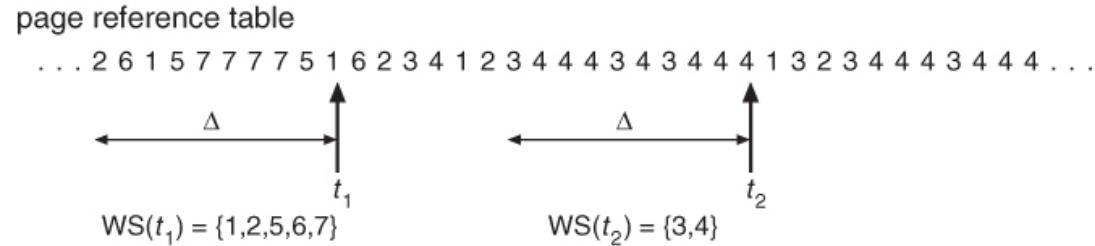
Matching the Working Set

- **Goal →** Give each process enough frames to contain its working set
 - Informally, the working set is the set of pages the process is using "right now"
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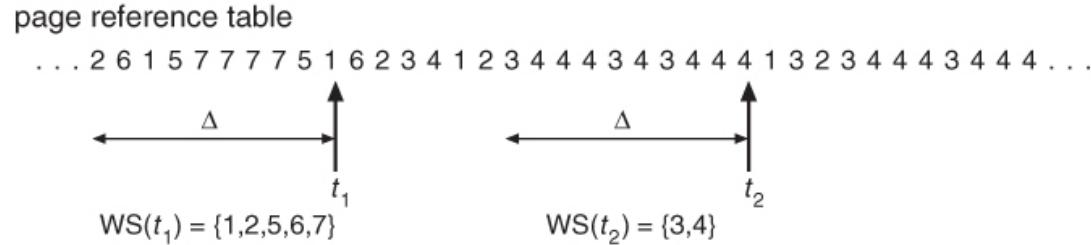
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- How does the OS pick T ?
 - 1 page fault takes order of 10 msecs to be served
 - 10 msecs ~ 10 million instructions
 - T needs to account for a lot more than 10 million instructions

Determining the Working Set



The selection of Δ is critical to the success of the working set model

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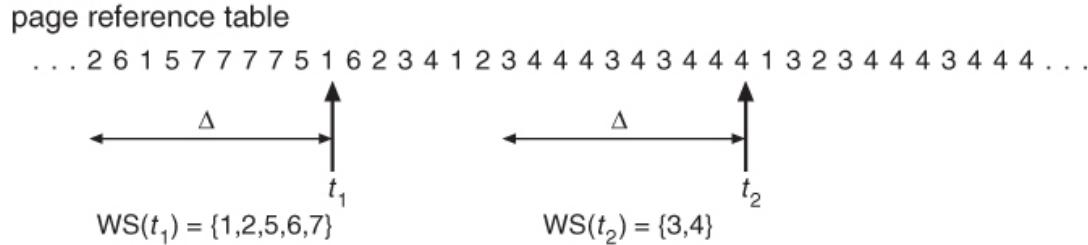


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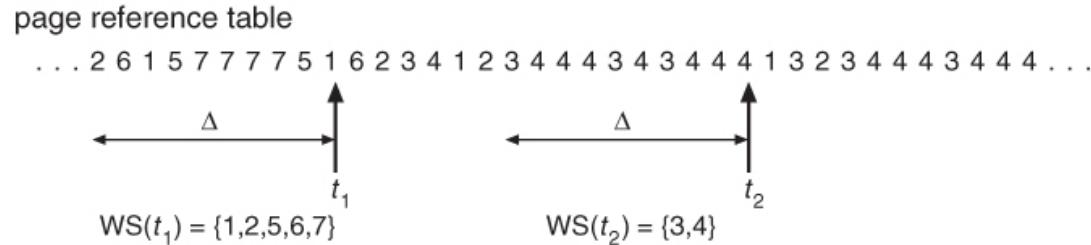
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Exact tracking is expensive: update the working set at each memory access

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- To avoid the overhead of keeping a list of the last Δ referenced pages, the working set is often implemented with **sampling**

Approximating the Working Set

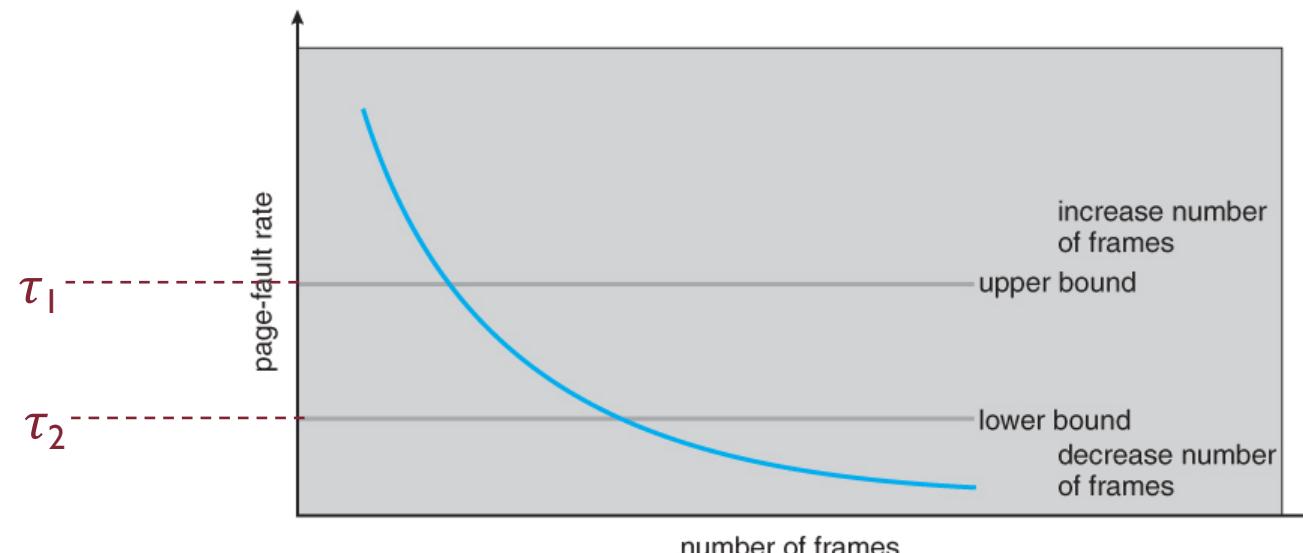
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- At each memory reference a new reference appears at one end and the oldest reference drops off the other end
- To avoid the overhead of keeping a list of the last Δ referenced pages, the working set is often implemented with **sampling**
- Every k memory references (e.g., $k = 1,000$), consider the working set to be all pages referenced within *that* period of time

Tracking Page Fault Rate

- Ultimately, our goal is to minimize the **page fault rate**

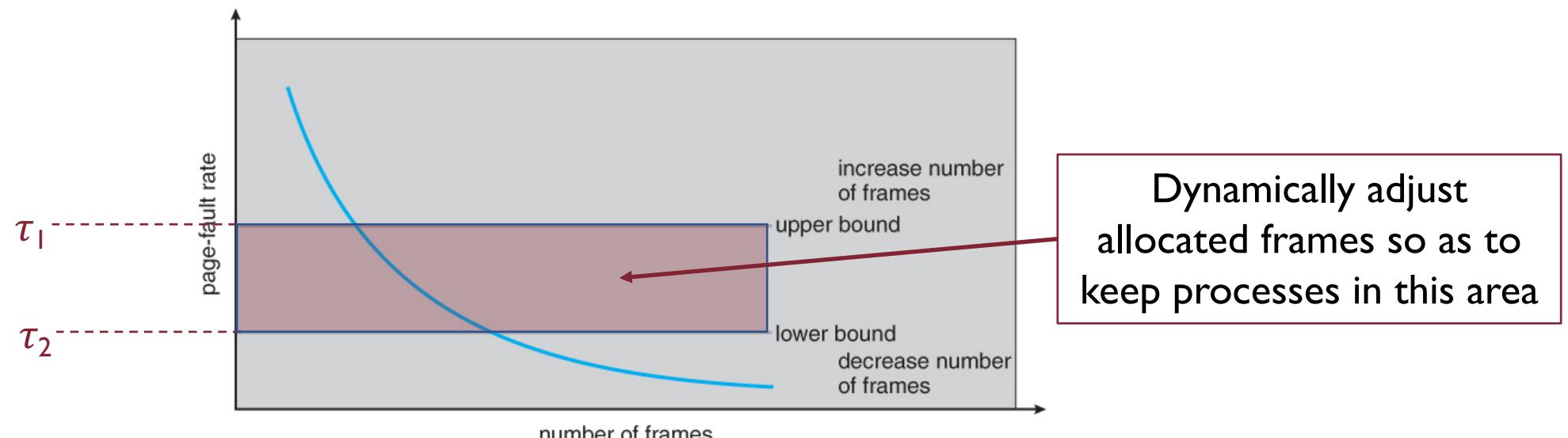
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- A more direct approach is to track page fault rate of each process:
 - If the page fault rate is above a given threshold τ_1 → give it more frames
 - If the page fault rate is below a given threshold τ_2 → reclaim some frames



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Kernel Memory

- So far, we only considered memory allocation for user processes
- But kernel needs memory to store things too: code and data structures like PCB, page tables, etc.
- Kernel does not use any of the advanced mechanisms seen so far
 - No paging → what if a page fault occurs for the kernel?

Kernel Memory: Buddy Allocator

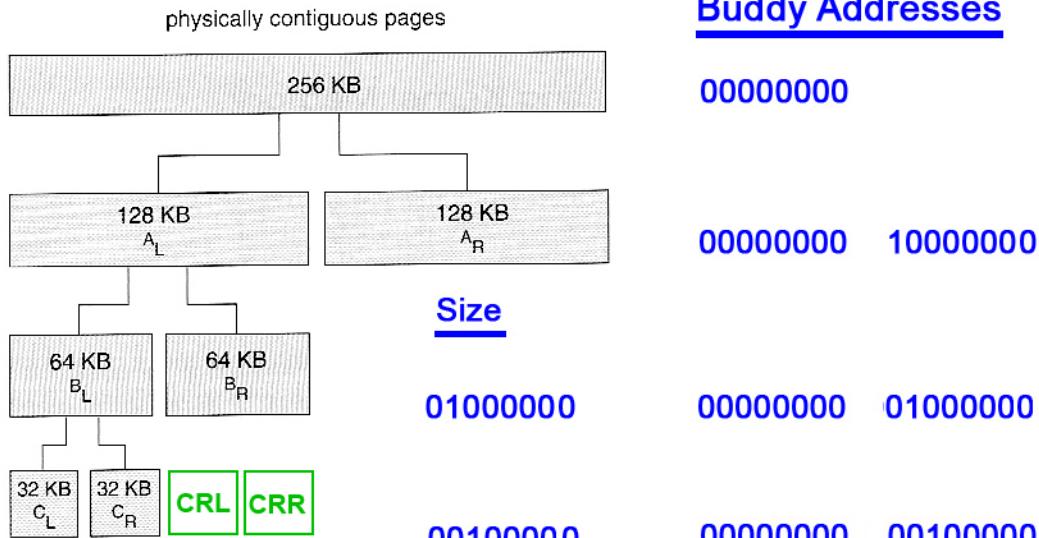
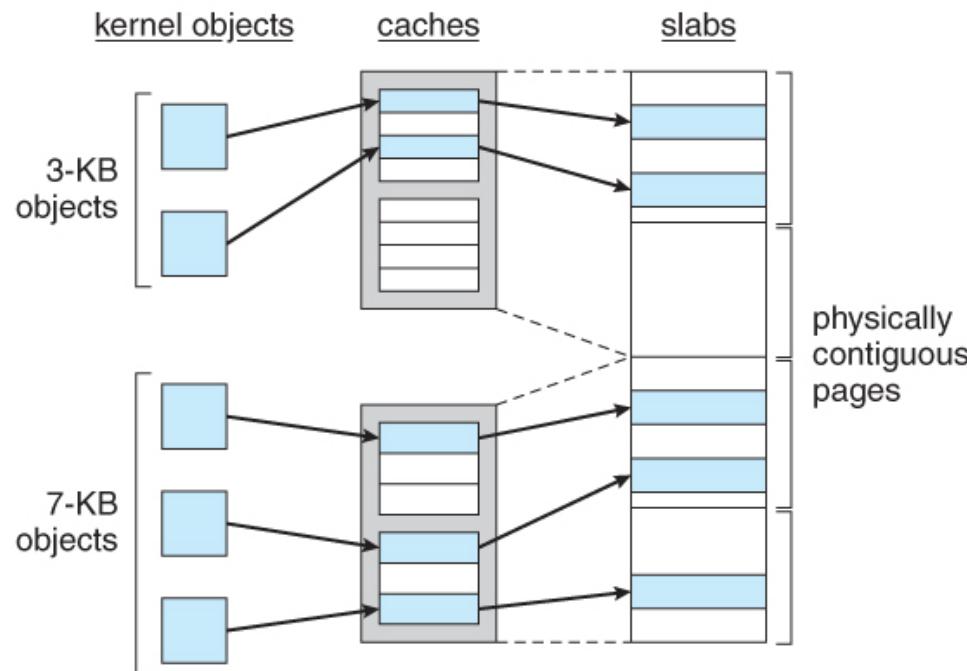


Figure 9.27 Buddy system allocation.

- Allocates memory using a power of 2 allocator (e.g., 4K, 8K, 16K), rounding up to the next nearest power of two if necessary
- If a block of the correct size is not available, then one is formed by (repeatedly) splitting the next larger block in two
- Can lead to internal fragmentation

Kernel Memory: Slab Allocator



- Group of objects of the same size in a **slab**
- Object cache points to one or more slabs
- Separate cache for each kernel data structure (e.g., PCB)
- No internal fragmentation
- Used in Solaris and Linux

Page Sizes

- Reasons for **small** pages?

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- Reasons for **small** pages?
 - Decreasing internal fragmentation
 - Higher degree of multiprogramming
- Reasons for **large** pages?
 - Smaller page table size (i.e., smaller number of page table entries)
 - Fewer page faults (locality reference)
 - Amortizes disk overhead (reading a 1KB page from disk takes approximately the same as reading an 8KB one)

Summary of Page Replacement

- The choice of page replacement algorithm is crucial when physical memory is limited
 - All algorithms approach to the optimum as the physical memory allocated to a process approaches to the virtual memory size

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- The choice of page replacement algorithm is crucial when physical memory is limited
 - All algorithms approach to the optimum as the physical memory allocated to a process approaches to the virtual memory size
- The more processes running concurrently, the less physical memory each one can have
- The OS must choose how many processes (and the number of frames per process) can share memory simultaneously