

# Systems and Networking – Unit I

B.Sc. in Applied Computer Science and Artificial Intelligence  
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# Paging + Segmentation

- Paging (OS' view of memory)
  - Divide memory into fixed-size pages and map them to physical frames
- Segmentation (compiler's view of memory)
  - Divide process into logical segments (e.g., code, data, stack, heap)
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  - Segmented Paging

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So far, the entire virtual address space of a process was assumed to fit and be all in memory

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**Virtual Memory** uses backing storage (i.e., disk) to store unused pages and give the illusion of infinite virtual address space

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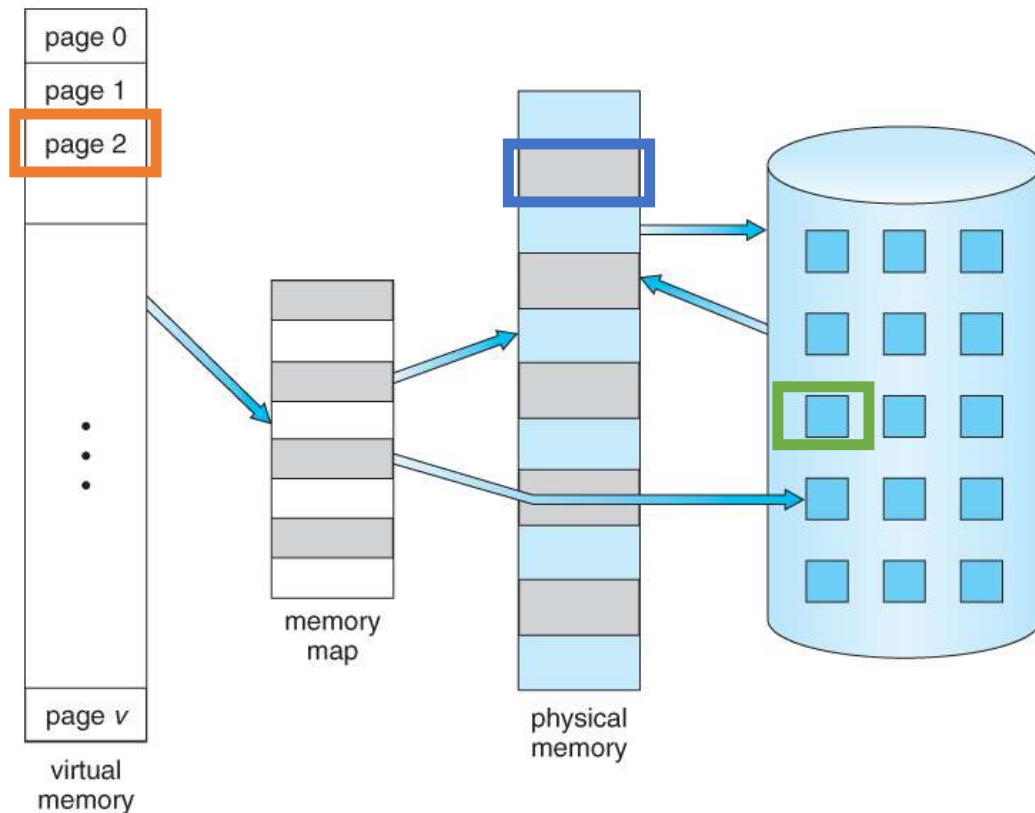
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  - Less I/O is needed for swapping processes in and out of memory, speeding things up

# Virtual Memory: The Big Picture

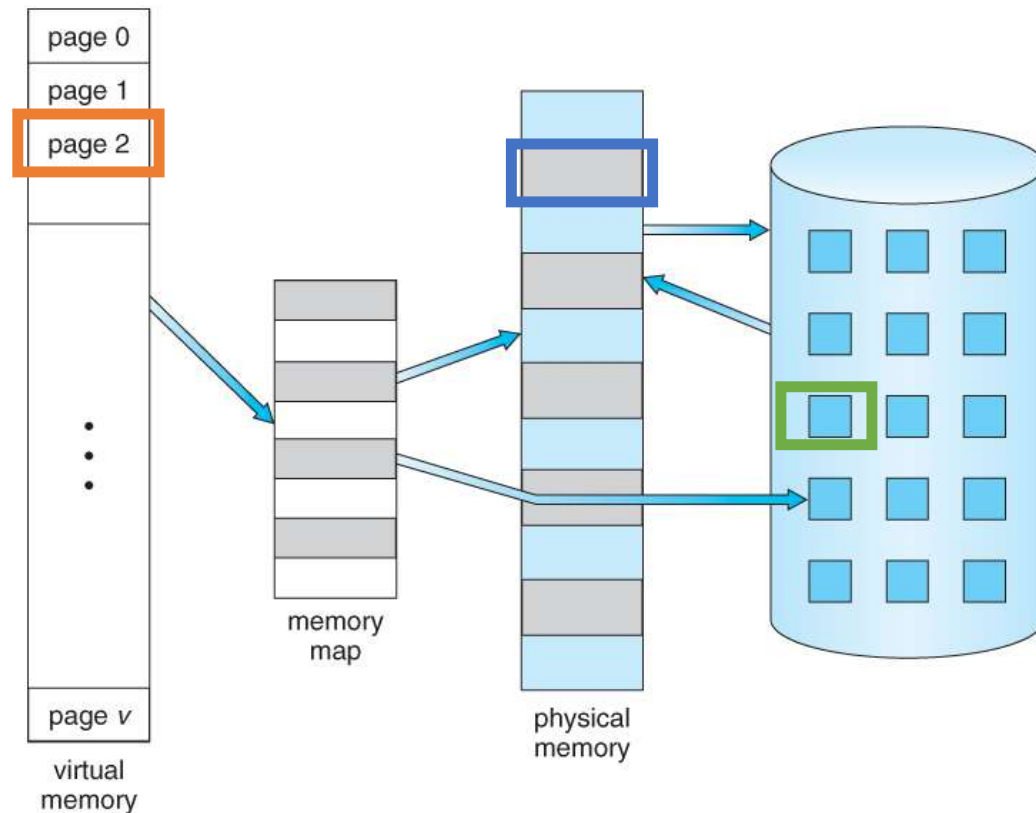


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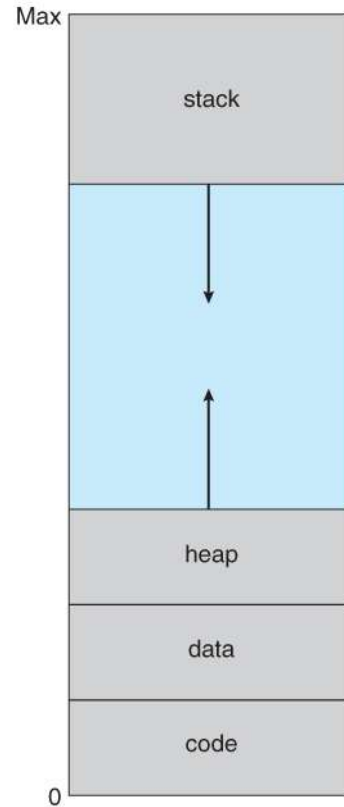
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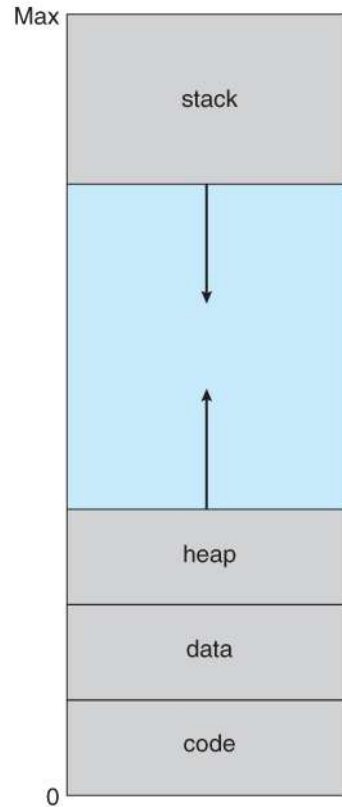
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# The Sparseness of Virtual Address Space



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A lot of virtual memory addresses remain  
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- Remember: access to disk is extremely slower than access to memory
- Therefore, memory accesses must reference pages that are in memory  
with high probability

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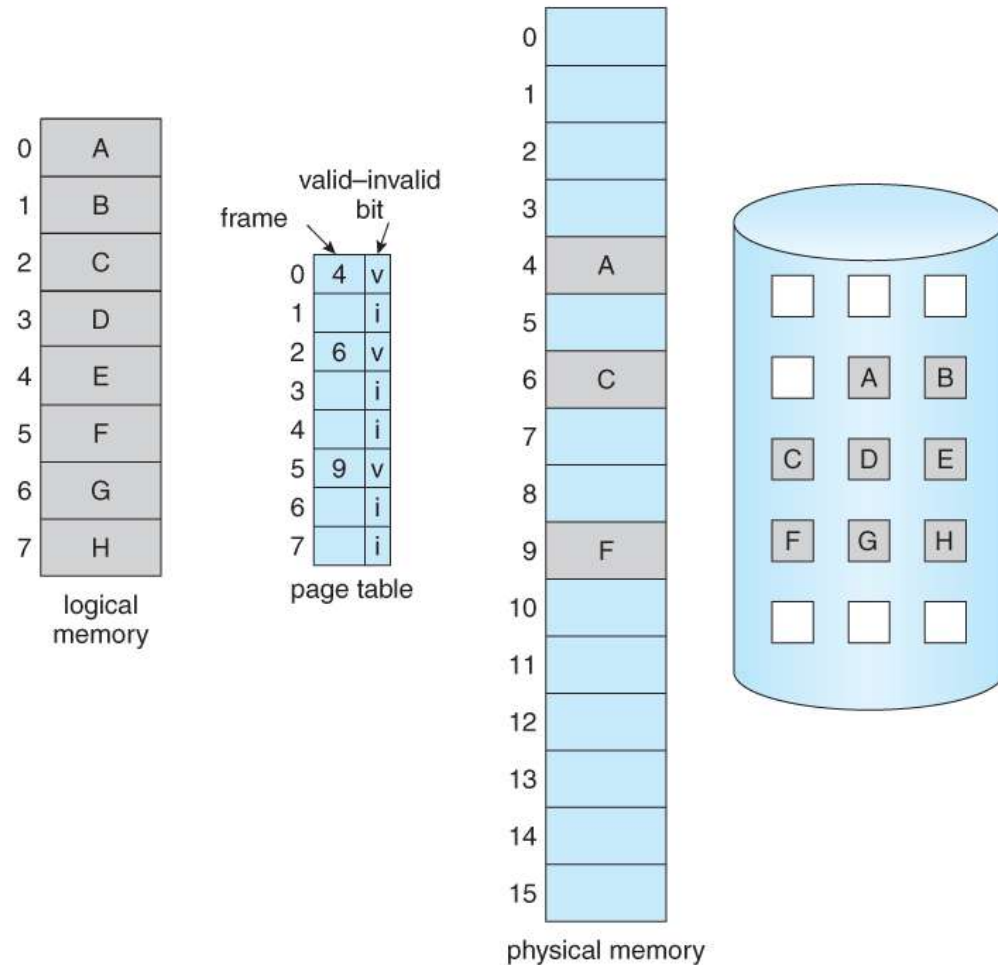
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- But in a reasonably small time frame, the working set stays the same

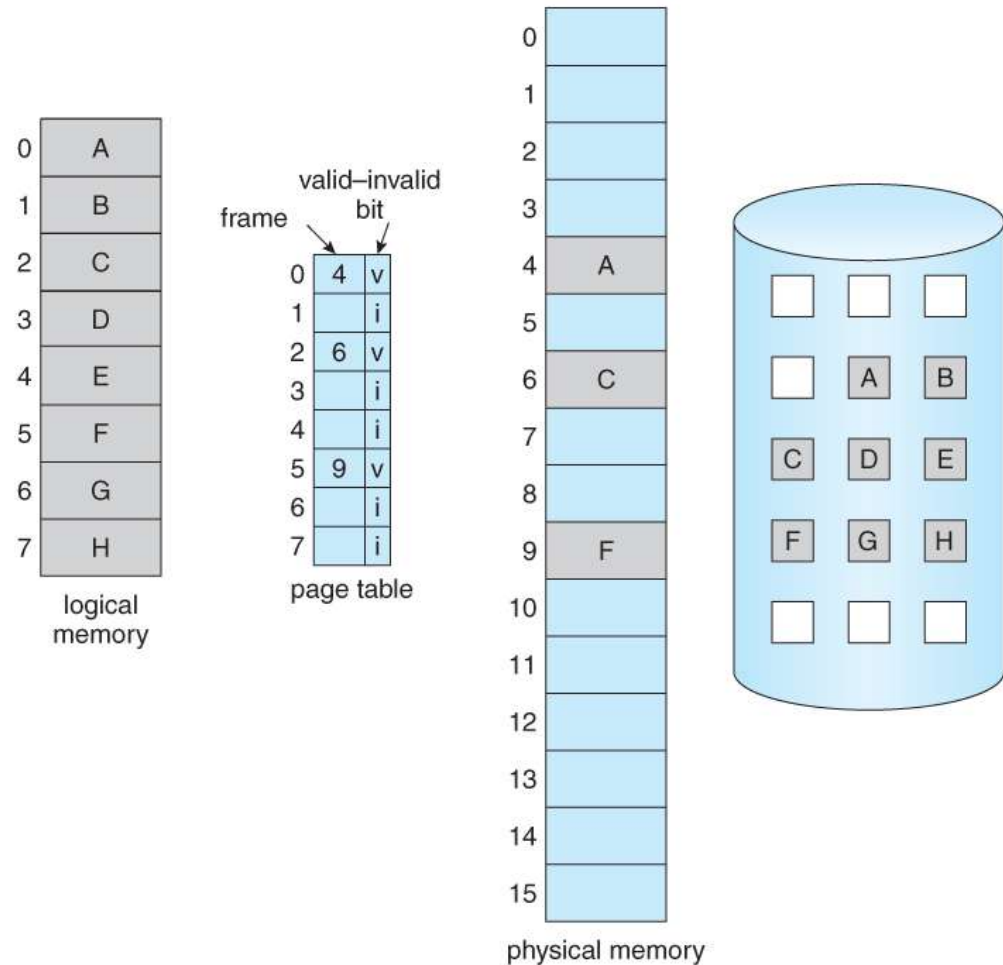


# Virtual Memory: Basic Concepts



At each logical memory reference, a page table lookup is performed as usual

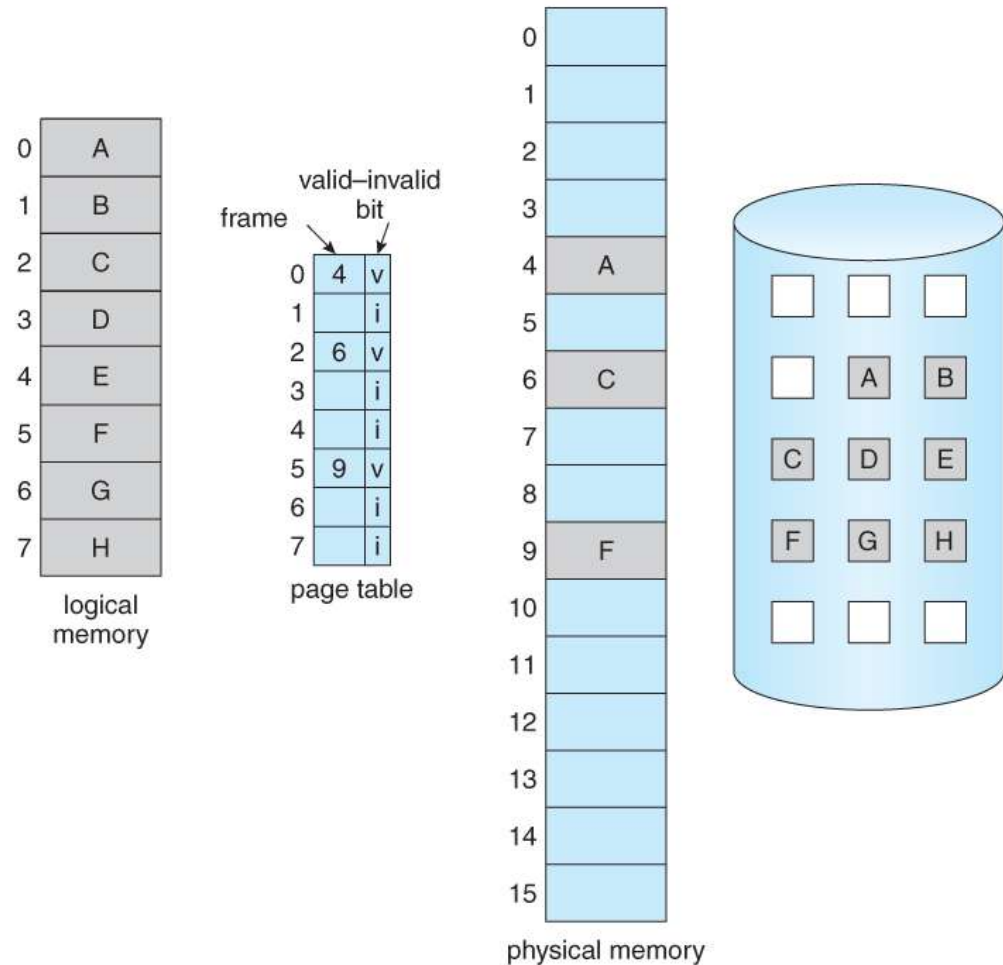
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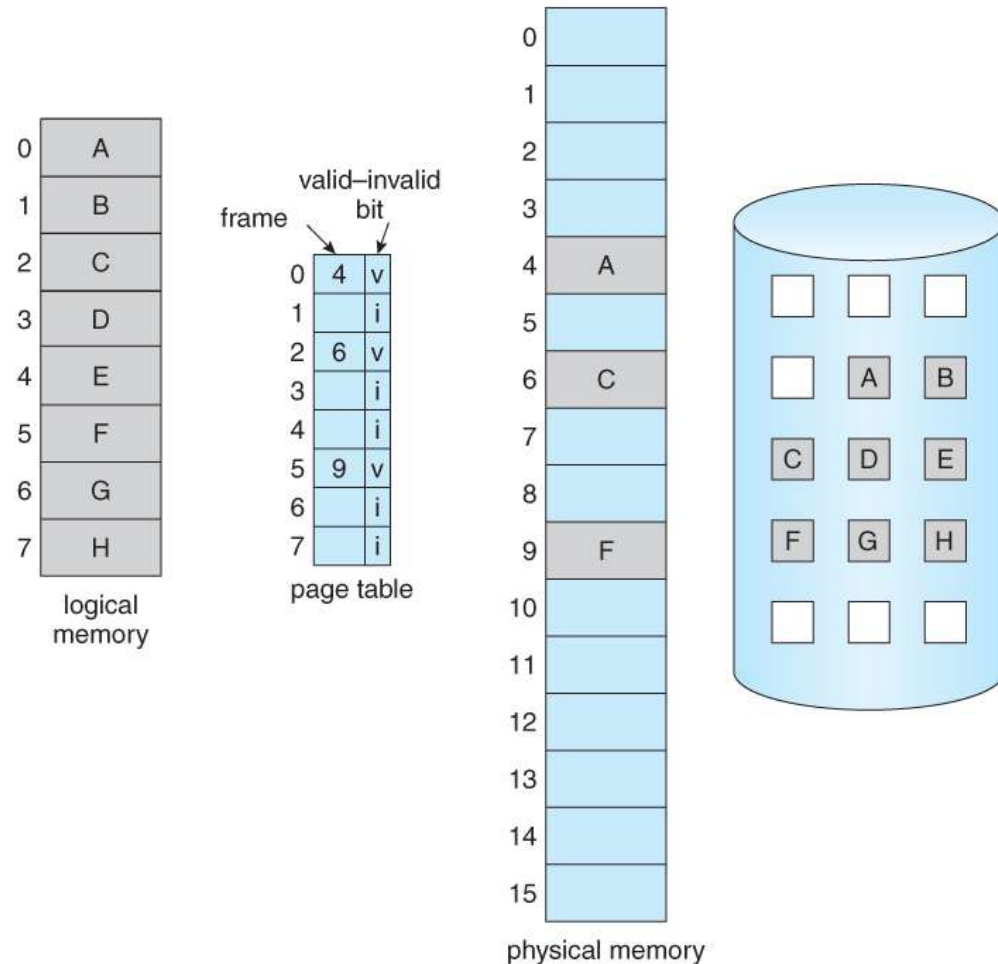


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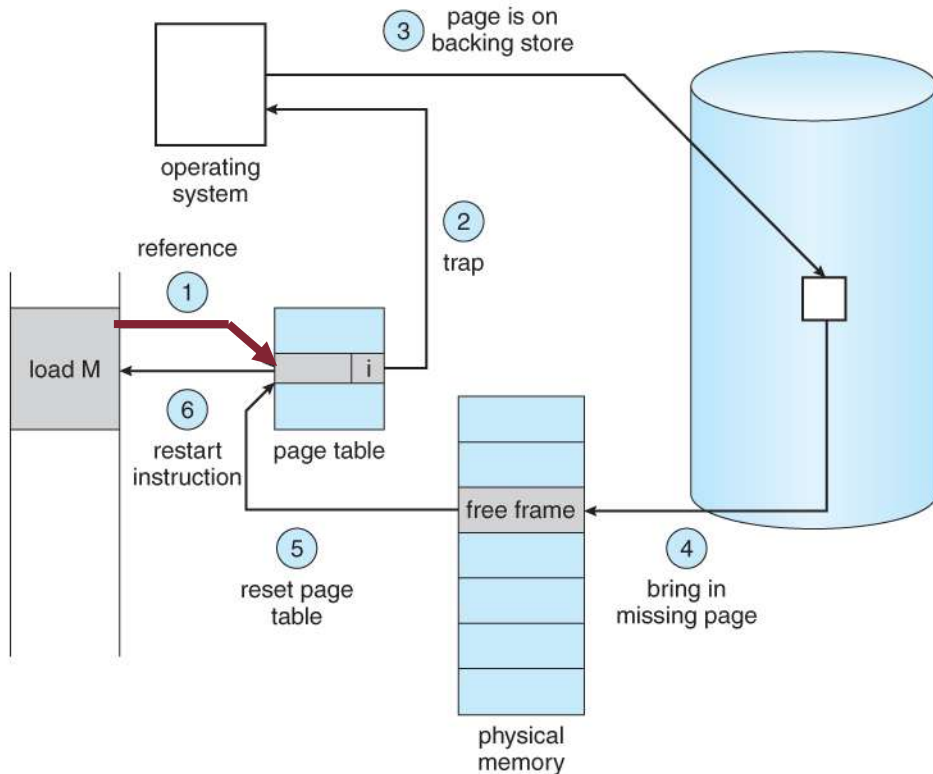
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Otherwise, a **page fault trap** occurs, and the page has to be loaded (i.e., fetched) from disk

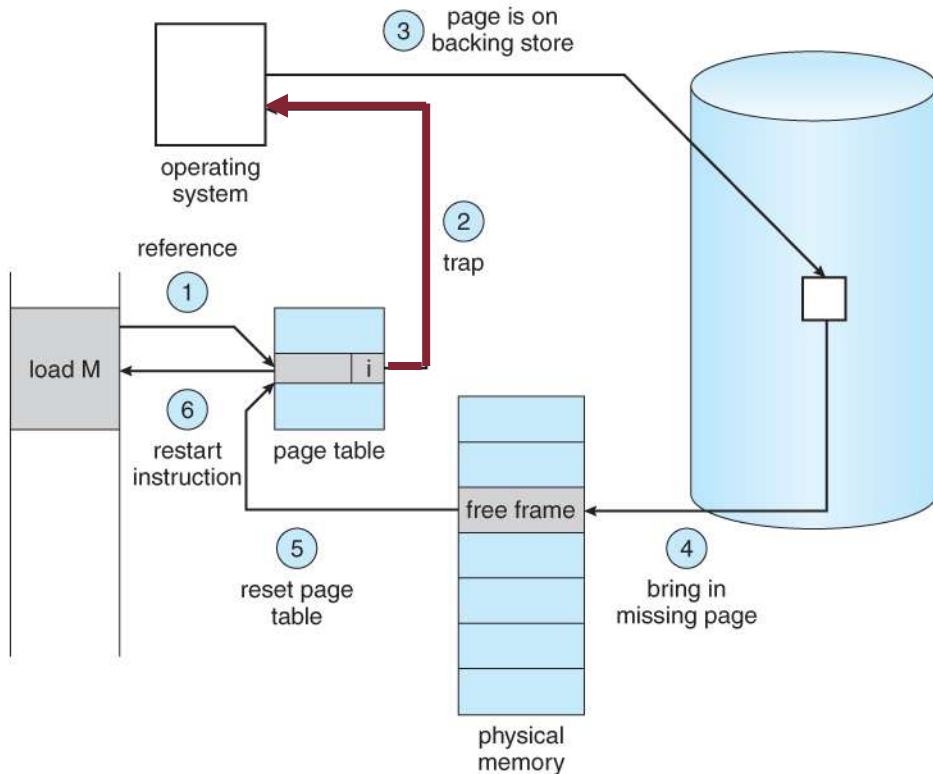
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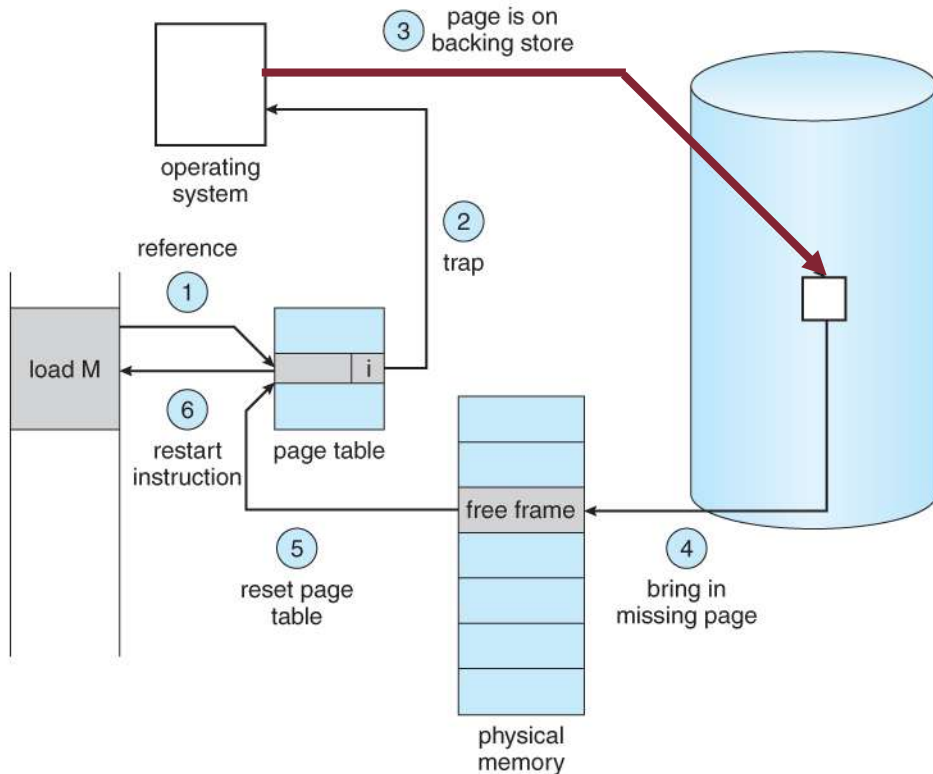


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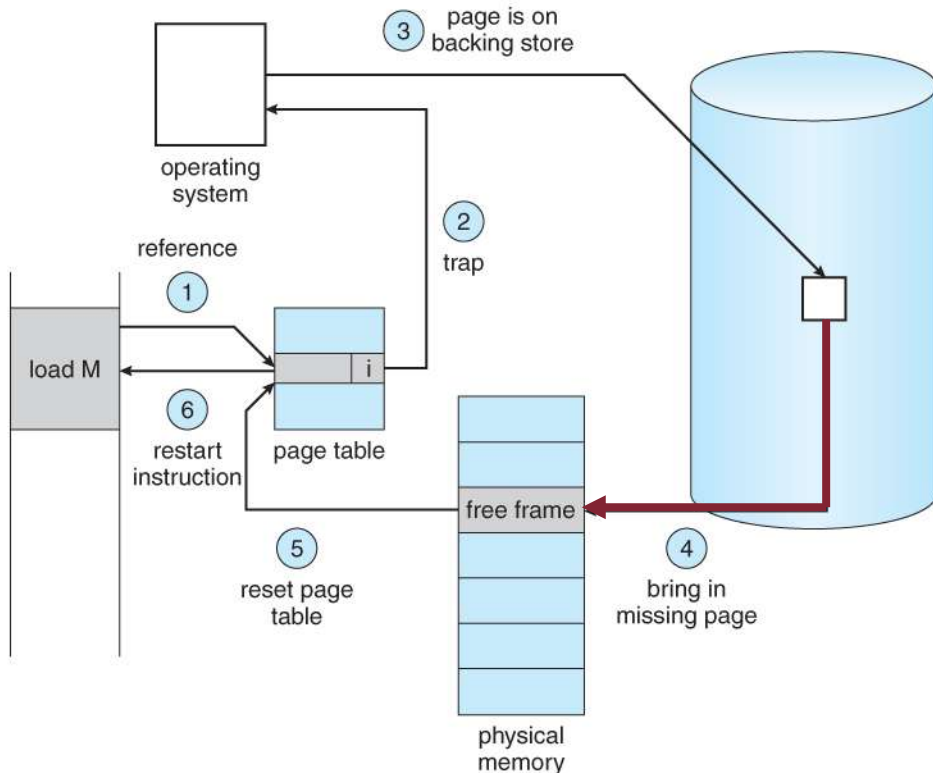


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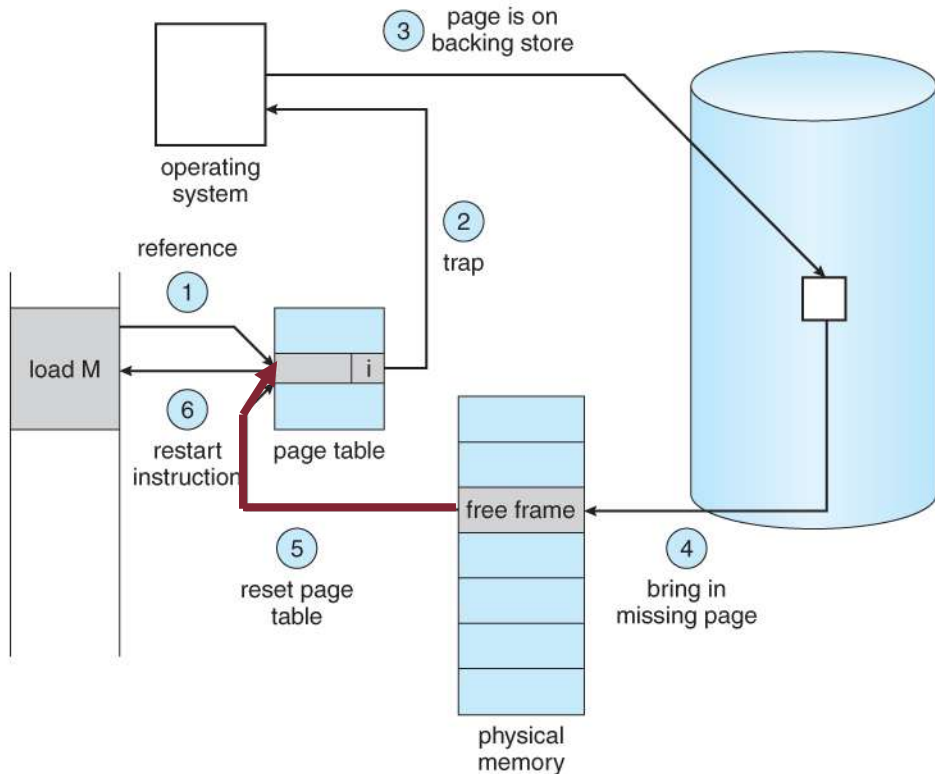
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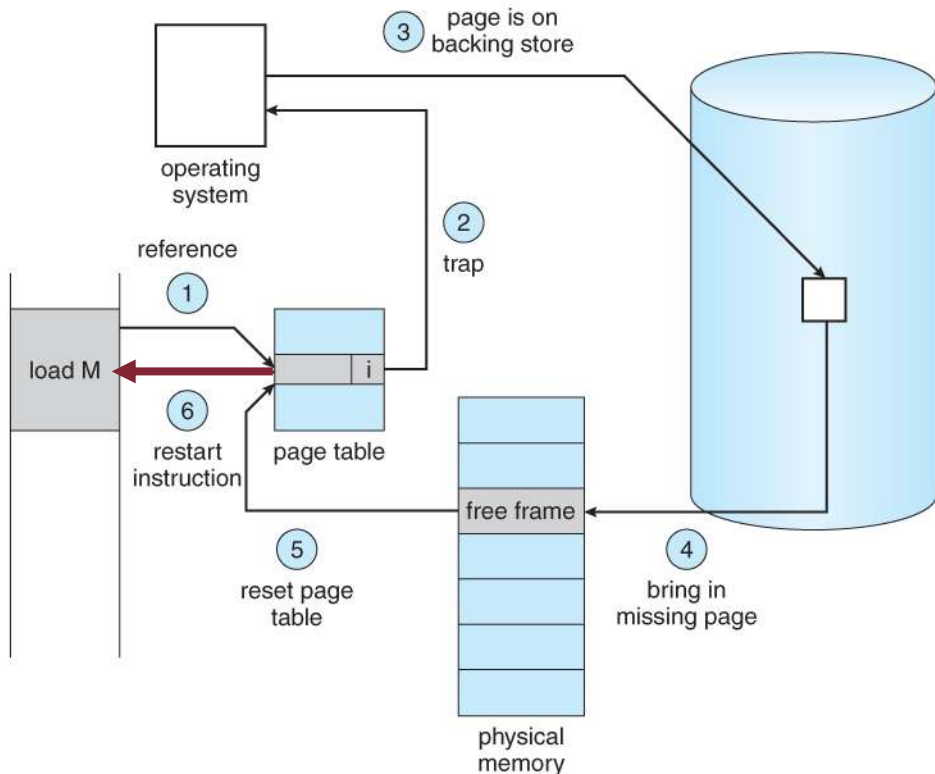


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5. When the I/O operation is complete, the process's page table is updated with the new frame number, and the bit is set to valid
6. The current process gets interrupted and the instruction that caused the page fault must be restarted from the beginning

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- If we get a TLB hit but the frame is not actually in main memory, we have to go fetch the page from disk anyway!
- TLB hit means the requested page entry is in the cache **and** the referenced frame is also in memory

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  - The OS picks a TLB entry to replace and fills it with the new entry as follows
    - invalidates the TLB entry
    - performs page fault trap operations
    - updates the TLB entry
    - restarts the faulting instruction

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- Architecture-dependent:
  - x86: hardware saves the virtual address that caused the fault (CR2 register)
  - On some platforms, OS gets only address of faulting instruction, must simulate the instruction and try every address to find the one that generated the fault

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- Transparently restarting process execution after a page fault is tricky, since the fault may have occurred in the middle of an instruction
- To restart (from scratch) a faulty instruction the OS needs hardware support for saving:
  - The faulting instruction
  - The CPU state

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- idempotent → just restart the faulting instruction (hardware saves instruction address during page fault)
- non-idempotent → much more difficult to restart
  - `MOV [ %R1 ], + ( %R2 )` → increment the value of R2 and store it to memory address in R1
  - What if memory address [ %R1 ] causes the page fault?
  - Cannot naively redo the instruction from scratch, otherwise R2 gets incremented twice



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- Even harder when using instructions that are not easily undoable
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How to unwind those complicated side-effects?

Make sure all the addresses within the block to be transferred are in memory before starting executing the instruction

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- Still, an overhead must be paid every time a page fault occurs as the OS needs to interact with slower disk



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$t_{MA}$  = physical memory access time

$t_{FAULT}$  = time to handle a page fault

$p \in [0, 1]$  = probability of page fault

$t_{ACCESS}$  = effective time for each memory reference

$$t_{ACCESS} = (1 - p) * t_{MA} + p * t_{FAULT}$$

Let's assume:  $t_{MA} = 100$  nsec and  $t_{FAULT} = 20$  msec = 20,000,000 nsec

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This heavily depends on  $p$ !

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The access time increases from just 100 nsec up to ~20.1 microsec

200 times slowdown factor

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$$\begin{aligned} 1.1 * 100 &= 100 - 100p + 20,000,000p = \\ 19,999,900p &= 110 - 100 = \end{aligned}$$

To achieve that goal, we can tolerate at most 1 page fault every about 2 million accesses!

$$p = \frac{10}{19,999,900} = \frac{1}{1,999,990} \approx 0,0000005 = 5 * 10^{-7}$$

# Virtual Memory: Performance Example

More generally, given  $t_{MA}$ ,  $t_{FAULT}$ , and a threshold  $\epsilon > 0$  if we want to find  $p$  s.t.:

$$t_{ACCESS} = (1 + \epsilon) * t_{MA}$$

We substitute  $t_{ACCESS}$  and solve for  $p$  the resulting equation:

$$\begin{aligned}(1 - p) * t_{MA} + p * t_{FAULT} &= (1 + \epsilon) * t_{MA} = \\ t_{MA} - p * t_{MA} + p * t_{FAULT} &= t_{MA} + \epsilon * t_{MA} \\ p(t_{FAULT} - t_{MA}) &= \epsilon * t_{MA} =\end{aligned}$$

$$p = \frac{\epsilon * t_{MA}}{t_{FAULT} - t_{MA}}$$



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- So far, we have described how the OS (with the support of HW) manages page faults
- Still, the OS has to answer 2 fundamental questions:
  - When to load process' pages into main memory (**page fetching**)
  - Which page to remove from memory if this gets filled (**page replacement**)

# Page Fetching Goals

- The overall goal is still to make physical memory look larger than it is
- Exploiting the locality reference of programs
- Keep in memory only those pages that is being used
- Keep on disk those pages that are unused
- Ideally, producing a memory system with the performance of main memory and the cost/capacity of disk!

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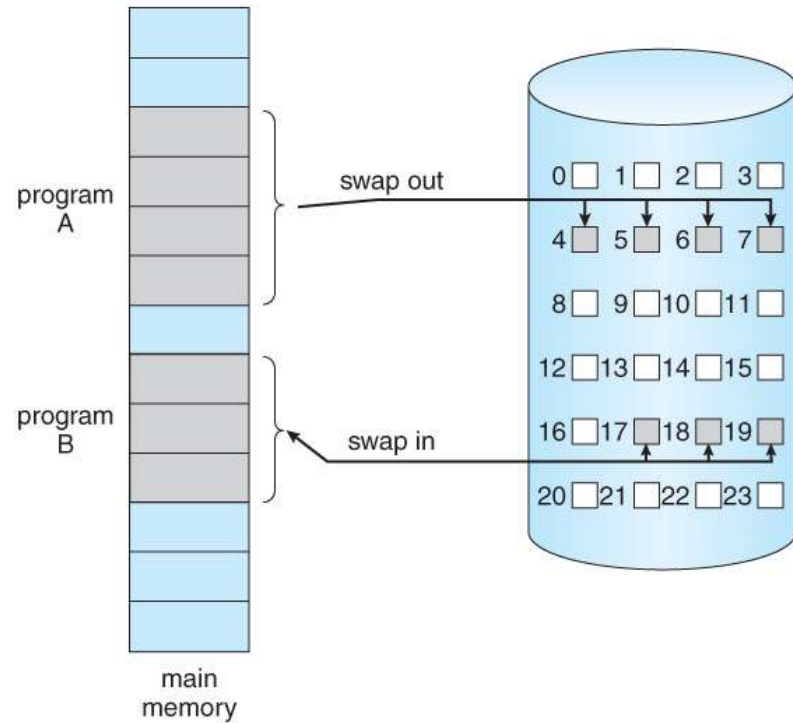
Most modern OSs use **demand fetching**



# (Pure) Demand Paging

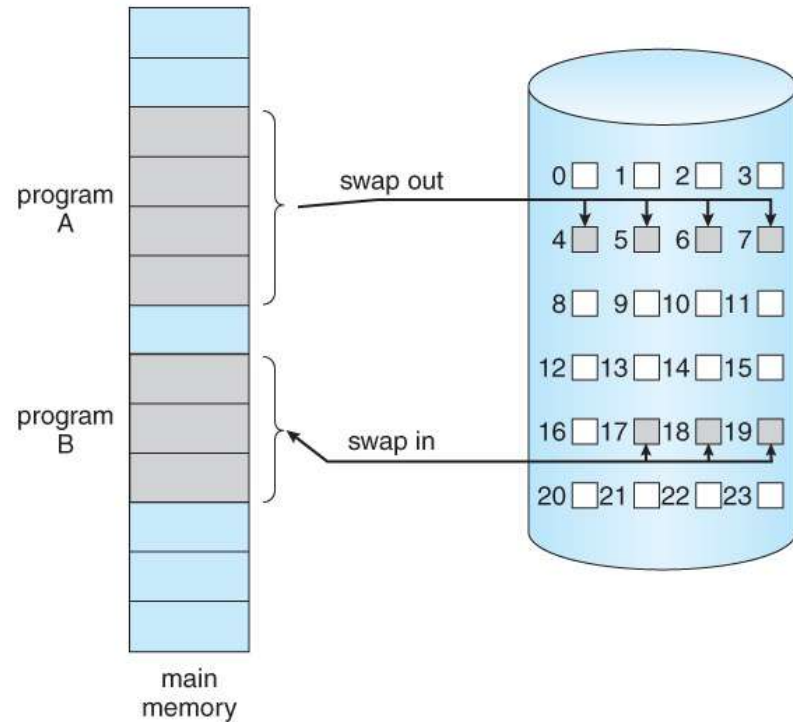
- When a process starts up, **none** of its pages are loaded
- Rather, a page is swapped in only when the process references it (upon a page fault)
- This is termed a **lazy swapper** or **pager**
- Opposite of loading all the pages at process startup!

# Prefetching



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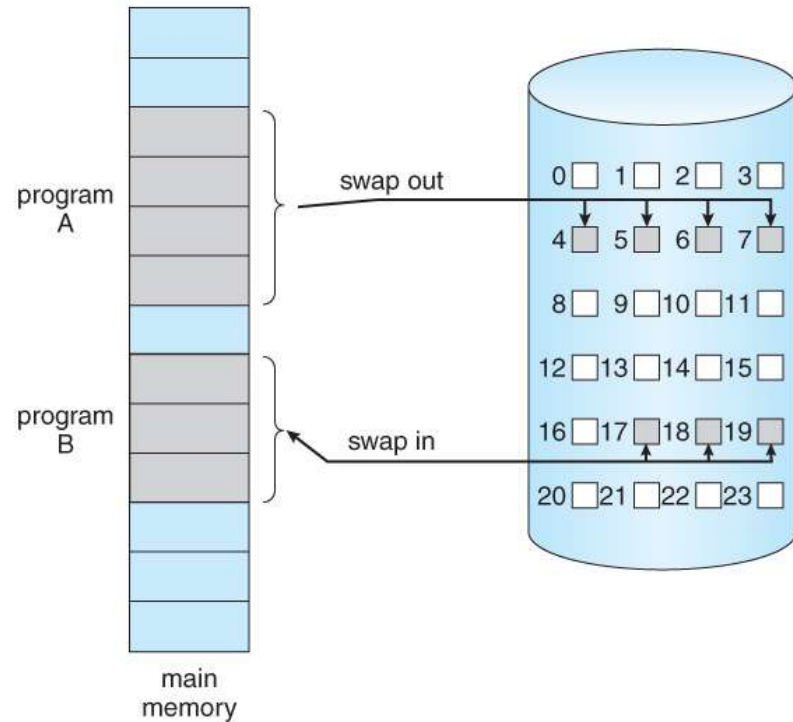
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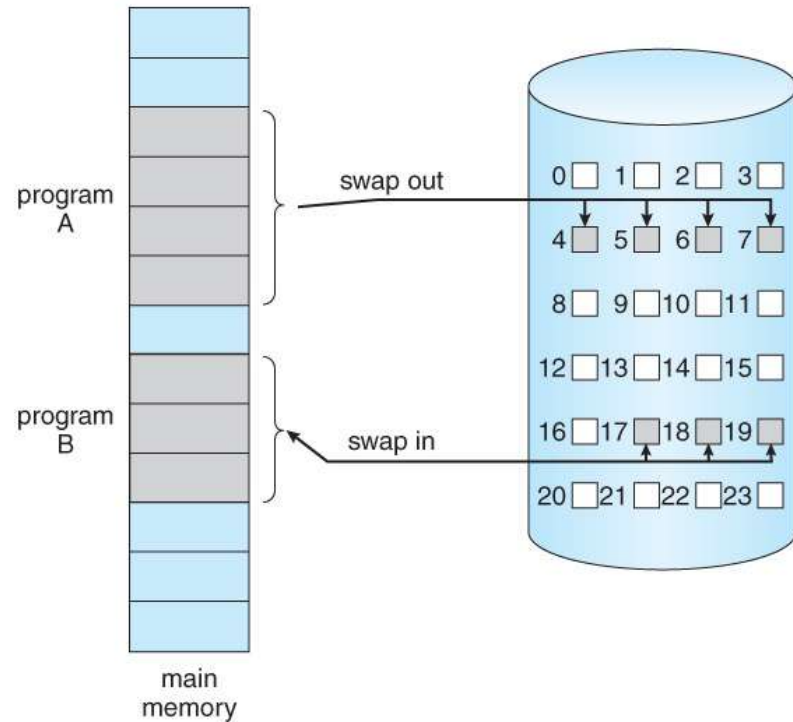


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Possible approach: upon page fault, load many pages instead of only the faulty one

works if program accesses memory sequentially

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- On Mac, instead, swap space is part of the file system
  - swapfiles

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  - **Data** (initialized/uninitialized)
- Depending on which kind of page is removed, different optimizations may apply upon page swap-out

# Swap Out Optimizations

- Code page (read-only):
  - Code content does not change!
  - Just remove it and re-load it back from executable file stored on disk
  - Make use of the filesystem

# Swap Out Optimizations

- **Code** page (read-only):
  - Code content does not change!
  - Just remove it and re-load it back from executable file stored on disk
  - Make use of the filesystem
- **Data** page:
  - Data content does actually change!
  - Save it to a separate paging file, so that no changes are lost when it will be loaded in the future
  - Need to use the dedicated swap space

# Page Replacement: Motivation

- On a page fault, we need to load a page from disk into memory
- If physical memory has still free frames, the page can be safely loaded into one of those
- However, if physical memory is full (i.e., all of its frames are loaded) a frame must be swapped out to make room for the swap-in page
- Several algorithms to select the page to evict from memory

# Page Replacement Algorithms

- **Random:** pick any page at random (works surprisingly well!)



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- **Random**: pick any page at random (works surprisingly well!)
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- **MIN (OPT)**: remove the page that will not be accessed for the longest time (provably optimal [Belady 1966])
  - Needs to predict the future → very hard!

# Page Replacement Algorithms

- **Random**: pick any page at random (works surprisingly well!)
- **FIFO (First-In-First-Out)**: throw out the page that has been in memory for longest time (i.e., the oldest)
  - Easy to implement but may remove frequently accessed pages
- **MIN (OPT)**: remove the page that will not be accessed for the longest time (provably optimal [Belady 1966])
  - Needs to predict the future → very hard!
- **LRU (Least Recently Used)**: approximation of MIN, remove the page that has not been used in the longest time
  - Assumes the past is a good predictor of the future (not always true!)

# FIFO Page Replacement: Example

3 physical frames:  $F_1, F_2, F_3$

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
$F_1$											
$F_2$											
$F_3$											

How many page faults (denoted by \*)?

# FIFO Page Replacement: Example

3 physical frames:  $F_1, F_2, F_3$

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
$F_1$											
$F_2$											
$F_3$											

Initially, no frame is loaded in memory at all  
(pure demand paging)

# FIFO Page Replacement: Example

3 physical frames:  $F_1, F_2, F_3$

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
$F_1$											
$F_2$											
$F_3$											

Virtual address within page A is referenced

# FIFO Page Replacement: Example

3 physical frames:  $F_1$ ,  $F_2$ ,  $F_3$

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
$F_1$											
$F_2$											
$F_3$											

Virtual address within page A is referenced



page fault

# FIFO Page Replacement: Example

3 physical frames:  $F_1, F_2, F_3$

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
$F_1$	A*										
$F_2$											
$F_3$											

Virtual address within page A is referenced



page fault



A loaded

FIFO = A



# FIFO Page Replacement: Example

3 physical frames:  $F_1, F_2, F_3$

4 virtual pages:  $A, B, C, D$

Reference sequence of pages:  $A, B, C, A, B, D, A, D, B, C, A$

	A	B	C	A	B	D	A	D	B	C	A
$F_1$	$A^*$	A									
$F_2$											
$F_3$											

Virtual address within page  $B$  is referenced

FIFO =  $A$

# FIFO Page Replacement: Example

3 physical frames:  $F_1, F_2, F_3$

4 virtual pages:  $A, B, C, D$

Reference sequence of pages:  $A, B, C, A, B, D, A, D, B, C, A$

	A	B	C	A	B	D	A	D	B	C	A
$F_1$	$A^*$	A									
$F_2$											
$F_3$											

Virtual address within page  $B$  is referenced



page fault

FIFO =  $A$

# FIFO Page Replacement: Example

3 physical frames:  $F_1, F_2, F_3$

4 virtual pages:  $A, B, C, D$

Reference sequence of pages:  $A, B, C, A, B, D, A, D, B, C, A$

	A	B	C	A	B	D	A	D	B	C	A
$F_1$	A*	A									
$F_2$		B*									
$F_3$											

Virtual address within page  $B$  is referenced



page fault



$B$  loaded

FIFO =  $A \rightarrow B$

# FIFO Page Replacement: Example

3 physical frames:  $F_1, F_2, F_3$

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
$F_1$	A*	A	A								
$F_2$		B*	B								
$F_3$											

Virtual address within page C is referenced

FIFO = A  $\rightarrow$  B

# FIFO Page Replacement: Example

3 physical frames:  $F_1, F_2, F_3$

4 virtual pages:  $A, B, C, D$

Reference sequence of pages:  $A, B, C, A, B, D, A, D, B, C, A$

	A	B	C	A	B	D	A	D	B	C	A
$F_1$	A*	A	A								
$F_2$		B*	B								
$F_3$											

Virtual address within page  $C$  is referenced



page fault

FIFO =  $A \rightarrow B$

# FIFO Page Replacement: Example

3 physical frames:  $F_1, F_2, F_3$

4 virtual pages:  $A, B, C, D$

Reference sequence of pages:  $A, B, C, A, B, D, A, D, B, C, A$

	A	B	C	A	B	D	A	D	B	C	A
$F_1$	A*	A	A								
$F_2$		B*	B								
$F_3$			C*								

Virtual address within page  $C$  is referenced



page fault



$C$  loaded

FIFO =  $A \rightarrow B \rightarrow C$

# FIFO Page Replacement: Example

3 physical frames:  $F_1, F_2, F_3$

4 virtual pages:  $A, B, C, D$

Reference sequence of pages:  $A, B, C, A, B, D, A, D, B, C, A$

	A	B	C	A	B	D	A	D	B	C	A
$F_1$	A*	A	A	A							
$F_2$		B*	B	B							
$F_3$			C*	C							

Virtual address within page  $A$  is referenced



$A$  is already loaded

FIFO =  $A \rightarrow B \rightarrow C$

# FIFO Page Replacement: Example

3 physical frames:  $F_1, F_2, F_3$

4 virtual pages:  $A, B, C, D$

Reference sequence of pages:  $A, B, C, A, B, D, A, D, B, C, A$

	A	B	C	A	B	D	A	D	B	C	A
$F_1$	A*	A	A	A	A						
$F_2$		B*	B	B	B						
$F_3$			C*	C	C						

Virtual address within page  $B$  is referenced



$B$  is already loaded

FIFO =  $A \rightarrow B \rightarrow C$



# FIFO Page Replacement: Example

3 physical frames:  $F_1, F_2, F_3$

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
$F_1$	A*	A	A	A	A	A					
$F_2$		B*	B	B	B	B					
$F_3$			C*	C	C	C					

Virtual address within page D is referenced

FIFO = A → B → C

# FIFO Page Replacement: Example

3 physical frames:  $F_1, F_2, F_3$

4 virtual pages:  $A, B, C, D$

Reference sequence of pages:  $A, B, C, A, B, D, A, D, B, C, A$

	A	B	C	A	B	D	A	D	B	C	A
$F_1$	A*	A	A	A	A	A					
$F_2$		B*	B	B	B	B					
$F_3$			C*	C	C	C					

Virtual address within page  $D$  is referenced



page fault

FIFO =  $A \rightarrow B \rightarrow C$

# FIFO Page Replacement: Example

3 physical frames:  $F_1, F_2, F_3$

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
$F_1$	A*	A	A	A	A	D*					
$F_2$		B*	B	B	B	B					
$F_3$			C*	C	C	C					

Virtual address within page D is referenced



page fault



A replaced  
D loaded

FIFO = B → C → D

# FIFO Page Replacement: Example

3 physical frames:  $F_1, F_2, F_3$

4 virtual pages:  $A, B, C, D$

Reference sequence of pages:  $A, B, C, A, B, D, A, D, B, C, A$

	A	B	C	A	B	D	A	D	B	C	A
$F_1$	A*	A	A	A	A	D*	D				
$F_2$		B*	B	B	B	B	B				
$F_3$			C*	C	C	C	C				

Virtual address within page  $A$  is referenced

FIFO =  $B \rightarrow C \rightarrow D$

# FIFO Page Replacement: Example

3 physical frames:  $F_1, F_2, F_3$

4 virtual pages:  $A, B, C, D$

Reference sequence of pages:  $A, B, C, A, B, D, A, D, B, C, A$

	A	B	C	A	B	D	A	D	B	C	A
$F_1$	A*	A	A	A	A	D*	D				
$F_2$		B*	B	B	B	B	B				
$F_3$			C*	C	C	C	C				

Virtual address within page  $A$  is referenced



page fault

FIFO =  $B \rightarrow C \rightarrow D$

# FIFO Page Replacement: Example

3 physical frames:  $F_1, F_2, F_3$

4 virtual pages:  $A, B, C, D$

Reference sequence of pages:  $A, B, C, A, B, D, A, D, B, C, A$

	A	B	C	A	B	D	A	D	B	C	A
$F_1$	A*	A	A	A	A	D*	D				
$F_2$		B*	B	B	B	B	A*				
$F_3$			C*	C	C	C	C				

Virtual address within page  $A$  is referenced



page fault



$B$  replaced  
 $A$  loaded

FIFO =  $C \rightarrow D \rightarrow A$

# FIFO Page Replacement: Example

3 physical frames:  $F_1, F_2, F_3$

4 virtual pages:  $A, B, C, D$

Reference sequence of pages:  $A, B, C, A, B, D, A, D, B, C, A$

	A	B	C	A	B	D	A	D	B	C	A
$F_1$	A*	A	A	A	A	D*	D	D			
$F_2$		B*	B	B	B	B	A*	A			
$F_3$			C*	C	C	C	C	C			

Virtual address within page  $D$  is referenced



$D$  is already loaded

FIFO =  $C \rightarrow D \rightarrow A$

# FIFO Page Replacement: Example

3 physical frames:  $F_1, F_2, F_3$

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
$F_1$	A*	A	A	A	A	D*	D	D	D	C*	C
$F_2$		B*	B	B	B	B	A*	A	A	A	A
$F_3$			C*	C	C	C	C	C	B*	B	B

Eventually, we get a total of 7 page faults



# MIN Page Replacement: Example

3 physical frames:  $F_1, F_2, F_3$

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
$F_1$											
$F_2$											
$F_3$											

How many page faults (denoted by \*)?

# MIN Page Replacement: Example

3 physical frames:  $F_1, F_2, F_3$

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
$F_1$											
$F_2$											
$F_3$											

Initially, no frame is loaded in memory at all  
(pure demand paging)

# MIN Page Replacement: Example

3 physical frames:  $F_1, F_2, F_3$

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
$F_1$	A*	A	A	A	A						
$F_2$		B*	B	B	B						
$F_3$			C*	C	C						

Up to this point, the same as FIFO

# MIN Page Replacement: Example

3 physical frames:  $F_1, F_2, F_3$

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
$F_1$	A*	A	A	A	A	A					
$F_2$		B*	B	B	B	B					
$F_3$			C*	C	C	C					

Virtual address within page D is referenced

# MIN Page Replacement: Example

3 physical frames:  $F_1, F_2, F_3$

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
$F_1$	A*	A	A	A	A	A					
$F_2$		B*	B	B	B	B					
$F_3$			C*	C	C	C					

Virtual address within page D is referenced



page fault

# MIN Page Replacement: Example

3 physical frames:  $F_1, F_2, F_3$

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
$F_1$	A*	A	A	A	A	A					
$F_2$		B*	B	B	B	B					
$F_3$			C*	C	C	C					

Virtual address within page D is referenced



page fault

What's the page that will be requested the furthest away?

# MIN Page Replacement: Example

3 physical frames:  $F_1, F_2, F_3$

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
$F_1$	A*	A	A	A	A	A					
$F_2$		B*	B	B	B	B					
$F_3$			C*	C	C	D*					

Virtual address within page D is referenced



page fault



C replaced  
D loaded

C is the page that will be requested the furthest away

# MIN Page Replacement: Example

3 physical frames:  $F_1, F_2, F_3$

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
$F_1$	A*	A	A	A	A	A	A	A	A		
$F_2$		B*	B	B	B	B	B	B	B		
$F_3$			C*	C	C	D*	D	D	D		

Up to this point, no more page faults



# MIN Page Replacement: Example

3 physical frames:  $F_1, F_2, F_3$

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
$F_1$	A*	A	A	A	A	A	A	A	A	A	
$F_2$		B*	B	B	B	B	B	B	B	B	
$F_3$			C*	C	C	D*	D	D	D	D	

Virtual address within page C is referenced

# MIN Page Replacement: Example

3 physical frames:  $F_1, F_2, F_3$

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
$F_1$	A*	A	A	A	A	A	A	A	A	A	
$F_2$		B*	B	B	B	B	B	B	B	B	
$F_3$			C*	C	C	D*	D	D	D	D	

Virtual address within page C is referenced



page fault

What's the page that will be requested the furthest away?

# MIN Page Replacement: Example

3 physical frames:  $F_1, F_2, F_3$

4 virtual pages:  $A, B, C, D$

Reference sequence of pages:  $A, B, C, A, B, D, A, D, B, C, A$

	A	B	C	A	B	D	A	D	B	C	A
$F_1$	A*	A	A	A	A	A	A	A	A	A	
$F_2$		B*	B	B	B	B	B	B	B	C*	
$F_3$			C*	C	C	D*	D	D	D	D	

Virtual address within page  $C$  is referenced



page fault



$B$  replaced  
 $C$  loaded

$B$  or  $D$  will be requested the furthest away (surely not  $A$ ): pick one (e.g.,  $B$ )

# MIN Page Replacement: Example

3 physical frames:  $F_1, F_2, F_3$

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
$F_1$	A*	A	A	A	A	A	A	A	A	A	A
$F_2$		B*	B	B	B	B	B	B	B	C*	C
$F_3$			C*	C	C	D*	D	D	D	D	D

Eventually, we get a total of 5 page faults

# LRU Page Replacement: Example

3 physical frames:  $F_1, F_2, F_3$

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
$F_1$											
$F_2$											
$F_3$											

How many page faults (denoted by \*)?

# LRU Page Replacement: Example

3 physical frames:  $F_1, F_2, F_3$

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
$F_1$											
$F_2$											
$F_3$											

Initially, no frame is loaded in memory at all  
(pure demand paging)

# LRU Page Replacement: Example

3 physical frames:  $F_1, F_2, F_3$

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
$F_1$	A*	A	A	A	A						
$F_2$		B*	B	B	B						
$F_3$			C*	C	C						

Up to this point, the same as FIFO

# LRU Page Replacement: Example

3 physical frames:  $F_1, F_2, F_3$

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
$F_1$	A*	A	A	A	A	A					
$F_2$		B*	B	B	B	B					
$F_3$			C*	C	C	C					

Virtual address within page D is referenced



page fault



# LRU Page Replacement: Example

3 physical frames:  $F_1, F_2, F_3$

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
$F_1$	A*	A	A	A	A	A					
$F_2$		B*	B	B	B	B					
$F_3$			C*	C	C	C					

Virtual address within page D is referenced



**page fault**

We can't look forward anymore!

# LRU Page Replacement: Example

3 physical frames:  $F_1, F_2, F_3$

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
$F_1$	A*	A	A	A	A	A					
$F_2$		B*	B	B	B	B					
$F_3$			C*	C	C	D*					

Virtual address within page D is referenced



page fault



C replaced  
D loaded

# LRU Page Replacement: Example

3 physical frames:  $F_1, F_2, F_3$

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
$F_1$	A*	A	A	A	A	A	A	A	A		
$F_2$		B*	B	B	B	B	B	B	B		
$F_3$			C*	C	C	D*	D	D	D		

Up to this point, no more page faults

# LRU Page Replacement: Example

3 physical frames:  $F_1, F_2, F_3$

4 virtual pages:  $A, B, C, D$

Reference sequence of pages:  $A, B, C, A, B, D, A, D, B, C, A$

	A	B	C	A	B	D	A	D	B	C	A
$F_1$	$A^*$	A	A	A	A	A	A	A	A	A	
$F_2$		$B^*$	B	B	B	B	B	B	B	B	
$F_3$			$C^*$	C	C	$D^*$	D	D	D	D	

Virtual address within page  $C$  is referenced

# LRU Page Replacement: Example

3 physical frames:  $F_1, F_2, F_3$

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
$F_1$	A*	A	A	A	A	A	A	A	A	A	
$F_2$		B*	B	B	B	B	B	B	B	B	
$F_3$			C*	C	C	D*	D	D	D	D	

Virtual address within page C is referenced



page fault

We can't look forward anymore!

# LRU Page Replacement: Example

3 physical frames:  $F_1, F_2, F_3$

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
$F_1$	A*	A	A	A	A	A	A	A	A	C*	
$F_2$		B*	B	B	B	B	B	B	B	B	
$F_3$			C*	C	C	D*	D	D	D	D	

Virtual address within page C is referenced



page fault



A replaced  
C loaded

# LRU Page Replacement: Example

3 physical frames:  $F_1, F_2, F_3$

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
$F_1$	A*	A	A	A	A	A	A	A	A	C*	C
$F_2$		B*	B	B	B	B	B	B	B	B	B
$F_3$			C*	C	C	D*	D	D	D	D	D

Virtual address within page A is referenced

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Reference sequence of pages:  $A, B, C, A, B, D, A, D, B, C, A$

	A	B	C	A	B	D	A	D	B	C	A
$F_1$	A*	A	A	A	A	A	A	A	A	C*	C
$F_2$		B*	B	B	B	B	B	B	B	B	B
$F_3$			C*	C	C	D*	D	D	D	D	D

Virtual address within page  $A$  is referenced



**page fault**

We can't look forward anymore!



# LRU Page Replacement: Example

3 physical frames:  $F_1, F_2, F_3$

4 virtual pages:  $A, B, C, D$

Reference sequence of pages:  $A, B, C, A, B, D, A, D, B, C, A$

	A	B	C	A	B	D	A	D	B	C	A
$F_1$	A*	A	A	A	A	A	A	A	A	C*	C
$F_2$		B*	B	B	B	B	B	B	B	B	B
$F_3$			C*	C	C	D*	D	D	D	D	A*

Virtual address within page  $A$  is referenced



page fault



$D$  replaced  
 $A$  loaded

# LRU Page Replacement: Example

3 physical frames:  $F_1, F_2, F_3$

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
$F_1$	A*	A	A	A	A	A	A	A	A	C*	C
$F_2$		B*	B	B	B	B	B	B	B	B	B
$F_3$			C*	C	C	D*	D	D	D	D	A*

Eventually, we get a total of 6 page faults

# LRU Page Replacement: (An Unlucky) Example

3 physical frames:  $F_1, F_2, F_3$

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, D, A, B, C, D, A, B, C

	A	B	C	D	A	B	C	D	A	B	C
$F_1$											
$F_2$											
$F_3$											

How many page faults (denoted by \*)?

# LRU Page Replacement: (An Unlucky) Example

3 physical frames:  $F_1, F_2, F_3$

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, D, A, B, C, D, A, B, C

	A	B	C	D	A	B	C	D	A	B	C
$F_1$	A*	A	A								
$F_2$		B*	B								
$F_3$			C*								

# LRU Page Replacement: (An Unlucky) Example

3 physical frames:  $F_1, F_2, F_3$

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, D, A, B, C, D, A, B, C

	A	B	C	D	A	B	C	D	A	B	C
$F_1$	A*	A	A	D*							
$F_2$		B*	B	B							
$F_3$			C*	C							

# LRU Page Replacement: (An Unlucky) Example

3 physical frames:  $F_1, F_2, F_3$

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	A	B	C	D	A	B	C	D	A	B	C
$F_1$	A*	A	A	D*	D						
$F_2$		B*	B	B	A*						
$F_3$			C*	C	C						

# LRU Page Replacement: (An Unlucky) Example

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	A	B	C	D	A	B	C	D	A	B	C
$F_1$	A*	A	A	D*	D	D					
$F_2$		B*	B	B	A*	A					
$F_3$			C*	C	C	B*					

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	A	B	C	D	A	B	C	D	A	B	C
$F_1$	A*	A	A	D*	D	D	C*				
$F_2$		B*	B	B	A*	A	A				
$F_3$			C*	C	C	B*	B				



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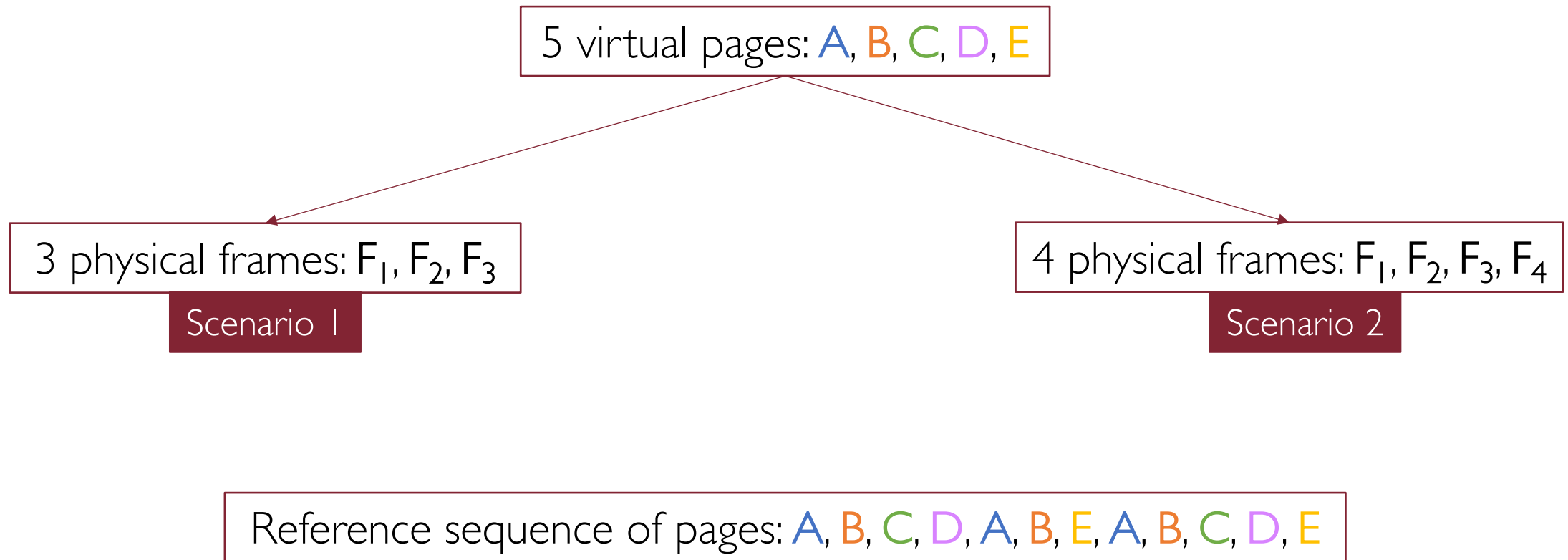
	A	B	C	D	A	B	C	D	A	B	C
$F_1$	A*	A	A	D*	D	D	C*	C	C	B*	B
$F_2$		B*	B	B	A*	A	A	D*	D	D	C*
$F_3$			C*	C	C	B*	B	B	A*	A	A

Eventually, we get a total of 11 page faults

# Page Replacement: What If We Add Memory?

- Does adding memory always reduce the number of page faults?
- Intuitively, it would seem so...
- The answer, in fact, depends on the page replacement algorithm
- Let's see this with an example, using FIFO page replacement

# FIFO Page Replacement: Example



# FIFO Page Replacement: Example

	A	B	C	D	A	B	E	A	B	C	D	E
F <sub>1</sub>	A*	A	A	D*	D	D	E*	E	E	E	E	E
F <sub>2</sub>		B*	B	B	A*	A	A	A	A	C*	C	C
F <sub>3</sub>			C*	C	C	B*	B	B	B	B	D*	D

F <sub>1</sub>	A*	A	A	A	A	A	E*	E	E	E	D*	D
F <sub>2</sub>		B*	B	B	B	B	B	A*	A	A	A	E*
F <sub>3</sub>			C*	C	C	C	C	C	B*	B	B	B
F <sub>4</sub>				D*	D	D	D	D	D	C*	C	C

# FIFO Page Replacement: Example

	A	B	C	D	A	B	E	A	B	C	D	E
--	---	---	---	---	---	---	---	---	---	---	---	---

F <sub>1</sub>	A*	A	A	D*	D	D	E*	E	E	E	E	E
F <sub>2</sub>		B*	B	B	A*	A	A	A	A	C*	C	C
F <sub>3</sub>			C*	C	C	B*	B	B	B	B	D*	D

10 page faults

F <sub>1</sub>	A*	A	A	A	A	A	E*	E	E	E	D*	D
F <sub>2</sub>		B*	B	B	B	B	B	A*	A	A	A	E*
F <sub>3</sub>			C*	C	C	C	C	C	B*	B	B	B
F <sub>4</sub>				D*	D	D	D	D	D	C*	C	C

11 page faults

## Belady's Anomaly

Adding page frames may cause more page faults with some algorithms

# LRU Page Replacement: Example

	A	B	C	D	A	B	E	A	B	C	D	E
F <sub>1</sub>	A*	A	A	D*	D	D	E*	E	E	C*	C	C
F <sub>2</sub>		B*	B	B	A*	A	A	A	A	A	D*	D
F <sub>3</sub>			C*	C	C	B*	B	B	B	B	B	B
F <sub>1</sub>	A*	A	A	A	A	A	A	A	A	A	A	E*
F <sub>2</sub>		B*	B	B	B	B	B	B	B	B	B	B
F <sub>3</sub>			C*	C	C	C	E*	E	E	E	D*	D
F <sub>4</sub>				D*	D	D	D	D	D	C*	C	C

9 page faults

8 page faults

With **LRU**, adding page frames **always** decreases the number of page faults

# LRU Page Replacement: Example

	A	B	C	D	A	B	E	A	B	C	D	E
F <sub>1</sub>	A*	A	A	D*	D	D	E*	E	E	C*	C	C
F <sub>2</sub>		B*	B	B	A*	A	A	A	A	A	D*	D
F <sub>3</sub>			C*	C	C	B*	B	B	B	B	B	B
F <sub>1</sub>	A*	A	A	A	A	A	A	A	A	A	A	E*
F <sub>2</sub>		B*	B	B	B	B	B	B	B	B	B	B
F <sub>3</sub>			C*	C	C	C	E*	E	E	E	D*	D
F <sub>4</sub>				D*	D	D	D	D	D	C*	C	C

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Why?

# LRU Page Replacement: Example

	A	B	C	D	A	B	E	A	B	C	D	E
F <sub>1</sub>	A*	A	A	D*	D	D	E*	E	E	C*	C	C
F <sub>2</sub>		B*	B	B	A*	A	A	A	A	A	D*	D
F <sub>3</sub>			C*	C	C	B*	B	B	B	B	B	B
F <sub>1</sub>	A*	A	A	A	A	A	A	A	A	A	A	E*
F <sub>2</sub>		B*	B	B	B	B	B	B	B	B	B	B
F <sub>3</sub>			C*	C	C	C	E*	E	E	E	D*	D
F <sub>4</sub>				D*	D	D	D	D	D	C*	C	C

At each point in time 4-frame memory contains a subset of 3-frame

Can't do any worst!



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- **FIFO** is easy to implement but may lead to too many page faults
  - May suffer from Belady's Anomaly
- **MIN** is the optimal choice but cannot be used in practice since future memory references are never known in advance
- **LRU** is a fair approximation of MIN assuming the past is a good predictor of the future
  - Exploits the locality reference (small working set that fits in memory)
  - Works poorly when the locality reference doesn't hold (large working set)

# LRU: Implementation Details

How could we implement LRU page replacement algorithm?

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Keep a timestamp for each page with the time it has been last accessed  
Remove the page with the highest difference w.r.t. current timestamp

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Every time a page is accessed its timestamp must be updated

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Remove the page with the highest difference w.r.t. current timestamp

## Problems?

Every time a page is accessed its timestamp must be updated

Linear scan of all the pages to select the one to be removed



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How could we implement LRU page replacement algorithm?



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Keep a list of pages with the most recently used in front and the least recently used at the end: every time a page is accessed move it to front

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How could we implement LRU page replacement algorithm?



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Keep a list of pages with the most recently used in front and the least recently used at the end: every time a page is accessed move it to front

## Problems?



Still too expensive as the OS must change multiple pointers on each memory access

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- Many systems provide some HW support which is enough to approximate LRU fairly well
- **Single-Reference Bit** → Maintain 1 bit for each page table entry
  - Initially, all bits for all pages are set to 0
  - On each access to a page, the HW sets the reference bit to 1
  - Enough to distinguish pages that have been accessed since the last clear
  - No total order of page access

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- The specific number of bits used and the frequency with which the reference byte is updated are adjustable

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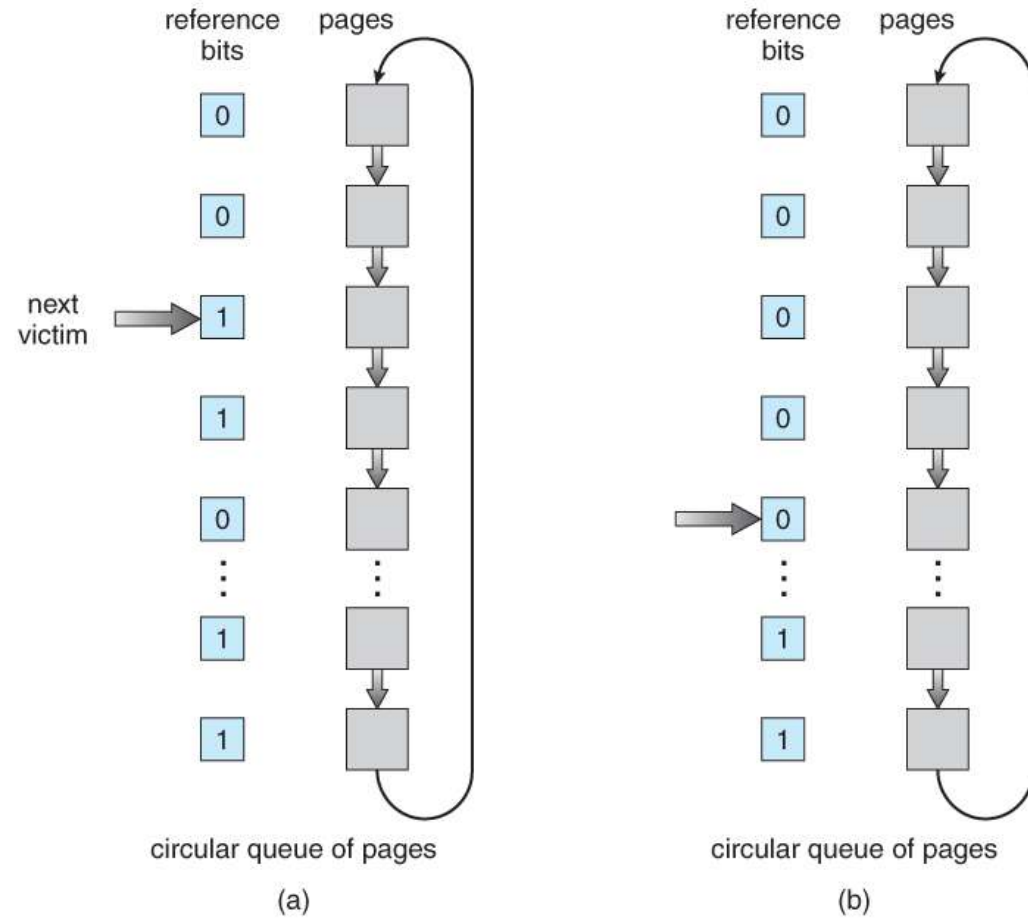
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- OS keeps frames in a FIFO circular list
- On every memory access, the reference bit is set to 1
- On a page fault, the OS scans the list of page table, checking the reference bit of the frame:
  - If this is 0, it replaces the page and sets it to 1
  - If this is 1, it sets it to 0 (second chance) and move to the next frame

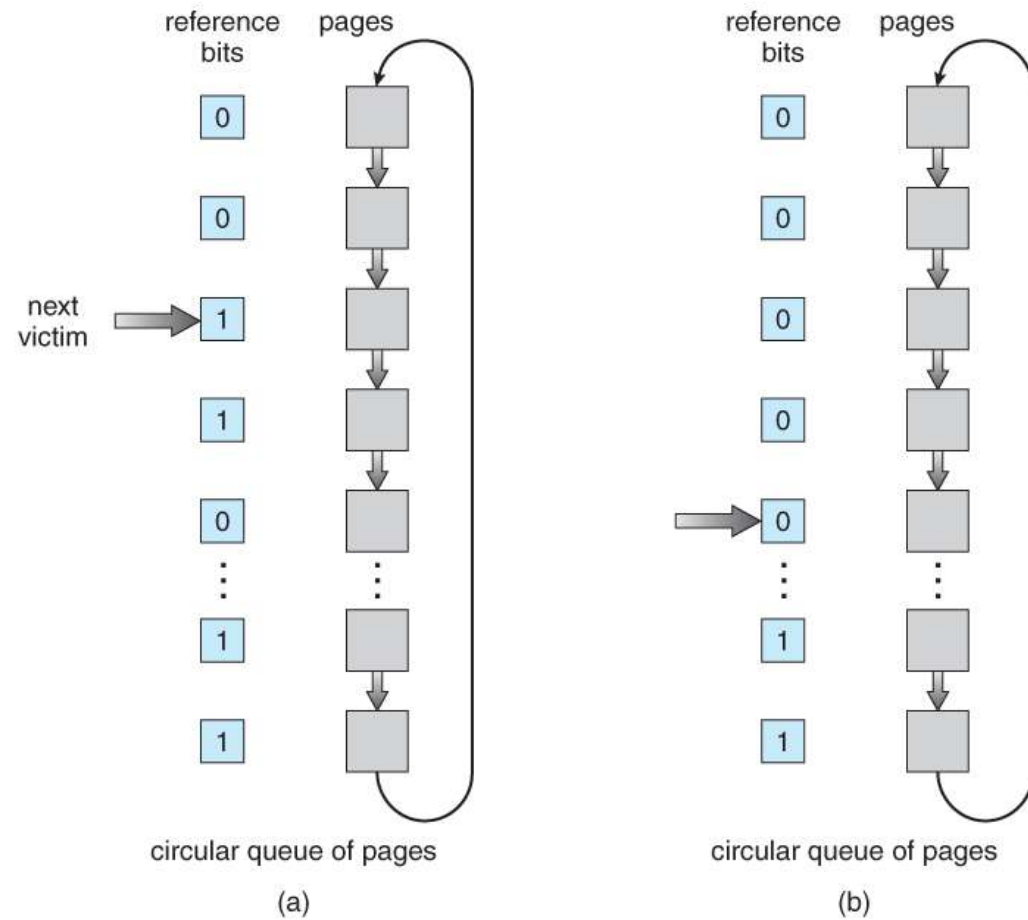
# Second Chance Algorithm (Clock)



A raw partitioning into: young vs. old frames



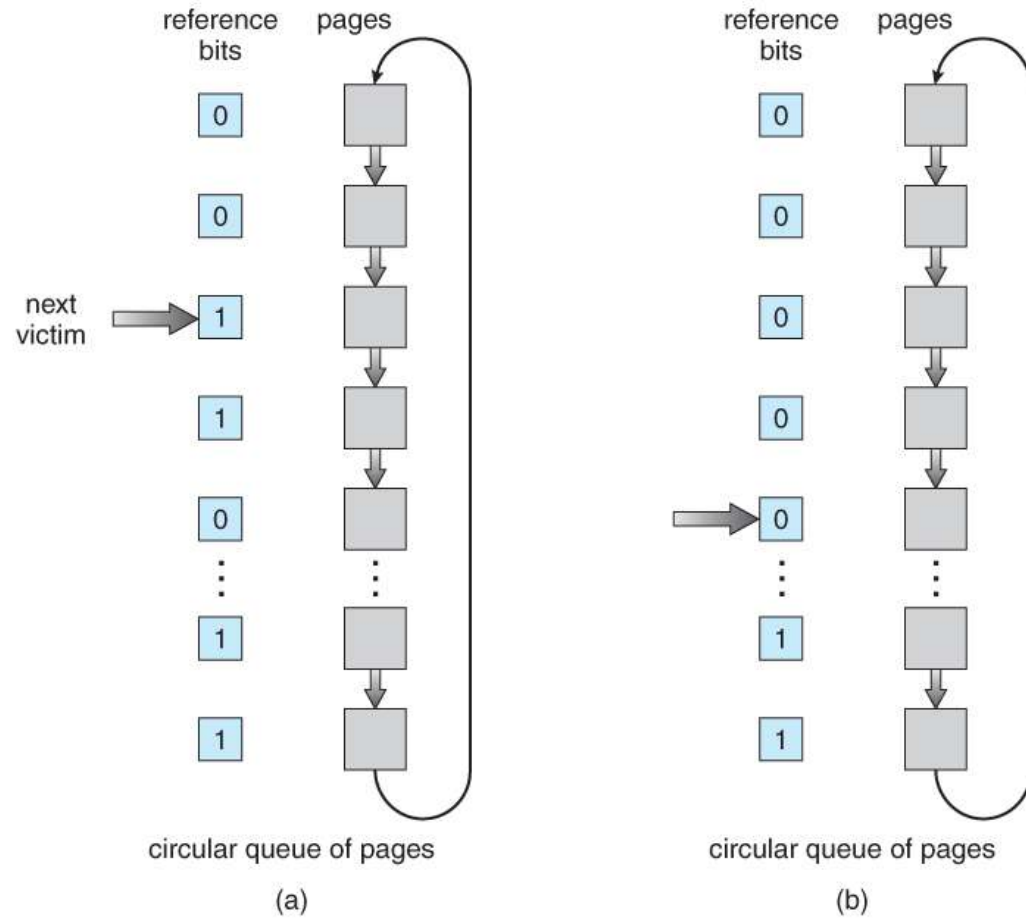
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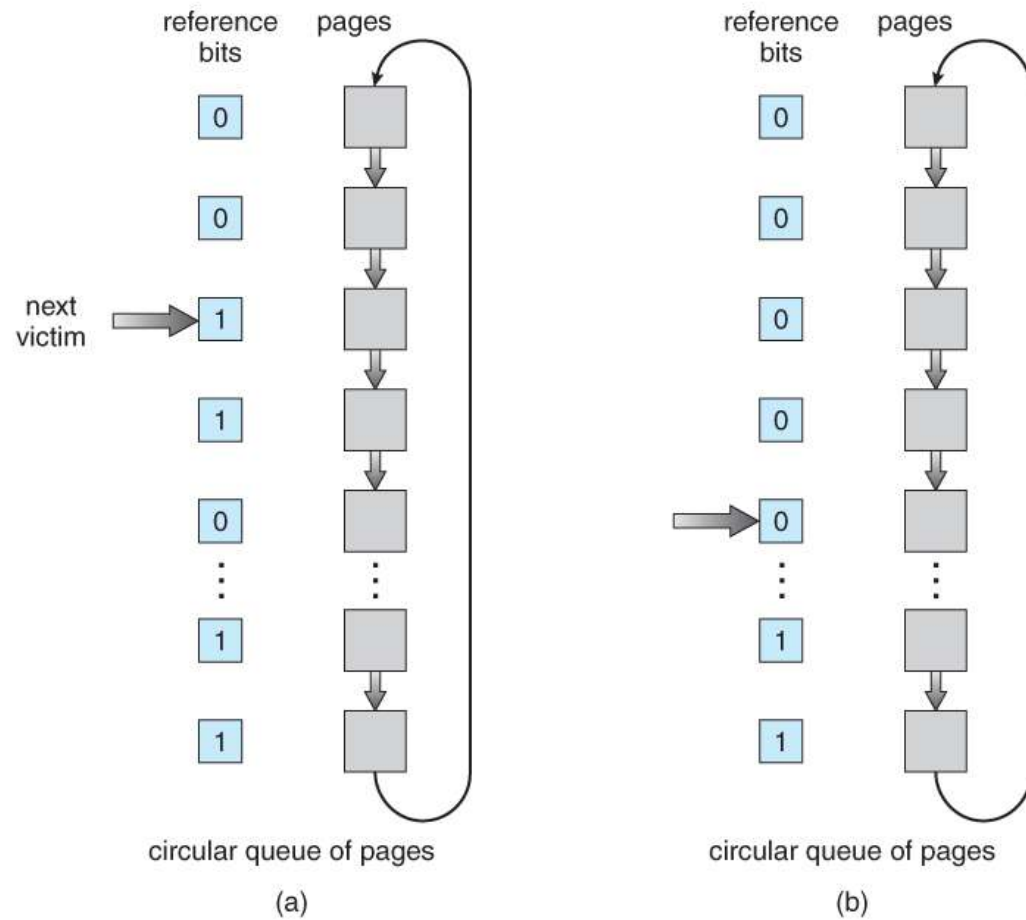


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Fast, since it needs only to set a single bit on each memory reference (no need for a shift)

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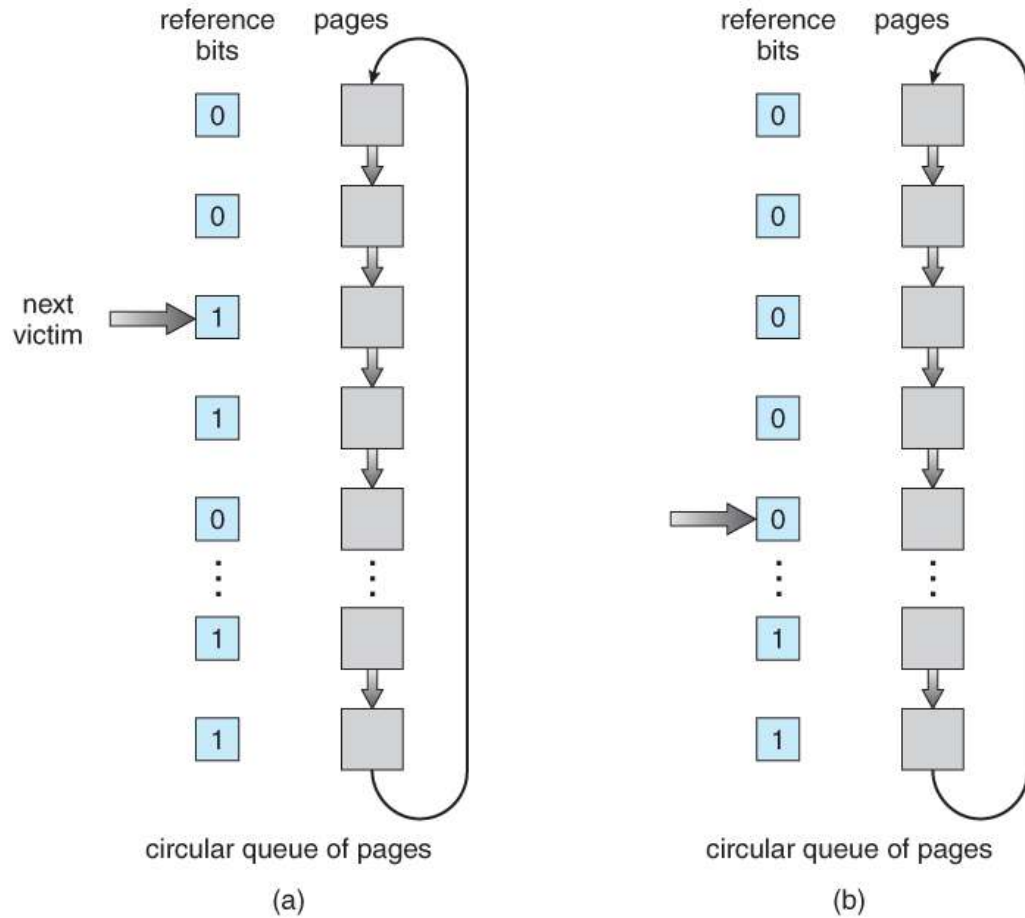
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This algorithm is also known as **clock** because it mimics the hands of a clock

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- Page replacement generally involves 2 I/O operations:
  - write the evicted page back to disk
  - read the newly referenced page from disk
- **Intuition:** It is cheaper to replace a page which has not been modified, since the OS does not need to write this back to disk
- OS should give preference to paging-out un-modified frames
  - Yet, it can proactively write to disk modified frames for later

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- HW keeps a modify bit (in addition to the reference bit)
  - 1 means the page has been modified (different from the copy on disk)
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# Enhanced Second Chance Algorithm

- HW keeps a modify bit (in addition to the reference bit)
  - 1 means the page has been modified (different from the copy on disk)
  - 0 means the page is the same as the one stored on disk
- Use both the reference and modify bits ( $r, m$ ) to classify pages into:
  - $(0, 0)$ : neither recently used nor modified;
  - $(0, 1)$ : not recently used, but modified;
  - $(1, 0)$ : recently used, but clean
  - $(1, 1)$ : recently used and modified

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- First, it looks for a (0, 0) frame, and if it can't find one, it makes another pass looking for a (0, 1), etc.
- The first page with the lowest numbered category is replaced
- The main difference between this algorithm and the standard clock is the preference for replacing clean pages if possible

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- Multiple processes can however run concurrently on a single-CPU system
- The degree of multiprogramming is not fixed apriori, yet it is driven by the locality reference (a.k.a. 90÷10 rule)
- This allows a system to load the **working set** (i.e., few pages) of many processes, thereby increasing the degree of multiprogramming

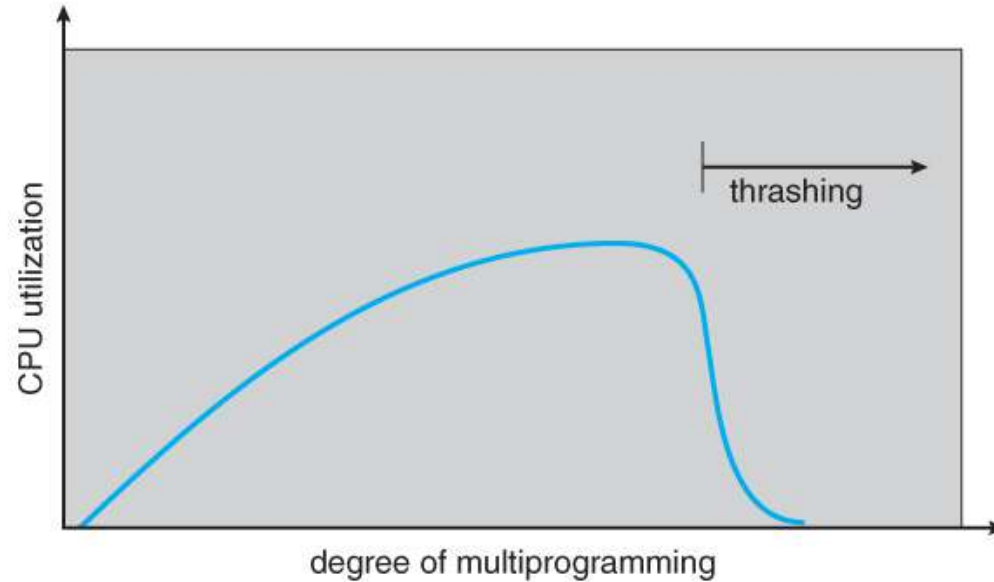
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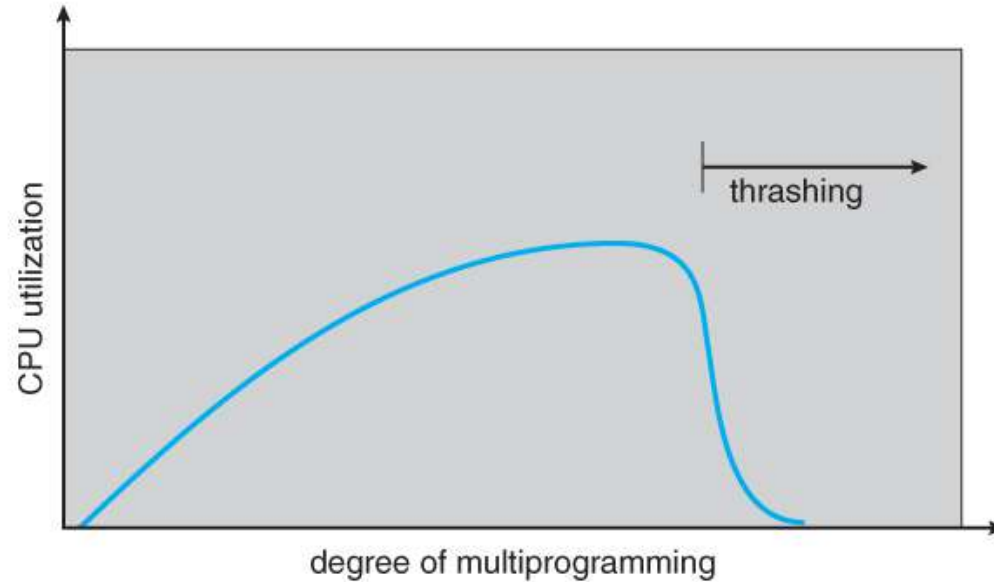
- When the degree of multiprogramming is too high, active working sets of running processes may saturate the whole memory capacity
- **Thrashing** → Memory is over-committed and pages are continuously tossed out while they are still in use
  - Memory access time approaches disk access time due to many page faults
  - Drastic degradation of performance

# Multiprogramming and Thrashing



CPU utilization drops after a certain degree of multiprogramming

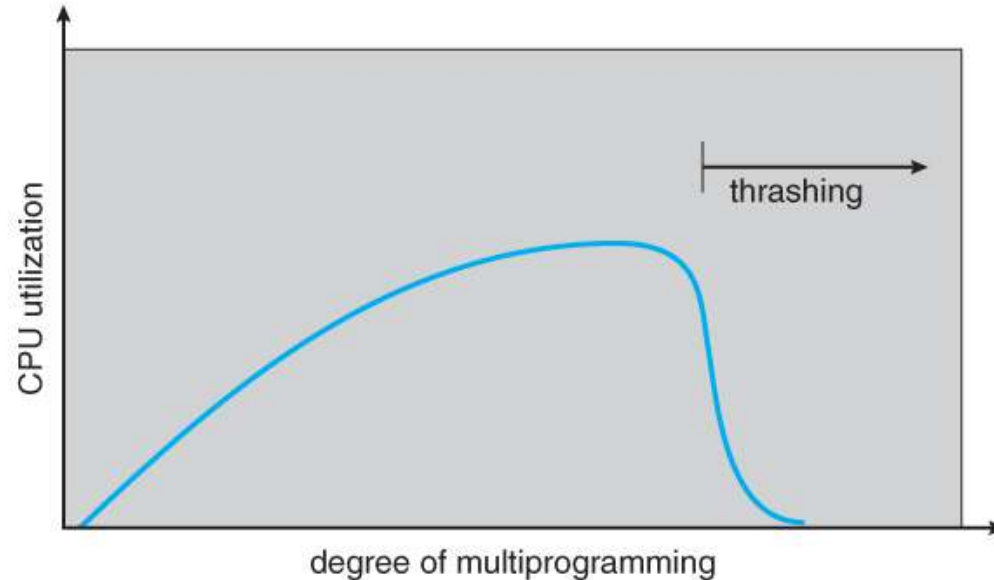
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Eventually, also CPU-bound processes turn into I/O-bound ones (due to page faults)

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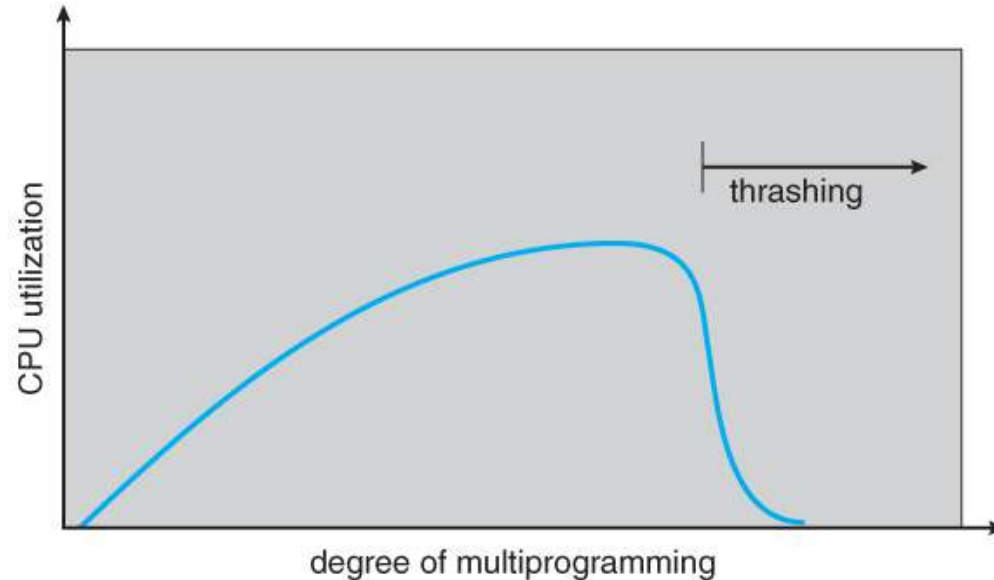


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Eventually, also CPU-bound processes turn into I/O-bound ones (due to page faults)

What can we do to limit thrashing in a multiprogrammed system?

Fixing the degree of multi-programming apriori may be a too inflexible option

# Allocation/Replacement Policies

Ultimately, we want to give each process enough memory so as to avoid thrashing



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## Global Allocation/Replacement

- All pages from all processes are in a single pool (single LRU queue)
- Upon page replacement, any page may be a potential victim, whether it currently belongs to the process seeking a free frame or not
- **PRO:** flexibility
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- **PRO:** flexibility
- **CON:** thrashing more likely (no isolation)

## Local Allocation/Replacement

- Each process has its own fixed pool of frames
- Run only group of processes that fits in memory
- LRU replacement affects only each process' frames
- **PRO:** isolation
- **CON:** performance (a process may not be given enough memory)

# Local Allocation/Replacement

$m$  = number of available physical page frames

$n$  = number of processes

$S_i$  = size of the  $i$ -th process;  $S = \sum_{i=1}^n S_i$  = total size of all processes

**Equal Allocation/Replacement:**  $\frac{m}{n}$

**Proportional Allocation/Replacement:**  $\frac{m * S_i}{S}$

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**Equal Allocation/Replacement:**  $\frac{m}{n}$

**Proportional Allocation/Replacement:**  $\frac{m * S_i}{S}$

Variations on proportional allocation could consider priority of process rather than just their size

# Local Allocation/Replacement

$m$  = number of available physical page frames

$n$  = number of processes

$S_i$  = size of the  $i$ -th process;  $S = \sum_{i=1}^n S_i$  = total size of all processes

**Equal Allocation/Replacement:**  $\frac{m}{n}$

**Proportional Allocation/Replacement:**  $\frac{m * S_i}{S}$

Variations on proportional allocation could consider priority of process rather than just their size

As allocations fluctuate over time, so does  $m$   
(processes must either be swapped out or not allowed to start if not enough frames)

# Proportional Allocation/Replacement

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- However, there might be cases where this is not true
  - e.g., a process allocates a 1 GB array but then only uses a small portion of it
- In other words, the working set of a process may not be correlated with its memory footprint

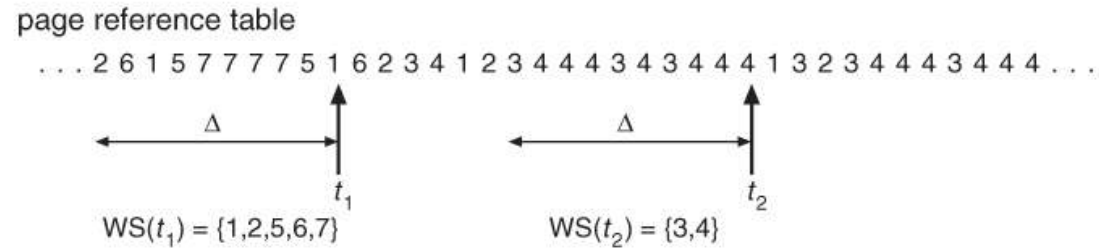
# Matching the Working Set

- **Goal** → Give each process enough frames to contain its working set
  - Informally, the working set is the set of pages the process is using "right now"
  - More formally, it is the set of all pages that the process has referenced during the past  $T$  units of time (e.g., seconds)

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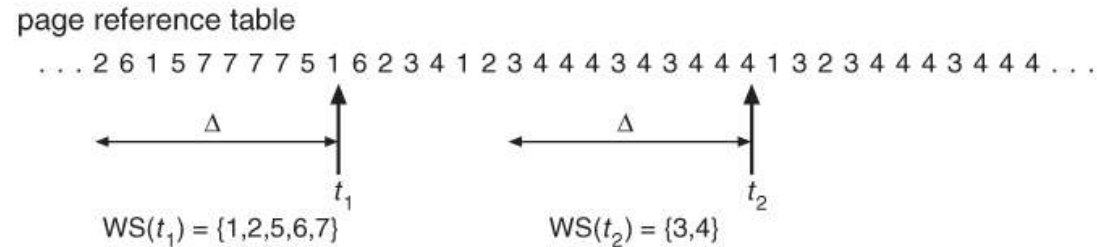
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  - Informally, the working set is the set of pages the process is using "right now"
  - More formally, it is the set of all pages that the process has referenced during the past  $T$  units of time (e.g., seconds)
- How does the OS pick  $T$ ?
  - 1 page fault takes order of 10 msecs to be served
  - 10 msecs  $\sim$  10 million instructions
  - $T$  needs to account for a lot more than 10 million instructions

# Determining the Working Set



The selection of  $\Delta$  is critical to the success of the working set model

# Determining the Working Set

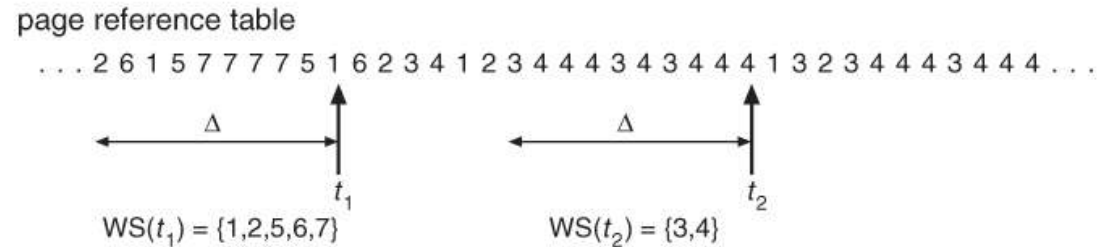


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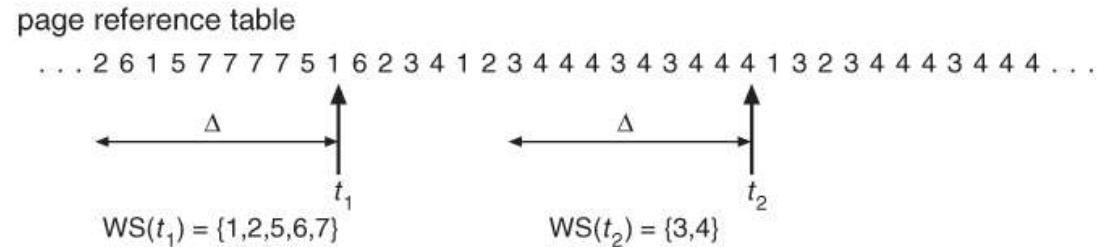
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Exact tracking is expensive: update the working set at each memory access

# Approximating the Working Set

- Computing the working set exactly requires to keep track of a moving window of size  $\Delta$



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# Approximating the Working Set

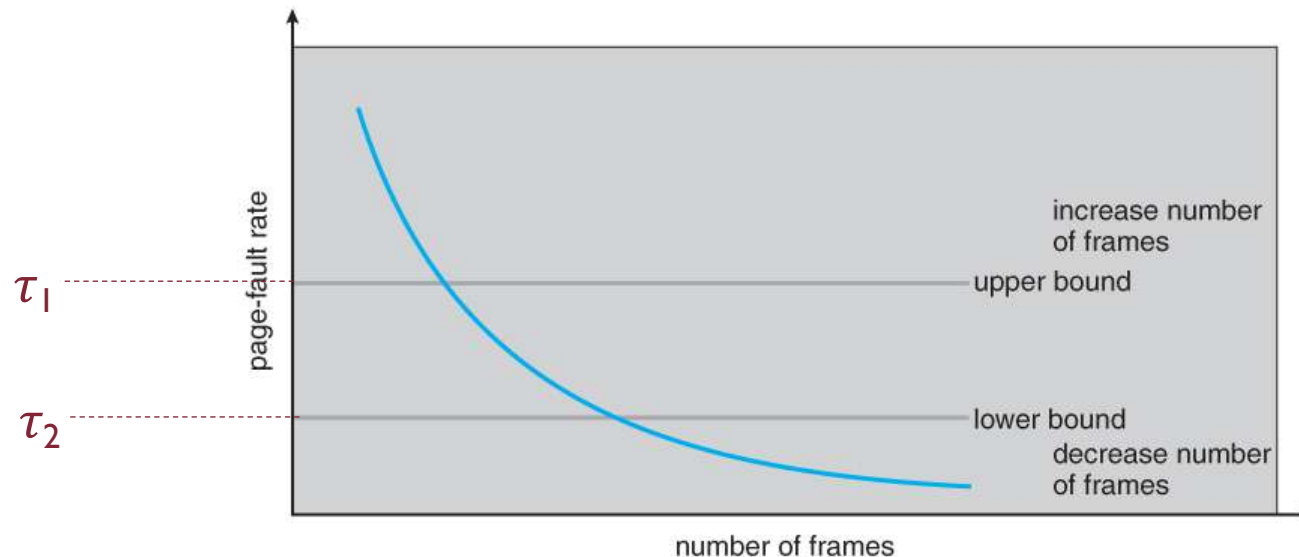
- Computing the working set exactly requires to keep track of a moving window of size  $\Delta$
- At each memory reference a new reference appears at one end and the oldest reference drops off the other end
- To avoid the overhead of keeping a list of the last  $\Delta$  referenced pages, the working set is often implemented with **sampling**
- Every  $k$  memory references (e.g.,  $k = 1,000$ ), consider the working set to be all pages referenced within *that* period of time

# Tracking Page Fault Rate

- Ultimately, our goal is to minimize the **page fault rate**

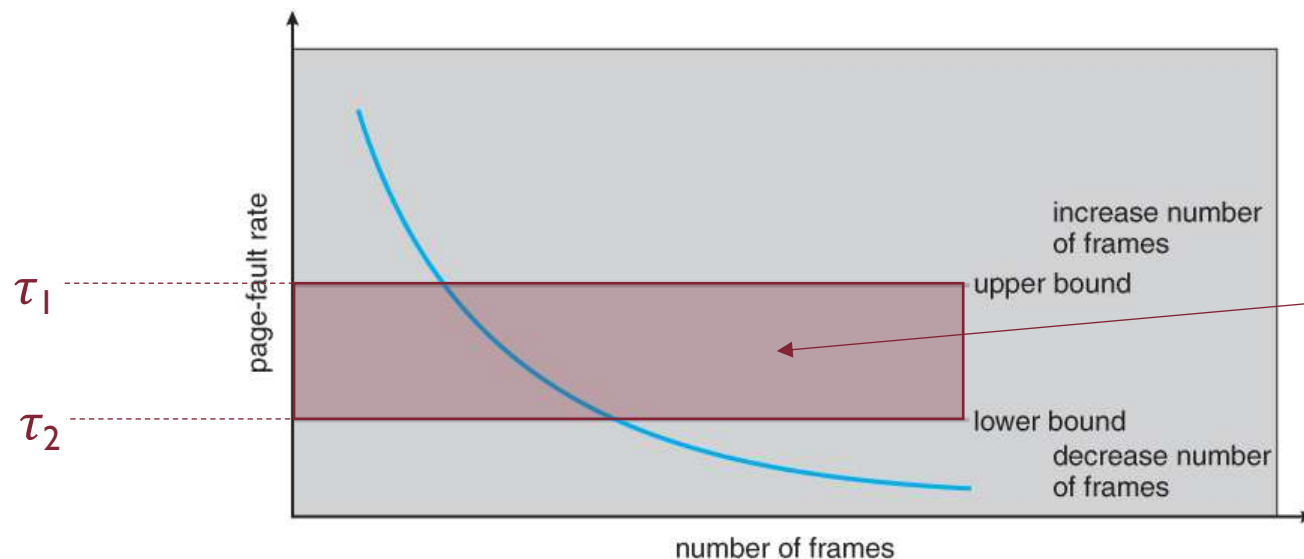
# Tracking Page Fault Rate

- Ultimately, our goal is to minimize the **page fault rate**
- A more direct approach is to track page fault rate of each process:
  - If the page fault rate is above a given threshold  $\tau_1 \rightarrow$  give it more frames
  - If the page fault rate is below a given threshold  $\tau_2 \rightarrow$  reclaim some frames



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Dynamically adjust allocated frames so as to keep processes in this area

# Kernel Memory

- So far, we only considered memory allocation for user processes
- But kernel needs memory to store things too: code and data structures like PCB, page tables, etc.
- Kernel does not use any of the advanced mechanisms seen so far
  - No paging → what if a page fault occurs for the kernel?

# Kernel Memory: Buddy Allocator

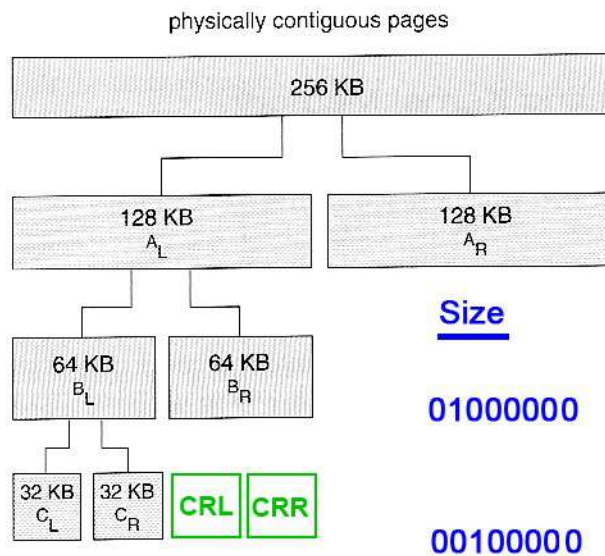


Figure 9.27 Buddy system allocation.

## Buddy Addresses

00000000

00000000 10000000

## Size

01000000

00000000 01000000

00100000

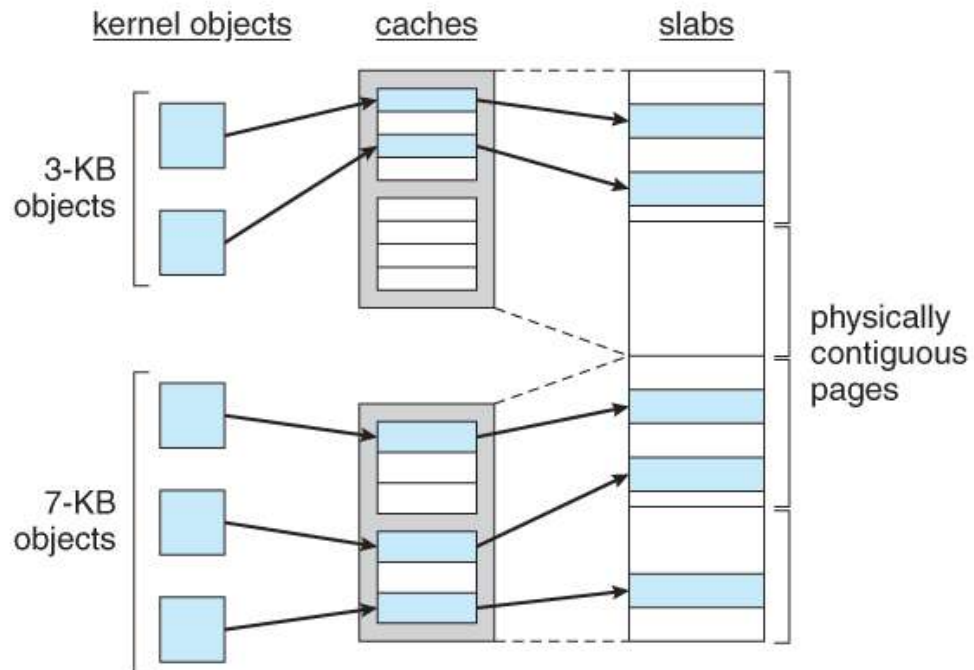
00000000 00100000

01000000 01100000

- Allocates memory using a power of 2 allocator (e.g., 4KiB, 8KiB, 16KiB), rounding up to the next nearest power of two if necessary
- If a block of the correct size is not available, then one is formed by (repeatedly) splitting the next larger block in two
- Can lead to internal fragmentation



# Kernel Memory: Slab Allocator



- Group of objects of the same size in a **slab**
- Object cache points to one or more slabs
- Separate cache for each kernel data structure (e.g., PCB)
- No internal fragmentation
- Used in Solaris and Linux

# Page Sizes

- Reasons for **small** pages?

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- Reasons for **small** pages?
  - Decreasing internal fragmentation
  - Higher degree of multiprogramming
- Reasons for **large** pages?
  - Smaller page table size (i.e., smaller number of page table entries)
  - Fewer page faults (locality reference)
  - Amortizes disk overhead (reading a 1 KiB page from disk takes approximately the same as reading an 8KiB one)

# Summary of Page Replacement

- The choice of page replacement algorithm is crucial when physical memory is limited
  - All algorithms approach to the optimum as the physical memory allocated to a process approaches to the virtual memory size

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# Summary of Page Replacement

- The choice of page replacement algorithm is crucial when physical memory is limited
  - All algorithms approach to the optimum as the physical memory allocated to a process approaches to the virtual memory size
- The more processes running concurrently, the less physical memory each one can have
- The OS must choose how many processes (and the number of frames per process) can share memory simultaneously