



**MathsNET**

A joined up approach to  
teaching and learning  
mathematics

# The limiting stationary distribution

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- Complete the following sentence: If all the elements in the transition matrix of a Markov chain are positive all the states in the chain are ...”
- Explain the meaning of the terms eigenvalue and eigenvector and write an equation involving a square matrix, one of its eigenvectors and one of its eigenvalues.
- How is the inverse of a matrix defined? Can a non-square matrix have an inverse?
- Explain what we mean when we state that a matrix is diagonalisable? Can non-square matrices be diagonalized?



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- Explain how you would go about calculating the fourth power of a diagonal matrix?
- The Perron-Frobenius theorem tells us that square matrices with all positive real elements always have a unique largest eigenvalue,  $\lambda_1$ . What does this tell us about the following limit  $\lim_{n \rightarrow \infty} \left( \frac{\lambda_2}{\lambda_1} \right)^n$  if  $\lambda_2$  is the second largest eigenvalue?
- Consider a  $2 \times 2$  diagonal matrix,  $\mathbf{A}$ , that has  $A(1,1) = \lambda$  and calculate the following product of three matrices  $\mathbf{VAB}$ . Use  $v_{11}$ ,  $v_{12}$ ,  $v_{21}$  and  $v_{22}$  for the elements of  $\mathbf{V}$  and similar names for the elements of  $\mathbf{B}$ . What do you notice about the rows of the final product matrix. Explain the significance of this result in the context of Markov chains.