Introduction to Bayes theorem

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- For the first problem in the video - the one about the female engineers - is the quantity you are being asked to calculate a conditional or an absolute probability?

- For the female engineers problem you are given two conditional probabilities and one absolute probability. What are these the probabilities of?

- For the question about the medical test there are two random variables: there is a Bernoulli random variable that tells you whether you have the disease and a Bernoulli random variable that tells you whether or not the test result was positive. Are these random variables independent of each other? Explain your reasoning.

- Give a statement of Bayes theorem.

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- Suppose you now administered two tests to the patient to determine whether or not they have the disease. Test A comes up positive in $P(T_1 = 1|D = 1)$ percent of diseased patients and positive in $P(T_1 = 1|D = 0)$ percent of healthy patients. Test B by contrast comes up positive in $P(T_2 = 1|D = 1)$ percent of diseased patients and positive in $P(T_2 = 1|D = 0)$ percent of healthy patients. Draw a Venn diagram showing the various possible categories each individual could be in. If we can calculate the probability that a person has the disease given they had positive results for the two tests using $P(D = 1|T_1 = 1 \wedge T_2 = 1) = \frac{P(T_1=1|D=1)P(T_2=1|D=1)P(D=1)}{P(T_1=1|D=1)P(T_2=1|D=1)P(D=1)+P(T_1=0|D=1)P(T_2=1|D=1)P(D=1)+P(T_1=1|D=1)P(T_2=0|D=1)P}$ what further assumption have we made? How would we use Bayes theorem to calculate this conditional probability with this assumption relaxed?

- Consider a pair of discrete random variables, X and Y, that can both take values between 0 and n. Use Bayes theorem to derive an expression for the conditional probability $P(X = a|Y = y)$ in terms of the set of absolute probabilities, $P(X = x_j)$, for getting each possible value for X and the set of conditional probabilities $P(Y = y|X = x_j)$ for getting $Y = y$ given that $X = x_j$. In doing this you will need to use summation notation.