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# The generalised partition function

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- How is the average energy,  $\langle E \rangle$ , calculated from the probabilities of being in the various microstates  $P_i$  and the energies of the various microstates  $E_i$
- Explain what two types of constraints are introduced on extensive thermodynamic variables when constructing thermodynamic states from the various microstates in phase space.
- Complete the following sentence: We can determine the probability of being in any given microstate by...
- Write down the function for which we are finding the constrained minimum and the two constraints on this function



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- Write down the extended function whose unconstrained optimum is found in order to find the required constrained optimum.
  
  
  
  
  
  
  
  
  
  
- Write down the partial derivative of the function you have just written down with respect to  $P_j$ .
  
  
  
  
  
  
  
  
  
  
- Explain why the derivative of  $f = \sum_i P_i$  with respect to  $P_j$  is equal to one.
  
  
  
  
  
  
  
  
  
  
- Give an expression for the derivative of  $g = \sum_i \lambda B_i P_i$  with respect to  $P_j$ .



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- Give an expression for the derivative of  $h = \sum_i P_i \ln P_i$  with respect to  $P_j$ .
- Explain (by making reference to the results that you have written down to the previous questions) why the probability of being in a microstate is given by:  $P_j = \frac{e^{-\sum_k \lambda_k B_j^{(k)}}}{e^\Psi}$
- Give an expression for the generalised partition function and explain how this is derived.