

# An introduction to Monte Carlo integration

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## 0.1 Level 1

This exercise should be revision: Use the block below to generate 20 pairs of uniform random variables  $X_i$  and  $Y_i$  that each lie between 0 and 1. Then plot each of these pairs of random variables at  $(X_i, Y_i)$ . All the points you show should have  $0 < X_i < 1$  and  $0 < Y_i < 1$ .

## 0.2 Level 2

Now use the blocks below to generate 25 points on the graph shown on the right. These points should be at  $(0.1 + i * 0.2, 0.1 + j * 0.2)$  with  $i \in \{0, 1, 2, 3, 4\}$  and  $j \in \{0, 1, 2, 3, 4\}$ . [Click here](#) if you want to watch the explanatory video.

## 0.3 Level 3

Use the blocks below to generate 25 points on the graph shown on the right. These points should again be at  $(0.1 + i * 0.2, 0.1 + j * 0.2)$  with  $i \in \{0, 1, 2, 3, 4\}$  and  $j \in \{0, 1, 2, 3, 4\}$ . Now though I only want you to display the points if they lie within a circle of radius one that is centered on the origin. [Click here](#) if you want to watch the explanatory video.

## 0.4 Level 4

Let's now reintroduce the random variables. Select a random point  $(X, Y)$  by generating two uniform random variables between 0 and 1,  $X$  and  $Y$ , and determine whether the point you selected is within a circle of radius one that is centred on the origin. Set the bernoulli random variable  $Z$  equal to one if  $(X, Y)$  is within the circle and zero otherwise. Use the blocks below to generate a sample of  $Z$  values and draw a graph showing how the sample mean,  $\mu_n = \frac{1}{n} \sum_{i=1}^n Z_i$  for the random variable  $Z$  changes as the number of independent random variables in the sample (the number of  $Z_i$  values) increases. In other words, plot a graph with points at  $(n, \mu_n)$  for  $n$  values between 0 and 20. [Click here](#) if you want to watch the explanatory video.

## 0.5 Level 5

Compute a set of 10 samples of the integral that you computed during the last exercise. Calculate each estimate of the sample mean by generating 5  $Z$  values. Use the plotting to display the mean of your 10 samples of the integral together with an error bar indicating the 90% confidence interval. [Click here](#) if you want to watch the explanatory video.

## 0.6 Level 6

Compute the integral that you computed during the last exercise by sampling only once. Calculate the 80% confidence interval. [Click here](#) if you want to watch the explanatory video.