

# Estimating probability mass functions and probability density functions



## 0.1 Level 1

Use the blocks below to generate 20 random variables each of which represents the outcome from rolling two dice. You should plot each of these random variables on the graph. The  $i$ th random variable you generate,  $X_i$ , should be at the coordinate  $(i, X_i)$ .

## 0.2 Level 2

Use the blocks below to generate 20 random variables from the distribution described in the previous exercise. Each of these random variables will take a value of either 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 or 12. Now instead of plotting each of the random variables on the graph I would like you to use the blocks below to count the number of times each of the random variables come up and to plot how these counts of the number times a particular number comes up changes with time.

## 0.3 Level 3

In the last exercise you should have plotted the number of times each of the random variables came up. What I would now like you to do is to plot the fraction of times each of the random variables came up in the experiment. If you do this correctly the sum of all the fractions will be equal to one.

## 0.4 Level 4

We are now going to repeat the exercise of calculating a histogram that we just performed but we are going to do the exercise for a normal random variable. Divide the range between -4 and +4 into 20 intervals of length 0.4 and create a list with one scalar for each of the sub-ranges in this range. Now use the blocks below to generate 20 random variables from a normal distribution with mean 0 and variance 1. Use the list that you created to count the number of times the random variable falls into each of the sub-ranges. Once you have generated all the random variables plot a bar graph showing the fraction of times that the random variable fell into each of the intervals of interest on the  $y$  axis and the value at the center of the sub-range on the  $x$ .

## 0.5 Level 5

Lets add one final complication to add to the process and learn to deal with weighted data. Repeat the previous exercise, however, now generate two normal random variables,  $X_i$  and  $W_i$ , between 0 and 1 during each of your 20 iterations. The first of these variables,  $X_i$ , should be used as you used the normal random variable in the previous exercise. The second random variable,  $W_i$ , however, should be used as a weight for this particular variable. You should thus compute the elements of a histogram,  $\{h_j\}$ , using 
$$h_j = \frac{1}{\sum_{i=1}^N W_i} \sum_{i=1}^N W_i H(X_i - a_j) [1 - H(X_i - a_{j+1})].$$
 In this expression  $N$  is the number of random variables you generate,  $a_j$  is the lower bound for the  $j$ th bin of the histogram and  $H(x)$  is a step function that equals one when  $x \geq 0$  and that is zero otherwise.