

# Ehrenfest urns and stationary Markov chains

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## 0.1 Level 1

Let's start by modelling a simplified version of the problem. We will suppose that there are initially 3 balls in container A and 3 balls in container B. After completing the dice roll there will thus be either 2 balls in container A or 4 balls in container A. Use the blocks below to generate a random variable,  $X_i$ , that tells us how many balls there are in container A after the dice is rolled and we do the appropriate moving of balls about. Suppose for this initial exercise that the balls are reset so that there are 3 balls in container A once more when the dice is rolled again. Generate 10 instances of your random variable and plot the values of each random variable on the graph at the point  $(i, X_i)$ , where the  $i$ s are the integers between 1 and 10.

## 0.2 Level 2

Let's continue modelling simplified versions of the problem. This time we will suppose that there are initially 2 balls in container A and 4 balls in container B. After completing the dice roll there will thus be either 1 ball in container A or 3 balls in container A. Use the blocks below to generate a random variable,  $X_i$ , that tells us how many balls there are in container A after the dice is rolled and we do the appropriate moving of balls about. Suppose for this initial exercise that the balls are reset so that there are 2 balls in container A once more when the dice is rolled again. Generate 10 instances of your random variable and plot the values of each random variable on the graph at the point  $(i, X_i)$ , where the  $i$ s are the integers between 1 and 10.

## 0.3 Level 3

Let's now try to bring everything we have learnt together and to model the actual Markov chain. Let's once again start with 2 balls in container A and 4 balls in container B. Now though the system will not be reset after the dice has been rolled. Instead the Markov chain will continue from a situation in which there is either 1 ball or 3 balls in container A. Use the blocks below to model 200 steps of this Markov chain and draw a graph showing how the number of balls in container A with the number of rolls of the dice.