High-dimensional local optimization in variational inference

Otte Heinävaara

Antti Honkela
Probabilistic Inference and Computational Biology (PROBIC)
Statistical Machine Learning and Bioinformatics
HIIT

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Variational inference

Bayes' rule	–	$P(Z\|E) = \frac{P(Z)}{P(E)} P(E\|Z)$
$\Downarrow$		
Bayesian inference	–	$P(Z\|E) = \frac{P(Z)}{\sum_i P(E\|Z_i) P(Z_i)} P(E\|Z)$
$\Downarrow$		
Variational inference	–	$P(Z\|E) \approx Q(Z)$

Topic models

How does a machine discover a topic?

This study is about the science of food. Do you like pizza or pasta? Is there way to determine how tasty food is without tasting or smelling it?

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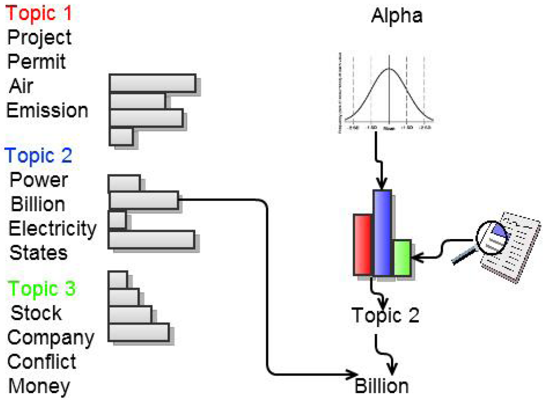
Words related to **food** **science**

LDA - Latent Dirichlet Allocation

D. M. Blei, A. Y. Ng, and M. I. Jordan. Latent Dirichlet allocation. *The Journal of Machine Learning Research*, 3:993-1022, 2003.

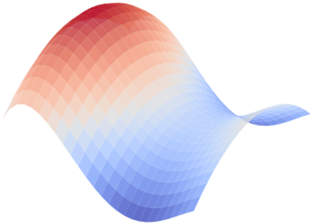


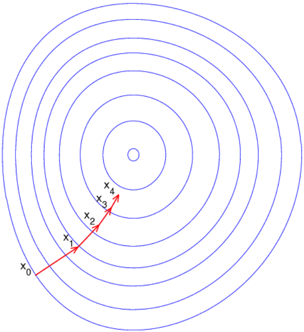
Picture: Amogh Mahapatra, Nisheeth Srivastava and Jaideep Srivastava. Contextual Anomaly Detection in Text Data, *Algorithms 2012*, pages 469-489, figure 3, 2012.

Underlying math (essentially)

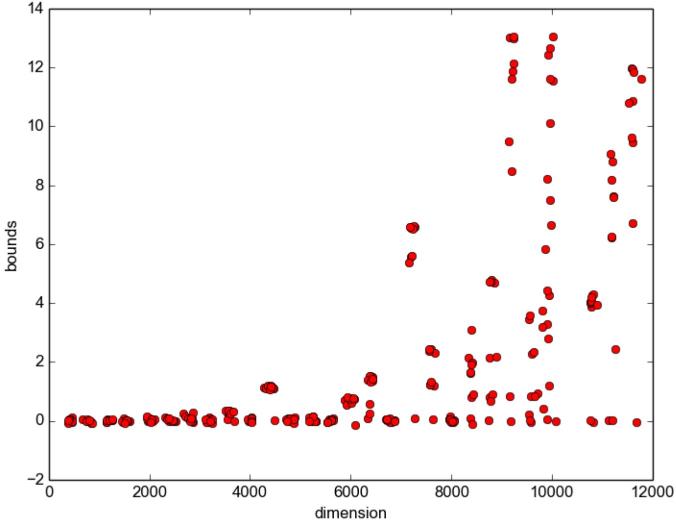
1. Use LDA to interpret the topic assignment as a problem in Bayesian inference
2. Variational inference:
   - Using suitable family of approximations and metric, the problem is turned to optimization in a metric space
   - Doing suitable estimations and assumptions, problem is moved to R^n and the target function (i.e. the metric) becomes tractable to compute

High-dimensional optimization

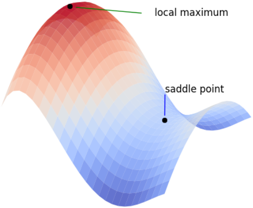




Effect of dimension



Where do we end up?

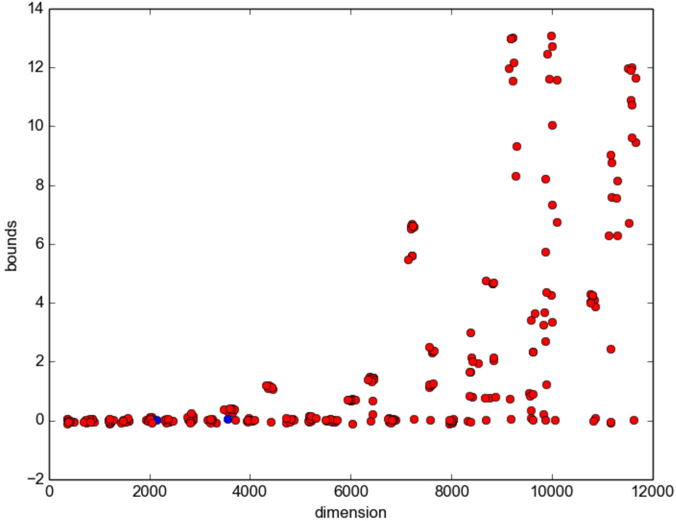


Curse of dimensionality

"...in high dimensions, the chance that all the directions around a critical point lead upward (positive curvature) is exponentially small..." (Dauphin et al. 2014[1])

[1] Yann N Dauphin, Razvan Pascanu, Caglar Gulcehre, Kyunghyun Cho, Surya Ganguli, and Yoshua Bengio. Identifying and attacking the saddle point problem in high-dimensional non-convex optimization. *In Advances in Neural Information Processing Systems*, pages 2933-2941, 2014.

Effect of dimension, eigenvalues



What next?

- More info about the eigenvalues/local behaviour
- Improving methods

The main things

To sum up:

- Turning topic assignment to high-dimensional optimization via

    1. building a model
    2. using variational ideas to deal with Bayesian inference

- In high-dimensional spaces, many simple ideas lose their edges