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Tiivistelmä — Beferat — Abstract			

This thesis is about Loewner's theory of matrix monotone functions. Matrix monotone functions of order n are real functions which, when lifted via functional calculus, preserve Loewner order on $n \times n$ Hermitian matrices. In his seminal 1934 paper Loewner proved two famous results on matrix monotone functions. The first characterizes matrix monotone functions of order n by positivity of certain matrices. The second characterizes functions that are matrix monotone of all orders as certain functions extending analytically to upper half-plane. The goal of this thesis is to reinterpret and give new proofs for these results.

Odd numbered chapters 1,3,5 and 7 follow the route towards the aforementioned goals. Chapters 1 and 3 discuss various basic facts and heuristics on matrix monotone functions. Chapters 5 and 7 contain the proofs of the main results of this thesis.

Even numbered chapters 2, 4 and 6 offer supplementary information. Chapter 2 discusses fundamentals of Loewner order and functional calculus. Chapter 4 introduces divided differences and their key properties. Chapter 6 explores the basic properties of Pick–Nevanlinna functions.

Avainsanat — Nyckelord — Keywords

Matrix monotone functions, Loewner's theorem

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