

Contents

1	Basic	1
1.1	.vimrc	1
1.2	Default Bear	1
1.3	IO Optimize	1
1.4	PBDS	1
1.5	Set Comperator	1
1.6	Random	1
1.7	Python	1
2	Graph	1
2.1	2 SAT	1
2.2	Bellman Ford	1
2.3	Biconnected Component	1
2.4	Bridge Connected Component	2
2.5	Bridge	2
2.6	C3C4	2
2.7	Centroid Decomposition	2
2.8	Close Vertices	3
2.9	Disjoint Set	3
2.10	Heavy Light Decomposition	4
2.11	KSP	4
2.12	LCA	5
2.13	Maximum Clique	5
2.14	SCC Kosaraju	5
2.15	SCC Tarjan	5
2.16	Tree Centroid	6
2.17	Virtual Tree	6
3	Data Structure	6
3.1	2D BIT	6
3.2	2D Segment Tree	6
3.3	BIT	7
3.4	chtholly tree	7
3.5	LiChaoST	7
3.6	persistent	7
3.7	Sparse Table	8
3.8	Treap	8
3.9	ZKW Segment Tree	8
4	Flow	8
4.1	Bipartite Matching	8
4.2	Bounded Flow	9
4.3	Dinic	9
4.4	KM	9
4.5	Maximum Simple Graph Matching	10
4.6	MCMF	10
4.7	Mimum Vertex Cover	10
4.8	Theorem	11
5	Geometry	11
5.1	Basic 2D	11
5.2	Convex Hull	12
5.3	Dynamic Convex Hull	12
5.4	Segmentation Intersection	12
5.5	Theorem	13
6	Math	13
6.1	Big Int	13
6.2	Chinese Remainder	14
6.3	Extgcd	15
6.4	FFT	15
6.5	Gauss Elimination	15
6.6	Gauss Elimination2	15
6.7	Karatsuba	15
6.8	Linear Sieve	15
6.9	Matrix	16
6.10	Miller Rabin	16
6.11	Mobius	16
6.12	NTT	16
6.13	Pollard Rho	16
6.14	Primes	17
6.15	Primitive Root	17
7	String	17
7.1	AC	17
7.2	Hash	17
7.3	KMP	17
7.4	Manacher	17
7.5	SA	18
7.6	SA2	18
7.7	SAIS	18
7.8	Suffix Automaton	19
7.9	Trie	19
7.10	Z	19
8	Others	19
8.1	Aliens	19
8.2	Knapsack on Tree	20
8.3	Mo	20
8.4	Mono Slope	20
8.5	Partial Ordering	21
8.6	Xor Basis	21

1 Basic

1.1 .vimrc

```
"This file should be placed at ~/.vimrc"
se nu ai hls et ru ic is sc cul
se re=1 ts=4 sts=4 sw=4 ls=2 mouse=a
syntax on
hi cursorline cterm=none ctermbg=89
set bg=dark
inoremap {<CR> {<CR>}<Esc>ko<tab>
```

1.2 Default Bear

```
#include <bits/stdc++.h>
using namespace std;
typedef long long ll;
#define int ll
typedef pair<int, int> pii;
#define X first
#define Y second
#define pb push_back
#define All(a) a.begin(), a.end()
#define SZ(a) ((int)a.size())
#define endl '\n'

signed main() {
    ios_base::sync_with_stdio(0);
    cin.tie(0);
}
```

1.3 IO Optimize

```
bool rit(auto& x) {
    x = 0; char c = cin.rdbuf()->sgetc(); bool neg = 0;
    while (!isdigit(c)) {
        if (c == EOF) return 0;
        if (c == '-') neg = 1;
        c = cin.rdbuf()->sgetc();
    }
    while (isdigit(c))
        x = x * 10 + c - '0', c = cin.rdbuf()->sgetc();
    return x = neg ? -x : x, 1;
}

void wit(auto x) {
    if (x < 0) cout.rdbuf()->sputc('-'), x = -x;
    char s[20], len = 0;
    do s[len++] = x % 10 + '0'; while (x /= 10);
    while (len) cout.rdbuf()->sputc(s[--len]);
}
```

1.4 PBDS

```
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
// #include <bits/extc++.h>
#include <bits/stdc++.h>
using namespace __gnu_pbds;
using namespace std;
template <typename T>
using rbtree = tree<T, null_type, less<T>
    >, rb_tree_tag, tree_order_statistics_node_update>;
// less<T> : increasing, greater<T> : decreasing
// rb_tree_tag, splay_tree_tag, ov_tree_tag

int main() {
    int x;
    rbtree<int> t, rhs, rhs2;
    t.insert(x);
    t.erase(x); // return 1 or 0
    cout << t.order_of_key(x) << '\n'; // rank
    cout << *t.find_by_order(x) << '\n'; // x-th
    cout << *t.lower_bound(x) << '\n'; // iterator >= x
    cout << *t.upper_bound(x) << '\n'; // iterator > x
    t.join(rhs);
    // merge // same type, no duplicate elements
    t.split(x, rhs2);
    // tree : elements <= x, rhs : elements > x
}
```

1.5 Set Comperator

```
auto cmp = [](int a, int b) {
    return a > b;
};
set<int, decltype(cmp)> s = {1, 2, 3, 4, 5};
cout << *s.begin() << '\n';
```

1.6 Random

```
#include <random>
#include <chrono>
#include <algorithm>
mt19937 rng(chrono
    ::system_clock::now().time_since_epoch().count());
int randint(int lb, int ub) {
    return uniform_int_distribution<int>(lb, ub)(rng);
}
// shuffle(v.begin(), v.end(), rng);
```

1.7 Python

```
from decimal import *
setcontext(Context(prec
    =MAX_PREC, Emax=MAX_EMAX, rounding=ROUND_FLOOR))
print(Decimal(input()) * Decimal(input()))
from fractions import Fraction
Fraction
    ('3.14159').limit_denominator(10).numerator # 22
```

1.8 Stress Test

```
#!/bin/bash
while true; do
    ./random > input
    ./a.out < input > out1
    ./brute < input > out2
    if diff -w out1 out2 > /dev/null; then
        echo "OK"
    else
        echo "WA:"
        cat input
        break
    fi
done
```

2 Graph

2.1 2SAT

```
struct TwoSAT {
    int n;
    Scc scc;
    void init(int _n) {
        // (0,1),(2,3),...
        n = _n; scc.init(n * 2);
    }
    void add_disjunction(int a, int na, int b, int nb) {
        a = 2 * a ^ na, b = 2 * b ^ nb;
        scc.addEdge(a ^ 1, b);
        scc.addEdge(b ^ 1, a);
    }
    vector<int> solve() {
        scc.solve();
        vector<int> assignment(n, 0);
        for (int i = 0; i < n; i++) {
            if (scc
                .bln[2 * i] == scc.bln[2 * i ^ 1]) return {};
            assignment
                [i] = scc.bln[2 * i] > scc.bln[2 * i ^ 1];
        }
        return assignment;
    }
};
```

2.2 Bellman Ford

```
struct edge{
    int u, v;
    int cost;
};
vector<int> d(n, inf);
bool bellman_ford(vector<edge> &ee, int n, int s){
    d[s] = 0;
    auto relax = [&](edge e){
        if(d[e.v] > d[e.u] + e.cost){
            d[e.v] = d[e.u] + e.cost;
            return 1;
        }
        return 0;
    }
    for(int t = 1; t <= n; ++t){
        bool update = 0;
        for(auto &e: ee)
            update |= relax(e);
        if(t == n && update) return 0;
    }
    return 1;
}
```

2.3 Biconnected Component

```
// beware of multiple inputs
#define ep emplace
#define eb emplace_back
const int N = 2e5 + 5;

int d[N], low[N];
vector<int> g[N];
vector<vector<int>> bcc;
stack<int> st;

void dfs(int x, int p) {
    d[x] = ~p ? d[p] + 1 : 1, low[x] = d[x];
    st.ep(x);
    for (const auto& i : g[x]) {
        if (i == p) continue;
        if (!d[i]) {
            dfs(i, x);
            low[x] = min(low[x], low[i]);
            if (d[x] <= low[i]) {
                int tmp;
                bcc.eb();
                do tmp = st.top(), st.pop()
                    (), bcc.back().eb(tmp); while (tmp != x);
                st.ep(x);
            }
        }
        low[x] = min(low[x], d[i]);
    }
}
```

2.4 Bridge Connected Component

```
#define ep emplace
constexpr int N = 2e5 + 1;

int d[N], low[N], bcc[N], nbcc;
vector<int> g[N];
stack<int> st;

void dfs(int x, int p) {
    d[x] = ~p ? d[p] + 1 : 1, low[x] = d[x];
    st.ep(x);
    for (const auto& i : g[x]) {
        if (i == p) continue;
        if (!d[i]) {
            dfs(i, x);
            low[x] = min(low[x], low[i]);
        }
        low[x] = min(low[x], d[i]);
    }
    if (low[x] == d[x]) {
        nbcc++;
        int tmp;
        do tmp = st.top()
            , st.pop(), bcc[tmp] = nbcc; while (tmp != x);
    }
}
```

2.5 Bridge

```
#define eb emplace_back
using pii = pair<int, int>;
const int N = 2e5 + 5;

int d[N], low[N];
vector<int> g[N];
vector<int> ap; // articulation point
vector<pii> bridge;

void dfs(int x, int p) {
    d[x] = ~p ? d[p] + 1 : 1, low[x] = d[x];
    int cnt = 0;
    bool isap = 0;
    for (const auto& i : g[x]) {
        if (i == p) continue;
        if (!d[i]) {
            dfs(i, x), cnt++;
            if (d[x] <= low[i]) isap = 1;
            if (d[x] < low[i]) bridge.eb(x, i);
            low[x] = min(low[x], low[i]);
        }
        low[x] = min(low[x], d[i]);
    }
    if (p == -1 && cnt < 2) isap = 0;
    if (isap) ap.eb(x);
}
```

2.6 C3C4

```
#include <bits/stdc++.h>
using namespace std;

signed main() {
    cin.tie(0) -> sync_with_stdio(0);

    int N, M;
    cin >> N >> M;

    vector<int> deg(N);
    vector<array<int, 2>> e(M);
    for (auto &[u, v] : e) {
        cin >> u >> v;
        --u, --v;
        ++deg[u], ++deg[v];
    }

    vector<int> ord(N), rk(N);
    iota(all(ord), 0);
    sort(all(ord), [&](int x, int y) { return deg[x] > deg[y]; });
    for (int i = 0; i < N; i++) rk[ord[i]] = i;

    vector<vector<int>> D(N), adj(N);
    for (auto [u, v] : e) {
        if (rk[u] > rk[v]) swap(u, v);
        D[u].emplace_back(v);
        adj[u].emplace_back(v);
        adj[v].emplace_back(u);
    }

    vector<int> vis(N);

    int64_t c3 = 0, c4 = 0;
    // ord = sort by deg decreasing, rk[ord[i]] = i
    // D[i] = edge point from rk small to rk big
    for (int x : ord) { // c3
        for (int y : D[x]) vis[y] = 1;
        for (int y : D[x]) for (int z : D[y]) c3 += vis[z];
        for (int y : D[x]) vis[y] = 0;
    }
    for (int x : ord) { // c4
        for (int y : D[x]) for (int z : adj[y])
            if (rk[z] > rk[x]) c4 += vis[z]++;
        for (int y : D[x]) for (int z : adj[y])
            if (rk[z] > rk[x]) --vis[z];
    } // both are O(M*sqrt(M)), test @ 2022 CCPC guangzhou

    cout << c4 * 8 << '\n';
}
```

2.7 Centroid Decomposition

```
const int MAXN = 1e5 + 5;
int n, q, vis[MAXN], sz[MAXN];
vector<int> adj[MAXN], pa[MAXN], mx[MAXN], dis[MAXN];

void dfs_sz(int x, int p) {
    sz[x] = 1;
    for (int i : adj[x]) {
        if (i == p or vis[i]) continue;
        dfs_sz(i, x);
        sz[x] += sz[i];
    }
}

int cen;
void dfs_cen(int x, int p, int all) {
    int tmp = all - sz[x];
    for (int i : adj[x]) {
        if (i == p or vis[i]) continue;
        dfs_cen(i, x, all);
        chmax(tmp, sz[i]);
    }
    if (tmp * 2 <= all) cen = x;
}

void dfs(int x, int p, int d) {
    pa[x].pb(cen);
    dis[x].pb(d);
    if (d >= mx[cen].size()) mx[cen].pb(x);
    else chmax(mx[cen][d], x);
    for (int i : adj[x]) {
        if (i == p or vis[i]) continue;
        dfs(i, x, d + 1);
    }
}
```

```
}

void deco(int x, int d) {
    dfs_sz(x, x);
    dfs_cen(x, x, sz[x]);
    vis[cen] = 1;
    dfs(cen, cen, 0);
    for (int i = 1; i < mx[cen].size(); i++) {
        chmax(mx[cen][i], mx[cen][i - 1]);
    }
    for (int i : adj[cen]) {
        if (vis[i]) continue;
        deco(i, d + 1);
    }
}

int get(int x, int k) {
    if (!mx[x].size() or k < 0) return 0;
    return k >= mx[x].size() ? mx[x].back() : mx[x][k];
}

int query(int x, int k) {
    int res = get(x, k);
    for (int i = 0; i < pa[x].size(); i++) {
        int p = pa[x][i];
        int d = dis[x][i];
        chmax(res, get(p, k - d));
    }
    return res;
}

signed main() {
    WOSHAOJI
    cin >> n >> q;
    for (int i = 1, u, v; i < n; i++) {
        cin >> u >> v;
        adj[u].pb(v);
        adj[v].pb(u);
    }
    deco(1, 0);
    while (q--) {
        int x, k; cin >> x >> k;
        cout << query(x, k) << '\n';
    }
}
```

2.8 Close Vertices

```
#include <iostream>
#include <vector>
#include <bitset>
#include <algorithm>
#include <cstring>
using namespace std;
int l, w;
vector<pair<int, short>> tree[100000];
bitset<100000> removed;
int current_centroid, BIT[100000];
// Return subtree size internally
// and place the discovered centroid in current_centroid

int find_centroid(
    const int n, const int u, const int p = -1) {
    if (n == 1) { current_centroid = u; return 0; }
    int subtree_sum = 0;
    for (const auto &[v, w] : tree[u]) if (v != p && !removed[v]) {
        subtree_sum += find_centroid(n, v, u);
        if (current_centroid > -1) return 0;
        if (subtree_sum >= n >> 1) { current_centroid = u; return 0; }
    }
    return subtree_sum + 1;
}

void DFS(const int u, const int p, const int length,
    const int weight, vector<pair<int, int>> &record) {
    record.emplace_back(weight, length);
    for (const auto &[v, w] : tree[u]) if (v != p && !removed[v])
        DFS(v, u, length + 1, weight + w, record);
}

bool greater_size(const vector<pair<int, int>> &v, const vector<pair<int, int>> &w) {
    return v.size() > w.size();
}

long long centroid_decomposition(const int n, int u) {
    long long ans = 0;
    // Step 1: find the centroid
    current_centroid = -1; find_centroid(n, u);
    removed[u = current_centroid] = true;
```

```
// Step 2: DFS from the centroid (again)
// and continue the centroid decomposition
vector<vector<pair<int, int>>> root2subtree_paths;
for (const auto &[v, w] : tree[u]) if (!removed[v]) {
    root2subtree_paths.emplace_back();
    DFS(v, u, 1, w, root2subtree_paths.back());
    // Sort mainly according to weight
    ranges::sort(root2subtree_paths.back());
    ans += centroid_decomposition
        (root2subtree_paths.back().size(), v);
}
for (const auto &v : root2subtree_paths)
    for (const auto &[weight, length] : v)
        if (length <= l && weight <= w) ++ans;
// Step 3: optimal merging
ranges::make_heap(root2subtree_paths, greater_size);
while (root2subtree_paths.size() > 1) {
    ranges::pop_heap(root2subtree_paths, greater_size);
    // Merge
    front() (with maybe larger size) and back()
    // Count cross-centroid paths
    memset(BIT, 0, root2subtree_paths
        .back().size() * sizeof(int));
    auto p = root2subtree_paths.front().cbegin();
    for (auto q = root2subtree_paths.back().cbegin();
        q != root2subtree_paths.back().cend(); ++q) {
        int L;
        while (p != root2subtree_paths.front().crend()
            && p->first + q->first > w) {
            L = min(L - p->second,
                static_cast<int>(
                    root2subtree_paths.back().size()));
            while
                (L > 0) { ans += BIT[L - 1]; L -= L & -L; }
            ++p;
        }
        L = q->second;
        while (L <= static_cast
            <int>(root2subtree_paths.back().size()))
            ++BIT[L - 1]; L += L & -L;
    }
    while (p != root2subtree_paths.front().crend()) {
        int L = min(L - p++->second, static_cast
            <int>(root2subtree_paths.back().size()));
        while (L > 0) { ans += BIT[L - 1]; L -= L & -L; }
    }
    // Actually merge the lists
    vector<pair<int, int>> buffer;
    buffer.reserve(root2subtree_paths.front
        ().size() + root2subtree_paths.back().size());
    ranges::merge
        (root2subtree_paths.front(), root2subtree_paths
            .back(), back_inserter(buffer));

    root2subtree_paths.pop_back();
    ranges::pop_heap(root2subtree_paths, greater_size);
    root2subtree_paths.back() = move(buffer);
    ranges
        ::push_heap(root2subtree_paths, greater_size);
}
return ans;
}
int main() {
    ios_base::sync_with_stdio(false);
    int n; cin >> n >> l >> w;
    for (int i = 1; i < n; ++i) {
        int p; short w; cin >> p >> w;
        tree[--p].emplace_back(i, w);
        tree[i].emplace_back(p, w);
    }
    cout << centroid_decomposition(n, 0) << endl;
}
```

2.9 Disjoint Set

```
#include <bits/stdc++.h>
using namespace std;

struct disjoint_set {
    static const int maxn = (int)5e5 + 5;
    int n, fa[maxn], sz[maxn];
    vector<pair<int*, int>> h;
    vector<int> sp;
    void init(int n) {
        n = _n;
```

```
for (int i = 0; i < n; ++i)
    fa[i] = i, sz[i] = 1;
sp.clear(); h.clear();
}
void assign(int *k, int v) {
    h.push_back({k, *k});
    *k = v;
}
void save() { sp.push_back((int)h.size()); }
void undo() {
    assert(!sp.empty());
    int last = sp.back(), cnt = 0; sp.pop_back();
    while (h.size() > last) {
        auto x = h.back(); h.pop_back();
        *x.first = x.second;
        cnt++;
    }
    n += cnt / 2;
}
int f(int x) {
    while (fa[x] != x) x = fa[x];
    return x;
}
bool merge(int x, int y) {
    x = f(x); y = f(y);
    if (x == y) return 0;
    if (sz[x] < sz[y]) swap(x, y);
    assign(&sz[x], sz[x] + sz[y]);
    assign(&fa[y], x);
    n--;
    return 1;
}
} djs;
```

2.10 Heavy Light Decomposition

```
#include <bits/stdc++.h>
using namespace std;
const int N = 2e5 + 5;
#define eb emplace_back
int t, n, q, seg[N << 1]; // t := time-stamp
int sz[N], fa[N], dep[N], to[N], fr[N], dfn[N], arr[N];
// size, father, depth
// to-heavy-child, from-head, dfs-order, a_i value
vector<int> g[N];
void upd(int x, int v) {
    for (seg[x += n] = v; x > 1; x >>= 1)
        seg[x >> 1] = max(seg[x], seg[x ^ 1]);
}
int qry(int l, int r) { // [l, r]
    int ret = -1e9; // -max
    for (l += n, r += n + 1; l < r; l >>= 1, r >>= 1) {
        if (l & 1) ret = max(ret, seg[l++]);
        if (r & 1) ret = max(ret, seg[--r]);
    }
    return ret;
}
void dfs(int x, int p) {
    sz[x] = 1, fa[x] = p, to[x] = -1, dep[x] = ~p ? dep[p] + 1 : 0;
    for (auto i : g[x])
        if (i != p) {
            dfs(i, x);
            if (to[x] == -1 || sz[i] > sz[to[x]]) to[x] = i;
            sz[x] += sz[i];
        }
}
void dfs2(int x, int f) {
    fr[x] = f, dfn[x] = ++t, upd(dfn[x], arr[x]);
    if (to[x] != -1) dfs2(to[x], f);
    for (auto i : g[x])
        if (i != fa[x] && i != to[x]) dfs2(i, i);
}
int qry2(int u, int v) { // query on tree
    int fu = fr[u], fv = fr[v], ret = -1e9;
    while (fu != fv) {
        if (dep[fu] < dep[fv]) swap(fu, fv), swap(u, v);
        ret = max(ret, qry(dfn
            [fu], dfn[u])); // interval: [dfn[fu], dfn[u]]
        u = fa[fu], fu = fr[u];
    }
    if (dep[u] > dep[v]) swap(u, v);
    // u is the LCA
    ret = max(ret, qry(dfn[u], dfn[v]));
    return ret;
}
int main() {
```

```

ios::sync_with_stdio(false), cin.tie(nullptr);
cin >> n >> q;
for (int i = 1; i <= n; i++) cin >> arr[i];
for (int i = 1, a, b; i < n; i++)
    cin >> a >> b, g[a].eb(b), g[b].eb(a);
dfs(1, -1), dfs2(1, 1);
while (q--) {
    int op; cin >> op;
    if (op == 1) {
        int x,
            v; cin >> x >> v, arr[x] = v, upd(dfn[x], v);
    }
    else {
        int a, b; cin >> a >> b;
        cout << qry2(a, b) << '\n';
    }
}
}

```

2.11 KSP

```

// from CRyptoGRapheR
// time: O(|E| \lg |E| + |V| \lg |V| + K)
// memory: O(|E| \lg |E| + |V|)
struct KSP { // 1-base
    struct nd {
        int u, v; ll d;
        nd(int ui = 0, int vi
            = 0, ll di = INF) { u = ui; v = vi; d = di; }
    };
    struct heap { nd* edge; int dep; heap* chd[4]; };
    static int cmp(heap
        * a, heap* b) { return a->edge->d > b->edge->d; }
    struct node {
        int v; ll d; heap* H; nd* E;
        node() {}
        node(ll
            _d, int _v, nd* _E) { d = _d; v = _v; E = _E; }
        node(heap* _H, ll _d) { H = _H; d = _d; }
        friend bool operator<(node a, node b)
        { return a.d > b.d; }
    };
    int n, k, s, t, dst[N]; nd *nxt[N];
    vector<nd*> g[N], rg[N]; heap *nullNd, *head[N];
    void init(int _n, int _k, int _s, int _t) {
        n = _n; k = _k; s = _s; t = _t;
        for (int i = 1; i <= n; i++) {
            g[i].clear(); rg[i].clear();
            nxt[i] = NULL; head[i] = NULL; dst[i] = -1;
        }
    }
    void addEdge(int ui, int vi, ll di) {
        nd* e = new nd(ui, vi, di);
        g[ui].push_back(e); rg[vi].push_back(e);
    }
    queue<int> dfsQ;
    void dijkstra() {
        while (dfsQ.size()) dfsQ.pop();
        priority_queue<node> Q; Q.push(node(0, t, NULL));
        while (!Q.empty()) {
            node p = Q
                .top(); Q.pop(); if (dst[p.v] != -1) continue;
            dst[p.v] = p.d; nxt[p.v] = p.E; dfsQ.push(p.v);
            for (auto e
                : rg[p.v]) Q.push(node(p.d + e->d, e->u, e));
        }
    }
    heap* merge(heap* curNd, heap* newNd) {
        if (curNd == nullNd) return newNd;
        heap* root
            = new heap; memcpy(root, curNd, sizeof(heap));
        if (newNd->edge->d < curNd->edge->d) {
            root->edge = newNd->edge;
            root->chd[2] = newNd->chd[2];
            root->chd[3] = newNd->chd[3];
            newNd->edge = curNd->edge;
            newNd->chd[2] = curNd->chd[2];
            newNd->chd[3] = curNd->chd[3];
        }
        if (root->chd[0]->dep < root->chd[1]->dep)
            root->chd[0] = merge(root->chd[0], newNd);
        else root->chd[1] = merge(root->chd[1], newNd);
        root->dep = max(root->chd[0]->dep,
            root->chd[1]->dep) + 1;
        return root;
    }
}
vector<heap*> V;

```

```

void build() {
    nullNd = new
        heap; nullNd->dep = 0; nullNd->edge = new nd;
    fill(nullNd->chd, nullNd->chd + 4, nullNd);
    while (not dfsQ.empty()) {
        int u = dfsQ.front(); dfsQ.pop();
        if (!nxt[u]) head[u] = nullNd;
        else head[u] = head[nxt[u]->v];
        V.clear();
        for (auto && e : g[u]) {
            int v = e->v;
            if (dst[v] == -1) continue;
            e->d += dst[v] - dst[u];
            if (nxt[u] != e) {
                heap* p = new
                    heap; fill(p->chd, p->chd + 4, nullNd);
                p->dep = 1; p->edge = e; V.push_back(p);
            }
        }
        if (V.empty()) continue;
        make_heap(V.begin(), V.end(), cmp);
#define L(X) ((X<<1)+1)
#define R(X) ((X<<1)+2)
        for (size_t i = 0; i < V.size(); i++) {
            if (L(i) < V.size()) V[i]->chd[2] = V[L(i)];
            else V[i]->chd[2] = nullNd;
            if (R(i) < V.size()) V[i]->chd[3] = V[R(i)];
            else V[i]->chd[3] = nullNd;
        }
        head[u] = merge(head[u], V.front());
    }
}
vector<ll> ans;
void first_K() {
    ans.clear(); priority_queue<node> Q;
    if (dst[s] == -1) return;
    ans.push_back(dst[s]);
    if (head[s] != nullNd)
        Q.push(node(head[s], dst[s] + head[s]->edge->d));
    for (int _ = 1; _ < k and not Q.empty(); _++) {
        node p = Q.top(); q; Q.pop(); ans.push_back(p.d);
        if (head[p.H->edge->v] != nullNd) {
            q.H = head
                [p.H->edge->v]; q.d = p.d + q.H->edge->d;
            Q.push(q);
        }
        for (int i = 0; i < 4; i++)
            if (p.H->chd[i] != nullNd) {
                q.H = p.H->chd[i];
                q.d = p
                    .d - p.H->edge->d + p.H->chd[i]->edge->d;
                Q.push(q);
            }
    }
}
void
    solve() { // ans[i] stores the i-th shortest path
        dijkstra(); build();
        first_K(); // ans.size() might less than k
    }
} solver;

```

2.12 LCA

```

#define eb emplace_back
const int N = 2e5 + 5, logN = __lg(N) + 1, inf = 1e9;
int n, q, logn;
int dep[N], fa[N][logN];
vector<int> g[N];

void dfs(int x, int p) {
    dep[x] = ~p ? dep[p] + 1 : 0;
    fa[x][0] = p;
    for (int i = 1; (1 << i) <= dep[x]; i++)
        fa[x][i] = fa[fa[x][i-1]][i-1];
    for (const auto& u : g[x])
        if (u != p) dfs(u, x);
}

int LCA(int u, int v) {
    if (dep[u] > dep[v]) swap(u, v);
    for (int i = 0; i < logn; i++)
        if ((dep[v] - dep[u]) >> i & 1) v = fa[v][i];
    if (u == v) return u;
    for (int i = logn - 1; i >= 0; i--)
        if (fa[u][i] != fa[v][i])
            u = fa[u][i], v = fa[v][i];
}

```

```

return fa[u][0];
}
// logn = __lg(n) + 1
// g[a].eb(b)
// dfs(root, -1)
// query -> LCA(u, v)
// distance
of (u, v) = dep[u] + dep[v] - 2 * dep[LCA(u, v)]

```

2.13 Maximum Clique

```

struct Maximum_Clique {
    typedef bitset<MAXN> bst;
    bst N[MAXN], empty;
    int p[MAXN], n, ans;
    void BronKerbosch2(bst R, bst P, bst X) {
        if (P == empty && X == empty)
            return ans = max(ans, (int)R.count()), void();
        bst tmp = P | X;
        int u;
        if ((R | P | X).count() <= ans) return;
        for (int uu = 0; uu < n; ++uu) {
            u = p[uu];
            if (tmp[u] == 1) break;
        }
        // if (double(clock())/CLOCKS_PER_SEC > .999)
        // return;
        bst now2 = P & ~N[u];
        for (int vv = 0; vv < n; ++vv) {
            int v = p[vv];
            if (now2[v] == 1) {
                R[v] = 1;
                BronKerbosch2(R, P & N[v], X & N[v]);
                R[v] = 0, P[v] = 0, X[v] = 1;
            }
        }
    }
    void init(int _n) {
        n = _n;
        for (int i = 0; i < n; ++i) N[i].reset();
    }
    void add_edge(int u, int v) {
        N[u][v] = N[v][u] = 1;
    }
    int solve() { // remember srand
        bst R, P, X;
        ans = 0, P.flip();
        for (int i = 0; i < n; ++i) p[i] = i;
        random_shuffle(p, p + n), BronKerbosch2(R, P, X);
        return ans;
    }
};

```

2.14 SCC Kosaraju

```

#define eb emplace_back
const int N = 2e5 + 5;
vector<int> g[N], rg[N], ord;
int scc[N];
bool v[N];
void rddfs(int x) {
    v[x] = 1;
    for (const auto& i : rg[x])
        if (!v[i]) rddfs(i);
    ord.eb(x);
}
void dfs(int x, int nscc) {
    scc[x] = nscc;
    for (const auto& i : g[x])
        if (scc[i] == -1) dfs(i, nscc);
}
void kosaraju(int n) {
    memset(v, 0, sizeof(v));
    memset(scc, -1, sizeof(scc));
    for (int i = 0; i < n; i++)
        if (!v[i]) rddfs(i);
    int nscc = 0;
    for (int i = n - 1; i >= 0; i--) {
        int x = ord[i];
        if (scc[x] == -1)
            dfs(x, nscc++);
    }
}

```

2.15 SCC Tarjan

```

#define ep emplace

```

```

const int N = 2e5 + 5;
int d[N], low[N], scc[N], ins[N], nscc;
vector<int> g[N];
stack<int> st;
void dfs(int x, int p) {
    d[x] = ~p ? d[p] + 1 : 1, low[x] = d[x];
    st.ep(x), ins[x] = 1;
    for (const auto& i : g[x]) {
        if (!d[i]) dfs(i, x), low[x] = min(low[x], low[i]);
        else if (ins[i]) low[x] = min(low[x], d[i]);
    }
    if (d[x] == low[x]) {
        nscc++;
        int tmp;
        do tmp = st.top(), st.pop(), scc
            [tmp] = nscc, ins[tmp] = 0; while (tmp != x);
    }
}

```

2.16 Tree Centroid

```

const int N = 2e5 + 5, inf = 1e9;
vector<int> g[N];
int n, sz[N], center, csize; // set csize = inf
void dfs(int x, int p) {
    int mxsub = 0;
    sz[x] = 1;
    for (const auto& i : g[x])
        if (i != p) dfs(i, x), sz[x] += sz[i], mxsub = max(mxsub, sz[i]);
    mxsub = max(mxsub, n - sz[x]);
    if (mxsub < csize) center = x, csize = mxsub;
}

```

2.17 Virtual Tree

```

vector<int> vG[N];
int top, st[N];
void insert(int u) {
    if (top == -1) return st[++top] = u, void();
    int p = LCA(st[top], u);
    if (p == st[top]) return st[++top] = u, void();
    while (top >= 1 && dep[st[top - 1]] >= dep[p])
        vG[st[top - 1]].pb(st[top]), --top;
    if (st[top] != p)
        vG[p].pb(st[top]), --top, st[++top] = p;
    st[++top] = u;
}
void reset(int u) {
    for (int i : vG[u]) reset(i);
    vG[u].clear();
}
void solve(vector<int> &v) {
    top = -1;
    sort(ALL(v),
        [&](int a, int b) { return dfn[a] < dfn[b]; });
    for (int i : v) insert(i);
    while (top > 0) vG[st[top - 1]].pb(st[top]), --top;
    // do something
    reset(v[0]);
}

```

3 Data Structure

3.1 2D BIT

```

const int N = 1000 + 5;
int a[N][N];
struct BIT { // 1-based
    ll bit[N][N];
    int n, m;
    void init(int _n, int _m) { // O(nm)
        n = _n, m = _m;
        for (int i = 1; i <= n; i++)
            for (int j = 1; j <= m; j++)
                bit[i][j] = a[i][j];
        for (int b = 1; b <= 1 <= max(n, m); b <= 1) {
            for (int i = b; i + b <= n; i += b <= 1)
                for (int j = 1; j <= m; j++)
                    bit[i + b][j] += bit[i][j];
            for (int i = 1; i <= n; i++)

```

```

        for (int j = b; j + b <= m; j += b << 1)
            bit[i][j + b] += bit[i][j];
    }
}
void upd(int x, int y, int v) {
    for (int i = x; i <= n; i += i & -i)
        for (int j = y; j <= m; j += j & -j)
            bit[i][j] += v;
}
ll qry(int x, int y) {
    ll ret = 0;
    for (int i = x; i; i -= i & -i)
        for (int j = y; j; j -= j & -j)
            ret += bit[i][j];
    return ret;
}
ll qry(int
    x1, int y1, int x2, int y2) { // closed-interval
    return qry(x2, y2) - qry(x1 -
        1, y2) - qry(x2, y1 - 1) + qry(x1 - 1, y1 - 1);
}
} tree;
// tree.init(n, m)

```

3.2 2D Segment Tree

```

const int inf = 1e9;
#define lc(x) (x << 1)
#define rc(x) (x << 1 | 1)
int N, M; // N : row max, M : col max
struct seg {
    vector<int> st;
    void pull(int);
    void merge(const seg& a, const seg& b, int, int, int);
    void build(int, int, int);
    void upd(int, int, int, int, int);
    int qry(int, int, int, int, int);
    seg(int size): st(size << 2 | 1) {}
};
void seg::pull(int id) {
    st[id] = max(st[lc(id)], st[rc(id)]);
}
void seg::merge(const seg& a
    , const seg& b, int id = 1, int l = 1, int r = M) {
    st[id] = max(a.st[id], b.st[id]);
    if (l == r) return;
    int m = (l + r) >> 1;
    merge(a,
        b, lc(id), l, m), merge(a, b, rc(id), m + 1, r);
}
void seg::build(int id = 1, int l = 1, int r = M) {
    if (l == r) {cin >> st[id]; return;}
    int m = (l + r) >> 1;
    build(lc(id), l, m), build(rc(id), m + 1, r);
    pull(id);
}
void seg::upd
    (int x, int v, int id = 1, int l = 1, int r = M) {
    if (l == r) {st[id] = v; return;}
    int m = (l + r) >> 1;
    if (x <= m) upd(x, v, lc(id), l, m);
    else upd(x, v, rc(id), m + 1, r);
    pull(id);
}
int seg::qry(
    int ql, int qr, int id = 1, int l = 1, int r = M) {
    if (ql <= l && r <= qr) return st[id];
    int m = (l + r) >> 1, ret = -inf;
    if (ql
        <= m) ret = max(ret, qry(ql, qr, lc(id), l, m));
    if (qr >
        m) ret = max(ret, qry(ql, qr, rc(id), m + 1, r));
    return ret;
}
}
struct segseg {
    vector<seg> st;
    void pull(int, int);
    void build(int, int, int);
    void upd(int, int, int, int, int, int);
    int qry(int, int, int, int, int, int);
    segseg(int n, int m): st(n << 2 | 1, seg(m)) {}
};
void segseg::pull(int id, int x) {
    st[id].upd(x,
        max(st[lc(id)].qry(x, x), st[rc(id)].qry(x, x)));
}

```

```

void segseg::build(int id = 1, int l = 1, int r = N) {
    if (l == r) {st[id].build(); return;}
    int m = (l + r) >> 1;
    build(lc(id), l, m), build(rc(id), m + 1, r);
    st[id].merge(st[lc(id)], st[rc(id)]);
}
void segseg::upd(int y
    , int x, int v, int id = 1, int l = 1, int r = N) {
    if (l == r) {st[id].upd(x, v); return;}
    int m = (l + r) >> 1;
    if (y <= m) upd(y, x, v, lc(id), l, m);
    else upd(y, x, v, rc(id), m + 1, r);
    pull(id, x);
}
int segseg::qry(int y1, int y2,
    int x1, int x2, int id = 1, int l = 1, int r = N) {
    if (y1 <= l && r <= y2) return st[id].qry(x1, x2);
    int m = (l + r) >> 1, ret = -inf;
    if (y1 <= m) ret
        = max(ret, qry(y1, y2, x1, x2, lc(id), l, m));
    if (y2 > m) ret =
        max(ret, qry(y1, y2, x1, x2, rc(id), m + 1, r));
    return ret;
}
}

```

3.3 BIT

```

const int N = 2e5 + 5;
int n, a[N];

struct BIT { // 1-based
    ll bit1[N], bit2[N];
    ll sum(ll* bit, int x) {
        ll ret = 0;
        for (; x; x -= x & -x) ret += bit[x];
        return ret;
    }
    void upd(ll* bit, int x, ll v) {
        for (; x <= n; x += x & -x) bit[x] += v;
    }
    ll qry(int x) {
        return (x + 1) * sum(bit1, x) - sum(bit2, x);
    }
    ll qry(int l, int r) { // [l, r]
        return qry(r) - qry(l - 1);
    }
    void upd(int l, int r, ll v) { // [l, r]
        upd(bit1, l, v), upd(bit2, l, l * v);
        upd(bit1
            , r + 1, -v), upd(bit2, r + 1, (r + 1) * -v);
    }
    BIT() {
        fill_n(bit1, N, 0), fill_n(bit2, N, 0);
    }
    BIT(int* a) { // O(n) build
        fill_n(bit1, N, 0), fill_n(bit2, N, 0);
        for (int i = 1;
            i <= n; i++) bit1[i] = a[i] - a[i - (i & -i)];
        for (int i = n; i; i--) a[i] -= a[i - 1];
        for (int
            i = 1; i <= n; i++) a[i] = a[i - 1] + a[i] * i;
        for (int i = 1;
            i <= n; i++) bit2[i] = a[i] - a[i - (i & -i)];
    }
};

```

3.4 chtholly tree

```

// 存 {x, v} , 從 x 開始到下一個位置前都是v
map<int, int> s;
// [l, r)
void ins(int l, int r, int i) {
    auto it1 = s.find(l);
    auto it2 = s.find(r);
    for (auto it = it1; it != it2; it++) {
        }
    s.erase(it1, it2); // [it', it2)
    s[l] = ;
}
void split(int pos) {
    auto it = s.lower_bound(pos);
    if (it == s.end() or it->F != pos) {
        s[pos] = prev(it)->S;
    }
}

```


3.5 LiChaoST

```

struct LiChao_min {
    struct line {
        LL m, c;
        line(LL _m = 0, LL _c = 0) {
            m = _m;
            c = _c;
        }
        LL eval(LL x) { return m * x + c; }
    };
    struct node {
        node *l, *r;
        line f;
        node(line v) {
            f = v;
            l = r = NULL;
        }
    };
    typedef node *pnode;
    pnode root;
    int sz;
#define mid ((l + r) >> 1)
    void insert(line &v, int l, int r, pnode &nd) {
        if (!nd) {
            nd = new node(v);
            return;
        }
        LL trl = nd->f.eval(l), trr = nd->f.eval(r);
        LL vl = v.eval(l), vr = v.eval(r);
        if (trl <= vl && trr <= vr) return;
        if (trl > vl && trr > vr) {
            nd->f = v;
            return;
        }
        if (trl > vl) swap(nd->f, v);
        if (nd->f.eval(mid) < v.eval(mid))
            insert(v, mid + 1, r, nd->r);
        else swap(nd->f, v), insert(v, l, mid, nd->l);
    }
    LL query(int x, int l, int r, pnode &nd) {
        if (!nd) return LLONG_MAX;
        if (l == r) return nd->f.eval(x);
        if (mid >= x)
            return min(
                nd->f.eval(x), query(x, l, mid, nd->l));
        return min(
            nd->f.eval(x), query(x, mid + 1, r, nd->r));
    }
    /* -sz <= query_x <= sz */
    void init(int _sz) {
        sz = _sz + 1;
        root = NULL;
    }
    void add_line(LL m, LL c) {
        line v(m, c);
        insert(v, -sz, sz, root);
    }
    LL query(LL x) { return query(x, -sz, sz, root); }
};

```

3.6 persistent

```

const int MAXN = 2e5 + 5;
int a[MAXN];

int sum[MAXN * 25], lc[MAXN * 25], rc[MAXN * 25];
int add_node() {
    static int now = 0;
    return ++now;
}
void pull(int x) {
    sum[x] = sum[lc[x]] + sum[rc[x]];
}
void init(int &x, int lx, int rx) {
    if (!x) x = add_node();
    if (lx + 1 == rx) return;
    int mid = (lx + rx) / 2;
    init(lc[x], lx, mid);
    init(rc[x], mid, rx);
}
void update(int fa, int &x, int lx, int rx, int i) {
    if (!x) x = add_node();
    if (lx + 1 == rx) return sum[x]++, void();
    int mid = (lx + rx) / 2;
    if (i < mid) {
        rc[x] = rc[fa];
    }
    else {
        lc[x] = lc[fa];
        update(rc[fa], lc[x], lx, mid, i);
    }
    pull(x);
}
int query(int x, int lx, int rx, int l, int r) {
    if (lx >= r || rx <= l) return 0;
    if (lx >= l && rx <= r) return sum[x];
    int mid = (lx + rx) / 2;
    return query(lc[x], lx, mid, l, r) + query(rc[x], mid, rx, l, r);
}

```

```

    update(lc[fa], lc[x], lx, mid, i);
}
else {
    lc[x] = lc[fa];
    update(rc[fa], rc[x], mid, rx, i);
}
pull(x);
}
int query(int x, int lx, int rx, int l, int r) {
    if (lx >= r || rx <= l) return 0;
    if (lx >= l && rx <= r) return sum[x];
    int mid = (lx + rx) / 2;
    return query(lc[x], lx, mid, l, r) + query(rc[x], mid, rx, l, r);
}

```

3.7 Sparse Table

```

const int N = 5e5 + 5, logN = __lg(N) + 1;
int a[N];
struct sparse_table { // 0-based
    int st[logN][N];
    void init(int n) {
        copy(a, a + n, st[0]);
        for (int i = 1; (1 << i) <= n; i++)
            for (int j = 0; j + (1 << i) - 1 <= n; j++)
                st[i][j] = max(st[i - 1][j], st[i - 1][j + (1 << (i - 1))]);
    }
    int qry(int l, int r) {
        int k = __lg(r - l + 1);
        return max(st[k][l], st[k][r - (1 << k) + 1]);
    }
} st;
// st.init(n)
// st.qry(l - 1, r - 1)

```

3.8 Treap

```

#include <bits/stdc++.h>
using namespace std;
mt19937 rng;
struct node {
    node *l, *r;
    int v, p, s; bool t; // val, pri, size, tag
    void pull() {
        s = 1;
        for (auto x : {l, r})
            if (x) s += x->s;
    }
    void push() {
        if (t) {
            swap(l, r), t = 0;
            for (auto& x : {l, r})
                if (x) x->t ^= 1;
        }
    }
    node(int _v
        = 0): v(_v), p(rng()), s(1), t(0), l(0), r(0) {}
};
int sz(node* o) {return o ? o->s : 0;}
node* merge(node* a, node* b) {
    if (!a || !b) return a ? a : b;
    if (a->p < b->p) return
        a->push(), a->r = merge(a->r, b), a->pull(), a;
    else return
        b->push(), b->l = merge(a, b->l), b->pull(), b;
}
void split(node
    * o, node*& a, node*& b, int k) { // a < k, b >= k
    if (!o) return a = b = nullptr, void();
    o->push();
    if (o->v < k) a = o, split(o->r, a->r, b, k);
    else b = o, split(o->l, a, b->l, k);
    o->pull();
}
void insert(node*& o, int k) {
    node *a, *b;
    split(
        o, a, b, k), o = merge(a, merge(new node(k), b));
}
void ssplit(node* o, node
    *& a, node*& b, int k) { // split first k things
    if (!o) return a = b = nullptr, void();
    o->push();
    if (sz(o->l) + 1 <= k
        ) a = o, ssplit(o->r, a->r, b, k - sz(o->l) - 1);
    else
        split(o->l, a, b, k), o = merge(a, merge(new node(o->v), b));
}

```



```

else b = o, ssplit(o->l, a, b->l, k);
o->pull();
}
void reverse(node* o, int l, int r) { // [l, r]
    node *a, *b, *c;
    ssplit(o, a, b, l - 1), ssplit(b, b, c, r - l + 1);
    b->t ^= 1, o = merge(a, merge(b, c));
}
/*
node* root = nullptr;
for (int i = 0; i < n; i++)
    root = merge(root, new node(x));
*/

```

3.9 ZKW Segment Tree

```

const int N = 5e5 + 5;
int a[N];

struct seg_tree { // 0-based
    int seg[N << 1], n;
    void upd(int x, int v) {
        for (seg[x += n] = v; x > 1; x >>= 1)
            seg[x >> 1] = max(seg[x], seg[x ^ 1]);
    }
    int qry(int l, int r) { // [ql, qr]
        int ret = -1e9;
        for (l += n, r += n + 1; l < r; l >>= 1, r >>= 1) {
            if (l & 1) ret = max(ret, seg[l++]);
            if (r & 1) ret = max(ret, seg[--r]);
        }
        return ret;
    }
    void init(int _n) {
        n = _n;
        copy(a, a + n, seg + n);
        for (int i = n - 1; i >= 0; i--)
            seg[i] = max(seg[i << 1], seg[i << 1 | 1]);
    }
} tree;
// tree.init(n)
// tree.qry(l - 1, r - 1)

```

4 Flow

4.1 Bipartite Matching

```

// O(E * sqrt(V))
struct Bipartite_Matching { // 0-base
    int l, r;
    int mp[MAXN], mq[MAXN];
    int dis[MAXN], cur[MAXN];
    vector<int> G[MAXN];
    bool dfs(int u) {
        for (int &i = cur[u]; i < SZ(G[u]); ++i) {
            int e = G[u][i];
            if (!mq[e] || (dis[mq[e]] == dis[u] + 1 && dfs(mq[e])))
                return mp[mq[e] = u] = e, 1;
        }
        dis[u] = -1;
        return 0;
    }
    bool bfs() {
        int rt = 0;
        queue<int> q;
        fill_n(dis, l, -1);
        for (int i = 0; i < l; ++i)
            if (!mp[i])
                q.push(i), dis[i] = 0;
        while (!q.empty()) {
            int u = q.front();
            q.pop();
            for (int e : G[u])
                if (!mq[e])
                    rt = 1;
                else if (!dis[mq[e]]) {
                    q.push(mq[e]);
                    dis[mq[e]] = dis[u] + 1;
                }
        }
        return rt;
    }
    int matching() {
        int rt = 0;
        fill_n(mp, l, -1);
        fill_n(mq, r, -1);
    }
}

```

```

while (bfs()) {
    fill_n(cur, l, 0);
    for (int i = 0; i < l; ++i)
        if (!mp[i] && dfs(i))
            ++rt;
}
return rt;
}
void add_edge(int s, int t) {
    G[s].pb(t);
}
void init(int _l, int _r) {
    l = _l, r = _r;
    for (int i = 0; i < l; ++i)
        G[i].clear();
}
}
};

```

4.2 Bounded Flow

```

// time complexity: same as Dinic
struct BoundedFlow { // 0-base
    struct edge {
        int to, cap, flow, rev;
    };
    vector<edge> G[N];
    int n, s, t, dis[N], cur[N], cnt[N];
    void init(int _n) {
        n = _n;
        for (int i = 0; i < n + 2; ++i)
            G[i].clear(), cnt[i] = 0;
    }
    void add_edge(int u, int v, int lcap, int rcap) {
        cnt[u] -= lcap, cnt[v] += lcap;
        G[u].pb(edge{v, rcap, lcap, SZ(G[v])});
        G[v].pb(edge{u, 0, 0, SZ(G[u]) - 1});
    }
    void add_edge(int u, int v, int cap) {
        G[u].pb(edge{v, cap, 0, SZ(G[v])});
        G[v].pb(edge{u, 0, 0, SZ(G[u]) - 1});
    }
    int dfs(int u, int cap) {
        if (u == t || !cap) return cap;
        for (int &i = cur[u]; i < SZ(G[u]); ++i) {
            edge &e = G[u][i];
            if (dis[e.to] == dis[u] + 1 && e.cap != e.flow) {
                int df = dfs(e.to, min(e.cap - e.flow, cap));
                if (df) {
                    e.flow += df, G[e.to][e.rev].flow -= df;
                    return df;
                }
            }
        }
        dis[u] = -1;
        return 0;
    }
    bool bfs() {
        fill_n(dis, n + 3, -1);
        queue<int> q;
        q.push(s), dis[s] = 0;
        while (!q.empty()) {
            int u = q.front();
            q.pop();
            for (edge &e : G[u])
                if (!dis[e.to] && e.flow != e.cap)
                    q.push(e.to), dis[e.to] = dis[u] + 1;
        }
        return dis[t] != -1;
    }
    int maxflow(int _s, int _t) {
        s = _s, t = _t;
        int flow = 0, df;
        while (bfs()) {
            fill_n(cur, n + 3, 0);
            while ((df = dfs(s, INF))) flow += df;
        }
        return flow;
    }
    bool solve() {
        int sum = 0;
        for (int i = 0; i < n; ++i)
            if (cnt[i] > 0)
                add_edge(n + 1, i, cnt[i]), sum += cnt[i];
            else if (cnt[i] < 0) add_edge(i, n + 2, -cnt[i]);
        if (sum != maxflow(n + 1, n + 2)) sum = -1;
        for (int i = 0; i < n; ++i)
            if (cnt[i] > 0)

```

```

    G[n + 1].pop_back(), G[i].pop_back();
    else if (cnt[i] < 0)
        G[i].pop_back(), G[n + 2].pop_back();
    return sum != -1;
}
int solve(int _s, int _t) {
    add_edge(_t, _s, INF);
    if (!solve()) return -1; // invalid flow
    int x = G[_t].back().flow;
    return G[_t].pop_back(), G[_s].pop_back(), x;
}
};

```

4.3 Dinic

```

// O(V^2 * E)
// O(min(V^(2/3), E^(1/2)) * E) for unit graph (all cap are same)
// O(E * sqrt(V)) for bipartite matching
struct MaxFlow { // 0-base
    struct edge {
        int to, cap, flow, rev;
    };
    vector<edge> G[MAXN];
    int s, t, dis[MAXN], cur[MAXN], n;
    int dfs(int u, int cap) {
        if (u == t || !cap) return cap;
        for (int &i = cur[u]; i < (int)G[u].size(); ++i) {
            edge &e = G[u][i];
            if (dis[e.to] == dis[u] + 1 && e.flow != e.cap) {
                int df = dfs(e.to, min(e.cap - e.flow, cap));
                if (df) {
                    e.flow += df;
                    G[e.to][e.rev].flow -= df;
                    return df;
                }
            }
        }
        dis[u] = -1;
        return 0;
    }
    bool bfs() {
        fill_n(dis, n, -1);
        queue<int> q;
        q.push(s), dis[s] = 0;
        while (!q.empty()) {
            int tmp = q.front();
            q.pop();
            for (auto &u : G[tmp])
                if (!dis[u.to] && u.flow != u.cap) {
                    q.push(u.to);
                    dis[u.to] = dis[tmp] + 1;
                }
        }
        return dis[t] != -1;
    }
    int maxflow(int _s, int _t) {
        s = _s, t = _t;
        int flow = 0, df;
        while (bfs()) {
            fill_n(cur, n, 0);
            while ((df = dfs(s, INF))) flow += df;
        }
        return flow;
    }
    void init(int _n) {
        n = _n;
        for (int i = 0; i < n; ++i) G[i].clear();
    }
    void reset() {
        for (int i = 0; i < n; ++i)
            for (auto &j : G[i]) j.flow = 0;
    }
    void add_edge(int u, int v, int cap) {
        G[u].pb(edge{v, cap, 0, (int)G[v].size()});
        G[v].pb(edge{u, 0, 0, (int)G[u].size() - 1});
    }
};

```

4.4 KM

```

// O(n^3), where n is the number
// of vertices on one side of the bipartite graph
// Finds
// the maximum weight matching in a bipartite graph
struct KM { // 0-base
    int w[MAXN][MAXN], hl[MAXN], hr[MAXN], slk[MAXN], n;

```

```

    int fl[MAXN], fr[MAXN], pre[MAXN], qu[MAXN], ql, qr;
    bool vl[MAXN], vr[MAXN];
    void init(int _n) {
        n = _n;
        for (int i = 0; i < n; ++i)
            for (int j = 0; j < n; ++j) w[i][j] = -INF;
    }
    void add_edge(int a, int b, int wei) {
        w[a][b] = wei;
    }
    bool Check(int x) {
        if (vl[x] = 1, ~fl[x])
            return vr[qu[qr++]] = fl[x] = 1;
        while (~x) swap(x, fr[fl[x] = pre[x]]);
        return 0;
    }
    void Bfs(int s) {
        fill(slk, slk + n, INF);
        fill(vl, vl + n, 0), fill(vr, vr + n, 0);
        ql = qr = 0, qu[qr++] = s, vr[s] = 1;
        while (1) {
            int d;
            while (ql < qr)
                for (int x = 0, y = qu[ql++]; x < n; ++x)
                    if (!vl[x] &&
                        slk[x] >= (d = hl[x] + hr[y] - w[x][y]))
                        if (pre[x] = y, d) slk[x] = d;
                        else if (!Check(x)) return;
            d = INF;
            for (int x = 0; x < n; ++x)
                if (!vl[x] && d > slk[x]) d = slk[x];
            for (int x = 0; x < n; ++x) {
                if (vl[x]) hl[x] += d;
                else slk[x] -= d;
                if (vr[x]) hr[x] -= d;
            }
            for (int x = 0; x < n; ++x)
                if (!vl[x] && !slk[x] && !Check(x)) return;
        }
    }
    int Solve() {
        fill(fl, fl + n, -1), fill(fr, fr + n, -1),
        fill(hr, hr + n, 0);
        for (int i = 0; i < n; ++i)
            hl[i] = *max_element(w[i], w[i] + n);
        for (int i = 0; i < n; ++i) Bfs(i);
        int res = 0;
        for (int i = 0; i < n; ++i) res += w[i][fl[i]];
        return res;
    }
};

```

4.5 Maximum Simple Graph Matching

```

// O(V^3), where V is the number of vertices
struct Matching { // 0-base
    queue<int> q; int n;
    vector<int> fa, s, vis, pre, match;
    vector<vector<int>> G;
    int Find(int u) {
        return u == fa[u] ? u : fa[u] = Find(fa[u]);
    }
    int LCA(int x, int y) {
        static int tk = 0; tk++; x = Find(x); y = Find(y);
        for (; swap(x, y)) if (x != n) {
            if (vis[x] == tk) return x;
            vis[x] = tk;
            x = Find(pre[match[x]]);
        }
    }
    void Blossom(int x, int y, int l) {
        for (; Find(x) != l; x = pre[y]) {
            pre[x] = y, y = match[x];
            if (s[y] == 1) q.push(y), s[y] = 0;
            for (int z : {x, y}) if (fa[z] == z) fa[z] = l;
        }
    }
    bool Bfs(int r) {
        iota(ALL(fa), 0); fill(ALL(s), -1);
        q = queue<int>(); q.push(r); s[r] = 0;
        for (; !q.empty(); q.pop()) {
            for (int x = q.front(); int u : G[x])
                if (s[u] == -1) {
                    if (pre[u] = x, s[u] = 1, match[u] == n) {
                        for (int a = u, b = x, last;
                            b != n; a = last, b = pre[a])
                            last = match[b], match[b] = a, match[a] = b;

```

```

        return true;
    }
    q.push(match[u]); s[match[u]] = 0;
} else if (!s[u] && Find(u) != Find(x)) {
    int l = LCA(u, x);
    Blossom(x, u, l); Blossom(u, x, l);
}
}
return false;
}
Matching(int _n) : n(_n), fa(n + 1), s(n + 1), vis
(n + 1), pre(n + 1, n), match(n + 1, n), G(n) {}
void add_edge(int u, int v)
{ G[u].pb(v), G[v].pb(u); }
int solve() {
    int ans = 0;
    for (int x = 0; x < n; ++x)
        if (match[x] == n) ans += Bfs(x);
    return ans;
} // match[x] == n means not matched
};

```

4.6 MCMF

```

// O(E * logV), where F is
// the maximum flow, E is edges, and V is vertices.
struct MinCostMaxFlow { // 0-base
    struct Edge {
        ll from, to, cap, flow, cost, rev;
    } *past[N];
    vector<Edge> G[N];
    int inq[N], n, s, t;
    ll dis[N], up[N], pot[N];
    bool BellmanFord() {
        fill_n(dis, n, INF), fill_n(inq, n, 0);
        queue<int> q;
        auto relax = [&](int u, ll d, ll cap, Edge * e) {
            if (cap > 0 && dis[u] > d) {
                dis[u] = d, up[u] = cap, past[u] = e;
                if (!inq[u]) inq[u] = 1, q.push(u);
            }
        };
        relax(s, 0, INF, 0);
        while (!q.empty()) {
            int u = q.front();
            q.pop(), inq[u] = 0;
            for (auto &e : G[u]) {
                ll d2 = dis[u] + e.cost + pot[u] - pot[e.to];
                relax
                    (e.to, d2, min(up[u], e.cap - e.flow), &e);
            }
        }
        return dis[t] != INF;
    }
    void solve(int _s
, int _t, ll &flow, ll &cost, bool neg = true) {
        s = _s, t = _t, flow = 0, cost = 0;
        if (neg) BellmanFord(), copy_n(dis, n, pot);
        for (; BellmanFord(); copy_n(dis, n, pot)) {
            for (int
                i = 0; i < n; ++i) dis[i] += pot[i] - pot[s];
            flow += up[t], cost += up[t] * dis[t];
            for (int i = t; past[i]; i = past[i]->from) {
                auto &e = *past[i];
                e.flow += up[t], G[e.to][e.rev].flow -= up[t];
            }
        }
    }
    void init(int _n) {
        n = _n, fill_n(pot, n, 0);
        for (int i = 0; i < n; ++i) G[i].clear();
    }
    void add_edge(ll a, ll b, ll cap, ll cost) {
        G[a].pb(Edge{a, b, cap, 0, cost, SZ(G[b])});
        G[b].pb(Edge{b, a, 0, 0, -cost, SZ(G[a]) - 1});
    }
};

```

4.7 Mimum Vertex Cover

```

// O(VE)
struct Maximum_cardinality_matching {
    int n, k;
    int match[1005]; //right
    int vis[1005]; // left
    vector<int> adj[1005]; // left
    int dfs(int x) {

```

```

        vis[x] = 1;
        for (int i : adj[x]) {
            if (match[i] ==
                -1 or (!vis[match[i]] and dfs(match[i]))) {
                match[i] = x;
                return true;
            }
        }
        return false;
    }
    int paired[1005];
    int vis2[1005];
    void dfs2(int x) {
        vis[x] = 1;
        for (int i : adj[x]) {
            if (!vis2[i]) {
                vis2[i] = 1;
                dfs2(match[i]);
            }
        }
    }
    void matching() {
        fill(match + 1, match + 1 + k, -1);
        int res = 0;
        FOR (i, 1, k) {
            fill(vis + 1, vis + 1 + k, 0);
            res += dfs(i);
        }
        FOR (i, 1, k) {
            if (match[i] != -1) {
                paired[match[i]] = 1;
            }
        }
        fill(vis + 1, vis + 1 + k, 0);
        fill(vis2 + 1, vis2 + 1 + k, 0);
        FOR (i, 1, k) {
            if (!paired[i]) {
                dfs2(i);
            }
        }
        vector<int> a, b;
        FOR (i, 1, k) {
            if (!vis[i]) a.pb(i);
            if (vis2[i]) b.pb(i);
        }
        cout << SZ(a) << ' ' << SZ(b) << '\n';
        for (int i : a) cout << i << ' '; cout << '\n';
        for (int i : b) cout << i << ' '; cout << '\n';
        assert(SZ(a) + SZ(b) == res);
    }
};

```

4.8 Theorem

- Maximum Independent Set: A largest set of non-adjacent vertices.
- Maximum Matching: A largest set of edges with no shared vertices.
- Minimum Vertex Cover: A smallest set of vertices that covers all edges.
- Minimum Edge Cover: A smallest set of edges that covers all vertices.
- Maximum Clique: A largest complete subgraph.
- Properties:
 - $|\text{Maximum Matching}| = |\text{Minimum Vertex Cover}|$
 - $|\text{Maximum Matching}| + |\text{Minimum Edge Cover}| = |V|$
 - $|\text{Maximum Independent Set}| + |\text{Minimum Vertex Cover}| = |V|$
 - $|\text{Maximum Independent Set}| = |V| - |\text{Maximum Matching}|$
 - $|\text{Maximum Clique}| = |\text{Maximum Independent Set in the Complement Graph}|$

5 Geometry

5.1 Basic 2D

```

// Courtesy of Jinkela
const double PI = atan2(0.0, -1.0);
template<typename T>
struct point {
    T x, y;
    point() {}
    point(const T&x, const T&y): x(x), y(y) {}
    point operator+(const point &b)const {
        return point(x + b.x, y + b.y);
    }
    point operator-(const point &b)const {
        return point(x - b.x, y - b.y);
    }
    point operator*(const T &b)const {
        return point(x * b, y * b);
    }
    point operator/(const T &b)const {
        return point(x / b, y / b);
    }
};

```

```

bool operator==(const point &b) const {
    return x == b.x && y == b.y;
}
T dot(const point &b) const { return x * b.x + y * b.y; }
T cross(const point &b) const { return x * b.y - y * b.x; }
point normal() const { //求法向量
    return point(-y, x);
}
T abs2() const { return dot(*this); }
T rad(const point &b) const { //兩向量的弧度
    return fabs(atan2(fabs(cross(b)), dot(b)));
}
T getA() const { //對x軸的弧度
    T A = atan2(y, x); //超過180度會變負的
    if (A <= -PI / 2) A += PI * 2;
    return A;
}
};
template<typename T>
struct line {
    line() {}
    point<T> p1, p2;
    T a, b, c; //ax+by+c=0
    line(const point<T>&x, const point<T>&y): p1(x), p2(y) {}
    void pton() { //轉成一般式
        a = p1.y - p2.y;
        b = p2.x - p1.x; c = -a * p1.x - b * p1.y;
    }
    T ori(const point<T> &p) const { //點和有向直線的關聯, >0左邊、=0在線上<0右邊
        return (p2 - p1).cross(p - p1);
    }
    T btw(const point<T> &p) const { //點投影落在線段上<=0
        return (p1 - p).dot(p2 - p);
    }
    bool point_on_segment(const point<T>&p) const {
        return ori(p) == 0 && btw(p) <= 0;
    }
    T dis2(const point<T> &p, bool is_segment = 0) const { //點跟直線/線段的距離平方
        point<T> v = p2 - p1, v1 = p - p1;
        if (is_segment) {
            point<T> v2 = p - p2;
            if (v.dot(v1) <= 0) return v1.abs2();
            if (v.dot(v2) >= 0) return v2.abs2();
        }
        T tmp = v.cross(v1); return tmp * tmp / v.abs2();
    }
    T seg_dis2(const line<T> &l) const { //兩線段距離平方
        return min({dis2(l.p1, 1), dis2(l.p2, 1), l.dis2(p1, 1), l.dis2(p2, 1)});
    }
    point<T> projection(const point<T> &p) const { //點對直線的投影
        point<T> n = (p2 - p1).normal();
        return p - n * (p - p1).dot(n) / n.abs2();
    }
    point<T> mirror(const point<T> &p) const { //點對直線的鏡射, 要先呼叫pton轉成一般式
        point<T> R; T d = a * a + b * b;
        R.x = (b * b * p.x - a * a * p.x - 2 * a * b * p.y - 2 * a * c) / d;
        R.y = (a * a * p.y - b * b * p.y - 2 * a * b * p.x - 2 * b * c) / d;
        return R;
    }
    bool parallel(const line &l) const {
        return (p1 - p2).cross(l.p1 - l.p2) == 0;
    }
};
template<typename T>
struct polygon {
    polygon() {}
    vector<point<T>> p; //逆時針順序
    T double_signed_area() const {
        T ans = 0;
        for (int i = 0; i < p.size() - 1; i++)
            ans += p[i].cross(p[i+1]);
        return ans;
    }
};

```

```

point<T> center_of_mass() const {
    T cx = 0, cy = 0, w = 0;
    for (int i = 0; i < p.size(); i++) {
        T a = p[i].cross(p[i+1]);
        cx += (p[i].x + p[i+1].x) * a; cy += (p[i].y + p[i+1].y) * a;
        w += a;
    }
    return point<T>(cx / 3 / w, cy / 3 / w);
}
int ahas(const point<T>&t) const { //點是否在簡單多邊形上, 是的話回傳1、在邊上回傳-1、否則回傳0
    int c = 0; //Works for clockwise input as well
    for (int i = 0; i < p.size(); i++) {
        if (line<T>(p[i], p[i+1]).point_on_segment(t)) return -1;
        if ((p[i].y > t.y) != (p[i+1].y > t.y)) {
            T L = (t.x - p[i].x) * (p[i+1].y - p[i].y);
            T R = (p[i+1].x - p[i].x) * (t.y - p[i].y);
            if (p[i+1].y < p[i].y) { L = -L; R = -R; }
            if (L < R) c = !c;
        }
    }
    return c;
}
int point_in_convex(const point<T>&x) const {
    int l = 1, r = (int)p.size() - 2;
    while (l <= r) { //點是否在凸多邊形上, 是的話回傳1、在邊上回傳-1、否則回傳0
        int mid = (l + r) / 2;
        T a1 = (p[mid] - p[0]).cross(x - p[0]);
        T a2 = (p[mid + 1] - p[0]).cross(x - p[0]);
        if (a1 >= 0 && a2 <= 0) {
            T res = (p[mid + 1] - p[mid]).cross(x - p[mid]);
            return res > 0 ? 1 : (res >= 0 ? -1 : 0);
        }
        if (a1 < 0) r = mid - 1; else l = mid + 1;
    }
    return 0;
}
vector<T> getA() const { //凸包邊對x軸的夾角
    vector<T> res; //一定是遞增的
    for (size_t i = 0; i < p.size(); ++i)
        res.push_back((p[(i + 1) % p.size()] - p[i]).getA());
    return res;
}
bool line_intersect(const vector<T>&A, const line<T> &l) const { //O(logN)
    int f1 = upper_bound(A.begin(), A.end(), (l.p1 - l.p2).getA()) - A.begin();
    int f2 = upper_bound(A.begin(), A.end(), (l.p2 - l.p1).getA()) - A.begin();
    return l.cross_seg(p[f1], p[f2]);
}
T diam() {
    int n = p.size(), t = 1;
    T ans = 0; p.push_back(p[0]);
    for (int i = 0; i < n; i++) {
        point<T> now = p[i + 1] - p[i];
        while (now.cross(p[t + 1] - p[i]) > now.cross(p[t] - p[i])) t = (t + 1) % n;
        ans = max(ans, (p[i] - p[t]).abs2());
    }
    return p.pop_back(), ans;
}
T min_cover_rectangle() {
    int n = p.size(), t = 1, r = 1, l;
    if (n < 3) return 0; //也可以做最小周長矩形
    T ans = 1e99; p.push_back(p[0]);
    for (int i = 0; i < n; i++) {
        point<T> now = p[i + 1] - p[i];
        while (now.cross(p[t + 1] - p[i]) > now.cross(p[t] - p[i])) t = (t + 1) % n;
        while (now.dot(p[r + 1] - p[i]) > now.dot(p[r] - p[i])) r = (r + 1) % n;
        if (!i) l = r;
        while (now.dot(p[l + 1] - p[i]) <= now.dot(p[l] - p[i])) l = (l + 1) % n;
        T d = now.abs2();
        T tmp = now.cross(p[t] - p[i]) * (now.dot(p[r] - p[i]) - now.dot(p[l] - p[i])) / d;
        ans = min(ans, tmp);
    }
    return p.pop_back(), ans;
}
T dis2(polygon &p) { //凸包最近距離平方

```

```

vector<point<T> > &P = p, &Q = pl.p;
int n = P.size(), m = Q.size(), l = 0, r = 0;
for (int i = 0; i < n; ++i) if (P[i].y < P[l].y) l = i;
for (int i = 0; i < m; ++i) if (Q[i].y < Q[r].y) r = i;
P.push_back(P[0]), Q.push_back(Q[0]);
T ans = 1e99;
for (int i = 0; i < n; ++i) {
    while ((P[l] - P[l + 1])
        .cross(Q[r + 1] - Q[r]) < 0) r = (r + 1) % m;
    ans = min(ans, line<T>(P[l],
        P[l + 1]).seg_dis2(line<T>(Q[r], Q[r + 1])));
    l = (l + 1) % n;
} return P.pop_back(), Q.pop_back(), ans;
}
static int sign(const point<T>&t) {
    return (t.y ? t.y : t.x) < 0;
}
static bool
    angle_cmp(const line<T>& A, const line<T>& B) {
    point<T> a = A.p2 - A.p1, b = B.p2 - B.p1;
    return sign(a) < sign
        (b) || (sign(a) == sign(b) && a.cross(b) > 0);
}
int halfplane_intersection(vector<line<T> > &s) {
    sort(s.begin()
        , s.end(), angle_cmp); // 區段左側 該區段半平面
    int L, R, n = s.size();
    vector<point<T> > px(n);
    vector<line<T> > q(n);
    q[L = R = 0] = s[0];
    for (int i = 1; i < n; ++i) {
        while (L < R && s[i].ori(px[R - 1]) <= 0) --R;
        while (L < R && s[i].ori(px[L]) <= 0) ++L;
        q[++R] = s[i];
        if (q[R].parallel(q[R
            - 1]) && q[--R].ori(s[i].p1) > 0) q[R] = s[i];
        if (L < R)
            px[R - 1] = q[R - 1].line_intersection(q[R]);
    }
    while (L < R && q[L].ori(px[R - 1]) <= 0) --R;
    p.clear();
    if (R - L <= 1) return 0;
    px[R] = q[R].line_intersection(q[L]);
    for (int i = L; i <= R; ++i) p.push_back(px[i]);
    return R - L + 1;
}
};

```

5.2 Convex Hull

```

#define f first
#define s second
#define ALL(x) (x).begin(), (x).end()
template <typename T>
pair<T, T> operator
    -(const pair<T, T>& a, const pair<T, T>& b) {
    return {a.f - b.f, a.s - b.s};
}
template <typename T>
int cross(const pair<T,
    T>& o, const pair<T, T>& a, const pair<T, T>& b) {
    auto p = a - o, q = b - o;
    return p.f * q.s - q.f * p.s;
}
template <typename T>
vector
    <pair<T, T>> convex_hull(vector<pair<T, T>> hull) {
    if (hull.size() <= 2) return hull;
    sort(ALL(hull));
    vector<pair<T, T>> stk;
    int n = hull.size();
    for (int i = 0; i < n; ++i) {
        while (stk.size() >= 2 && cross
            (stk.end()[-2], stk.end()[-1], hull[i]) <= 0)
            stk.pop_back();
        stk.push_back(hull[i]);
    }
    for (
        int i = n - 2, t = stk.size() + 1; i >= 0; i--) {
        while ((int)stk.size() >= t && cross
            (stk.end()[-2], stk.end()[-1], hull[i]) <= 0)
            stk.pop_back();
        stk.push_back(hull[i]);
    }
    return stk.pop_back(), stk;
}

```

```

}

```

5.3 Dynamic Convex Hull

```

struct Line {
    ll a, b, l = MIN, r = MAX;
    Line(ll a, ll b): a(a), b(b) {}
    ll operator()(ll x) const {
        return a * x + b;
    }
    bool operator<(Line b) const {
        return a < b.a;
    }
    bool operator<(ll b) const {
        return r < b;
    }
};

ll iceil(ll a, ll b) {
    if (b < 0) a *= -1, b *= -1;
    if (a > 0) return (a + b - 1) / b;
    else return a / b;
}

ll intersect(Line a, Line b) {
    return iceil(a.b - b.b, b.a - a.a);
}

struct DynamicConvexHull {
    multiset<Line, less<>> ch;

    void add(Line ln) {
        auto it = ch.lower_bound(ln);
        while (it != ch.end()) {
            Line tl = *it;
            if (tl(tl.r) <= ln(tl.r)) {
                it = ch.erase(it);
            }
            else break;
        }
        auto it2 = ch.lower_bound(ln);
        while (it2 != ch.begin()) {
            Line tl = *prev(it2);
            if (tl(tl.l) <= ln(tl.l)) {
                it2 = ch.erase(prev(it2));
            }
            else break;
        }
        it = ch.lower_bound(ln);
        if (it != ch.end()) {
            Line tl = *it;
            if (tl(tl.l) >= ln(tl.l)) ln.r = tl.l - 1;
            else {
                ll pos = intersect(ln, tl);
                tl.l = pos;
                ln.r = pos - 1;
                ch.erase(it);
                ch.insert(tl);
            }
        }
        it2 = ch.lower_bound(ln);
        if (it2 != ch.begin()) {
            Line tl = *prev(it2);
            if (tl(tl.r) >= ln(tl.r)) ln.l = tl.r + 1;
            else {
                ll pos = intersect(tl, ln);
                tl.r = pos - 1;
                ln.l = pos;
                ch.erase(prev(it2));
                ch.insert(tl);
            }
        }
        if (ln.l <= ln.r) ch.insert(ln);
    }

    ll query(ll pos) {
        auto it = ch.lower_bound(pos);
        if (it == ch.end()) return 0;
        return (*it)(pos);
    }
};

```

5.4 Segmentation Intersection

```

int sign(ll x) {
    return (x > 0 ? 1 : (x < 0 ? -1 : 0));
}

```

```

ll cross
(pair<ll, ll> o, pair<ll, ll> a, pair<ll, ll> b) {
    return (a.first - o.first) * (b.second - o.second
        ) - (a.second - o.second) * (b.first - o.first);
}

bool intersect1D(ll a, ll b, ll c, ll d) {
    if (a > b) swap(a, b);
    if (c > d) swap(c, d);
    return max(a, c) <= min(b, d);
}

bool intersect2D(pair<ll, ll> a
    , pair<ll, ll> b, pair<ll, ll> c, pair<ll, ll> d) {
    return
        intersect1D(a.first, b.first, c.first, d.first)
        && intersect1D
            (a.second, b.second, c.second, d.second)
        && sign(cross
            (a, b, c)) * sign(cross(a, b, d)) <= 0
        && sign(cross
            (c, d, a)) * sign(cross(c, d, b)) <= 0;
}

```

5.5 Theorem

- Pick's Theorem:
 - If a polygon has vertices with integer coordinates (lattice points), then the area is given by:

$$\text{Area}(P) = i + \frac{1}{2}p - 1$$

where i is the number of lattice points inside the polygon, and p is the number of lattice points on the perimeter of the polygon.

6 Math

6.1 Big Int

```

#include <bits/stdc++.h>
using namespace std;

template<typename T>
inline string to_string(const T& x) {
    stringstream ss;
    return ss << x, ss.str();
}

using ll = long long;
struct bigN: vector<ll> {
    const static
        int base = 1000000000, width = log10(base);
    bool negative;
    bigN(const_iterator
        a, const_iterator b): vector<ll>(a, b) {}
    bigN(string s) {
        if (s.empty()) return;
        if (s[0] == '-') negative = 1, s = s.substr(1);
        else negative = 0;
        for (int
            i = int(s.size()) - 1; i >= 0; i -= width) {
            ll t = 0;
            for (int j = max(0, i - width + 1); j <= i; ++j)
                t = t * 10 + s[j] - '0';
            push_back(t);
        }
        trim();
    }
    template<typename T>
    bigN(const T &x): bigN(to_string(x)) {}
    bigN(): negative(0) {}
    void trim() {
        while (size() && !back()) pop_back();
        if (empty()) negative = 0;
    }
    void carry(int _base = base) {
        for (size_t i = 0; i < size(); ++i) {
            if (at(i) >= 0 && at(i) < _base) continue;
            if (i + 1u == size()) push_back(0);
            int r = at(i) % _base;
            if (r < 0) r += _base;
            at(i + 1) += (at(i) - r) / _base;
            at(i) = r;
        }
    }
    int abscmp(const bigN &b) const {
        if (size() > b.size()) return 1;
        if (size() < b.size()) return -1;

```

```

        for (int i = int(size()) - 1; i >= 0; --i) {
            if (at(i) > b[i]) return 1;
            if (at(i) < b[i]) return -1;
        }
        return 0;
    }
    int cmp(const bigN &b) const {
        if (negative
            != b.negative) return negative ? -1 : 1;
        return negative ? -abscmp(b) : abscmp(b);
    }
    bool operator
        <(const bigN&b) const {return cmp(b) < 0;}
    bool operator
        >(const bigN&b) const {return cmp(b) > 0;}
    bool operator
        <=(const bigN&b) const {return cmp(b) <= 0;}
    bool operator
        >=(const bigN&b) const {return cmp(b) >= 0;}
    bool operator==(const bigN&b) const {return !cmp(b);}
    bool operator
        !=(const bigN&b) const {return cmp(b) != 0;}
    bigN abs() const {
        bigN res = *this;
        return res.negative = 0, res;
    }
    bigN operator-() const {
        bigN res = *this;
        return res.negative = !negative, res.trim(), res;
    }
    bigN operator+(const bigN &b) const {
        if (negative) return -(-(*this) + (-b));
        if (b.negative) return *this - (-b);
        bigN res = *this;
        if (b.size() > size()) res.resize(b.size());
        for (size_t
            i = 0; i < b.size(); ++i) res[i] += b[i];
        return res.carry(), res.trim(), res;
    }
    bigN operator-(const bigN &b) const {
        if (negative) return -(-(*this) - (-b));
        if (b.negative) return *this + (-b);
        if (abscmp(b) < 0) return -(b - (*this));
        bigN res = *this;
        if (b.size() > size()) res.resize(b.size());
        for (size_t
            i = 0; i < b.size(); ++i) res[i] -= b[i];
        return res.carry(), res.trim(), res;
    }
    bigN convert_base
        (int old_width, int new_width) const {
        vector<
            long long> p(max(old_width, new_width) + 1, 1);
        for (size_t
            i = 1; i < p.size(); ++i) p[i] = p[i - 1] * 10;
        bigN ans;
        long long cur = 0;
        int cur_id = 0;
        for (size_t i = 0; i < size(); ++i) {
            cur += at(i) * p[cur_id];
            cur_id += old_width;
            while (cur_id >= new_width) {
                ans.push_back(cur % p[new_width]);
                cur /= p[new_width];
                cur_id -= new_width;
            }
        }
        return ans.push_back(cur), ans.trim(), ans;
    }
    bigN karatsuba(const bigN &b) const {
        bigN res; res.resize(size() * 2);
        if (size() <= 32) {
            for (size_t i = 0; i < size(); ++i)
                for (size_t j = 0; j < size(); ++j)
                    res[i + j] += at(i) * b[j];
            return res;
        }
        size_t k = size() / 2;
        bigN a1(begin(), begin() + k);
        bigN a2(begin() + k, end());
        bigN b1(b.begin(), b.begin() + k);
        bigN b2(b.begin() + k, b.end());
        bigN a1b1 = a1.karatsuba(b1);
        bigN a2b2 = a2.karatsuba(b2);
        for (size_t i = 0; i < k; ++i) a2[i] += a1[i];
        for (size_t i = 0; i < k; ++i) b2[i] += b1[i];

```



```

bigN r = a2.karatsuba(b2);
for (size_t i = 0; i < a1b1.size(); ++i) r[i] -= a1b1[i];
for (size_t i = 0; i < a2b2.size(); ++i) r[i] -= a2b2[i];
for (size_t i = 0; i < r.size(); ++i) res[i + k] += r[i];
for (size_t i = 0; i < a1b1.size(); ++i) res[i] += a1b1[i];
for (size_t i = 0; i < a2b2.size(); ++i) res[i + size()] += a2b2[i];
return res;
}
bigN operator*(const bigN &b) const {
    const static int mul_base
        = 1000000, mul_width = log10(mul_base);
    bigN A = convert_base(width, mul_width);
    bigN B = b.convert_base(width, mul_width);
    int n = max(A.size(), B.size());
    while (n & (n - 1)) ++n;
    A.resize(n), B.resize(n);
    bigN res = A.karatsuba(B);
    res.negative = negative != b.negative;
    res.carry(mul_base);
    res = res.convert_base(mul_width, width);
    return res.trim(), res;
}
bigN operator*(long long b) const {
    bigN res = *this;
    if (b < 0) res.negative = !negative, b = -b;
    for (size_t i = 0, is = 0; i < res.size() || is; ++i) {
        if (i == res.size()) res.push_back(0);
        long long a = res[i] * b + is;
        is = a / base;
        res[i] = a % base;
    }
    return res.trim(), res;
}
bigN operator/(const bigN &b) const {
    int norm = base / (b.back() + 1);
    bigN x = abs() * norm;
    bigN y = b.abs() * norm;
    bigN q, r;
    q.resize(x.size());
    for (int i = int(x.size()) - 1; i >= 0; --i) {
        r = r * base + x[i];
        int s1 = r.size() <= y.size() ? 0 : r[y.size()];
        int s2
            = r.size() < y.size() ? 0 : r[y.size() - 1];
        int d = (ll(base) * s1 + s2) / y.back();
        r = r - y * d;
        while (r.negative) r = r + y, --d;
        q[i] = d;
    }
    q.negative = negative != b.negative;
    return q.trim(), q;
}
bigN operator%(const bigN &b) const {
    return *this - (*this / b) * b;
}
friend istream& operator>>(istream &ss, bigN &b) {
    string s;
    return ss >> s, b = s, ss;
}
friend
    ostream& operator<<(ostream &ss, const bigN &b) {
        if (b.negative) ss << '-';
        ss << (b.empty() ? 0 : b.back());
        for (int i = int(b.size()) - 2; i >= 0; --i)
            ss << setw(width) << setfill('0') << b[i];
        return ss;
    }
template<typename T>
operator T() {
    stringstream ss;
    ss << *this;
    T res;
    return ss >> res, res;
}
};

```

6.2 Chinese Remainder

```

int solve(int n, vector<int> &a, vector<int> &m){
    int M = 1;
    for(auto i : m) M *= i;

```

```

    int ans = 0;
    for(int i = 0; i < n; ++i){
        int m1 = M / m[i], m2 = extgcd(m1, m[i]).X;
        ans += (a[i] * m1 * m2) % M;
    }
    ans = ans % M + M;
    ans %= M;
    return ans;
}

```

6.3 Extgcd

```

pair<ll, ll> extgcd(ll a, ll b) {
    if (b == 0) return {1, 0};
    auto [xp, yp] = extgcd(b, a % b);
    return {yp, xp - a / b * yp};
}

```

6.4 FFT

```

// Remember not to output -0
/*
    polynomial multiply:
    DFT(a, len); DFT(b, len);
    for(int i=0;i<len;i++) c[i] = a[i]*b[i];
    iDFT(c, len);
    (len must be 2^k and >= 2*(max(a, b)))
    Hand written Cplx would be 2x faster
*/
Cplx omega[2][N];
void init_omega(int n) {
    static constexpr llf PI = acos(-1);
    const llf arg = (PI + PI) / n;
    for (int i = 0; i < n; ++i)
        omega[0][i] = {cos(arg * i), sin(arg * i)};
    for (int i = 0; i < n; ++i)
        omega[1][i] = conj(omega[0][i]);
}
void tran(Cplx arr[], int n, Cplx omg[]) {
    for (int i = 0, j = 0; i < n; ++i) {
        if (i > j) swap(arr[i], arr[j]);
        for (int l = n >> 1; (j ^= l) < l; l >>= 1);
    }
    for (int l = 2; l <= n; l <= 1) {
        int m = l >> 1;
        for (auto p = arr; p != arr + n; p += l) {
            for (int i = 0; i < m; ++i) {
                Cplx t = omg[n / l * i] * p[m + i];
                p[m + i] = p[i] - t; p[i] += t;
            }
        }
    }
}
void DFT(Cplx arr[], int n) {tran(arr, n, omega[0]);}
void iDFT(Cplx arr[], int n) {
    tran(arr, n, omega[1]);
    for (int i = 0; i < n; ++i) arr[i] /= n;
}

```

6.5 Gauss Elimination

```

#include <bits/stdc++.h>

std::bitset<1000> a[500];

int main() {
    int n; std::cin >> n;
    for (int i = 0; i < n; ++i) {
        for (int j = 0, t; j < n; ++j)
            std::cin >> t, a[i][j] = t;
        a[i][i + n] = 1;
    }
    for (int i = 0; i < n; ++i) {
        int t;
        for (t = i; t < n; ++t) if (a[t][i]) break;
        if (t == n) return std::cout << "-1\n", 0;
        std::swap(a[i], a[t]);
        for (int j
            = i + 1; j < n; ++j) if (a[j][i]) a[j] ^= a[i];
    }
    for (int i = n - 1; i >= 0; --i)
        for (int j = i - 1; j >= 0; --j)
            if (a[j][i]) a[j] ^= a[i];
    for (int i = 0; i < n; ++i) {
        std::vector<int> ans;
        for (int j = n; j < 2 *
            n; ++j) if (a[i][j]) ans.push_back(j - n + 1);
    }
}

```



```

    for (size_t j = 0; j < ans.size(); ++j)
        std::cout << ans[j] << " \n"[j == ans.size()];
}
return 0;
}

```

6.6 Gauss Elimination2

```

using ll = long long;
const ll mod = 998244353;
ll fp(ll a, ll b) {
    ll ret = 1;
    for (; b; b >>= 1, a = a * a % mod)
        if (b & 1) ret = ret * a % mod;
    return ret;
}
vector<ll> gauss_elimination
(vector<vector<ll>>& a) { // n * (n+1)
    // if a[i][j] < 0, a[i][j] += mod
    int n = a.size();
    bool swp = 0;
    for (int i = 0; i < n; i++) {
        for (int k = i; k < n; k++) {
            if (a[i][i] == 0 && a[k][i] != 0) {
                swap(a[i], a[k]), swp ^= 1; // det = -det
                break;
            }
        }
        if (a[i][i] == 0) return {}; // 0
        ll inv = fp(a[i][i], mod - 2);
        for (int j = 0; j < n; j++) {
            if (i != j) {
                ll tmp = a[j][i] * inv % mod;
                for (int k = i; k <= n; k++)
                    a[j][k] = (a[j][k] - tmp * a[i][k] % mod + mod) % mod;
            }
        }
    }
    // general solution
    vector<ll> ans(n);
    for (int i = 0; i < n; i++)
        ans[i] = a[i][n] * fp(a[i][i], mod - 2) % mod;
    return ans;
    // det
    // ll ret = 1;
    // for (
    //     int i = 0; i < n; i++) ret = ret * a[i][i] % mod;
    // return swp ? mod - ret : ret;
}

```

6.7 Karatsuba

```

const ll base = 10000000;
void karatsuba(const vector<ll>
    & f, const vector<ll>& g, vector<ll>& c, int n) {
    if (n <= 32) {
        for (int i = 0; i < n; i++)
            for (int j = 0; j < n; j++)
                c[i + j] += f[i] * g[j];
        return;
    }
    vector<ll> f1(n / 2), f2(n / 2), g1(n / 2), g2(n / 2);
    copy(f.begin(), f.begin() + n / 2, f1.begin());
    copy(f.begin() + n / 2, f.end(), f2.begin());
    copy(g.begin(), g.begin() + n / 2, g1.begin());
    copy(g.begin() + n / 2, g.end(), g2.begin());
    vector<ll> t1(n), t2(n), t3(n);
    karatsuba(f1, g1, t1, n / 2);
    karatsuba(f2, g2, t2, n / 2);
    for (int i = 0; i < n / 2; i++) f1[i] += f2[i];
    for (int i = 0; i < n / 2; i++) g1[i] += g2[i];
    karatsuba(f1, g1, t3, n / 2);
    for (int i = 0; i < n; i++) t3[i] -= t1[i] + t2[i];
    for (int i = 0; i < n; i++)
        c[i] += t1[i], c[i + n] += t2[i], c[i + n / 2] += t3[i];
}
void mul(const vector<ll>& a, const vector<ll>& b, vector<ll>& c) {
    int n = a.size(), m = b.size(), t = max(n, m), p = 1;
    while (p < t) p <= 1;
    vector<ll> aa(p), bb(p);
    copy(a.begin(), a.end(), aa);
    copy(b.begin(), b.end(), bb);
    c.assign(p << 1, 0), karatsuba(aa, bb, c, p);
}

```

```

p = n + m - 1;
for (int i = 0; i < p; i++)
    c[i + 1] += c[i] / base, c[i] %= base;
if (c[p]) p++;
c.resize(p);
}

```

6.8 Linear Sieve

```

vector<bool> isp;
vector<int> p;
void sieve(int n) {
    p.clear(), isp.assign(n + 1, 1);
    isp[0] = isp[1] = 0;
    for (int i = 2; i <= n; i++) {
        if (isp[i]) p.pb(i);
        for (const auto& x : p) {
            if (1LL * i * x > n) break;
            isp[i * x] = 0;
            if (i % x == 0) break;
        }
    }
}

```

6.9 Matrix

```

template <typename T> using vec = vector<T>;
template <typename T> using matrix = vec<vec<T>>;
constexpr int mod = 1e9 + 7;
template <typename T>
matrix<T>
operator*(const matrix<T>& a, const matrix<T>& b) {
    int n = a.size(), r = b.size(), m = b.front().size();
    matrix<T> ret(n, vec<T>(m));
    for (int i = 0; i < n; i++)
        for (int j = 0; j < m; j++)
            for (int k = 0; k < r; k++)
                ret[i][j] += 1LL *
                    a[i][k] * b[k][j] % mod, ret[i][j] %= mod;
    return ret;
}

```

6.10 Miller Rabin

```

using ll = long long;
ll mod_mul(ll a, ll b, ll m) {
    a %= m, b %= m;
    ll y = (ll)((
        double)a * b / m + 0.5); // fast for m < 2^58
    ll r = (a * b - y * m) % m;
    return r < 0 ? r + m : r;
}
template <typename T>
T pow(T a, T b, T mod) { // a^b % mod
    T ans = 1;
    for (; b; a = mod_mul(a, a, mod), b >>= 1)
        if (b & 1) ans = mod_mul(ans, a, mod);
    return ans;
}
int sprp[3] = {2, 7, 61}; // range of int
int llsp[7] = {2, 325, 9375, 28178, 450775,
    9780504, 1795265022}; // range of unsigned ll
template <typename T>
bool isprime(T n, int *sprp, int num) {
    if (n == 2) return 1;
    if (n < 2 || n % 2 == 0) return 0;
    int t = 0;
    T u = n - 1;
    for (; u % 2 == 0; ++t) u >>= 1;
    for (int i = 0; i < num; ++i) {
        T a = sprp[i] % n;
        if (a == 0 || a == 1 || a == n - 1) continue;
        T x = pow(a, u, n);
        if (x == 1 || x == n - 1) continue;
        for (int j = 1; j < t; ++j) {
            x = mod_mul(x, x, n);
            if (x == 1) return 0;
            if (x == n - 1) break;
        }
        if (x == n - 1) continue;
        return 0;
    }
    return 1;
}

```

6.11 Mobius

```
int mu[MAXN], lp[MAXN];
void build() {
    mu[1] = 1;
    FOR (i, 2, MAXN - 1) {
        if (!lp[i]) {
            for (int j = i; j < MAXN; j += i) {
                lp[j] = i;
            }
            if (i / lp[i] % lp[i])
                mu[i] = -mu[i / lp[i]];
        }
    }
}
```

6.12 NTT

```
const int G = 3, P = 998244353;
const int sval = 100, split = log10(sval);
int fpow(int x, int y) {
    int ret = 1;
    for (; y; y >>= 1, x = 1LL * x * x % P)
        if (y & 1) ret = 1LL * ret * x % P;
    return ret;
}
void ntt(vector<int>& x, int lim, int opt) {
    for (int i = 1, j = 0; i < lim; i++) {
        for (int k = lim >> 1; !(j ^= k) & k; k >>= 1);
        if (i < j) swap(x[i], x[j]);
    }
    for (int m = 2; m <= lim; m <= 1) {
        int k = m >> 1;
        int gn = fpow(G, (P - 1) / m);
        for (int i = 0; i < lim; i += m) {
            int g = 1;
            for (int j = 0; j < k; ++j, g = 1LL * g * gn % P) {
                int tmp = 1LL * x[i + j + k] * g % P;
                x[i + j + k] = (x[i + j] - tmp + P) % P;
                x[i + j] = (x[i + j] + tmp) % P;
            }
        }
    }
    if (opt == -1) {
        reverse(x.begin() + 1, x.begin() + lim);
        int inv = fpow(lim, P - 2);
        for (int i = 0; i < lim; ++i)
            x[i] = 1LL * x[i] * inv % P;
    }
}
vector<int> mul(vector<int> a, vector<int> b) {
    int lim = 1, n = a.size(), m = b.size();
    while (lim < (n + m - 1)) lim <= 1;
    a.resize(lim + 1), b.resize(lim + 1);
    ntt(a, lim, 1), ntt(b, lim, 1);
    for (int i = 0; i < lim; ++i)
        a[i] = 1LL * a[i] * b[i] % P;
    ntt(a, lim, -1);
    int len = 0;
    for (int i = 0; i < lim; ++i) {
        if (a[i] >= sval) len
            = i + 1, a[i + 1] += a[i] / sval, a[i] %= sval;
        if (a[i]) len = max(len, i);
    }
    while (a[len] >= sval) a[
        len + 1] += a[len] / sval, a[len] %= sval, len++;
    return a.resize(len + 1), a;
}
void print(const vector<int>& v) {
    if (!v.size()) return;
    cout << v.back();
    for (int i = v.size() - 2; ~i; --i)
        cout << setfill('0') << setw(split) << v[i];
    cout << '\n';
}
int main() {
    ios::sync_with_stdio(false), cin.tie(nullptr);
    string stra, strb;
    while (cin >> stra >> strb) {
        vector<int> a((stra.size() + split - 1) / split);
        vector<int> b((strb.size() + split - 1) / split);
        int tmp = stra.size();
        for (auto& i : a)
            tmp -= split, i = atoi(stra.substr(max
                (0, tmp), min(split, split + tmp)).data());
        tmp = strb.size();
```

```
        for (auto& i : b)
            tmp -= split, i = atoi(strb.substr(max
                (0, tmp), min(split, split + tmp)).data());
        print(mul(a, b));
    }
    return 0;
}
```

6.13 Pollard Rho

```
// does not work when n is prime
ll add(ll a, ll b, ll m) {return (a += b) > m ? a - m : a;}
ll mul(ll a, ll b, ll m) {
    a %= m, b %= m;
    ll y = (ll)((
        double)a * b / m + 0.5); /* fast for m < 2^58 */
    ll r = (a * b - y * m) % m;
    return r < 0 ? r + m : r;
}
ll f(ll x, ll mod) {return add(mul(x, x, mod), 1, mod);}
ll pollard_rho(ll n) {
    if (!(n & 1)) return 2;
    while (true) {
        ll y =
            2, x = rand() % (n - 1) + 1, res = 1, tmp = 1;
        for (int sz = 2; res == 1; sz *= 2, y = x) {
            for (int i = 0, t = 0; i < sz && res <= 1; i++, t++) {
                x = f(x, n); tmp = mul(tmp, abs(x - y), n);
                if (!(t & 31) || i + 1 == sz) res = __gcd(tmp, n), tmp = 1;
            }
        }
        if (res != 0 && res != n) return res;
    }
}
```

6.14 Primes

```
/* 12721 13331 14341 75577 123457 222557
   556679 999983 1097774749 1076767633 100102021
   999997771 1001010013 1000512343 987654361 999991231
   999888733 98789101 987777733 999991921 1010101333
   1010102101 1000000000039 1000000000000037
   2305843009213693951 4611686018427387847
   9223372036854775783 18446744073709551557 */
```

6.15 Primitive Root

```
// g is O(log^6 n).
// Runtime is O(ans * log
// phi(n) * log n), which is approximately O(log^8 n).
// #define int long long
int fp(int a, int b, int p) {
    int ret = 1;
    for (; b; b >>= 1, a = a * a % p)
        if (b & 1) ret = ret * a % p;
    return ret;
}
int generator(int p) {
    vector<int> fact;
    int phi = p - 1, n = phi;
    for (int i = 2; i * i <= n; ++i)
        if (n % i == 0) {
            fact.push_back(i);
            while (n % i == 0) n /= i;
        }
    if (n > 1) fact.push_back(n);
    for (int res = 2; res <= p; ++res) {
        bool ok = true;
        for (size_t i = 0; i < fact.size() && ok; ++i)
            ok &= fp(res, phi / fact[i], p) != 1;
        if (ok) return res;
    }
    return -1;
}
```

7 String

7.1 AC

```
struct ACautomata {
    struct Node {
        int cnt;
```

```

Node *go[26], *fail, *dic;
Node () {
    cnt = 0, fail = 0, dic = 0;
    memset(go, 0, sizeof(go));
}
} pool[1048576], *root;
int nMem;
Node* new_Node() {
    pool[nMem] = Node();
    return &pool[nMem++];
}
void init() { nMem = 0, root = new_Node(); }
void add(const string &str) { insert(root, str, 0); }
void insert(Node *cur, const string &str, int pos) {
    for (int i = pos; i < str.size(); i++) {
        if (!cur->go[str[i] - 'a'])
            cur->go[str[i] - 'a'] = new_Node();
        cur = cur->go[str[i] - 'a'];
    }
    cur->cnt++;
}
void make_fail() {
    queue<Node*> que;
    que.push(root);
    while (!que.empty()) {
        Node* fr = que.front(); que.pop();
        for (int i = 0; i < 26; i++) {
            if (fr->go[i]) {
                Node *ptr = fr->fail;
                while (ptr && !ptr->go[i]) ptr = ptr->fail;
                fr->go[i]->
                    fail = ptr = (ptr ? ptr->go[i] : root);
                fr->go[i]->dic = (ptr->cnt ? ptr : ptr->dic);
                que.push(fr->go[i]);
            }
        }
    }
}
} AC;

```

7.2 Hash

```

struct Hash {
    vector<ll> h;
    vector<ll> f;
    Hash(string s, int p = 127) {
        h.assign((int)s.size() + 1, 0);
        f.assign((int)s.size() + 1, 0);
        f[0] = 1;
        for (int i = 1; i <=
            (int)s.size(); ++i) f[i] = f[i - 1] * p % MOD;
        for (int i = 1; i <= (int)s.size(); ++i)
            h[i] = (h[i - 1] * p + s[i - 1]) % MOD;
    }
    int query(int l, int r) { // 0-based [l, r]
        if (r < l) return 0;
        return ((h[r
            + 1] - h[l] * f[r - l + 1]) % MOD + MOD) % MOD;
    }
};

```

7.3 KMP

```

#define pb push_back
const int N = 1e6 + 5;
int F[N];
vector<int> match(string A, string B) {
    vector<int> ans;
    F[0] = -1, F[1] = 0;
    for (int
        i = 1, j = 0; i < (int)B.size(); F[++i] = ++j) {
        if (B[i] == B[j]) F[i] = F[j]; // optimize
        while (j != -1 && B[i] != B[j]) j = F[j];
    }
    for (int i = 0, j = 0; i < (int)A.size(); ++i) {
        while (j != -1 && A[i] != B[j]) j = F[j];
        if (++j
            == (int)B.size()) ans.pb(i + 1 - j), j = F[j];
    }
    return ans;
}

```

7.4 Manacher

```

// P[2i] := max 2j+1: s[i-j, i+j] is palindromic
// P[2i-1] := max 2j: s[i-j, i+j] is palindromic
// maximal
// palindrome: s.substr((1 + i - P[i]) >> 1, P[i])

```

```

vector<unsigned> Manacher(const string &s) {
    unsigned L = 0, R = 1;
    vector<unsigned> P; P.reserve((s.size() << 1) - 1);
    P.push_back(1);
    for (unsigned i = 1; i < s.size(); ++i)
        for (int j = 0; j < 2; ++j) {
            if (i < R) {
                const int k = ((L + R - i) << 1) - j - 1;
                if (P[k] >> 1 <
                    R - i - j) { P.push_back(P[k]); continue; }
                L = (i << 1) - R + j;
            }
            else R = (L = i) + j;
            while (L > 0 &&
                R < s.size() && s[L - 1] == s[R]) {--L; ++R;}
            P.push_back(R - L);
        }
    return P;
}

```

7.5 SA

```

const int N = 2e5 + 5;

string s;
int sa[N], tmp[2][N], c[N], rk[N], h[N];
// lcp(sa[i], sa[j]) = min{h[k]} where i <= k <= j

void suffix_array() {
    int *x = tmp[0], *y = tmp[1], m = 256, n = s.size();
    fill(c, c + m, 0);
    for (int i = 0; i < n; i++) c[x[i] = s[i]]++;
    partial_sum(c, c + m, c);
    for (int i = n - 1; i >= 0; i--) sa[--c[x[i]]] = i;
    for (int k = 1; k < n; k <= 1) {
        fill(c, c + m, 0);
        for (int i = 0; i < n; i++) c[x[i]]++;
        partial_sum(c, c + m, c);
        int p = 0;
        for (int i = n - k; i < n; i++) y[p++] = i;
        for (int i = 0; i < n; i++)
            if (sa[i] >= k) y[p++] = sa[i] - k;
        for (int i
            = n - 1; i >= 0; i--) sa[--c[x[y[i]]]] = y[i];
        y[sa[0]] = p = 0;
        for (int i = 1; i < n; i++) {
            int a = sa[i], b = sa[i - 1];
            if (x[a] != x[b] || a + k >=
                n || b + k >= n || x[a + k] != x[b + k]) p++;
            y[sa[i]] = p;
        }
        if (n == p + 1) break;
        swap(x, y), m = p + 1;
    }
}

void LCP() {
    int n = s.size(), val = 0;
    for (int i = 0; i < n; i++) rk[sa[i]] = i;
    for (int i = 0; i < n; i++) {
        if (rk[i] == 0) h[rk[i]] = 0;
        else {
            if (val) val--;
            int p = sa[rk[i] - 1];
            while (val + i < n && val
                + p < n && s[val + i] == s[val + p]) val++;
            h[rk[i]] = val;
        }
    }
}
// cin >> s, suffix_array(), LCP();

```

7.6 SA2

```

void counting_sort
    (vector<int> &dest, const vector<int> &src
    , int bucket_count, function<int(const int)> f) {
    int *bucket_begin = new
        int[bucket_count], *buf = new int[src.size()];
    fill(bucket_begin, bucket_begin + bucket_count, 0);
    for (int i = 0; i < src.size(); ++i)
        if ((buf[i] = f(src[i])) + 1 < bucket_count)
            ++bucket_begin[buf[i] + 1];
    partial_sum(bucket_begin
        , bucket_begin + bucket_count, bucket_begin);
    dest.resize(src.size());
    for (int i = 0; i < src.size(); ++i)

```

```

    dest[bucket_begin[buf[i]]++] = src[i];
    delete[] bucket_begin; delete[] buf;
}
#define a 'a' // The smallest character in the alphabet
#define sz 26 // The size of the alphabet. The alphabet is [a, a + sz)
vector<int> suffix_array(const string &s) {
    vector<int> SA, sa(s.size());
    SA.reserve(s.size()); iota(sa.begin(), sa.end(), 0);
    counting_sort(SA,
        sa, sz, [&](const int &i) { return s[i] - a; });
    int *R = new int[SA.size()], *r = new int[SA.size()];
    R[SA[0]] = 1; // R = 0 is reserved for the empty string
    for (int i = 1; i < SA.size(); ++i)
        R[SA[i]] = s[SA[i] - 1] ? R[SA[i] - 1] : i + 1;
    int L = 1;
    while (L < s.size()) {
        auto R2 = [&](const int &i) {
            if (i + L < SA.size()) return R[i + L];
            return 0; // so that when L = 1, "a" is ordered before "aa"
        };
        counting_sort(sa, SA, SA.size() + 1, R2);
        counting_sort(SA, sa, SA.size(), [&](const int &i) { return R[i] - 1; });
        r[SA[0]] = 1;
        for (int i = 1; i < SA.size(); ++i)
            if (R[SA[i]] == R[SA[i - 1]] && R2(SA[i]) == R2(SA[i - 1]))
                r[SA[i]] = r[SA[i - 1]];
            else r[SA[i]] = i + 1;
        swap(R, r); L <= 1;
    }
    delete[] R; delete[] r; return SA;
}
#undef a
#undef sz

```

7.7 SAIS

```

const int N = 300010;
struct SA {
#define REP(i,n) for(int i=0;i<int(n);i++)
#define REP1(i,a,b) for(int i=(a);i<=int(b);i++)
    bool _t[N * 2]; int _s[N * 2], _sa[N * 2];
    int _c[N * 2], x[N], _p[N], _q[N * 2], hei[N], r[N];
    int operator [](int i) { return _sa[i]; }
    void build(int *s, int n, int m) {
        memcpy(_s, s, sizeof(int)*n);
        sais(_s, _sa, _p, _q, _t, _c, n, m); mkhei(n);
    }
    void mkhei(int n) {
        REP(i, n) r[_sa[i]] = i;
        hei[0] = 0;
        REP(i, n) if (r[i]) {
            int ans = i > 0 ? max(hei[r[i] - 1] - 1, 0) : 0;
            while (_s[i + ans] == _s[_sa[r[i] - 1] + ans]) ans++;
            hei[r[i]] = ans;
        }
    }
    void sais(int *s, int *sa,
        int *p, int *q, bool *t, int *c, int n, int z) {
        bool uniq = t[n - 1] = true, neq;
        int nn = 0, nmz = -1, *nsa = s + n, *ns = s + n, lst = -1;
#define MS0(x,n) memset((x),0,n*sizeof(*(x)))
#define MAGIC(XD) MS0(sa,n);\
        memcpy(x,c,sizeof(int)*z); XD;\
        memcpy(x+1,c,sizeof(int)*(z-1));\
        REP(i,n) if (sa[i]&&!t[sa[i]-1]) sa[x[s[sa[i]-1]]++] = sa[i]-1;\
        memcpy(x,c,sizeof(int)*z);\
        for(int i=n-1;i>=0;i--)
            if(sa[i]&&t[sa[i]-1]) sa[--x[s[sa[i]-1]]] = sa[i]-1;
            MS0(c, z); REP(i, n) uniq &= ++c[s[i]] < 2;
            REP(i, z - 1) c[i + 1] += c[i];
            if (uniq) { REP(i, n) sa[--c[s[i]]] = i; return; }
            for (int i = n - 2; i >= 0; i--)
                t[i] = (s[i] == s[i + 1] ? t[i + 1] : s[i] < s[i + 1]);
            MAGIC(REP1(i, 1, n - 1) if (t[i] && !t[i - 1]) sa[--x[s[i]]] = p[q[i] = nn++] = i);
            REP(i, n) if (sa[i] && t[sa[i]] && !t[sa[i] - 1]) {

```

```

                neq = lst < 0 || memcmp(s + sa[i], s + lst,
                    (p[q[sa[i]] + 1] - sa[i]) * sizeof(int));
                ns[q[lst = sa[i]]] = nmz += neq;
            }
            sais(ns, nsa, p + nn, q + n, t + n, c + z, nn, nmz + 1);
            MAGIC(for (int i = nn - 1; i >= 0; i--) sa[--x[s[p[nsa[i]]]]] = p[nsa[i]]);
        }
    } sa;
    int H[N], SA[N], RA[N];
    void suffix_array(int* ip, int len) {
        // should padding a zero in the back
        // ip is int array, len is array length
        // ip[0..n-1] != 0, and ip[len]=0
        ip[len++] = 0; sa.build(ip, len, 128);
        memcpy(H, sa.hei + 1, len < 2); memcpy(SA, sa._sa + 1, len < 2);
        for (int i = 0; i < len; i++) RA[i] = sa.r[i] - 1;
        // resulting height, sa array \in [0,len)
    }
}

```

7.8 Suffix Automaton

```

// O(n)
// find all suffix substrings in lexicographical order
#include <bits/stdc++.h>
class SuffixAutomaton {
public:
    static const int MAXN = 500 << 1;
    static const int MAXC = 26;
    struct Node {
        Node *next[MAXC], *pre;
        int step;
        Node() {
            pre = NULL, step = 0;
            memset(next, 0, sizeof(next));
        }
    } _mem[MAXN];
    int size;
    Node *root, *tail;
    void init() {
        size = 0;
        root = tail = newNode();
    }
    Node* newNode() {
        Node *p = &_mem[size++];
        *p = Node();
        return p;
    }
    int toIndex(char c) { return c - 'A'; }
    char toChar(int c) { return c + 'A'; }
    void add(char c, int len) {
        c = toIndex(c);
        Node *p, *q, *np, *nq;
        p = tail, np = newNode();
        np->step = len;
        for (; p && p->next[c] == NULL; p = p->pre)
            p->next[c] = np;
        tail = np;
        if (p == NULL) {
            np->pre = root;
        } else {
            if (p->next[c]->step == p->step + 1) {
                np->pre = p->next[c];
            } else {
                q = p->next[c], nq = newNode();
                *nq = *q;
                nq->step = p->step + 1;
                q->pre = np->pre = nq;
                for (; p && p->next[c] == q; p = p->pre)
                    p->next[c] = nq;
            }
        }
    }
    void build(const char *s) {
        init();
        for (int i = 0; s[i]; i++)
            add(s[i], i + 1);
    }
    void dfs(Node *u, int idx, char path[]) {
        for (int i = 0; i < MAXC; i++) {
            if (u->next[i]) {
                path[idx] = toChar(i);
                path[idx + 1] = '\0';
                puts(path);
                dfs(u->next[i], idx + 1, path);
            }
        }
    }
}

```

```

    }
}
}
void print() {
    char s[1024];
    dfs(root, 0, s);
}
} SAM;
int main() {
    char s[1024];
    while (scanf("%s", s) == 1) {
        SAM.build(s);
        SAM.print();
    }
    return 0;
}

```

7.9 Trie

```

int trie[MAXN * 31][2], node;
int tag[MAXN * 31];

void add(int x) {
    int now = 0;
    for (int i = 30; i >= 0; i--) {
        if (!trie[now][x >> i & 1]) = ++node;
        now = trie[now][x >> i & 1];
        tag[now]++;
    }
}

void del(int x) {
    int now = 0;
    for (int i = 30; i >= 0; i--) {
        now = trie[now][x >> i & 1];
        tag[now]--;
    }
}

int qry(int x) {
    int now = 0, res = 0;
    for (int i = 30; i >= 0; i--) {
        int id = (x >> i & 1) ^ 1;
        if (!tag[trie[now][id]]) id ^= 1;
        now = trie[now][id];
        res = res * 2 + id;
    }
    return res;
}

```

7.10 Z

```

void z_value(const char *s, int len, int *z) {
    z[0] = len;
    for (int i = 1, l = 0, r = 0; i < len; i++) {
        z[i] = i < r ? (i - l + z[i - l]) < z[l] ? z[i - l] : r - i : 0;
        while (i + z[i] < len && s[i + z[i]] == s[z[i]]) ++z[i];
        if (i + z[i] > r) l = i, r = i + z[i];
    }
}

```

8 Others

8.1 Aliens

```

/*
實際上如果這邊根本是平的，那我們只要讓二分歧找到最
小的P讓他的切點不超過K，那就保證了這條ㄅ會貼在上面
ll mid = (l+r < 0 ? (l + r) / 2 : (l + r + 1) / 2)
while(l < r){
    int m = (l + r) / 2;
    if(calc(m) <= K) r = m;
    else l = m + 1;
}
*/
#include <bits/stdc++.h>
#define F first
#define S second
#define int long long
using namespace std;

bool operator<(
    const pair<int, int> &a, const pair<int, int> &b) {
    return a.F < b.F or (a.F == b.F and a.S > b.S);
}
#define chmax(a, b) a = (a) < (b) ? (b) : (a)

```

```

int n, k;
int a[100005];
pair<int, int> dp[100005];
vector<int> last(100005, 0);

pair<int, int> DP(int penalty) {
    last.assign(100005, 0);
    pair<int, int> ans = {0, 0};
    int l = 0;
    for (int i = 1; i <= n; i++) {
        while (l < last[a[i]]) {
            l++;
            chmax(ans, dp[l]);
        }
        dp[i] = {ans.F + i - l - penalty, ans.S + 1};
        last[a[i]] = i;
    }
    while (l < n) {
        l++;
        chmax(ans, dp[l]);
    }
    return ans;
}

signed main() {
    ios_base::sync_with_stdio(0), cin.tie(0);
    cin >> n >> k;
    for (int i = 1; i <= n; i++) cin >> a[i];
    int l = -1, r = 2000000;
    while (l < r - 1) {
        int m = (l + r) / 2;
        pair<int, int> res = DP(m);
        if (res.S <= k) {
            r = m;
        } else {
            l = m;
        }
    }
    auto res = DP(r);
    cout << res.F + k * r << '\n';
}

```

8.2 Knapsack on Tree

```

#include <bits/stdc++.h>
#define F first
#define S second
#define pb push_back
#define all(x) begin(x), end(x)
#ifdef LOCAL
#define HEHE freopen("in.txt", "r", stdin);
#else
#define HEHE ios_base::sync_with_stdio(0), cin.tie(0);
#endif
using namespace std;

#define chmax(a, b) (a) = (a) < (b) ? (b) : (a)
#define chmin(a, b) (a) = (a) < (b) ? (a) : (b)

#define ll long long

#define FOR(i, a, b) for (int i = a; i <= b; i++)

int N, W, cur;
vector<int> w, v, sz;
vector<vector<int>> adj, dp;

void dfs(int x) {
    sz[x] = 1;
    for (int i : adj[x]) dfs(i), sz[x] += sz[i];
    cur++;
    // choose x
    FOR (i, w[x], W) {
        dp[cur][i] = dp[cur - 1][i - w[x]] + v[x];
    }
    // not choose x
    FOR (i, 0, W) {
        chmax(dp[cur][i], dp[cur - sz[x]][i]);
    }
}

signed main() {
    HEHE
    cin >> N >> W;
    adj.resize(N + 1);
    w.assign(N + 1, 0);
    v.assign(N + 1, 0);
    sz.assign(N + 1, 0);
}

```

```

    dp.assign(N + 2, vector<int>(W + 1, 0));
    FOR (i, 1, N) {
        int p; cin >> p;
        adj[p].pb(i);
    }
    FOR (i, 1, N) cin >> w[i];
    FOR (i, 1, N) cin >> v[i];
    dfs(0);
    cout << dp[N + 1][W] << '\n';
}

```

8.3 Mo

```

#include <bits/stdc++.h>
using namespace std;

const int N = 2e5 + 5, sqN = sqrt(N) + 5;
int a[N], ans[N], n, q, sz; // maybe need blk[sqN];

struct Query {
    int ql, qr, id;
    bool operator<(const Query& b) const {
        int aa = ql / sz, bb = b.ql / sz;
        if (aa != bb) return aa < bb;
        else return qr < b.qr;
    }
} Q[N];

void add(int x) {}
void sub(int x) {}
int qry(int k) {}

int main() {
    ios::sync_with_stdio(false), cin.tie(nullptr);
    cin >> n >> q, sz = sqrt(n);
    for (int i = 0; i < n; i++) cin >> a[i];
    for (int i = 0, ql, qr; i < q; i++)
        cin >> ql >> qr, Q[i] = {ql - 1, qr - 1, i};
    // Mo's algorithm
    sort(Q, Q + q); /* remember initialize arrays */
    int l = 0, r = -1;
    for (int i = 0; i < q; i++) {
        auto [ql, qr, k, id] = Q[i];
        while (r < qr) add(a[++r]);
        while (r > qr) sub(a[r--]);
        while (l < ql) sub(a[l++]);
        while (l > ql) add(a[--l]);
        ans[id] = qry(k);
    }
    for (int i = 0; i < q; i++) cout << ans[i] << '\n';
}

```

8.4 Mono Slope

```

struct Line{
    ll a, b;
    ll l = MIN, r = MAX;
    Line(ll a, ll b): a(a), b(b) {}
    ll operator()(ll x){
        return a * x + b;
    }
};

deque<Line> dq;

ll iceil(ll a, ll b){
    if(b < 0) a *= -1, b *= -1;
    if(a > 0) return (a + b - 1) / b;
    else return a / b;
}

ll intersect(Line a, Line b){
    return iceil(a.b - b.b, b.a - a.a);
}

void add(Line ln){
    while(!dq.empty()
        () && ln(dq.back().l) >= dq.back()(dq.back().l)){
        dq.pob();
    }
    if(dq.empty()){
        dq.eb(ln);
        return;
    }
    ll pos = intersect(ln, dq.back());
    if(pos > dq.back().r){
        if(dq.back().r != MAX){

```

```

            ln.l = dq.back().r + 1;
            dq.eb(ln);
        }
        return;
    }
    dq.back().r = pos - 1;
    ln.l = pos;
    dq.eb(ln);
}

```

```

ll query(ll x){
    while(dq.front().r < x) dq.pof();
    return dq.front()(x);
}

```

8.5 Partial Ordering

```

// O(n log^2 n)
#include <bits/stdc++.h>
using namespace std;
using ll = long long;
const int N = 1e5 + 5, M = 2e5 + 5;
int n, K, cnt, ans[N];
struct node {
    int x, y, z, v, ans, tag, id;
    node() { ans = tag = v = x = y = z = 0; }
    friend
        bool operator==(const node &a, const node &b) {
            return
                (a.x == b.x) && (a.y == b.y) && (a.z == b.z);
        }
} a[N], t[N];
bool cmp1(const node &a, const node &b) {
    if (a.x != b.x) return a.x < b.x;
    if (a.y != b.y) return a.y < b.y;
    return a.z < b.z;
}
bool cmp2(const node &a, const node &b) {
    if (a.y != b.y) return a.y < b.y;
    if (a.tag != b.tag) return a.tag < b.tag;
    return a.id < b.id;
}
#define lowbit(x) (x & -x)
int bit[M];
void add(int p, int x) {
    for (; p <= K; p += lowbit(p)) bit[p] += x;
}
int query(int p) {
    int ret = 0;
    for (; p; p -= lowbit(p)) ret += bit[p];
    return ret;
}

void CDQ(int l, int r) {
    if (l == r) return;
    int mid = (l + r) >> 1;
    CDQ(l, mid); CDQ(mid + 1, r);
    for (int i = l; i <= r; ++i) a[i].id = i;
    for (int i = l; i <= mid; ++i) a[i].tag = 0;
    for (int i = mid + 1; i <= r; ++i) a[i].tag = 1;
    sort(a + l, a + r + 1, cmp2);
    for (int i = l; i <= r; ++i) {
        if (!a[i].tag) add(a[i].z, a[i].v);
        else a[i].ans += query(a[i].z);
    }
    for (int i = l; i <= r; ++i)
        if (!a[i].tag) add(a[i].z, -a[i].v);
}

int main() {
    cin >> n >> K;
    for (int i = 1; i <= n; ++i)
        cin >> a[i].x >> a[i].y >> a[i].z, a[i].v = 1;
    sort(a + 1, a + n + 1, cmp1);
    cnt = 1;
    for (int i = 2; i <= n; ++i) {
        if (a[i] == a[cnt]) ++a[cnt].v;
        else a[++cnt] = a[i];
    }
    CDQ(1, cnt);
    // let ans[i] denote that the
    // number of (aj<=ai && bj<=bi && cj<=ci) for i != j
    for (int i = 1; i <= cnt; ++i)
        ans[a[i].ans + a[i].v - 1] += a[i].v;
    for (int i = 0; i < n; ++i) cout << ans[i] << '\n';
    return 0;
}

```

8.6 Xor Basis

```
int basis[20]
bool add(int x) {
    for (int i = 19; i >= 0; i--) {
        if (!(x >> i & 1)) continue;
        if (!basis[i]) {
            basis[i] = x;
            return true;
        }
        else x ^= basis[i];
    }
    return false;
}
// 維持 basis[i] 的最高位是 i
```






