YERDEĞİŞTİRME VEKTÖRÜNÜN MOD AÇILIMI

Birbirinden bağımsız N tane moda bağlı olarak yerdeğiştirme vektörü tanımlanabilir.

$$\mathbf{u} = \sum_{r=1}^{N} \phi_r q_r = \mathbf{\Phi q}$$

q_r mod koordinatları veya olağan koordinatlar.

$\mathbf{q}=\{q_1, q_2, \ldots, q_n\}^T$

ϕ_r bilindiğinde, belli bir **u** vektörüne karşı gelen q_r'leri bulmak için yukarıdaki denklem $\phi_n^T m$ ile çarpılır.

$$\phi_n^T \mathbf{mu} = \sum_{r=1}^{N} (\phi_n^T \mathbf{m} \phi_r) q_r$$

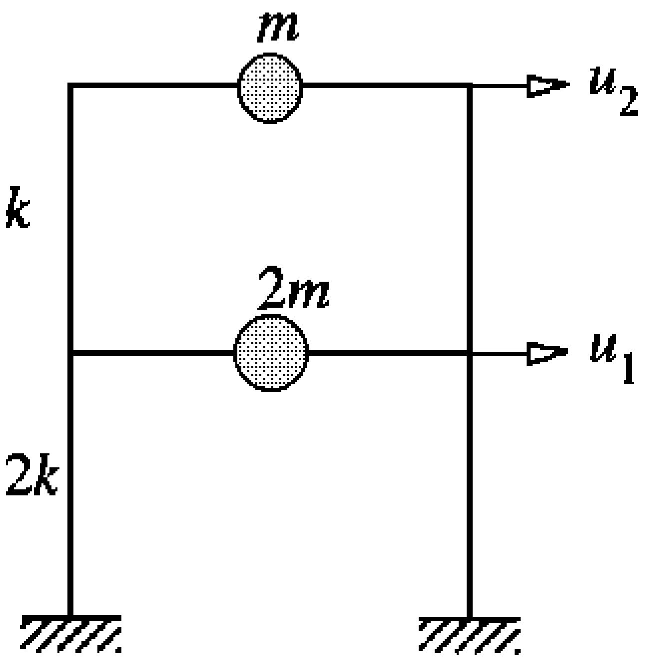
Diklik koşulundan dolayı, r=n olan terimlerin dışındaki tüm terimler sıfır olur.

$$\boldsymbol{\phi}_n^T \mathbf{m}\mathbf{u} = (\boldsymbol{\phi}_n^T \mathbf{m}\boldsymbol{\phi}_n) q_n$$

Skaler bir sayı

$$q_n = \frac{\boldsymbol{\phi}_n^T \mathbf{m}\mathbf{u}}{\boldsymbol{\phi}_n^T \mathbf{m}\boldsymbol{\phi}_n} = \frac{\boldsymbol{\phi}_n^T \mathbf{m}\mathbf{u}}{M_n}$$

Örnek: Şekildeki iki katlı kayma çerçevesi için $u=\{1,1\}^T$ biçimindeki yerdeğiştirme vektörünün mod açılımını bulunuz.

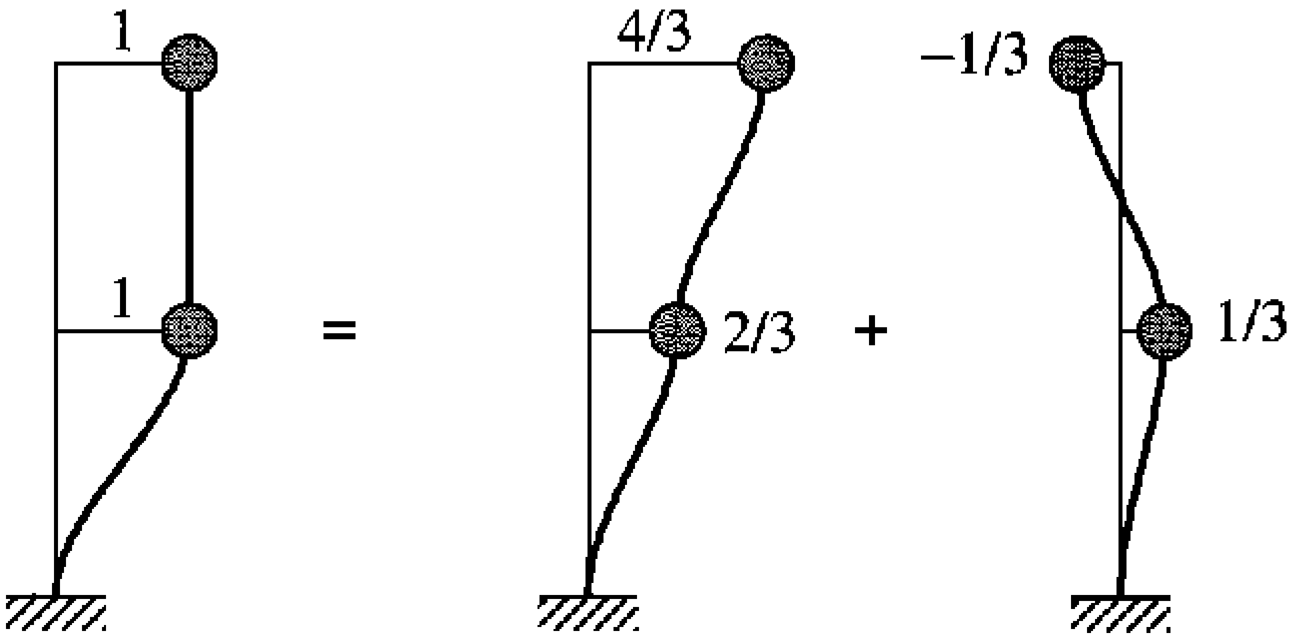


$$\phi_1 = \langle \tfrac{1}{2} \quad 1 \rangle^T$$

$$\phi_2 = \langle -1 \quad 1 \rangle^T$$

$$q_1 = \frac{\langle \frac{1}{2} \quad 1 \rangle \begin{bmatrix} 2m & \\ & m \end{bmatrix} \begin{Bmatrix} 1 \\ 1 \end{Bmatrix}}{\langle \frac{1}{2} \quad 1 \rangle \begin{bmatrix} 2m & \\ & m \end{bmatrix} \begin{Bmatrix} \frac{1}{2} \\ 1 \end{Bmatrix}} = \frac{2m}{3m/2} = \frac{4}{3}$$

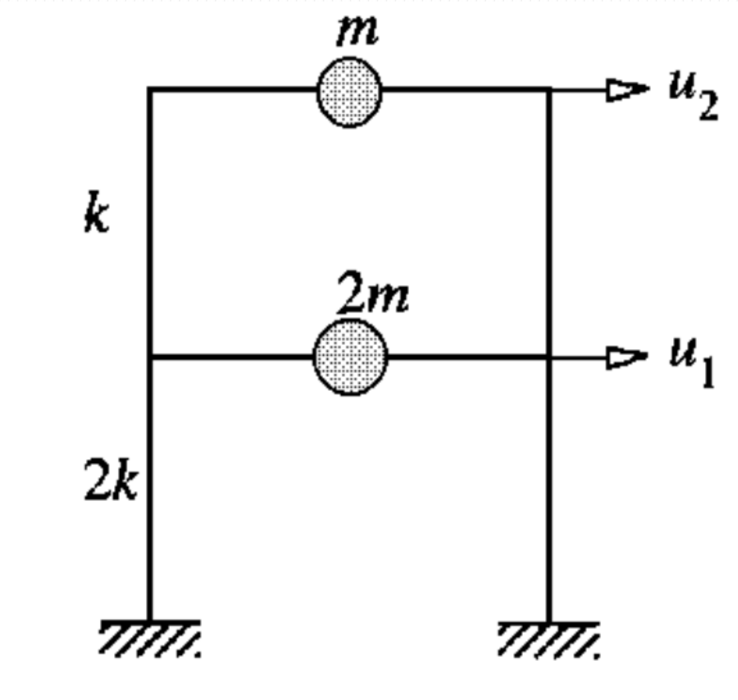
$$q_2 = \frac{\langle -1 \quad 1 \rangle \begin{bmatrix} 2m & \\ & m \end{bmatrix} \begin{Bmatrix} 1 \\ 1 \end{Bmatrix}}{\langle -1 \quad 1 \rangle \begin{bmatrix} 2m & \\ & m \end{bmatrix} \begin{Bmatrix} -1 \\ 1 \end{Bmatrix}} = \frac{-m}{3m} = -\frac{1}{3}$$



$$\mathbf{u} = (4/3)\phi_1 + (-1/3)\phi_2$$

SERBEST TİRTEŞİM

Serbest Titreşim Denklemlerinin Çözümü: Sönümsüz Sistemler



$$\mathbf{m}\ddot{\mathbf{u}} + \mathbf{k}\mathbf{u} = 0$$

$$\mathbf{u} = \mathbf{u}(0) \qquad \dot{\mathbf{u}} = \dot{\mathbf{u}}(0)$$

- Bu denklemin çözümü aranıyor.
- Önce özdeğer-özvektör problemi çözülüp
- Doğal frekanslar ve mod vektörleri bulunur.
- Genel çözüm (burada homojen çözüme eşittir) modal etkilerin toplamından bulunur.

$$\mathbf{u}(t) = \sum_{n=1}^{N} \phi_n (A_n \cos \omega_n t + B_n \sin \omega_n t)$$

A_n ve B_n (integrasyon sabitleri) başlangıç koşullarına bağlı (2N) tane sabittir.

$$\dot{\mathbf{u}}(t) = \sum_{n=1}^{N} \phi_n \omega_n (-A_n \sin \omega_n t + B_n \cos \omega_n t)$$

t=0 için $$\mathbf{u}(0) = \sum_{n=1}^{N} \phi_n A_n \qquad \dot{\mathbf{u}}(0) = \sum_{n=1}^{N} \phi_n \omega_n B_n$$

Başlangıç yerdeğiştirmesi ve hızları bilindiğine göre An ve Bn için N'şer adet cebirsel denklem elde edilir.

Aşağıdaki denklemler başlangıç deplasman ve hızlarının mod açılımları olarak ifade edilebilir.

$$\mathbf{u}(0) = \sum_{n=1}^{N} \phi_n q_n(0) \qquad \dot{\mathbf{u}}(0) = \sum_{n=1}^{N} \phi_n \dot{q}_n(0)$$

$$q_n = \frac{\phi_n^T \mathbf{m}\mathbf{u}}{\phi_n^T \mathbf{m} \phi_n} = \frac{\phi_n^T \mathbf{m}\mathbf{u}}{M_n}$$

Daha önce çıkarılan yandaki denklem kullanılarak başlangıç anındaki mod davranışları aşağıdaki gibi hesaplanır.

$$q_n(0) = \frac{\phi_n^T \mathbf{m}\mathbf{u}(0)}{M_n} \qquad \dot{q}_n(0) = \frac{\phi_n^T \mathbf{m}\dot{\mathbf{u}}(0)}{M_n}$$

$$\mathbf{u}(0) = \sum_{n=1}^{N} \phi_n A_n \qquad \dot{\mathbf{u}}(0) = \sum_{n=1}^{N} \phi_n \omega_n B_n$$

$$\mathbf{u}(0) = \sum_{n=1}^{N} \phi_n q_n(0) \qquad \dot{\mathbf{u}}(0) = \sum_{n=1}^{N} \phi_n \dot{q}_n(0)$$

Eşdeğer denklemlerdir.

Dolayısıyla ilk verilen denklem aşağıdaki gibi yazılabilir.

$$\mathbf{u}(t) = \sum_{n=1}^{N} \phi_n \left[q_n(0) \cos \omega_n t + \frac{\dot{q}_n(0)}{\omega_n} \sin \omega_n t \right]$$

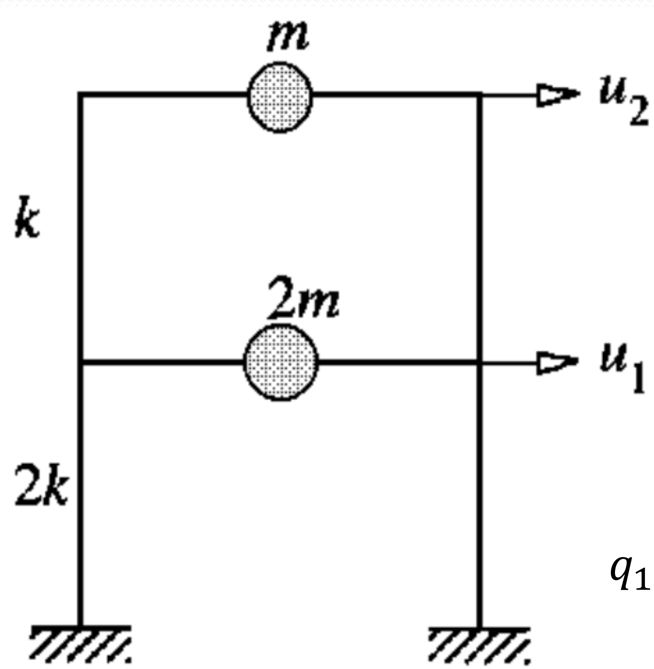
Alternatif olarak aşağıdaki gibi de yazılabilir.

$$\mathbf{u}(t) = \sum_{n=1}^{N} \phi_n q_n(t)$$

$$q_n(t) = q_n(0) \cos \omega_n t + \frac{\dot{q}_n(0)}{\omega_n} \sin \omega_n t$$

- Mod koordinatlarının zamana bağlı değişimini gösterir.
- Tek serbestlik dereceli sistemin serbest titreşim tepkisi ile benzeşir.
- **u**(t) değerleri modların nasıl ölçeklendirildiğinden bağımsız olsa da $q_n(t)$ değerleri bu ölçeklendirmeden bağımsız değildir.
- Frekanslar ve mod vektörleri hesaplandıktan sonra $q_n(0)$ v e $\dot{q}_n(0)$ değerleri iki sayfa önce verilen denklemlerden hesaplanır .
- **u** deplasman vektörü sistemin ayrık modlarının toplamı olarak hesaplanmış olur.

Örnek 1: Şekildeki iki katlı kayma çerçevesi için u(0)={1,2}^{T} biçimindeki başlangıç yerdeğiştirmeleri sonrasında serbest titreşimini hesaplayın.



$$\mathbf{u}(0) = \begin{Bmatrix} 1 \\ 2 \end{Bmatrix} \qquad \dot{\mathbf{u}}(0) = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

$$\omega_1 = \sqrt{\frac{k}{2m}} \qquad \omega_2 = \sqrt{\frac{2k}{m}} \qquad \phi_1 = \langle \tfrac{1}{2} \quad 1 \rangle^T \qquad \phi_2 = \langle -1 \quad 1 \rangle^T$$

$$q_1(0) = \frac{\emptyset_1^T \boldsymbol{mu}(\mathbf{0})}{M_1} = \frac{\{1/2 \quad 1\} \begin{bmatrix} 2m & 0 \\ 0 & m \end{bmatrix} \begin{Bmatrix} 1 \\ 2 \end{Bmatrix}}{\{1/2 \quad 1\} \begin{bmatrix} 2m & 0 \\ 0 & m \end{bmatrix} \begin{Bmatrix} 1/2 \\ 1 \end{Bmatrix}} = \frac{3m}{\frac{3}{2}m} = 2$$

$$q_2(0) = \frac{\emptyset_2^T \boldsymbol{mu}(\mathbf{0})}{M_2} = \frac{\{-1 \quad 1\} \begin{bmatrix} 2m & 0 \\ 0 & m \end{bmatrix} \begin{Bmatrix} 1 \\ 2 \end{Bmatrix}}{\{-1 \quad 1\} \begin{bmatrix} 2m & 0 \\ 0 & m \end{bmatrix} \begin{Bmatrix} -1 \\ 1 \end{Bmatrix}} = \frac{0}{3m} = 0$$

$$\dot{q}_1(0)=\frac{\emptyset_1^T \boldsymbol{m}\dot{\boldsymbol{u}}(\mathbf{0})}{M_1}=\frac{\{1/2 \quad 1\}\begin{bmatrix}2m & 0\\ 0 & m\end{bmatrix}\begin{Bmatrix}0\\0\end{Bmatrix}}{\{1/2 \quad 1\}\begin{bmatrix}2m & 0\\ 0 & m\end{bmatrix}\begin{Bmatrix}1/2\\1\end{Bmatrix}}=\frac{0}{\frac{3}{2}m}=0$$

$$\dot{q}_2(0)=\frac{\emptyset_2^T \boldsymbol{m}\dot{\boldsymbol{u}}(\mathbf{0})}{M_2}=\frac{\{-1 \quad 1\}\begin{bmatrix}2m & 0\\ 0 & m\end{bmatrix}\begin{Bmatrix}0\\0\end{Bmatrix}}{\{-1 \quad 1\}\begin{bmatrix}2m & 0\\ 0 & m\end{bmatrix}\begin{Bmatrix}-1\\1\end{Bmatrix}}=\frac{0}{3m}=0$$

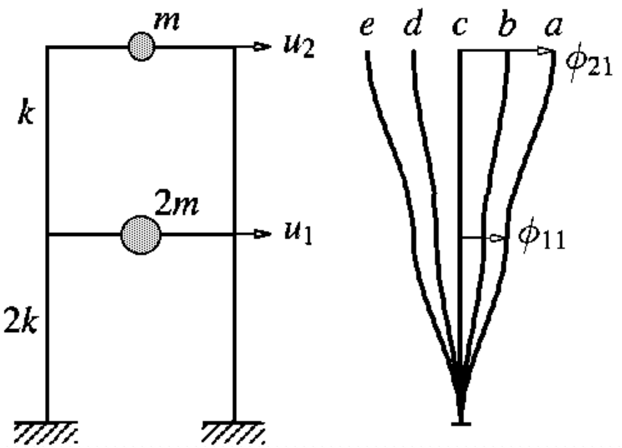
Modal koordinatlardaki çözümler aşağıdaki gibi olur.

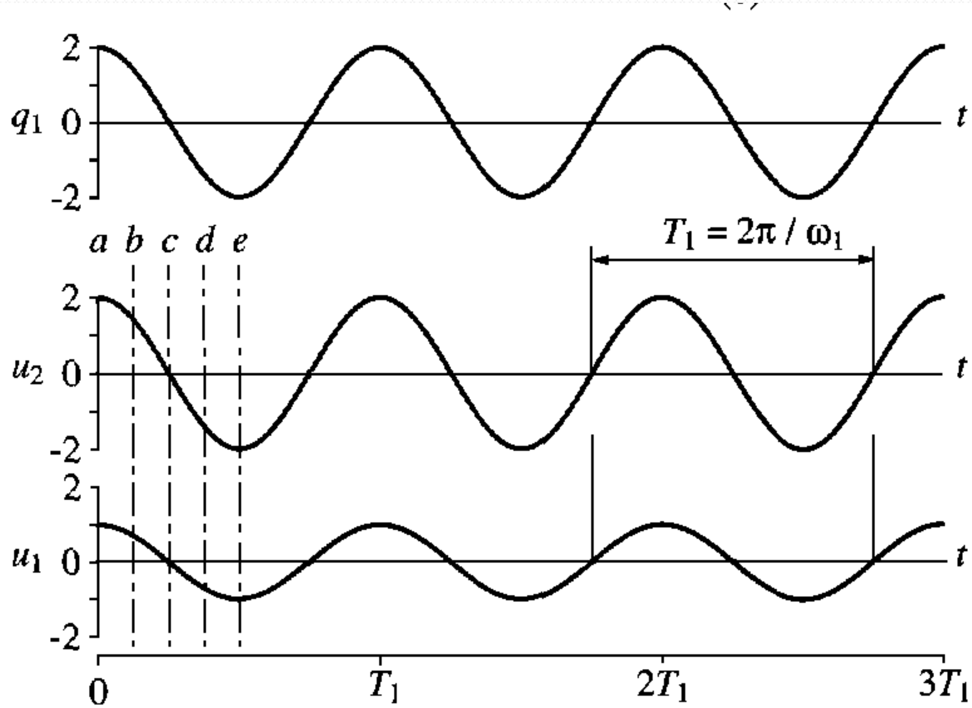
$$q_1(t)=q_1(0).Cos(\omega_1 t)+\frac{\dot{q}_1(0)}{\omega_1}.Sin(\omega_1 t)=2.Cos(\omega_1 t)+\frac{0}{\omega_1}.Sin(\omega_1 t)=2.Cos(\omega_1 t)$$

$$q_2(t)=q_2(0).Cos(\omega_2 t)+\frac{\dot{q}_2(0)}{\omega_2}.Sin(\omega_2 t)=0.Cos(\omega_2 t)+\frac{0}{\omega_2}.Sin(\omega_2 t)=0$$

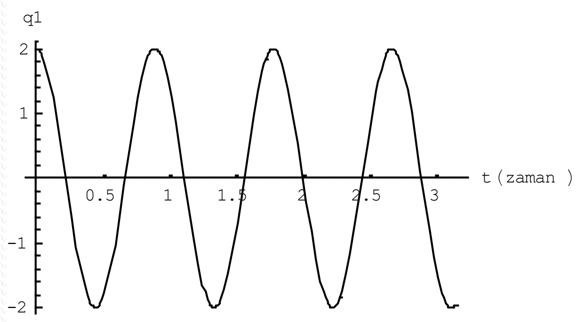
$$q_1(t) = 2\cos\omega_1 t \qquad q_2(t) = 0$$

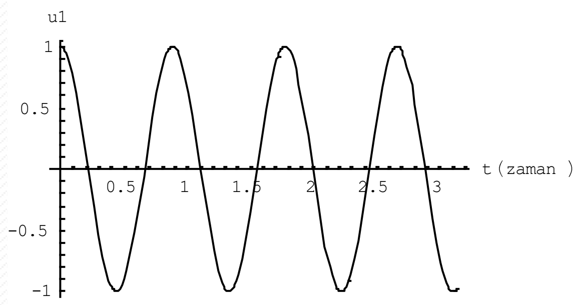
$$\begin{Bmatrix} u_1(t) \\ u_2(t) \end{Bmatrix} = \begin{Bmatrix} \frac{1}{2} \\ 1 \end{Bmatrix} 2\cos\omega_1 t = \begin{Bmatrix} 1 \\ 2 \end{Bmatrix} \cos\omega_1 t$$

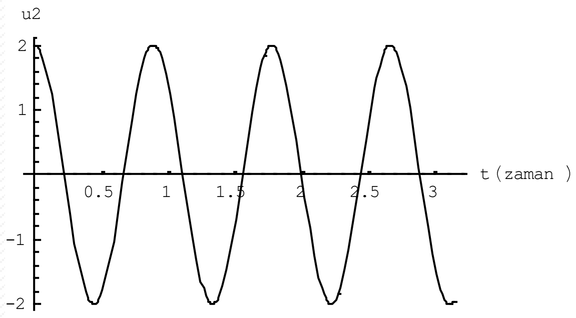




m=1, k=100 olması hali için grafikler







Örnek 2: Başlangıç değerleri aşağıdaki gibi olursa aynı problemi çözünüz.

$$u(0)=\begin{Bmatrix}0\\0\end{Bmatrix}\quad \dot{u}(0)=\begin{Bmatrix}10\\20\end{Bmatrix}$$

$$q_1(0)=\frac{\emptyset_1^T\boldsymbol{mu(0)}}{M_1}=\frac{\{1/2\quad 1\}\begin{bmatrix}2m & 0\\0 & m\end{bmatrix}\begin{Bmatrix}0\\0\end{Bmatrix}}{\{1/2\quad 1\}\begin{bmatrix}2m & 0\\0 & m\end{bmatrix}\begin{Bmatrix}1/2\\1\end{Bmatrix}}=\frac{0}{\frac{3}{2}m}=0$$

$$q_2(0)=\frac{\emptyset_2^T\boldsymbol{mu(0)}}{M_2}=\frac{\{-1\quad 1\}\begin{bmatrix}2m & 0\\0 & m\end{bmatrix}\begin{Bmatrix}0\\0\end{Bmatrix}}{\{-1\quad 1\}\begin{bmatrix}2m & 0\\0 & m\end{bmatrix}\begin{Bmatrix}-1\\1\end{Bmatrix}}=\frac{0}{3m}=0$$

$$\dot{q}_1(0)=\frac{\emptyset_1^T\boldsymbol{m\dot{u}(0)}}{M_1}=\frac{\{1/2\quad 1\}\begin{bmatrix}2m & 0\\0 & m\end{bmatrix}\begin{Bmatrix}10\\20\end{Bmatrix}}{\{1/2\quad 1\}\begin{bmatrix}2m & 0\\0 & m\end{bmatrix}\begin{Bmatrix}1/2\\1\end{Bmatrix}}=\frac{30m}{\frac{3}{2}m}=20$$

$$\dot{q}_2(0)=\frac{\emptyset_2^T\boldsymbol{m\dot{u}(0)}}{M_2}=\frac{\{-1\quad 1\}\begin{bmatrix}2m & 0\\0 & m\end{bmatrix}\begin{Bmatrix}10\\20\end{Bmatrix}}{\{-1\quad 1\}\begin{bmatrix}2m & 0\\0 & m\end{bmatrix}\begin{Bmatrix}-1\\1\end{Bmatrix}}=\frac{0}{3m}=0$$

Modal koordinatlardaki çözümler aşağıdaki gibi olur.

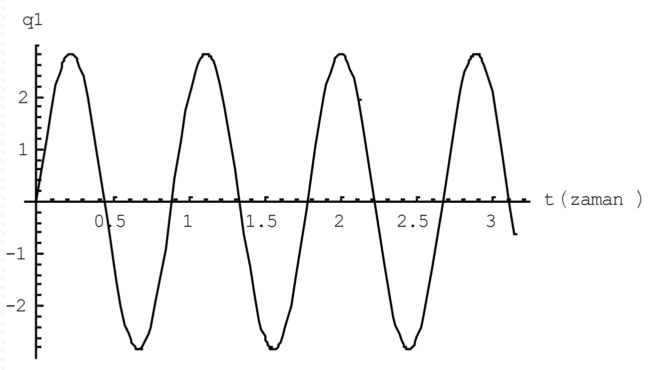
$$q_1(t) = q_1(0).Cos(\omega_1 t) + \frac{\dot{q}_1(0)}{\omega_1}.Sin(\omega_1 t) = 0.Cos(\omega_1 t) + \frac{20}{\omega_1}.Sin(\omega_1 t)$$
$$= \frac{20}{\omega_1}.Sin(\omega_1 t)$$

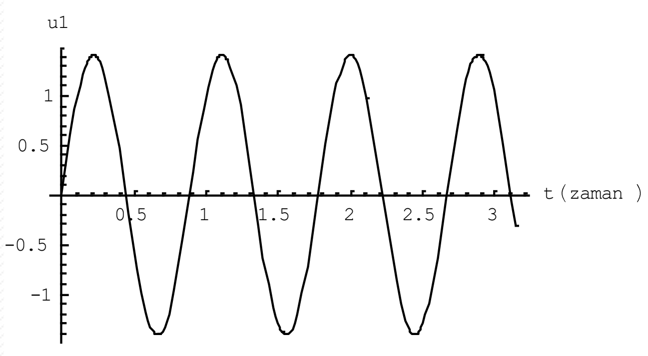
$$q_2(t) = q_2(0).Cos(\omega_2 t) + \frac{\dot{q}_2(0)}{\omega_2}.Sin(\omega_2 t) = 0.Cos(\omega_2 t) + \frac{0}{\omega_2}.Sin(\omega_2 t) = 0$$

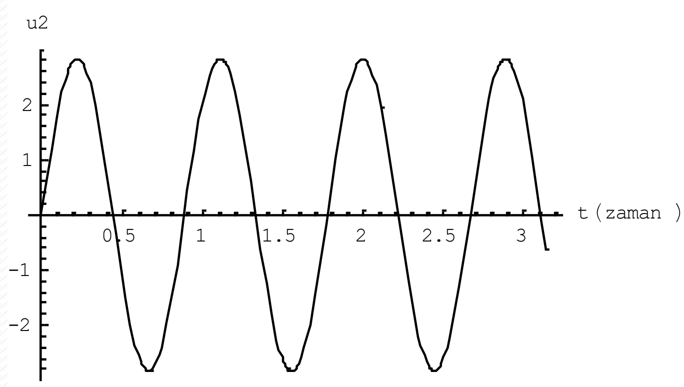
$$q_1(t) = \frac{20}{\omega_1}.Sin(\omega_1 t) \qquad\qquad q_2(t) = 0$$

$$\begin{Bmatrix} u_1(t) \\ u_2(t) \end{Bmatrix} = \emptyset_1 q_1 + \emptyset_2 q_2 = \begin{Bmatrix} 1/2 \\ 1 \end{Bmatrix} \frac{20}{\omega_1} Sin(\omega_1 t) = \begin{Bmatrix} 10 \\ 20 \end{Bmatrix} \frac{1}{\omega_1} Sin(\omega_1 t)$$

m=1, k=100 olması hali için grafikler







Örnek 3: Başlangıç değerleri aşağıdaki gibi olursa aynı problemi çözünüz.

$$u(0) = \begin{Bmatrix} 1 \\ 2 \end{Bmatrix} \quad \dot{u}(0) = \begin{Bmatrix} 10 \\ 20 \end{Bmatrix}$$

$$q_1(0) = \frac{\emptyset_1^T \boldsymbol{mu(0)}}{M_1} = \frac{\{1/2 \quad 1\} \begin{bmatrix} 2m & 0 \\ 0 & m \end{bmatrix} \begin{Bmatrix} 1 \\ 2 \end{Bmatrix}}{\{1/2 \quad 1\} \begin{bmatrix} 2m & 0 \\ 0 & m \end{bmatrix} \begin{Bmatrix} 1/2 \\ 1 \end{Bmatrix}} = \frac{0}{\frac{3}{2}m} = 2$$

$$q_2(0) = \frac{\emptyset_2^T \boldsymbol{mu(0)}}{M_2} = \frac{\{-1 \quad 1\} \begin{bmatrix} 2m & 0 \\ 0 & m \end{bmatrix} \begin{Bmatrix} 1 \\ 2 \end{Bmatrix}}{\{-1 \quad 1\} \begin{bmatrix} 2m & 0 \\ 0 & m \end{bmatrix} \begin{Bmatrix} -1 \\ 1 \end{Bmatrix}} = \frac{0}{3m} = 0$$

$$\dot{q}_1(0) = \frac{\emptyset_1^T \boldsymbol{m\dot{u}(0)}}{M_1} = \frac{\{1/2 \quad 1\} \begin{bmatrix} 2m & 0 \\ 0 & m \end{bmatrix} \begin{Bmatrix} 10 \\ 20 \end{Bmatrix}}{\{1/2 \quad 1\} \begin{bmatrix} 2m & 0 \\ 0 & m \end{bmatrix} \begin{Bmatrix} 1/2 \\ 1 \end{Bmatrix}} = \frac{30m}{\frac{3}{2}m} = 20$$

$$\dot{q}_2(0) = \frac{\emptyset_2^T \boldsymbol{m\dot{u}(0)}}{M_2} = \frac{\{-1 \quad 1\} \begin{bmatrix} 2m & 0 \\ 0 & m \end{bmatrix} \begin{Bmatrix} 10 \\ 20 \end{Bmatrix}}{\{-1 \quad 1\} \begin{bmatrix} 2m & 0 \\ 0 & m \end{bmatrix} \begin{Bmatrix} -1 \\ 1 \end{Bmatrix}} = \frac{0}{3m} = 0$$

$$q_1(t) = q_1(0).Cos(\omega_1 t) + \frac{\dot{q}_1(0)}{\omega_1}.Sin(\omega_1 t) = 2.Cos(\omega_1 t) + \frac{20}{\omega_1}.Sin(\omega_1 t)$$

$$q_2(t) = q_2(0).Cos(\omega_2 t) + \frac{\dot{q}_2(0)}{\omega_2}.Sin(\omega_2 t) = 0.Cos(\omega_2 t) + \frac{0}{\omega_2}.Sin(\omega_2 t) = 0$$

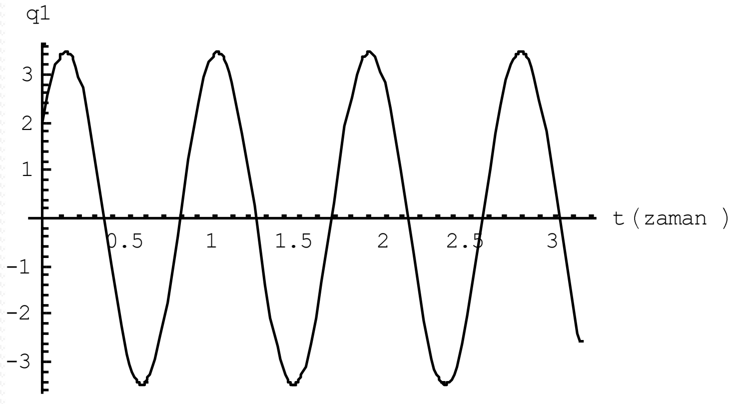
$$q_1(t) = 2.Cos(\omega_1 t) + \frac{20}{\omega_1}.Sin(\omega_1 t) \qquad q_2(t) = 0$$

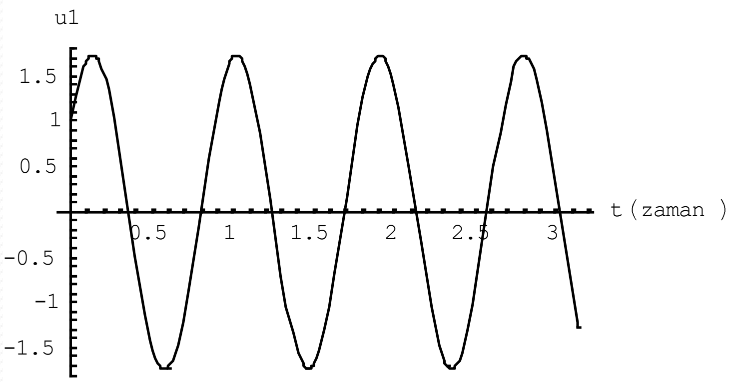
$$\begin{Bmatrix} u_1(t) \\ u_2(t) \end{Bmatrix} = \emptyset_1 q_1 + \emptyset_2 q_2 = \begin{Bmatrix} 1/2 \\ 1 \end{Bmatrix}\left(2.Cos(\omega_1 t) + \frac{20}{\omega_1}.Sin(\omega_1 t)\right) + \begin{Bmatrix} -1 \\ 1 \end{Bmatrix}(0)$$

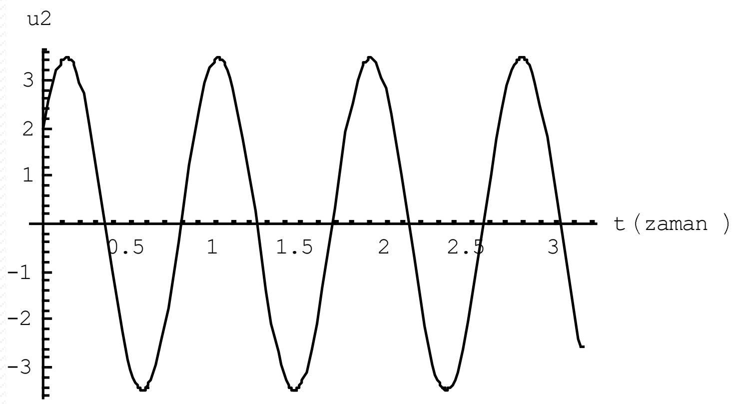
$$= \begin{Bmatrix} 1 \\ 2 \end{Bmatrix}.Cos(\omega_1 t) + \begin{Bmatrix} 10 \\ 20 \end{Bmatrix}\frac{1}{\omega_1}.Sin(\omega_1 t)$$

$$u_1(t) = 1.Cos(\omega_1 t) + 10\frac{1}{\omega_1}.Sin(\omega_1 t)$$

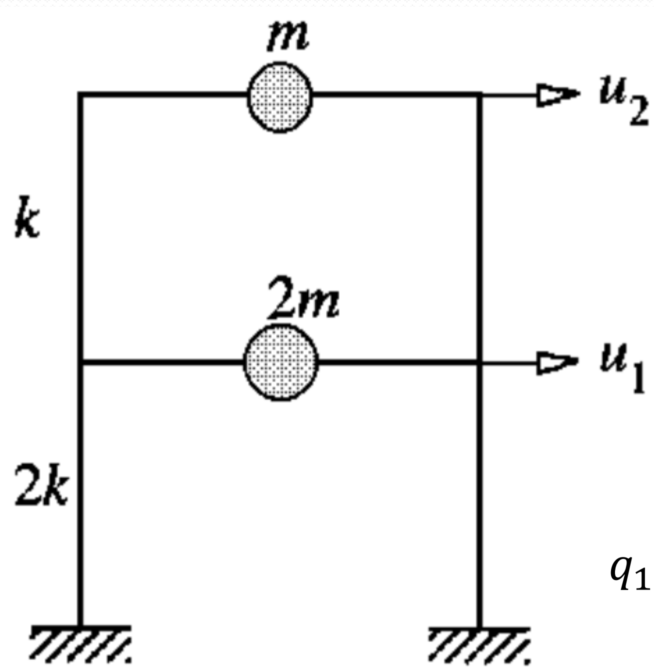
$$u_2(t) = 2.Cos(\omega_1 t) + 20\frac{1}{\omega_1}.Sin(\omega_1 t)$$


q1
3
2
1
-1
-2
-3
0.5
1
1.5
2
2.5
3
t (zaman)


u1
1.5
1
0.5
-0.5
-1
-1.5
0.5
1
1.5
2
2.5
3
t (zaman)


u2
3
2
1
-1
-2
-3
0.5
1
1.5
2
2.5
3
t (zaman)

Örnek 4: Başlangıç değerleri aşağıdaki gibi olursa aynı problemi çözünüz.



$$u(0)=\begin{Bmatrix}-1\\1\end{Bmatrix}\quad \dot{u}(0)=\begin{Bmatrix}0\\0\end{Bmatrix}$$

$$\omega_1=\sqrt{\frac{k}{2m}}\qquad \omega_2=\sqrt{\frac{2k}{m}}$$

$$\phi_1=\langle \tfrac{1}{2}\quad 1\rangle^T$$

$$\phi_2=\langle -1\quad 1\rangle^T$$

$$q_1(0)=\frac{\emptyset_1^T \boldsymbol{mu(0)}}{M_1}=\frac{\{1/2\quad 1\}\begin{bmatrix}2m & 0\\0 & m\end{bmatrix}\begin{Bmatrix}-1\\1\end{Bmatrix}}{\{1/2\quad 1\}\begin{bmatrix}2m & 0\\0 & m\end{bmatrix}\begin{Bmatrix}1/2\\1\end{Bmatrix}}=\frac{0}{\frac{3}{2}m}=0$$

$$q_2(0)=\frac{\emptyset_2^T \boldsymbol{mu(0)}}{M_2}=\frac{\{-1\quad 1\}\begin{bmatrix}2m & 0\\0 & m\end{bmatrix}\begin{Bmatrix}-1\\1\end{Bmatrix}}{\{-1\quad 1\}\begin{bmatrix}2m & 0\\0 & m\end{bmatrix}\begin{Bmatrix}-1\\1\end{Bmatrix}}=\frac{3m}{3m}=1$$

$$\dot{q}_1(0)=\frac{\emptyset_1^T \boldsymbol{m}\dot{\boldsymbol{u}}(\mathbf{0})}{M_1}=\frac{\{1/2 \quad 1\}\begin{bmatrix}2m & 0\\ 0 & m\end{bmatrix}\begin{Bmatrix}0\\0\end{Bmatrix}}{\{1/2 \quad 1\}\begin{bmatrix}2m & 0\\ 0 & m\end{bmatrix}\begin{Bmatrix}1/2\\1\end{Bmatrix}}=\frac{0}{\frac{3}{2}m}=0$$

$$\dot{q}_2(0)=\frac{\emptyset_2^T \boldsymbol{m}\dot{\boldsymbol{u}}(\mathbf{0})}{M_2}=\frac{\{-1 \quad 1\}\begin{bmatrix}2m & 0\\ 0 & m\end{bmatrix}\begin{Bmatrix}0\\0\end{Bmatrix}}{\{-1 \quad 1\}\begin{bmatrix}2m & 0\\ 0 & m\end{bmatrix}\begin{Bmatrix}-1\\1\end{Bmatrix}}=\frac{0}{3m}=0$$

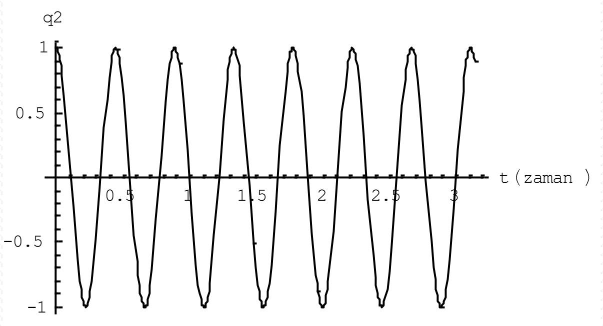
Modal koordinatlardaki çözümler aşağıdaki gibi olur.

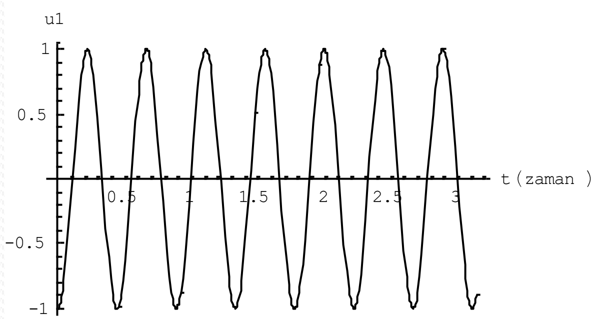
$$q_1(t)=q_1(0).Cos(\omega_1 t)+\frac{\dot{q}_1(0)}{\omega_1}.Sin(\omega_1 t)=0.Cos(\omega_1 t)+\frac{0}{\omega_1}.Sin(\omega_1 t)=0$$

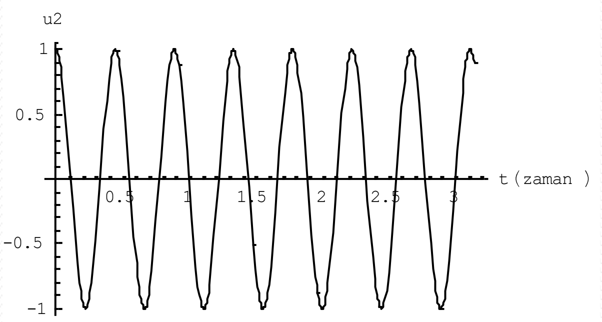
$$q_2(t)=q_2(0).Cos(\omega_2 t)+\frac{\dot{q}_2(0)}{\omega_2}.Sin(\omega_2 t)=1.Cos(\omega_2 t)+\frac{0}{\omega_2}.Sin(\omega_2 t)=1.Cos(\omega_2 t)$$

$$q_1(t) = 0 \qquad\qquad q_2(t) = 1.Cos(\omega_2 t)$$

$$\begin{Bmatrix} u_1(t) \\ u_2(t) \end{Bmatrix} = \emptyset_1 q_1 + \emptyset_2 q_2 = \begin{Bmatrix} 1/2 \\ 1 \end{Bmatrix}.(0) + \begin{Bmatrix} -1 \\ 1 \end{Bmatrix} 1.Cos(\omega_2 t) = \begin{Bmatrix} -1 \\ 1 \end{Bmatrix} Cos(\omega_2 t)$$

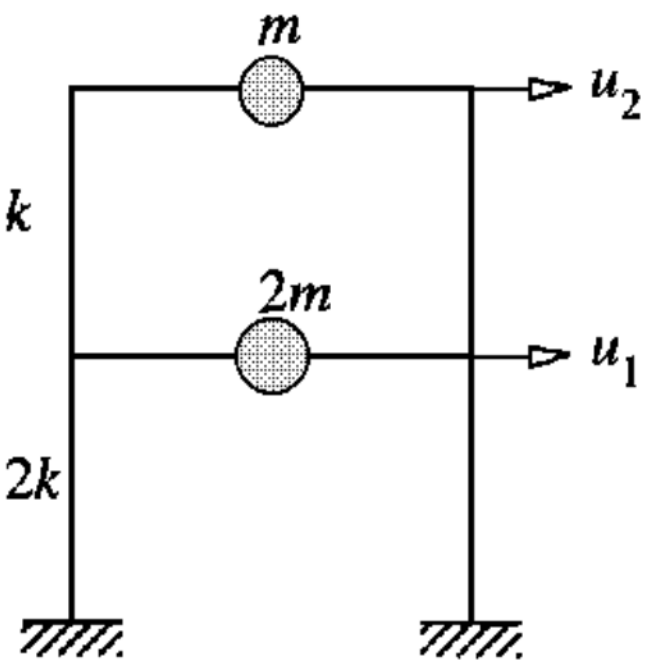






Örnek 4: Başlangıç değerleri aşağıdaki gibi olursa aynı problemi çözünüz.

$$u(0)=\begin{Bmatrix}-1/2\\ 2\end{Bmatrix}\quad \dot{u}(0)=\begin{Bmatrix}0\\ 0\end{Bmatrix}$$



$$\omega_1=\sqrt{\frac{k}{2m}}\qquad \omega_2=\sqrt{\frac{2k}{m}}\qquad \phi_1=\langle \tfrac{1}{2}\quad 1\rangle^T$$

$$\phi_2=\langle -1\quad 1\rangle^T$$

$$q_1(0)=\frac{\emptyset_1^T\boldsymbol{mu(0)}}{M_1}=\frac{\{1/2\quad 1\}\begin{bmatrix}2m & 0\\ 0 & m\end{bmatrix}\begin{Bmatrix}-1/2\\ 2\end{Bmatrix}}{\{1/2\quad 1\}\begin{bmatrix}2m & 0\\ 0 & m\end{bmatrix}\begin{Bmatrix}1/2\\ 1\end{Bmatrix}}=\frac{\frac{3}{2}m}{\frac{3}{2}m}=1$$

$$q_2(0)=\frac{\emptyset_2^T\boldsymbol{mu(0)}}{M_2}=\frac{\{-1\quad 1\}\begin{bmatrix}2m & 0\\ 0 & m\end{bmatrix}\begin{Bmatrix}-1/2\\ 2\end{Bmatrix}}{\{-1\quad 1\}\begin{bmatrix}2m & 0\\ 0 & m\end{bmatrix}\begin{Bmatrix}-1\\ 1\end{Bmatrix}}=\frac{3m}{3m}=1$$

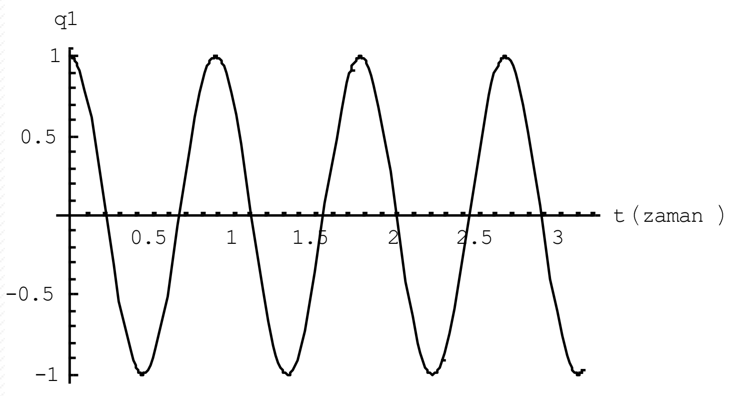
$$\dot{q}_1(0)=\frac{\emptyset_1^T\boldsymbol{m\dot{u}(0)}}{M_1}=\frac{\{1/2\quad 1\}\begin{bmatrix}2m & 0\\ 0 & m\end{bmatrix}\begin{Bmatrix}0\\ 0\end{Bmatrix}}{\{1/2\quad 1\}\begin{bmatrix}2m & 0\\ 0 & m\end{bmatrix}\begin{Bmatrix}1/2\\ 1\end{Bmatrix}}=\frac{0}{\frac{3}{2}m}=0$$

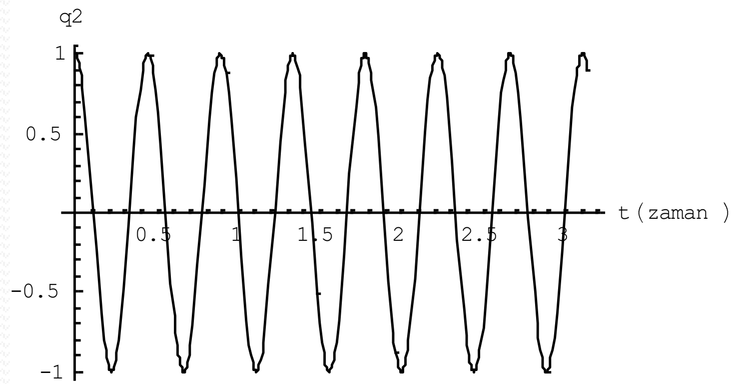
$$\dot{q}_2(0)=\frac{\emptyset_2^T\boldsymbol{m\dot{u}(0)}}{M_2}=\frac{\{-1\quad 1\}\begin{bmatrix}2m & 0\\ 0 & m\end{bmatrix}\begin{Bmatrix}0\\ 0\end{Bmatrix}}{\{-1\quad 1\}\begin{bmatrix}2m & 0\\ 0 & m\end{bmatrix}\begin{Bmatrix}-1\\ 1\end{Bmatrix}}=\frac{0}{3m}=0$$

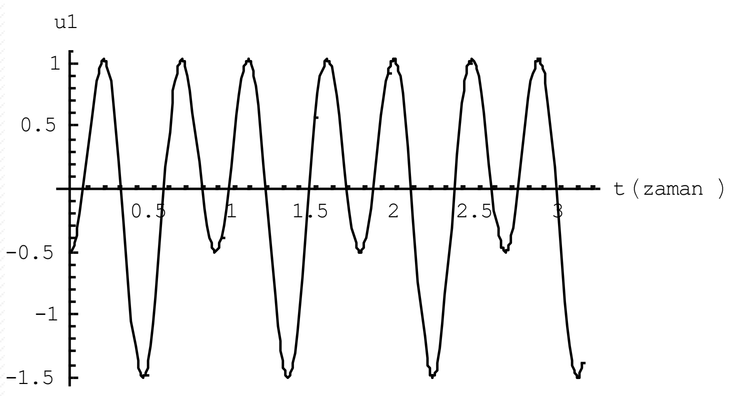
$$q_1(t) = q_1(0).Cos(\omega_1 t) + \frac{\dot{q}_1(0)}{\omega_1}.Sin(\omega_1 t) = 1.Cos(\omega_1 t) + \frac{0}{\omega_1}.Sin(\omega_1 t) = 1.Cos(\omega_1 t)$$

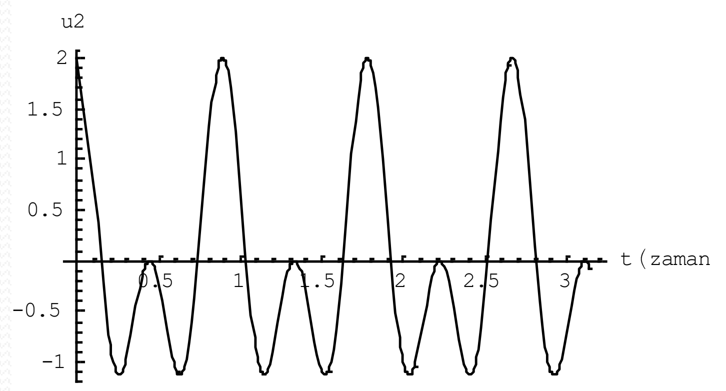
$$q_2(t) = q_2(0).Cos(\omega_2 t) + \frac{\dot{q}_2(0)}{\omega_2}.Sin(\omega_2 t) = 1.Cos(\omega_2 t) + \frac{0}{\omega_2}.Sin(\omega_2 t) = 1.Cos(\omega_2 t)$$

$$\begin{Bmatrix} u_1(t) \\ u_2(t) \end{Bmatrix} = \emptyset_1 q_1 + \emptyset_2 q_2 = \begin{Bmatrix} 1/2 \\ 1 \end{Bmatrix}.(1.Cos(\omega_1 t)) + \begin{Bmatrix} -1 \\ 1 \end{Bmatrix} 1.Cos(\omega_2 t)$$
$$= \begin{Bmatrix} 1/2 \\ 1 \end{Bmatrix} Cos(\omega_1 t) + \begin{Bmatrix} -1 \\ 1 \end{Bmatrix} Cos(\omega_2 t)$$

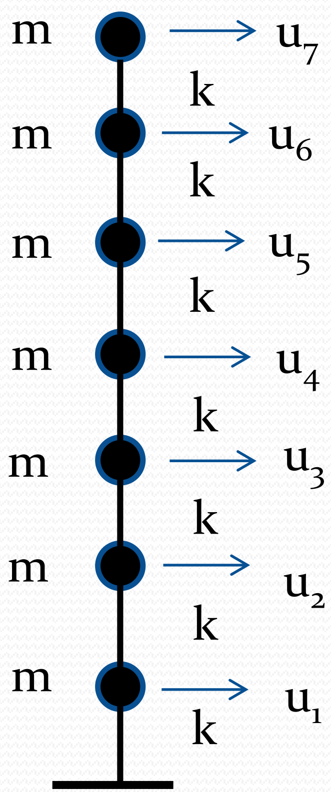

q1
1
0.5
-0.5
-1
0.5
1
1.5
2
2.5
3
t (zaman)


q2
1
0.5
-0.5
-1
0.5
1
1.5
2
2.5
3
t (zaman)


u1
1
0.5
-0.5
-1
-1.5
0.5
1
1.5
2
2.5
3
t (zaman)


u2
2
1.5
1
0.5
-0.5
-1
0.5
1
1.5
2
2.5
3
t (zaman)

Örnek 5: Şekildeki 7 katlı kayma çerçevesinde doğal frekans, periyot ve mod şekillerini hesaplayınız. Başlangıç deplasmanları u(0)={0, 0, 0, 0, 0, 0, 1} (ft) için modal ayrıklaştırma ile kat yerdeğiştirmelerini hesaplayınız.



m=100/32.17405 =3.10809 kip/ (ft/s2)

k=6000 kip/ft

$$M=\begin{pmatrix} 3.10809 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3.10809 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3.10809 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3.10809 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3.10809 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3.10809 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 3.10809 \end{pmatrix}$$

$$K=\begin{pmatrix} 12000 & -6000 & 0 & 0 & 0 & 0 & 0 \\ -6000 & 12000 & -6000 & 0 & 0 & 0 & 0 \\ 0 & -6000 & 12000 & -6000 & 0 & 0 & 0 \\ 0 & 0 & -6000 & 12000 & -6000 & 0 & 0 \\ 0 & 0 & 0 & -6000 & 12000 & -6000 & 0 \\ 0 & 0 & 0 & 0 & -6000 & 12000 & -6000 \\ 0 & 0 & 0 & 0 & 0 & -6000 & 6000 \end{pmatrix}$$

det[K−λ M]=0=

$$\begin{pmatrix} 12000-3.10809\lambda & -6000 & 0 & 0 & 0 & 0 & 0 \\ -6000 & 12000-3.10809\lambda & -6000 & 0 & 0 & 0 & 0 \\ 0 & -6000 & 12000-3.10809\lambda & -6000 & 0 & 0 & 0 \\ 0 & 0 & -6000 & 12000-3.10809\lambda & -6000 & 0 & 0 \\ 0 & 0 & 0 & -6000 & 12000-3.10809\lambda & -6000 & 0 \\ 0 & 0 & 0 & 0 & -6000 & 12000-3.10809\lambda & -6000 \\ 0 & 0 & 0 & 0 & 0 & -6000 & 6000-3.10809\lambda \end{pmatrix}$$

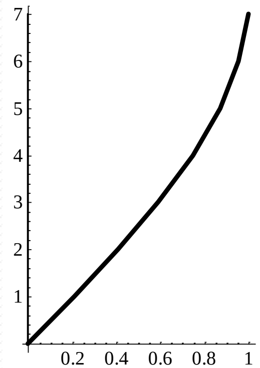
$$8.1716\times10^{18}\lambda^3+3.32594\times10^{15}\lambda^4-6.89156\times10^{11}\lambda^5+7.0317\times10^{7}\lambda^6-2801.95\,\lambda^7 == 0$$

$$\begin{pmatrix} \lambda\to 84.3696 \\ \lambda\to 737.364 \\ \lambda\to 1930.44 \\ \lambda\to 3457.31 \\ \lambda\to 5053.97 \\ \lambda\to 6444.32 \\ \lambda\to 7387.98 \end{pmatrix} \quad \omega_i(\mathrm{rad/s})=\begin{pmatrix} 9.18529 \\ 27.1544 \\ 43.9368 \\ 58.7989 \\ 71.0912 \\ 80.2765 \\ 85.9534 \end{pmatrix} \quad T_i(s)=\begin{pmatrix} 0.684048 \\ 0.231387 \\ 0.143005 \\ 0.106859 \\ 0.088382 \\ 0.0782693 \\ 0.0730999 \end{pmatrix}$$

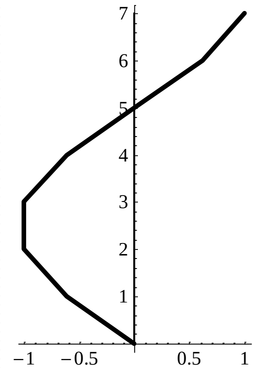
$$\phi{=}_1\begin{pmatrix} 0.209057 \\ 0.408977 \\ 0.591023 \\ 0.747238 \\ 0.870796 \\ 0.956295 \\ 1. \end{pmatrix} \quad \phi{=}_2\begin{pmatrix} -0.618034 \\ -1. \\ -1. \\ -0.618034 \\ 2.01474\times10^{-16} \\ 0.618034 \\ 1. \end{pmatrix} \quad \phi{=}_3\begin{pmatrix} 1. \\ 1. \\ -2.74277\times10^{-16} \\ -1. \\ -1. \\ -1.61365\times10^{-15} \\ 1. \end{pmatrix} \quad \phi{=}_4\begin{pmatrix} -1.33826 \\ -0.279773 \\ 1.27977 \\ 0.547318 \\ -1.16535 \\ -0.790943 \\ 1. \end{pmatrix}$$

$$\phi{=}_5\begin{pmatrix} 1.61803 \\ -1. \\ -1. \\ 1.61803 \\ 2.06798\times10^{-14} \\ -1.61803 \\ 1. \end{pmatrix} \quad \phi{=}_6\begin{pmatrix} -1.82709 \\ 2.44512 \\ -1.44512 \\ -0.51117 \\ 2.1292 \\ -2.33826 \\ 1. \end{pmatrix} \quad \phi{=}_7\begin{pmatrix} 1.9563 \\ -3.57433 \\ 4.57433 \\ -4.78339 \\ 4.16535 \\ -2.82709 \\ 1. \end{pmatrix}$$

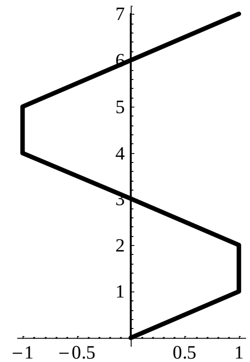
1. mod sekli



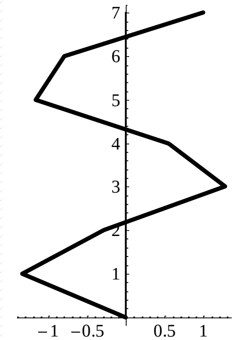
2. mod sekli



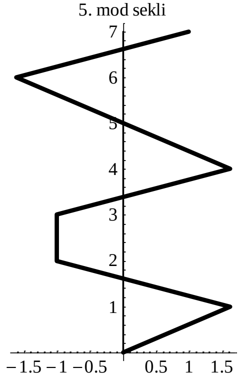
3. mod sekli



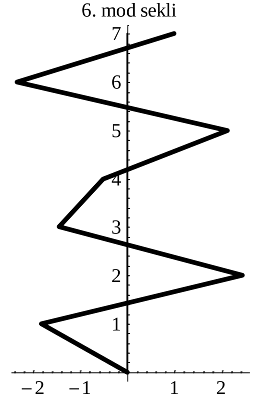
4. mod sekli



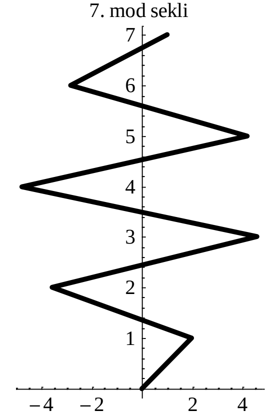
5. mod sekli



6. mod sekli



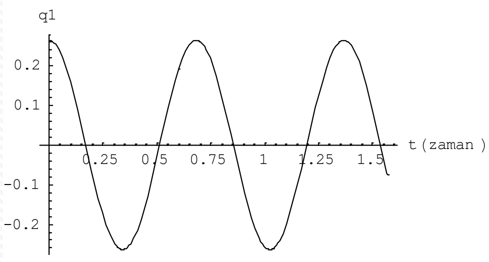
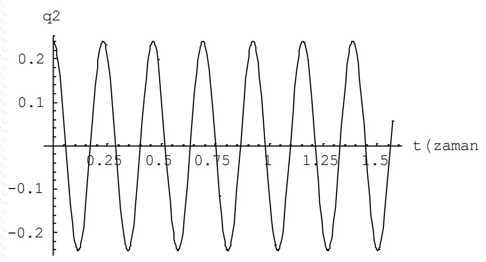
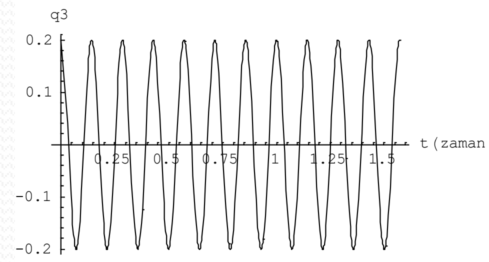
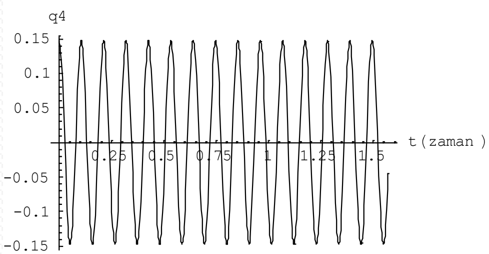
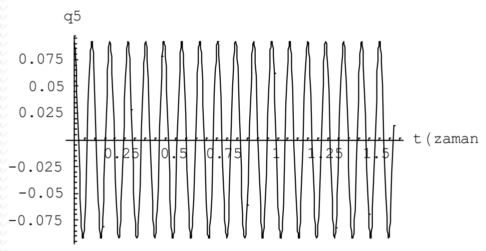
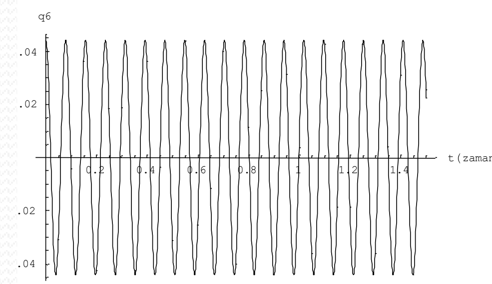
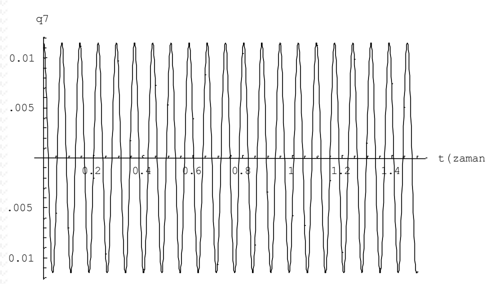
7. mod sekli

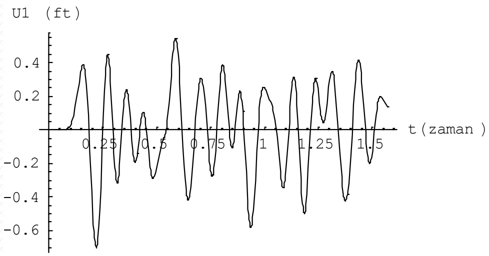


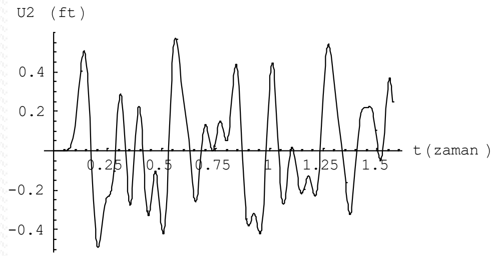
```
m = 100 / 32.17405;
(*kip/(ft/s2)*)
k1 = k2 = k3 = k4 = k5 = k6 = k7 = 6000;
(*kip/ft*)
M = m * IdentityMatrix[7]
K = {{k1 + k2, -k2, 0, 0, 0, 0, 0}, {-k2, k2 + k3, -k3, 0, 0, 0, 0}, {0, -k3, k3 + k4, -k4, 0, 0, 0},
  {0, 0, -k4, k4 + k5, -k5, 0, 0}, {0, 0, 0, -k5, k5 + k6, -k6, 0}, {0, 0, 0, 0, -k6, k6 + k7, -k7},
  {0, 0, 0, 0, 0, -k7, k7}}
As = Inverse[K].M;
v = √(1 / Eigenvalues[As] // N)
V = Eigenvectors[As] // N
Do[VMMi = V[[i]] / V[[i]][[7]], {i, 1, 7}]
Do[ωi = Part[v, i], {i, 1, 7}]
Table[VMMi, {i, 7}]
```

q1 (0) =0.263753	q1=0.263753 Cos[9.18529 t]
q2 (0) =0.241202	q2=0.241202 Cos[27.1544 t]
q3 (0) =0.2	q3=0.2 Cos[43.9368 t]
q4 (0) =0.14727	q4=0.14727 Cos[58.7989 t]
q5 (0) =0.0921311	q5=0.0921311 Cos[71.0912 t]
q6 (0) =0.0441159	q6=0.0441159 Cos[80.2765 t]
q7 (0) =0.0115273	q7=0.0115273 Cos[85.9534 t]

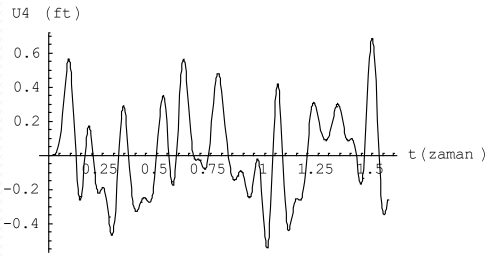
$$U=\begin{pmatrix}
0.0551394\cos[9.18529\,t]-0.149071\cos[27.1544\,t]+0.2\cos[43.9368\,t]-0.197086\cos[58.7989\,t]+0.149071\cos[71.0912\,t]-0.0806038\cos[80.2765\,t]+0.0225507\cos[85.9534\,t]\\
0.107869\cos[9.18529\,t]-0.241202\cos[27.1544\,t]+0.2\cos[43.9368\,t]-0.0412023\cos[58.7989\,t]-0.0921311\cos[71.0912\,t]+0.107869\cos[80.2765\,t]-0.0412023\cos[85.9534\,t]\\
0.155884\cos[9.18529\,t]-0.241202\cos[27.1544\,t]-5.48555\times10^{-17}\cos[43.9368\,t]+0.188473\cos[58.7989\,t]-0.0921311\cos[71.0912\,t]-0.063753\cos[80.2765\,t]+0.0527295\cos[85.9534\,t]\\
0.197086\cos[9.18529\,t]-0.149071\cos[27.1544\,t]-0.2\cos[43.9368\,t]+0.0806038\cos[58.7989\,t]+0.149071\cos[71.0912\,t]-0.0225507\cos[80.2765\,t]-0.0551394\cos[85.9534\,t]\\
0.229675\cos[9.18529\,t]+4.8596\times10^{-17}\cos[27.1544\,t]-0.2\cos[43.9368\,t]-0.171622\cos[58.7989\,t]+1.90525\times10^{-15}\cos[71.0912\,t]+0.0939318\cos[80.2765\,t]+0.0480151\cos[85.9534\,t]\\
0.252226\cos[9.18529\,t]+0.149071\cos[27.1544\,t]-3.22729\times10^{-16}\cos[43.9368\,t]-0.116483\cos[58.7989\,t]-0.149071\cos[71.0912\,t]-0.103155\cos[80.2765\,t]-0.0325886\cos[85.9534\,t]\\
0.263753\cos[9.18529\,t]+0.241202\cos[27.1544\,t]+0.2\cos[43.9368\,t]+0.14727\cos[58.7989\,t]+0.0921311\cos[71.0912\,t]+0.0441159\cos[80.2765\,t]+0.0115273\cos[85.9534\,t]
\end{pmatrix}$$

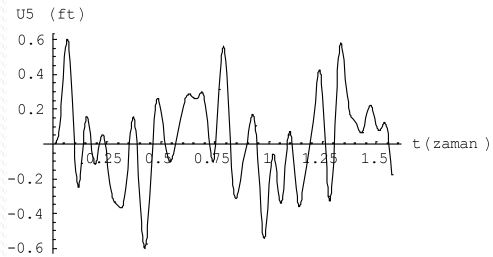

q1
0.2
0.1
-0.1
-0.2
0.25 0.5 0.75 1 1.25 1.5
t (zaman)

q2
0.2
0.1
-0.1
-0.2
0.25 0.5 0.75 1 1.25 1.5
t (zaman)

q3
0.2
0.1
-0.1
-0.2
0.25 0.5 0.75 1 1.25 1.5
t (zaman)

q4
0.15
0.1
0.05
-0.05
-0.1
-0.15
0.25 0.5 0.75 1 1.25 1.5
t (zaman)

q5
0.075
0.05
0.025
-0.025
-0.05
-0.075
0.25 0.5 0.75 1 1.25 1.5
t (zaman)

q6
.04
.02
.02
.04
0.2 0.4 0.6 0.8 1 1.2 1.4
t (zaman)

q7
0.01
.005
.005
0.01
0.2 0.4 0.6 0.8 1 1.2 1.4
t (zaman)

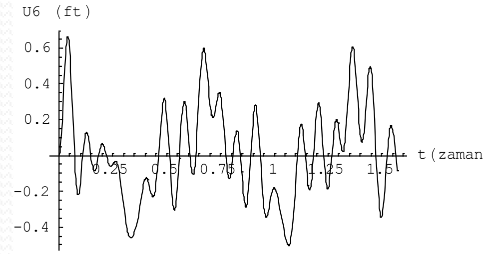

U1 (ft)
0.4
0.2
-0.2
-0.4
-0.6
0.25
0.5
0.75
1
1.25
1.5
t (zaman)

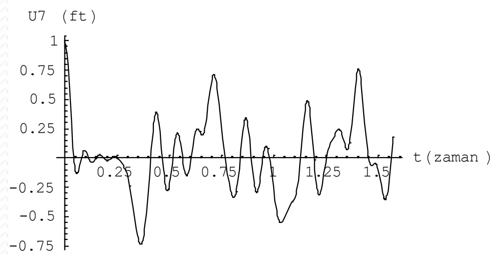

U2 (ft)
0.4
0.2
-0.2
-0.4
0.25
0.5
0.75
1
1.25
1.5
t (zaman)


U3 (ft)
0.6
0.4
0.2
-0.2
-0.4
0.25
0.5
0.75
1
1.25
1.5
t (zaman)


U4 (ft)
0.6
0.4
0.2
-0.2
-0.4
0.25
0.5
0.75
1
1.25
1.5
t (zaman)


U5 (ft)
0.6
0.4
0.2
-0.2
-0.4
-0.6
0.25
0.5
0.75
1
1.25
1.5
t (zaman)


U6 (ft)
0.6
0.4
0.2
-0.2
-0.4
0.25
0.5
0.75
1
1.25
1.5
t (zaman)


U7 (ft)
1
0.75
0.5
0.25
-0.25
-0.5
-0.75
0.25
0.5
0.75
1
1.25
1.5
t (zaman)

SERBEST TİRTEŞİM

Serbest Titreşim Denklemlerinin Çözümü: Sönümlü Sistemler

$$\mathbf{m\ddot{u}} + \mathbf{c\dot{u}} + \mathbf{ku} = \mathbf{0}$$

$$\mathbf{u} = \mathbf{u}(0) \qquad \mathbf{\dot{u}} = \mathbf{\dot{u}}(0)$$

Deplasmanlar modlara bağlı olarak yazılırsa,

$$\mathbf{m\Phi\ddot{q}} + \mathbf{c\Phi\dot{q}} + \mathbf{k\Phi q} = \mathbf{0}$$

Her taraf $\mathbf{\Phi}^T$ ile çarpılırsa,

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{C}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{0}$$

$$\mathbf{C} = \mathbf{\Phi}^T \mathbf{c} \mathbf{\Phi}$$

- Burada M ve K diyagonal matrislerdir.
- C ise sönümün dağılımına göre diyagonal olabilir veya olmayabilir.
- Eğer C diyagonal ise yukarıdaki dif. Denklem, modal koordinatlarda N adet girişimsiz dif. denklemi ifade eder iken, sistemin klasik sönüme sahip olduğu söylenebilir.
- Klasik modal analiz uygulanabilir.
- Bu sistemlerin doğal modları söümsüz sistemler ile aynıdır.
- Eğer C diyagonal değil ise bu sistemlere klasik modal analizi uygulayamayız.

Klasik Sönümlü Sistemler

$$\ddot{q}_n + 2\zeta_n\omega_n\dot{q}_n + \omega_n^2 q_n = 0$$

$$q_n(t) = e^{-\zeta_n\omega_n t}\left[q_n(0)\cos\omega_{nD}t + \frac{\dot{q}_n(0) + \zeta_n\omega_n q_n(0)}{\omega_{nD}}\sin\omega_{nD}t\right]$$

$$\omega_{nD} = \omega_n\sqrt{1-\zeta_n^2}$$

$$\mathbf{u}(t) = \sum_{n=1}^{N}\phi_n e^{-\zeta_n\omega_n t}\left[q_n(0)\cos\omega_{nD}t + \frac{\dot{q}_n(0) + \zeta_n\omega_n q_n(0)}{\omega_{nD}}\sin\omega_{nD}t\right]$$