Definition 1. A *category* $\mathcal{C}$, consists of the following data:

1. A collection of *objects* $\mathrm{ob}\,\mathcal{C}$,
2. For every two objects $x, y \in \mathrm{ob}\,C$ a collection of *morphisms* $\mathrm{Hom}_{\mathcal{C}}(x, y)$.
3. For every $x \in \mathcal{C}$, the identity morphism $\mathrm{id}_x \in \mathrm{Hom}_{\mathcal{C}}(x, x)$.
4. A composition map $\circ\colon \mathrm{Hom}_{\mathcal{C}}(y, z) \times \mathrm{Hom}_{\mathcal{C}}(x, y) \longrightarrow \mathrm{Hom}_{\mathcal{C}}(x, z)$

Such that, for all $x, y \in \mathcal{C}$ and $f \in \mathrm{Hom}_{\mathcal{C}}(x, y)$:

$$f \circ \mathrm{id}_x = f \;\; \mathrm{id}_y \circ f = f$$

And for all x, y, z, v with $f \in \mathrm{Hom}_{\mathcal{C}}(x, y), g \in \mathrm{Hom}_{\mathcal{C}}(y, z), h \in Homzv$:

$$h \circ (g \circ f) = (h \circ g) \circ f$$

Definition 2. A *functor* $F\colon \mathcal{C} \longrightarrow \mathcal{D}$ is a map $\mathrm{ob}\mathcal{C} \to \mathrm{ob}\mathcal{D}$ and a map of morphisms $\mathrm{Hom}_{\mathcal{C}}(x, y) \to \mathrm{Hom}_{\mathcal{D}}(F(x), F(y))$. Such that $F(\mathrm{id}_x) = \mathrm{id}_{F(x)}$ and $F(g \circ f) = F(g) \circ F(f)$.

Definition 3. $F\colon \mathcal{C} \to \mathcal{D}$ is *faithful* if for all $x, y \in \mathcal{C}$, $\mathrm{Hom}_{\mathcal{C}}(x, y) \to \mathrm{Hom}_{\mathcal{D}}(F(x), F(y))$ is injective. It is *full* if every such map is surjective.

Definition 4. A functor $F\colon \mathcal{C} \to \mathcal{D}$ is essentially surjective if for all $d \in \mathcal{D}$ there is $c \in \mathcal{C}$ such that $F(x) \cong d$.

Definition 5. For two functors $F, G\colon \mathcal{C} \to \mathcal{D}$, a *natural transformation* $\eta\colon F \Rightarrow D$ is a collection of morphisms $\eta_x \in \mathrm{Hom}_{\mathcal{D}}(F(x), G(x))$ such that for every $x \xrightarrow{f} y$, $\eta_y \circ F(f) = G(f) \circ \eta_x$.

It is a natural isomorphism if all morphisms η_x are isomorphisms.

Definition 6. Equivalence of categories: $F\colon \mathcal{C} \to \mathcal{D}$ and $G\colon \mathcal{D} \to \mathcal{C}$ with natural isomorphisms $e\colon \mathrm{id}_{\mathcal{C}} \Rightarrow GF$, $\epsilon\colon FG \Rightarrow \mathrm{id}_{\mathcal{D}}$. An adjoint equivalence is an equivalence where $F \dashv G$.

Proposition 7. The following are equivalent: $\mathcal{C}$ and $\mathcal{D}$ are equivalent, $\mathcal{C}$ and $\mathcal{D}$ are adjoint equivalent and there is $F\colon \mathcal{C} \to \mathcal{D}$ that is fully faithful and essentially surjective.