An Analysis of smart voting in liquid democracy

Giannis Tyrovolas

February 22, 2022

Contents

Chapter 1

Preliminaries

1.1 Notation

An election consists of a finite set of voters, voting on a finite set of issues. For each issue there is a finite set of alternatives. A special alternative is the abstention represented by $*$. Formally:

Definition 1.1.1 (Election)**.** An *election* consists of a tuple $\langle N, I, D\rangle$ where $N = \{1, ..., n\}$ is a finite non-empty set. The finite non-empty set I holds represents the issues of the election. For each $i \in I$, $D(i)$ is a finite non-empty set.

For single-issue elections we omit the reference to I and abbreviate $D(i)$ to D.

The model we will consider allows each voter to submit a smart ballot. A smart ballot is a preference list of smart votes. Each smart vote is a function with domain the other voters. A special requirement is that the final preference in the preference list is a direct vote on an alternative in D. Formally:

Definition 1.1.2 (Smart Ballots)**.** A smart ballot of an agent a on issue $i \in I$ is an ordering $((S^1, F^1) > \ldots > (S^k, F^k) > d)$ where $k \geq 0$. Each S^h for $h \leq k$ is a subset of N and $F^h \colon D(i)^{S^h} \longrightarrow D(i)$ is a resolute aggregation function. We also have that $d \in D(i)$.

Further when relevant we will consider F^{k+1} to be the constant function with output d. Now, in most cases the sets S^h are implicit and we will drop any mention to them. That is supported by the fact that we will treat two functions F, G as identical if they are extensionally equal. This is formalised by the following definition:

Definition 1.1.3 (Valid Smart Ballot)**.** A valid smart ballot of an agent a is a smart ballot B_a such that for all $1 \leq s < t \leq k+1$ F^s is not equivalent to F^t. Additionally $a \notin S_t$.

Chapter 2

Results

Chapter 3

Proposals