# An Analysis of smart voting in liquid democracy

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# Chapter 1

# Preliminaries

## 1.1 Notation

An election consists of a finite set of voters, voting on a finite set of issues. For each issue there is a finite set of alternatives. A special alternative is the abstention represented by $*$. Formally:

**Definition 1.1.1** (Election)**.** An *election* consists of a tuple $\langle N, I, D\rangle$ where $N = \{1, ..., n\}$ is a finite non-empty set. The finite non-empty set $I$ holds represents the issues of the election. For each $i \in I$, $D(i)$ is a finite non-empty set.

For single-issue elections we omit the reference to $I$ and abbreviate $D(i)$ to $D$.

The model we will consider allows each voter to submit a smart ballot. A smart ballot is a preference list of smart votes. Each smart vote is a function with domain the other voters. A special requirement is that the final preference in the preference list is a direct vote on an alternative in $D$. Formally:

**Definition 1.1.2** (Smart Ballots)**.** A smart ballot of an agent $a$ on issue $i \in I$ is an ordering $((S^1, F^1) > \ldots > (S^k, F^k) > d)$ where $k \geq 0$. Each $S^h$ for $h \leq k$ is a subset of $N$ and $F^h \colon D(i)^{S^h} \longrightarrow D(i)$ is a resolute aggregation function. We also have that $d \in D(i)$.

Further when relevant we will consider $F^{k+1}$ to be the constant function with output $d$. Now, in most cases the sets $S^h$ are implicit and we will drop any mention to them. That is supported by the fact that we will treat two functions $F, G$ as identical if they are extensionally equal. This is formalised by the following definition:

**Definition 1.1.3** (Valid Smart Ballot)**.** A valid smart ballot of an agent $a$ is a smart ballot $B_a$ such that for all $1 \leq s < t \leq k+1$ $F^s$ is not equivalent to $F^t$. Additionally $a \notin S_t$.

# Chapter 2

# Results

# Chapter 3

# Proposals