COMPUTER SCIENCE 349A

Handout Number 7

STABILITY OF AN ALGORITHM

Textbook (page 100 of the 7th ed.; page 97 of the 6th): a computation is numerically unstable if the uncertainty of the input values is greatly magnified by the numerical method. The following is a more precise definition.

Definition. An algorithm is said to be stable (for a class of problems) if it determines a computed solution (using floating-point arithmetic) that is close to the exact solution of some (small) perturbation of the given problem.

given problem, specified by data $\{d_i\}$ → floating-point computation → computed solution $\{r_i\}$

perturbed problem, data $\{\hat{d}_i\} = \{d_i + \varepsilon_i\}$ → exact computation → exact solution $\{\hat{r}_i\}$

with $\left|\frac{\varepsilon_i}{d_i}\right|$ small

If there exist data $\hat{d}_i \approx d_i$ (small ε_i for all i) such that $\hat{r}_i \approx r_i$ (for all *i*), then the algorithm is said to be **stable.**

If there exists no set of data $\{\hat{d}_i\}$ close to $\{d_i\}$ such that $\hat{r}_i \approx r_i$ for all *i*, then the algorithm is said to be **unstable**.

Meaning of numerical stability: the effect of uncertainty in the input data or of the floating-point arithmetic (the round-off error) is no worse than the effect of slightly perturbing the given problem, and solving the perturbed problem exactly.