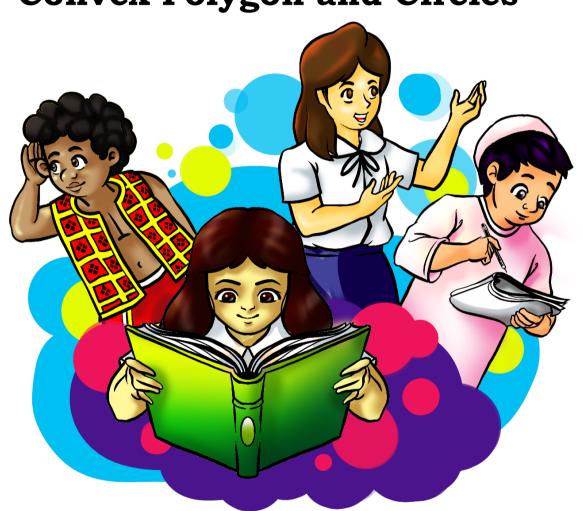




Mathematics

Quarter 3 – Module 6: Exterior and Interior Angles of a Convex Polygon and Circles





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Mathematics – Grade 7 Quarter 3 – Module 6: Exterior and Interior Angles of a Convex Polygon and Circles First Edition, 2020

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Published by the Department of Education – Region XI

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Printed in the Philippines by Davao City Division Learning Resources Management Development System (LRMDS)

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Mathematics

Quarter 3 – Module 6: Exterior and Interior Angles of a Convex Polygon and Circles



Introductory Message

For the facilitator:

As a facilitator, you are expected to orient the learners on how to use this module. You also need to keep track of the learners' progress while allowing them to manage their own learning at home. Furthermore, you are expected to encourage and assist the learners as they do the tasks included in the module.

For the learner:

As a learner, you must learn to become responsible of your own learning. Take time to read, understand, and perform the different activities in the module.

As you go through the different activities of this module be reminded of the following:

- 1. Use the module with care. Do not put unnecessary mark/s on any part of the module. Use a separate sheet of paper in answering the exercises.
- 2. Don't forget to answer *Let Us Try* before moving on to the other activities.
- 3. Read the instructions carefully before doing each task.
- 4. Observe honesty and integrity in doing the tasks and checking your answers.
- 5. Finish the task at hand before proceeding to the next.
- 6. Return this module to your teacher/facilitator once you are done.

If you encounter any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator. Always bear in mind that you are not alone. We hope that through this material, you will experience meaningful learning and gain deep understanding of the relevant competencies. You can do it!

Let Us Learn

After going through this module, you are expected to

- 1.) Derive inductively the relationship of exterior and interior angles of a convex polygon M7GE-IIIf-1, and
- 2.) Illustrate circles and the terms related to it: radius, diameter chord, center, arc, chord, central angle and inscribed angle. M7GE-IIIg-1

Specifically, you will:

- 1. define and identify interior and exterior angles of a convex polygon;
- 2. identify the number of triangles in a polygon
- 3. find the measures of the interior and exterior angles of a polygon
- 4. define, identify, and illustrate circles and the terms related to it:
 - a) Center
 - b) Radius
 - c) Diameter
 - d) Chord
 - e) secant
 - f) tangent
 - g) Arc
 - h) Central angle
 - i) Inscribed angle
- 5. Differentiate minor arc from a major arc.
- 6. Solve problems related to radius or diameter in a circle.

	Let Us Try
1	Read each item ca

arefully and write the letter of your answer on a separate sheet of paper.

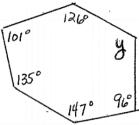
1. The	measure of each angle of a	regular polygon is always?	
	a. equal	c. less than	
	b. greater than	d. cannot be determined	
2. Wha	at is the sum of the measur	res of the interior angles of a triangle?	
	a. 120°	c. 180°	
	b. 150°	d. 160°	
3. Wha	at is the measure of each ir	terior angle of a regular hexagon?	
	a. 110°	c. 130°	
	b. 120°	d. 140°	
4. If th	ne sum of measures of the	e exterior angles of any polygon is 360°, then t	he
mea	sure of each exterior angle	of a regular decagon is?	
	a. 16°	c. 36°	
	b. 26°	d. 46°	

- 5. What is the measure of angle y in the given polygon?
 - a. 110°

c. 120°

b. 115°

d. 125°



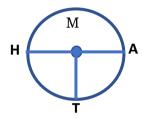
- 6. What do you call the segment from the center to any point of the circle
 - a. Radius
- c. Tangent

b.

d. Diameter

For items 7 - 9 refer to the given diagram below:

Chords



- 7. How many radii are there?
 - a. 1
 - b. 2

- c. 3
- d. 4
- 8. Which of the following is a semi-circle?
 - I. arc HT

II. arc TA

III. arc HA

a. I only

c. III only

b. II only

- d. I, II, and III
- 9. If $\overline{\text{HM}}$ measures 5 cm, what is the measurement of $\overline{\text{MT}}$?
 - a. 2.5cm

c. 5 cm

b. 5.5 cm

- d. 10 cm
- 10. A gardener constructed a 4.5 meters pathway in radius form of a circular flower garden, how long is the pathway if he decided to do it in diameter form?
 - a. 4m

c. 7m

b. 4.5 m

d. 9m



Let Us Study

"You Complete Me"

Consider each convex polygon with all possible diagonals drawn from one vertex (A). Each polygon is separated into triangle/s. You can find the sum of the measures of the interior angles of each polygon by adding the measures of the angles of the triangle/s formed. Complete the table below.

Polygon	Number of Sides	Number of Diagonals from one vertex	Number of Dissected Triangles	Sum of the Measures of Interior Angles
A C Triangle	3	0	1	180º
D C B Quadrilateral	4	1	2	360º
E D C Pentagon				
F C B Hexagon				
F E D C A B Octagon				
Any Polygon	n			

Guide Questions:

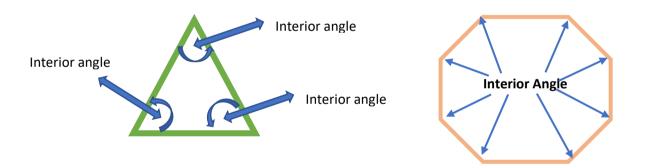
- 1. Do you see any connection between the number of sides and the number of triangles dissected through the diagonals made in a polygon? What do you think is their noteworthy connection?
- 2. Do you see any connection between the number of dissected triangles and the sum of the interior angles? What do you think is their noteworthy connection?
- 3. What will you do to find the number of dissected triangles from an n-sided polygon?
- 4. How will you find the sum of the interior angles of an n-sided polygon?

Exterior and Interior Angles of a Convex Polygon

People make use of angles everyday. Plumbers measure the angle in connecting the pipe to make sure of it having a good fitting; carpenters adjust their saw blade in order to cut the wood at a certain angle; a good billiard player must know their angle in order to plan for a perfect shot; and engineers compute the angles on a building in order for it to meet the specification as what is in the plan. These are just few examples where we do a lot of things that involve angles.

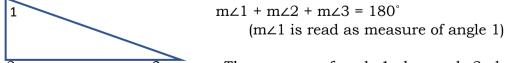
Interior Angles

An **interior angle** is an angle formed by two sides of a polygon with a common vertex. For example:



Based on the illustration above, the number of interior angles in a convex polygon is the same as the number of its side.

Triangle Sum Theorem: The sum of the measures of interior angles of a triangle is 180° .



The measure of angle 1 plus angle 2 plus angle 3 is always equal to 180°, regardless of the size and kind of triangle.

Finding the Number of Triangles in any Convex Polygon

In determining the number of triangles in any convex polygon, all you need to do is to subtract 2. In symbol, we use the formula:

n-2 where n is the number of sides of any polygon

Example: How many triangles are there in a 15-gon?

We know that a 15-gon has 15 sides, therefore, by using the formula, we have, n-2 = 15-2 = 13 Therefore, there are 13 triangles that can be

drawn from a 15-gon.

Finding the Sum of the Measures of Interior Angles of a Convex Polygon

To find the sum of the measures of the interior angles of a convex polygon, we multiply the number of triangles formed by the sum of the measures of the interior angles of a triangle.

Therefore, the formula is:

 $(n-2) \cdot 180$ where n is the number of sides of any polygon

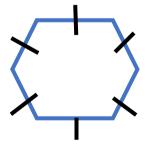
Example: What is the sum of the measures of interior angles of a 15 – gon?

Sum =
$$(n - 2) \cdot 180$$

= $(15 - 2) \cdot 180$
= $(13) \cdot 180$
= 2340°

Therefore, the sum of the measures of the interior angles of a 15-gon is 2340°

Finding the Measure of an Interior Angle



A regular polygon is a polygon having equal angles and equal sides. It is often indicated with a <u>marked line</u> to show their congruency.

In determining the measure of each interior angle of a regular polygon, we use the formula $\frac{(n-2) \, \mathbb{D} \, (180)}{n}$ where n is the number of sides.

Example 1: Find the measure of each interior angle of a regular octagon.

Solution: Since an octagon has 8 sides, measurement of each interior angle is obtained by:

Measurement of each interior angle = $\frac{(n-2) \, \mathbb{Z} \, (180)}{n}$



$$=\frac{(8-2) \ 2 \ (180)}{8}$$

$$=\frac{(6) \ 2 \ (180)}{8}$$

$$=\frac{1080}{8}$$

Each interior angle of a regular octagon measures 135°.

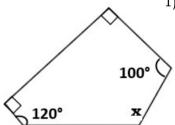
Example 2: What is the measure of the fifth angle of the pentagon?

Given: 90°, 90°, 100°, 120° = measures of the four interior angles

5 = number of sides

Solution:

1) Find the sum of the measures of the interior angles.



Sum of the measures =
$$(n-2) \cdot 180^{\circ}$$

= $(5-2) \cdot 180^{\circ}$
= $3 \cdot 180^{\circ}$
= 540°

2) Find the measure of the fifth angle.

$$x^{\circ} + 90^{\circ} + 90^{\circ} + 100^{\circ} + 120^{\circ} = 540^{\circ}$$
 $x^{\circ} + 400^{\circ} = 540^{\circ}$
 $x^{\circ} + 400^{\circ} - 400^{\circ} = 540^{\circ} - 400^{\circ}$
 $x^{\circ} = 140^{\circ}$

The measure of the fifth angle of the pentagon is 140°.

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Example 3:

Find the value of x.

a) 60° x°

Suppose that angles x are equal in the given diagram

Given: 60° = measure of one interior angle

4 = number of sides of the polygon

Solution:

1) Find the sum of the measures of the interior angles.

Sum of the measures = $(n-2) \cdot 180^{\circ}$ = $(4-2) \cdot 180^{\circ}$ = $2 \cdot 180^{\circ}$ = 360°

2) Find the measure of the other angles.

$$x^{\circ} + x^{\circ} + x^{\circ} + 60^{\circ} = 360^{\circ}$$
 $3x^{\circ} + 60^{\circ} = 360^{\circ}$
 $3x + 60 - 60 = 360 - 60$

$$\frac{3x}{3} = \frac{300}{3}$$

$$x = 100^{\circ}$$

Each angle x measures 100°.

Finding the Number of Sides of a Polygon

Example:

The sum of the interior angles of a polygon is 1620°. How many sides does it have?

Given: Sum of the measures of interior angles = 1620° Number of sides = ?

Solution:

Sum of Interior angle = $(n - 2) \circ 180$

 $1620 = (n-2) \cdot 180$

Distributive Property of Equality

1620 = 180n - 360

1620 + 360 = 180n - 360 + 360

Addition Property of Equality

 $\frac{1980}{180} = \frac{180n}{180}$

Dividing both sides by 180

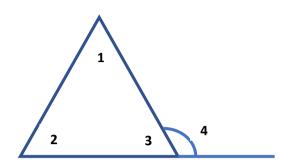
n = 11

The polygon has 11 sides.

Exterior Angles

An **exterior angle** is an angle between a side of a polygon and an extended adjacent side. For example ,

Let's examine the figure below



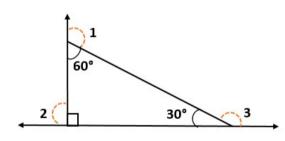
Angles 1, 2 and 3 are interior angles while angle 4 is an exterior angle.

In the <u>Triangle Sum Theorem</u>, it says that $m\angle 1 + m\angle 2 + m\angle 3 = \underline{180^\circ}$. Also, $m\angle 3 + m\angle 4 = 180^\circ$ because they are <u>supplementary</u> or <u>linear pair</u> and make a straight angle whose measure is also 180° .

Therefore by substitution $m\angle 1 + m\angle 2 + m\angle 3 = m\angle 3 + m\angle 4$ and subtracting $m\angle 3$ on both sides of the equation which paves to $m\angle 1 + m\angle 2 = m\angle 4$. This means that the measure of the exterior angle of a triangle is equal to the sum of the measures of the remote interior angles. (Exterior Angle Theorem)

In the given illustration above, if $m\angle 1$ is 60° and $m\angle 2$ is 70° , therefore, $m\angle 4=60^\circ+70^\circ$, which is 130° . Also, since $\angle 3$ and $\angle 4$ are supplementary; therefore, 180° - $m\angle 3=m\angle 4$, That is, 180° - $50^\circ=130^\circ$.

Take a closer look on the following example.



Exterior angles are $\angle 1$, $\angle 2$, and $\angle 3$.

Interior angles measures 90°, 60° and 30°.

Using the exterior angle theorem, we can deduce that the following measures of exterior angle 1, 2, and 3 can be obtained through the sum of its remote interior angle

$$m \angle 1 = 90^{\circ} + 30^{\circ}$$

$$m\angle 2 = 60^{\circ} + 30^{\circ}$$

$$m \angle 3 = 60^{\circ} + 90^{\circ}$$

Notice that the sum of exterior angle 1, 2 and 3, would give the sum of 360°. We can infer that $\underline{m} \angle 1 + \underline{m} \angle 2 + \underline{m} \angle 3 = 120^{\circ} + 90^{\circ} + 150^{\circ} = 360^{\circ}$

This leads to our next conjecture that the sum of exterior angle of any polygon having n sides would always be equal to **360**°

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Finding the Measure of Exterior Angles of a Polygon

To find the measure of each exterior angle of a regular polygon, the formula is:

$$\frac{360^{\circ}}{n}$$
 where **n** is the number of sides

Example 1:

Find the measure of each exterior angle of a regular 30-gon.

Given: 30 = number of sides

Solution:

Measure of each exterior angle =
$$\frac{360^{\circ}}{n}$$

$$= \frac{360^{\circ}}{30}$$

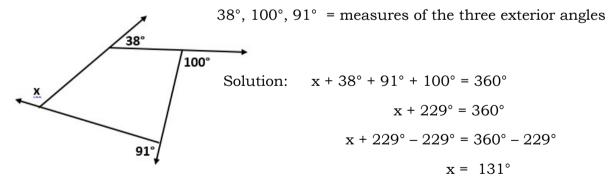
$$= 12$$

Each exterior angle of the regular 30-gon measures 12°.

Example 2:

Find the missing value of the fourth exterior angle \boldsymbol{x} .

Given:

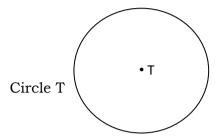


The measure of the fourth exterior angle is 131°.

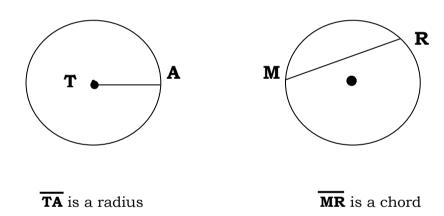
Circle

A **circle** is a set of all points that are at a fixed distance from a fixed point in the plane. The fixed point is the **center** and the fixed distance is the **radius**.

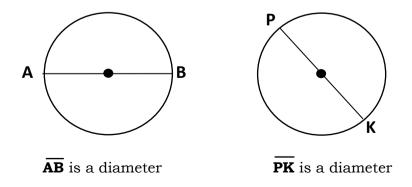
Circles are usually named by their centers. So, if the center of the circle is represented by letter T, then it is named as circle T (in symbol OT).



Radius (plural radii) is a segment from the center to any point on the circle while **chord** is a segment whose endpoints are points on the circle.

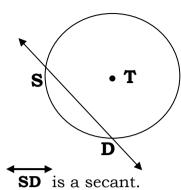


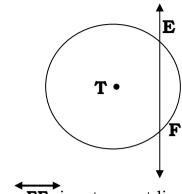
Diameter is a chord containing the center of the circle. It is known as the longest chord and divides the circle into two semicircles.



Secant is a line which intersects the circle at two distinct points. **Tangent** to a circle is a line, a ray or a segment in the plane of the circle which intersects the circle at exactly are point.

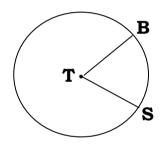
circle at exactly one point.

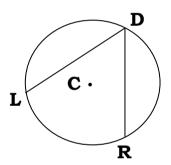




FE is a tangent line.

Central angle is an angle formed by two radii and with vertex at the center of the circle while an **inscribed angle** is an angle whose vertex is on the circle.

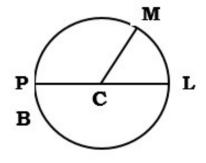




 \angle **BTS** is a central angle.

∠**LDR** is an inscribed angle.

An **arc** is a connected portion of a circle. If an arc is half a circle it is called as **semicircle**; if it is less than a semicircle it is called a **minor arc** but if it is more than a semi-circle, it is called a **major arc**.



PML (arc PML) is a semicircle.

PBL (arc PBL) is a semicircle.

ML (arc ML) is a minor arc.

PM (arc PM) is a minor arc.

PB (arc PB) is a minor arc.

BPL (arc BPL) is a major arc.

MLB (arc MLB) is a major arc.



"Find My Perfect Angle"

A. Use the given information to complete the table by filling the missing box with the correct answer. Write the answer on your answer sheet. The first one is already done for you.

Name of Polygon	No. of Sides	Sum of Interior angle	Measure of ONE Interior Angle (Regular Polygon)	Measure of ONE Exterior Angle (Regular Polygon)
Nonagon	9	1260°	180°	40°
Quadrilateral	4			
Pentagon	5			
Decagon	10			
Dodecagon	12			
15- gon	15			

B. Match each angle with its corresponding measure, given m∠1 = 130° and m∠7 = 70°. Indicate a match by writing the letter for the angle on the line in front of the corresponding angle measure. **Use the exterior angle theorem**

A. m∠2

_____ 50°

B. m∠3

60°

C. m∠4

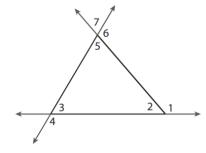
_____70°

D. m∠5

_____ 110°

E. m∠6

_____ 120°

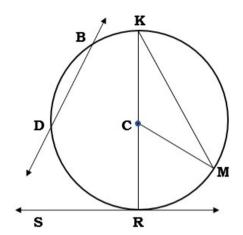




Let Us Practice More

A. Identify each of the following using the given circle C

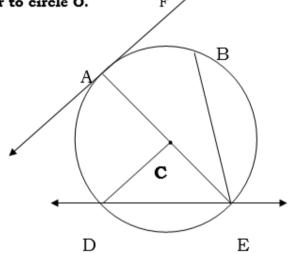
- 1) <u>CM</u>
- 2) MK
- 3) KR
- 4) ∠RCM
- 5) ∠RKM
- 6) KRM
- 7) RM
- 8) SR
- 9) DB
- 10) KMR



В.

Name each of the following. Refer to circle O.

- 1. Center
- 2. three radii
- 3. chord
- 4. diameter
- 5. secant
- 6. tangent
- 7. a semicircle
- 8. a minor arc
- 9. central angle
- 10. Inscribed Angle



Let Us Remember

Useful things to remember when dealing with interior and exterior angles of polygon.

Sum of Interior Angles of a Polygon	(n − 2) · 180
Sum of exterior Angles of a Polygon	360
Each Interior Angle of a Regular Polygon	$(n-2) \ 2 \ (180)$
where n is the number of sides	\overline{n}
Each Exterior Angle of a Regular Polygon	360
	\overline{n}

A circle is a set of points which are equidistant from a fixed point called the center. It is named through its center.

A radius (radii in plural form) is a segment from the center to any point of the circle. It is half of a diameter. All radii of the same circle are congruent to each other. Chord is a line segment whose endpoints are on the circle. Diameter is a chord that passes through the center of the circle. It is also known as the longest chord. It divides the circle into two semicircles. Secant is a line touches the circle at two points while tangent is a line touches the circle at exactly one point.

An arc is a curved line and connected part of a circle. If an arc is less than a semicircle it is called as minor arc, and if it is more than a semicircle it is called as a major arc.

A central angle is an angle whose vertex is located at the center and whose sides are radii of the circle. Inscribed angle is an angle whose vertex is on the circle and whose sides are chords of the circle.



Let Us Assess

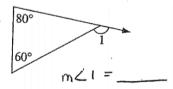
Let us assess what you have learned on the lesson by answering the problems below. Write the letter of your answer on a piece of paper.

- 1) What is the measure of angle 1 in the given illustration?
 - a. 120°

c. 140°

b. 130°

d. 150°



- 2) What is the measure of angle \boldsymbol{x} in the given polygon?
 - a. 100°

c. 150°

b. 110°

d. 168°



- 3) What is the measure of each interior angle of a regular octagon?
 - a. 125°
- b. 135°
- c. 152°
- d. 153°

4) We can find the measure of an exterior angle of any regular polygon by 360° by the number of sides. a. adding c. dividing b. multiplying d. subtracting 5) The sum of exterior of any polygon would always be equal to ____ b. 180° c. 270° d. 360° 6) What is the measure of angle \boldsymbol{a} in the given regular pentagon? a. 72° c. 108° b. 98° d. 120° 7) Find the missing value of the angle x in the given triangle. c. 55° 110° b. 45° d. 65° 8) Is it possible for a regular polygon to have each of its interior angles measure 142° a. Yes, it is a triangle b. No, a regular polygon has equal angles. If each interior angle is 142°, it would have 9.47 sides, which is not an exact number of sides. c. Yes, by using the sum of interior angle formula to get the number of sides d. No, it will be a concave polygon 9) Which of the following formula would you use to find each interior angle of a regular polygon? a. $(n-2) \circ 180$ c. n - 2 d. $\frac{(n-2) ? (180)}{n}$ 10) What do you call the segment from the center to any point of the circle? a. Radius c. Tangent b. Chords d. Diameter 11) Which of the following represents a circle? c. Badminton Racket a. Ball b. Ring d. Table Tennis Racket 12) What do you call a line which intersects a circle at two points? a. Radius c. Secant b. Minor Arc d. Tangent 13) A father bought a Hola hoop for his youngest daughter. The diameter of the Hola hoop measures 1 meter. What is the measurement (in cm) of its radius? a. 5cm c. 500 cm b. 50 cm d. 5000 cm 14) Which of the following is true about a diameter?

d. It has an endpoints located at the center and on the circle

a. It is less than a radius.

b. It is always in a horizontal position.c. It divides the circle into two semicircles.

- 15) A gardener constructed a 3.5 meters pathway in radius form of a circular flower garden, how long is the pathway if he decided to do it in diameter form?
 - a. 1.75 m

c. 7 m

b. 3.5 m

d. 7.5 m



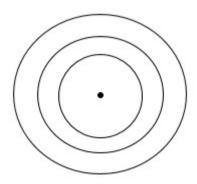
"My Polygon Adventure"

Look for 5 things within your house that resemble a convex polygon. Write the data below and have its corresponding data description be filled.

Object	Shape	Sum of the measures of Interior angles
Ex. Lunchbox	Rectangle	360°
1)		
2)		
3)		
4)		
5)		

"CONCENTRIC CIRCLES"

Look for 3 circular objects with different sizes in your house (glass, ring, etc), trace and cut the circles. Locate the center and measure the radius of each circle. Paste them in overlapping position in your answer sheet in such a way that they share with a common center. This is your performance task, illustrating a concentric circles or circles with different radii with the same center.

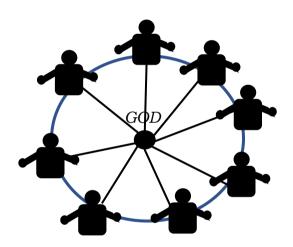


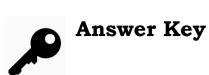
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Let Us Reflect

Angles are evident in our day to day living. The pillars of the building, the roadblocks and signages all consist angles. The most beautiful angle we make is the angle of our mouth as we use it to smile for others. Smile costs nothing but gives other people joy especially if they are dealing with some problems. It gives them hope and positive outlook in life, so smile and spread forth the joy and happiness to others like a perfume. Always remember "The most beautiful angle we make is the angle of our mouth as we use it to smile for others."

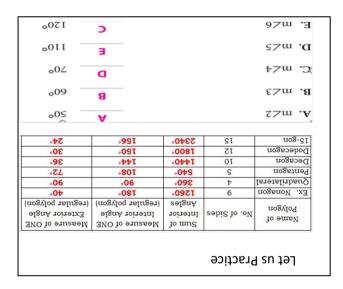
In addition, our life is like a circle with our Almighty God at the center. Just like any letter used to represent points and wherever its position on the circle, still the radii are congruent to each other. The concept is the same for us humans, we live in different types of environment. We have different economic status, physical appearances and academic attainment but still, our Almighty God treat us equally by giving us His unconditional love. Therefore, let us always be reminded, that we are all equal in God's eyes!

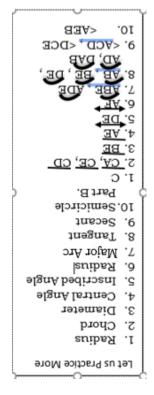




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