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Mathematics

Quarter 2 – Module 4:
Proving Theorems Related to Chords, Arcs, Central Angles, and Inscribed Angles

ALTERNATIVE DELIVERY MODE

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Alternative Delivery Mode
Quarter 2 – Module 4: Proving Theorems Related to Chords, Arcs, Central Angles, and Inscribed Angles
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Mathematics

Quarter 2 – Module 4:
Proving Theorems Related to Chords, Arcs, Central Angles, and Inscribed Angles

Introductory Message

This Self-Learning Module (SLM) is prepared so that you, our dear learners, can continue your studies and learn while at home. Activities, questions, directions, exercises, and discussions are carefully stated for you to understand each lesson.

Each SLM is composed of different parts. Each part shall guide you step-by-step as you discover and understand the lesson prepared for you.

Pre-tests are provided to measure your prior knowledge on lessons in each SLM. This will tell you if you need to proceed on completing this module or if you need to ask your facilitator or your teacher's assistance for better understanding of the lesson. At the end of each module, you need to answer the post-test to self-check your learning. Answer keys are provided for each activity and test. We trust that you will be honest in using these.

In addition to the material in the main text, Notes to the Teacher are also provided to our facilitators and parents for strategies and reminders on how they can best help you on your home-based learning.

Please use this module with care. Do not put unnecessary marks on any part of this SLM. Use a separate sheet of paper in answering the exercises and tests. And read the instructions carefully before performing each task.

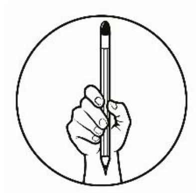
If you have any questions in using this SLM or any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator.

Thank you.

What I Need to Know

This module was designed and written with you in mind. It is here to help you prove theorems related to chords, arcs, central angles, and inscribed angles. The scope of this module permits it to be used in many different learning situations. The language used recognizes the diverse vocabulary level of students. The lessons are arranged to follow the standard sequence of the course but the order in which you read and answer this module is dependent on your ability.

This module contains Lesson 1: Theorems Related to Chords, Arcs, and Central Angles and Lesson 2: Theorems Related to Chords, Arcs, and Inscribed Angles. After going through this module, you are expected to prove theorems related to chords, arcs, central angles, and inscribed angles.

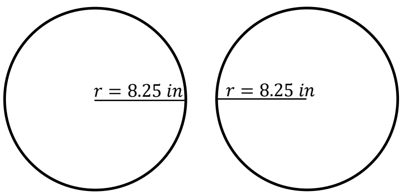


What I Know

Directions: Read and analyze each item very carefully. On your answer sheet, write the letter of the choice that corresponds to the correct answer.

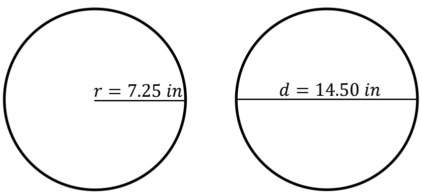
1. Which of the following illustrations do NOT show congruence?

a.



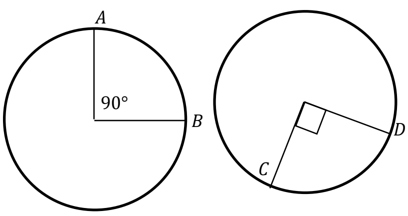
The two circles are congruent.

c.



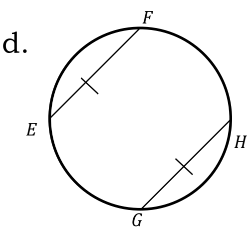
The two circles are congruent.

b.



$\widehat{AB} \cong \widehat{CD}$

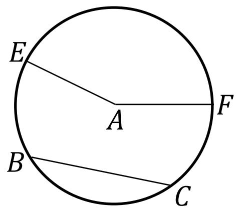
d.



$\widehat{EF} \cong \widehat{FH}$

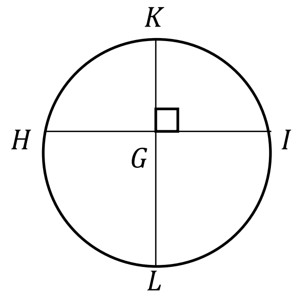
2. An inscribed angle is a right angle if it intercepts a __________.
   a. whole circle b. semicircle c. minor arc d. major arc

3. Consider $\odot A$ with $\widehat{EF} = 155°$. Which of the following statements is always true?
   a. In $\odot A$, $\angle EAF = 77.5°$.
   b. $\widehat{BC} = 155°$ if and only if $\widehat{EF} = 155°$.
   c. $\widehat{EB} \cong \widehat{FC}$ if and only if $\overline{EB} \cong \overline{FC}$.
   d. $\angle EAF \cong \angle BAF$ if and only if $\widehat{BCF} = 155°$.



Refer to $\odot G$ for items 4 to 6.

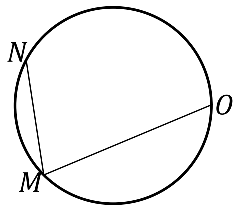
4. In $\odot G$, $\overline{KL}$ is a diameter that is perpendicular to chord HI. Which of the following is true?
   a. $\overline{KL} \cong \overline{HI}$
   b. $\widehat{HK} \cong \widehat{LI}$
   c. $\overline{KG} \cong \overline{GL}$
   d. $\overline{HG} \cong \overline{GI}$



5. Suppose $\overline{HI} = 15$ and $\widehat{IKH} = 120°$, then ____.
   a. $\overline{HG} = 15$, $\overline{GI} = 15$, $\widehat{KI} = 120°$, and $\widehat{KH} = 120°$
   b. $\overline{HG} = 7.5$, $\overline{GI} = 7.5$, $\widehat{KI} = 60°$, and $\widehat{KH} = 60°$
   c. $\overline{KG} = 7.5$, $\overline{GL} = 7.5$, $\widehat{HL} = 60°$, and $\widehat{LI} = 60°$
   d. Insufficient data, answer cannot be determined.

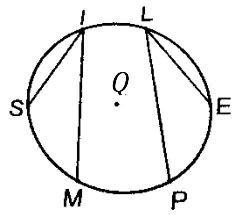
6. Suppose $\widehat{HLI} = 240°$, find the measure of $\widehat{KH}$.
   a. 240° b. 120° c. 60° d. 30°

7. What is the measure of the arc intercepted by inscribed angle $\angle NMO$ if $\angle NMO = 85°$?
   a. $\widehat{NO} = 170°$ c. $\widehat{NMO}$ =42.5°
   b. $\widehat{NMO} = 170°$ d. $\widehat{NO} = 42.5°$

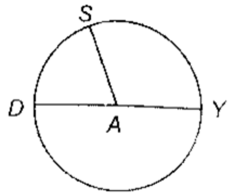


8. What phrase correctly completes the theorem, "*If a quadrilateral is _______________, then its opposite angles are supplementary.*"?
   a. inscribed in a circle c. inscribed in a semicircle
   b. circumscribed about a circle d. circumscribed about a semicircle

9. In $\odot Q$, $\widehat{MS} \cong \widehat{PE}$ and $\widehat{MS} = 40°$. Which of the following statements is NOT true?
   a. $\angle SIM$ and $\angle ELP$ both measure 20°.
   b. $\widehat{MS}$ and $\widehat{PE}$ both measure 40°.
   c. $\angle SIM$ and $\angle ELP$ both measure 40°.
   d. $\angle SIM$ and $\angle ELP$ intercepts arcs MS and EP respectively .

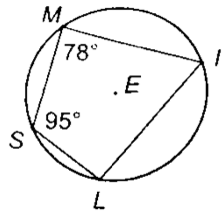


10. In $\odot A$, what is the measure of $\angle SAY$ if $\widehat{DSY}$ is a semicircle and $m\angle SAD = 50$?



a. 130° c. 100°
b. 110° d. 50°

11. Quadrilateral $SMIL$ is inscribed in $\odot E$.
If $m\angle SMI = 78$ and $m\angle MSL = 95$, find $m\angle SLI$.



a. 78° c. 95°
b. 85° d. 102°

12. The _______________ angles of a quadrilateral inscribed in a circle are supplementary.

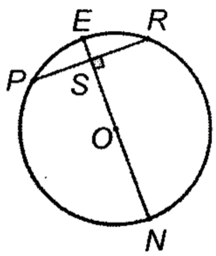
a. adjacent b. obtuse c. opposite d. vertical

13. All of the following parts from two congruent circles guarantee that two minor arcs from congruent circles are congruent except for one. Which one is it?

a. Their corresponding congruent chords.
b. Their corresponding central angles.
c. Their corresponding inscribed angles.
d. Their corresponding intercepted arcs.

Refer to ⊙O for items 14 and 15.

14. In ⊙O, what is $\overline{PR}$ if $\overline{NO} = 10$ units and $\overline{ES} = 4$ units?



a. 64 units c. 16 units
b. 32 units d. 8 units

15. In ⊙O, what is the measure of arc PR if $m\widehat{PE} = 40$?

a. 20° c. 60°
b. 40° d. 80°

What's In

Before we start, let us first have a recap on some parts of a circle.

Directions: Rearrange the jumbled letters to come up with a word that corresponds to the given definition. Write your answers on a separate sheet of paper.

1) C A R – A part of a circle between any two points and is measured in terms of degrees.
2) R O D C H – A line segment that has its endpoints on the circle.
3) E T E R M A D I – A chord that passes through the center of the circle.
4) L A R T N E C G A N E L – It is angle whose vertex is at the center of a circle and whose sides are radii of a circle.
5) B C D E I I N R S N E G A L – It is an angle whose vertex lies on the circle and its sides contain chords of the circle.

Lesson 1 Theorems Related to Chords, Arcs, and Central Angles

What's New

If and Then

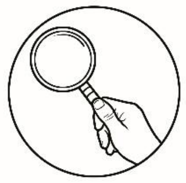
Read the following *If-then* statements. State whether you agree with the statement or not. Justify your answer.

1. If an arc measures 180°, then it is a semi-circle.

2. If all radii of a figure are congruent, then the figure is a circle.

3. If an angle is inscribed in a circle, then its measure is *one-half* the measure of its intercepted arc.

The activity that you just have done posed situations where a premise is presented and a conclusion is made. You shall be seeing more of these in lessons 1 and 2 where we will be proving theorems.



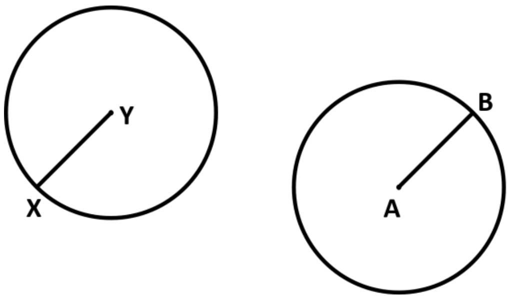
What is It

In the next set of activities, you are tasked to prove theorems you used in the previous module. We are going to review some of them and then provide the proofs to these theorems.

We will start with the following concepts. While doing so, note that all images are NOT drawn to scale.

Congruent Circles and Congruent Arcs

Congruent circles are circles with congruent radii.

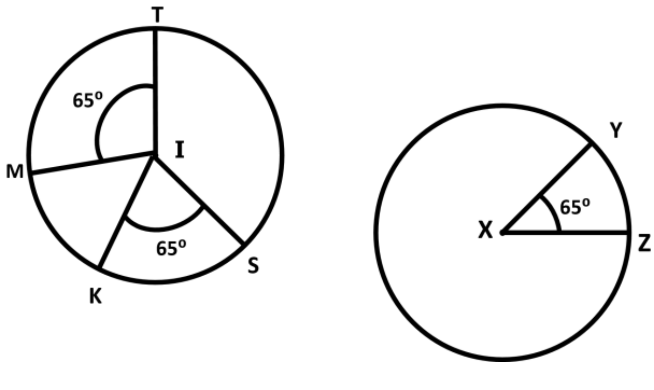


Example: $\overline{XY}$ is a radius of $\odot Y$.

$\overline{AB}$ is a radius of $\odot A$.

If $\overline{XY} \cong \overline{AB}$, then $\odot Y \cong \odot A$.

Congruent arcs are arcs of the same circle or of congruent circles with equal measures.

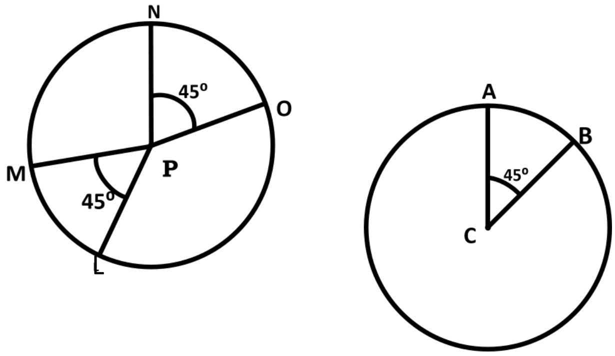


Example: In $\odot I$, if $m\widehat{TM} = m\widehat{KS}$, then $\widehat{TM} \cong \widehat{KS}$.

If $\odot I \cong \odot X$ and $m\widehat{TM} = m\widehat{KS} = m\widehat{YZ}$, then $\widehat{TM} \cong \widehat{KS} \cong \widehat{YZ}$.

Theorems on Central Angles, Arcs, and Chords

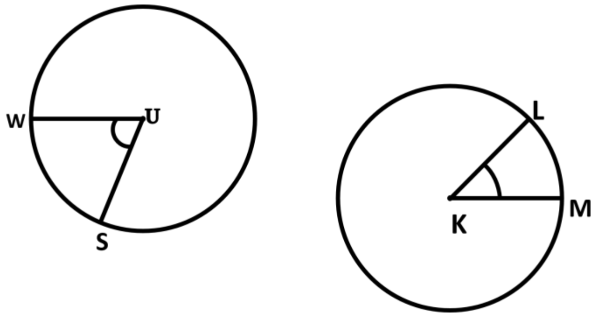
***Theorem 1**. In a circle or in congruent circles, two minor arcs are congruent if and only if their corresponding central angles are congruent.*



a. In $\odot P$, since $\angle LPM \cong \angle OPN$, then $\widehat{NO} \cong \widehat{ML}$.
b. If $\odot P \cong \odot C$ and $\angle LPM \cong \angle OPN \cong \angle ACB$, then $\widehat{LM} \cong \widehat{ON} \cong \widehat{AB}$.

Proof of the Theorem
Use a two-column proof to prove that the intercepted arcs of two corresponding congruent angles from two congruent circles are congruent.

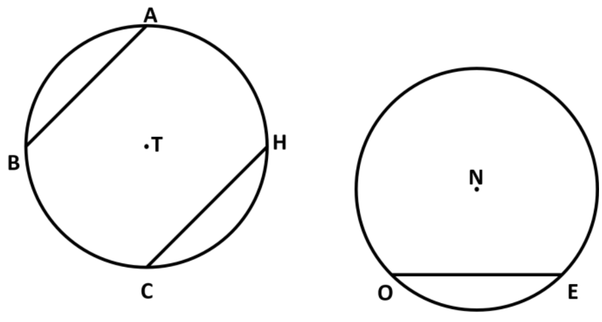
Given: $\odot U \cong \odot K$ and $\angle WUS \cong \angle LKM$
Prove: $\widehat{WS} \cong \widehat{LM}$



Proof:

Statement	Reason
1. $\odot U \cong \odot K$ $\angle WUS \cong \angle LKM$	1. Given
2. In $\odot U$, $m\angle WUS = m\widehat{WS}$ In $\odot K$, $m\angle LKM = m\widehat{LM}$	2. The measure of a central angle is equal to the degree measure of its intercepted arc.
3. $m\angle WUS = m\angle LKM$	3. Congruent angles have equal measures.
4. $m\widehat{WS} = m\widehat{LM}$	4. Substitution Property of Equality
5. $\boldsymbol{\widehat{WS} \cong \widehat{LM}}$	5. Two arcs are congruent if they have equal measures.

> ***Theorem 2***. *In a circle or in congruent circles, two minor arcs are congruent if and only if their corresponding chords are congruent.*

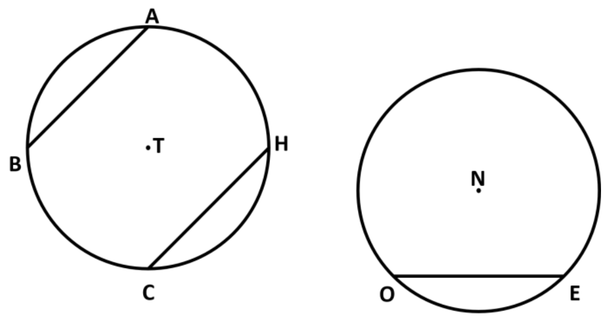


a. In $\odot T$, $\overline{BA} \cong \overline{CH}$. Since the two chords are congruent, then $\widehat{BA} \cong \widehat{CH}$.
b. If $\odot T \cong \odot N$ and $\overline{BA} \cong \overline{CH} \cong \overline{OE}$, then $\widehat{BA} \cong \widehat{CH} \cong \widehat{OE}$.

Proof of the Theorem

Given that $\odot T \cong \odot N$ and $\overline{AB} \cong \overline{OE}$, use a two-column proof to prove that $\widehat{AB}$ and $\widehat{OE}$ are congruent.

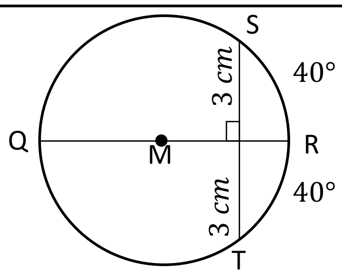
Given: $\odot T \cong \odot N$
$\overline{AB} \cong \overline{OE}$
Prove: $\widehat{AB} \cong \widehat{OE}$



Proof:

Statement	Reason
1. $\odot$ T $\cong$ $\odot$ N $\overline{AB} \cong \overline{OE}$	1. Given
2. $\overline{TA} \cong \overline{TB} \cong \overline{NO} \cong \overline{NE}$	2. Radii of the same circle or of congruent circles are congruent.
3. $\Delta ATB \cong \Delta ONE$	3. SSS Postulate
4. $\angle ATB \cong \angle ONE$	4. Corresponding Parts of Congruent Triangles are Congruent (CPCTC)
5. $\boldsymbol{\widehat{AB} \cong \widehat{OE}}$	5. In a circle or in congruent circles, two minor arcs are congruent if and only if their corresponding central angles are congruent.

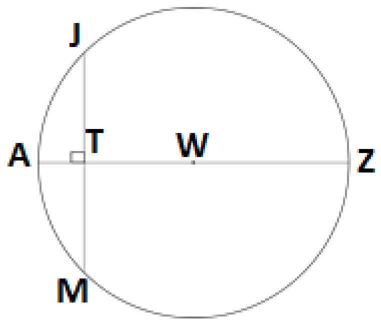
> ***Theorem 3**. In a circle, a diameter bisects a chord and an arc with the same endpoints if and only if it is perpendicular to the chord.*



In $\odot M$, diameter QR bisects chord ST and $\widehat{ST}$ since $\overline{QR} \perp \overline{ST}$.

Proof of the Theorem

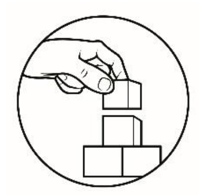
Use a two-column proof to prove that segments and arcs are congruent by showing that $\overline{AZ}$ bisects $\overline{JM}$ and $\widehat{JM}$.



Given: In $\odot$W, $\overline{AZ}$ is a diameter. $\overline{AZ} \perp \overline{JM}$ at T.
Prove: 1. $\overline{AZ}$ bisects $\overline{JM}$
2. $\overline{AZ}$ bisects $\widehat{JM}$

Proof:

Statements	Reasons
1. $\odot$W with diameter $\overline{AZ} \perp$ chord $\overline{JM}$	1. Given
2. $\angle JTW$ and $\angle MTW$ are right angles.	2. Definition of Perpendicular Lines
3. $\angle JTW \cong \angle MTW$	3. Right angles are congruent.
4. $\overline{WJ} \cong \overline{WM}$	4. Radii of the same circle are congruent.
5. $\overline{WT} \cong \overline{WT}$	5. Reflexive/Identity Property of Equality
6. $\Delta JTW \cong \Delta MTW$	6. HyL Theorem
7. $\overline{JT} \cong \overline{MT}$	7. Corresponding Parts of Congruent Triangles are Congruent (CPCTC)
8. **$\overline{AZ}$ bisects $\overline{JM}$**	8. Definition of Segment Bisector
9. $\angle JWA \cong m\angle MWA$	9. CPCTC
10. $m\angle JWA = m\angle MWA$	10. Congruent angles have equal measures.
11. $m\widehat{AJ} = m\angle JWA$ $m\widehat{AM} = m\angle MWA$	11. The degree measure of an arc and the central angle that intercepts it are equal.
12. $m\widehat{AM} = m\widehat{AJ}$	12. Substitution Property of Equality
13. $\widehat{AM} \cong \widehat{AJ}$	13. Definition of Congruent Arcs
14. **$\overline{AZ}$ bisects $\widehat{JM}$**	Definition of Segment Bisector

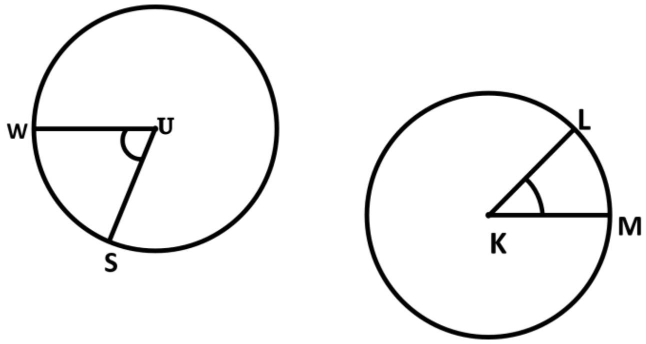


What's More

Activity 1. Prove Me Right

Complete the two-column proof to prove that the central angles intercepting two corresponding congruent minor arcs from corresponding congruent circles are congruent. Be guided by the statements and reasons already provided for you.

Given: $\odot U \cong \odot K$ and $\widehat{WS} \cong \widehat{LM}$
Prove: $\angle WUS \cong \angle LKM$



Proof:

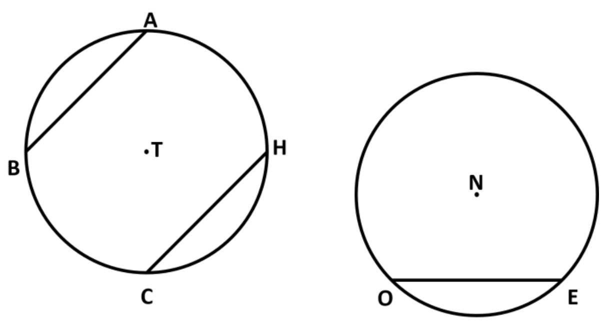
Statement	**Reason**
1.	1. Given
2. In $\odot U$, $m\widehat{WS} = m\angle WUS$. In $\odot K$, $m\widehat{LM} = m\angle LKM$.	2.
3.	3. Definition of Congruent Arcs
4. $m\angle WUS = m\angle LKM$	4.
5. $\boldsymbol{\angle WUS \cong \angle LKM}$	5.

Activity 2. Minor Arcs and Chords

Complete the two-column proof to prove that the chords from congruent circles with corresponding congruent minor arcs are congruent. Be guided by the statements and reasons already provided for you.

Given: $\odot T \cong \odot N$
$\widehat{AB} \cong \widehat{OE}$

Prove: $\overline{AB} \cong \overline{OE}$



Proof:

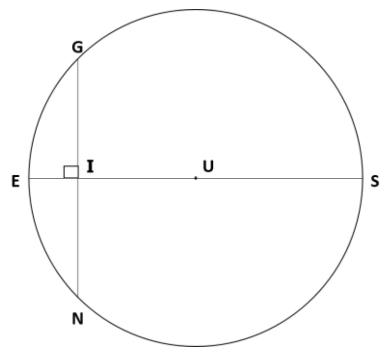
Statement	Reason
1.	1. Given
2. $m\widehat{AB} = m\widehat{OE}$	2.
3. $m\widehat{AB} = m\angle ATB$ and $m\widehat{OE} = m\angle ONE$	3.
4.	4. Substitution Property of Equality
5. $\angle ATB \cong \angle ONE$	5.
6. $\overline{TA} \cong \overline{TB} \cong \overline{NO} \cong \overline{NE}$	6.
7.	7. SAS Postulate
8. $\overline{\boldsymbol{AB}} \cong \overline{\boldsymbol{OE}}$	8.

Activity 3. Diameter Bisects Chords.

Complete the two-column proof to prove that the two chords are perpendicular if the diameter bisects the other chord. Be guided by the statements and reasons already provided for you.

Given: $\odot U$ with diameter $\overline{ES}$
$\overline{ES}$ bisects $\overline{GN}$ at I
$\overline{ES}$ bisects $\widehat{GN}$ at E

Prove : $\overline{ES} \perp \overline{GN}$



Proof:

Statement	Reasons
1.	1. Given
2. $\overline{GI} \cong \overline{NI}$ $\widehat{GE} = \widehat{NE}$	2.
3. $\overline{UI} \cong \overline{UI}$	3.
4.	4. Radii of the same circle are congruent.
5. $\Delta GIU \cong \Delta NIU$	5.
6. $\angle UIG \cong \angle UIN$	6.
7. $\angle UIG$ and $\angle UIN$ are right angles.	7. Angles which form a linear pair and are congruent are right angles.
8. $\overline{IU} \perp \overline{GN}$	8.
9. $\overline{ES} \perp \overline{GN}$	9. $\overline{IU}$ is on $\overline{ES}$

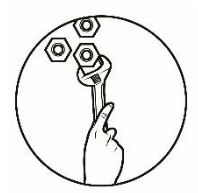
What I Have Learned

To summarize what you have learned, fill in the blanks with the correct terms.

1. If the radii of the two circles are ___________, then the circles are congruent.

2. Congruent arcs are arcs of the same circle and of congruent circles with ___________.

3. Minor arcs of congruent circles having corresponding congruent ___________ are congruent.



What I Can Do

On your birthday, your godparent gave you a thousand Pesos as a gift and told you to spend it wisely. Show, through a budget pie graph, how you would allocate this amount then answer the questions that follow. Your responses to the questions and your graph will be scored according to the given rubrics.

1. In which entry was the highest budget allocated? Why did you allot this item with the highest amount?
2. In which entry was the least budget allocated? Why did you allot this item with the least amount?
3. What is the degree measure of every entry in your pie graph?
4. How is the measure of the central angles related to the budget you have allocated for your entries?

Score	**Descriptors for the Content**
5	The justification is correct, substantial, specific, and convincing.
4	The justification is correct, substantial, and specific but not convincing.
3	The justification is correct and substantial but not specific and convincing.
2	The justification is correct but not substantial, specific, and convincing.
1	There is justification but it is not correct, substantial, specific, and convincing.

Score	**Descriptors for the Pie Graph** Criteria: a. The budgeting is logical and appropriate. b. The pie graph is accurately divided. c. The pie graph comes with clear and readable descriptions. d. The sum of the amounts in all the entries is ₱1,000.
5	The four criteria were met.
4	Three criteria were met.
3	Two criteria were met.
2	One criterion was met.
1	A pie graph is presented but none of these criteria were met.

Lesson 2 Theorems Related to Arcs, Chords, and Inscribed Angles

What's New

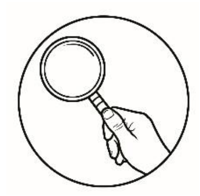
If and Then

Read the following *If-then* statements. State whether you agree with the statement or not. Justify your answer.

1. If the circle is intercepted by a diameter, then the arc measures 180^0.

2. If a square is inscribed in a circle, then it divides the circle into four congruent arcs.

The activity that you just have done posed situations where a premise (the if clause) is presented and a conclusion (the then clause) is made.



What is It

In the next set of activities, you are tasked to prove theorems you used in the previous module. We are going to review some of them and then provide the proofs to these theorems.

We will start with the following concepts.

An **inscribed angle** is an angle whose vertex is on the circle and whose sides contain chords of the circle. The arc that lies in the interior of an inscribed angle and has endpoints on the angle is called the intercepted arc of the angle.
An inscribed angle may contain the center of the circle in its interior, may have the center of the circle on one of its sides, or the center of the circle may be at the exterior of the circle.
Example:

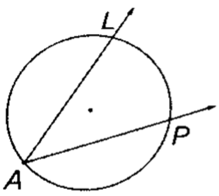


Figure 1

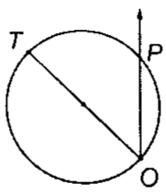


Figure 2

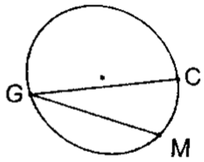


Figure 3

$\angle LAP$, $\angle TOP$, and $\angle CGM$ are inscribed angles. Their respective vertices, A, O and G are points on the circumference of the circles. Their respective sides, $\overline{AL}$ and $\overline{AP}$, $\overline{OT}$ and $\overline{OP}$, and $\overline{GC}$ and $\overline{GM}$, contain chords of the circles.

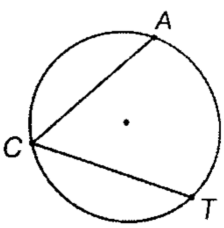
$\widehat{LP}$, $\widehat{TP}$, and $\widehat{CM}$ lie in the interior of inscribed angles $\angle LAP$, $\angle TOP$, and $\angle CGM$, respectively. Thus, $\widehat{LP}$, $\widehat{TP}$, and $\widehat{CM}$ are the intercepted arcs of these inscribed angles.

<u>Theorems on Inscribed Angles</u>

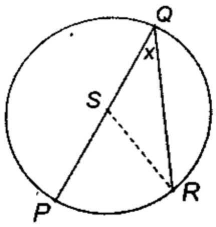
> ***Theorem 1***. *If an angle is inscribed in a circle, then the measure of the angle is equal to one-half the measure of its intercepted arc.*

- In the figure, $\angle ACT$ is an inscribed angle and $\widehat{AT}$ is its intercepted arc.

- If the measure of $\widehat{AT}$ is equal to 120^0, then the measure of $\angle ACT$ is equal to 60^0.



Proof of the Theorem



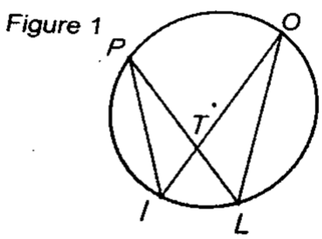
Given : $\angle PQR$ is inscribed in $\odot S$ and $\overline{PQ}$ is a diameter.
Prove: $m\angle PQR = \frac{1}{2} m\widehat{PR}$

Proof:

Draw $\overline{PQ}$ and let $m\angle PQR = x$

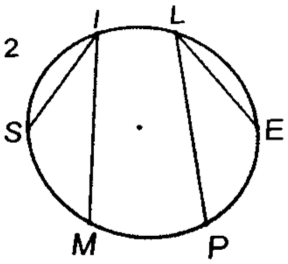
Statement	Reasons
1. $\angle PQR$ is inscribed in $\odot S$ and $\overline{PQ}$ is a diameter.	1. Given
2. $\overline{QS} \cong \overline{RS}$	2. Radii of a circle are congruent.
3. ΔQRS is an isosceles Δ.	3. Definition of Isosceles Triangle
4. $\angle PQR \cong \angle QRS$	4. The base angles of an isosceles triangle are congruent.
5. $m\angle PQR = m\angle QRS$	5. The measures of congruent angles are equal.
6. $m\angle QRS = x$	6. Transitive Property of Equality (If $m\angle PQR = x$ and $m\angle PQR = m\angle QRS$, then $m\angle QRS = x$.
7. $m\angle PSR = 2x$	7. The measure of an exterior angle of a triangle is equal to the sum of the measures of its remote interior angles.
8. $m\angle PSR = m\widehat{PR}$	8. The measure of a central angle is equal to the measure of its intercepted arc.
9. $m\widehat{PR} = 2x$	9. Transitive Property of Equality (from 7 & 8)
10. $m\widehat{PR} = 2(m\angle PQR)$	10. Substitution Property of Equality (from 5 & 6)
11. $\boldsymbol{m\angle PQR = \frac{1}{2} m\widehat{PR}}$	11. Multiplication Property of Equality

***Theorem 2**. If two inscribed angles of a circle (or of congruent circles) intercept congruent arcs or the same arc, then the angles are congruent.*



In figure 1 , $\angle PIO$ and $\angle PLO$ intercept $\widehat{PO}$. Since $\angle PIO$ and $\angle PLO$ intercept the same arc, then $\angle PIO \cong \angle PLO$.

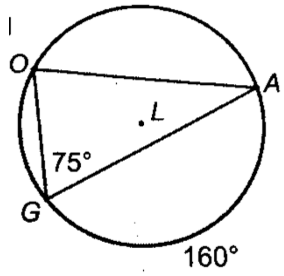
Figure 2



In figure 2, $\angle SIM$ and $\angle ELP$ intercept $\widehat{SM}$ and $\widehat{EP}$, respectively. If $\widehat{SM}$ is congruent to $\widehat{EP}$, then $\angle SIM \cong \angle ELP$.

The proof of the theorem is given as an exercise in activity 1 of what's more.

Example1. ΔGOA is inscribed in $\odot L$. If the measurement of $\angle OGA = 75$ and the measure of $\widehat{AG}$ is 160°, find:



a. $m\widehat{OA}$

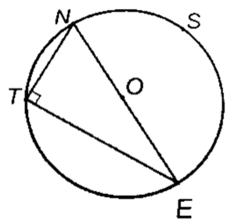
$$m\angle OGA = \frac{1}{2}\, m\widehat{OA}$$
$$75 = \frac{1}{2}\, m\widehat{OA}$$
$$150 = m\widehat{OA}$$
$$m\widehat{OA} = 150$$

b. $m\angle GOA$

$$m\angle GOA = \frac{1}{2}\, m\widehat{AG}$$
$$m\angle GOA = \frac{1}{2}\,(160)$$
$$m\angle GOA = 80$$

> ***Theorem 3****. If an inscribed angle of a circle intercepts a semicircle, then the angle is a right angle.*

In $\odot O$, $\angle NTE$ intercepts $\widehat{NSE}$. If $\widehat{NSE}$ is a semicircle, then $\angle NTE$ is a right angle.



Proof of the Theorem

Given: in circle O, $\angle NTE$ intercepts a semicircle NSE

Prove: $\angle NTE$ is a right angle

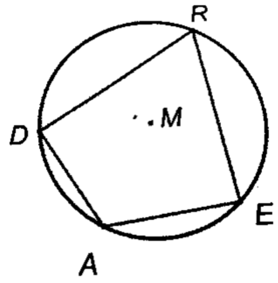
Proof:

Statements	Reasons
1. $\angle NTE$ intercepts a semicircle NSE	Given
2. $m\widehat{NSE} = 180°$	The degree measure of a semicircle is 180°.
3. $\angle NTE = 90°$	The degree measure of an inscribed angle is one-half the degree measure of its intercepted arc.
4. $\angle NTE$ is a right angle	If angle measures 90°, then it is a right angle.

Theorem 4. *If a quadrilateral is inscribed in a circle, then its opposite angles are supplementary.*

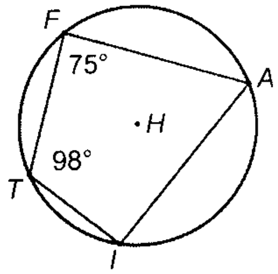
If Quadrilateral $DREA$ is inscribed in $\odot M$, then

- $m\angle RDA + m\angle REA = 180.$
- $m\angle DRE + m\angle DAE = 180.$



The proof of the theorem is given as an exercise in activity 1 of what's more.

Example 2. Quadrilateral $FAIT$ is inscribed in $\odot H$. If $m\angle AFT = 75$ and $m\angle FTI = 98$, find:

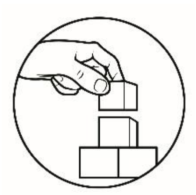


a. $m\angle TIA$

$180^0 = m\angle AFT + m\angle TIA$
$180^0 = 75^0 + m\angle TIA$
$180^0 - 75^0 = m\angle TIA$
$105^0 = m\angle TIA$
$m\angle TIA = 105$

b. $m\angle FAI$

$180^0 = m\angle FTI + m\angle FAI$
$180^0 = 98^0 + m\angle FAI$
$180^0 - 98^0 = m\angle FAI$
$82^0 = m\angle FAI$
$m\angle FAI = 82$

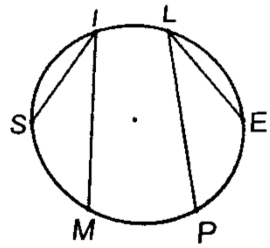


What's More

Activity 1. Prove me right. Write a proof for each of the following theorems.

a) If two inscribed angles of a circle (or of congruent circles) intercept congruent arcs or the same arc, then the angles are congruent.

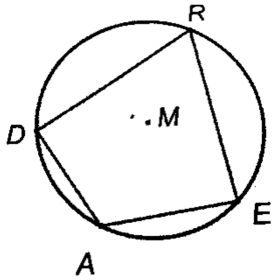
Given: In the circle at the right, $\widehat{SM}$ and $\widehat{PE}$ are the intercepted arcs of $\angle SIM$ and $\angle ELP$ respectively.
$\widehat{SM} \cong \widehat{PE}$



Prove: $\angle SIM \cong \angle ELP$

b) If a quadrilateral is inscribed in a circle, then its opposite angles are supplementary.

Given: Quadrilateral DREA is inscribed in $\odot M$.
Prove: $\angle RDA$ and $\angle REA$ are supplementary.



Activity 2. Read and Analyze

$\overline{DR}$ is a diameter of $\odot O$. If $m\widehat{MR} = 70$, find:

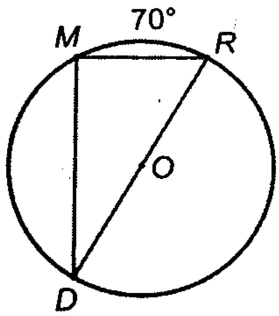
a. $m\angle RDM$

b. $m\angle DRM$

c. $m\angle DMR$

d. $m\widehat{DM}$

e. $m\widehat{RD}$

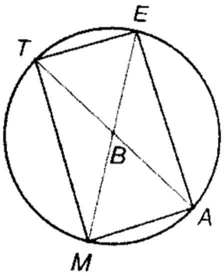


Activity 3. A Quad!

Rectangle $TEAM$ is inscribed in $\odot B$. If $m\widehat{TE} = 64$ and $m\angle TEM = 58$, find:

a. $m\widehat{TM}$ b. $m\widehat{MA}$ c. $m\widehat{AE}$

d. $m\angle MEA$ e. $m\angle TAM$



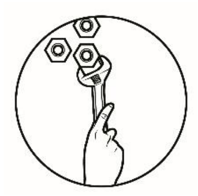
How did you do in the activity? What did you find out? I believe you learned something and discovered or proved that you are able to provide correct conclusions backed up by valid reasons.

What I Have Learned

After doing the activities, summarize what you have learned by filling in the blanks with the correct terms.

1. The measure of an inscribed angle is one-half the measure of its ___________.

2. ___________ of an inscribed quadrilateral are supplementary.

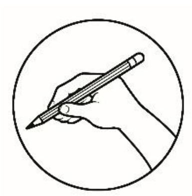


What I Can Do

In the previous activities, you have done proving using the two-column proof. Based on your daily activities, cite a situation with a justification, where the theorems on inscribed angles are applied.

____
____
____
____
____

Score	Descriptors for each Situation
4	The situation is correct with substantial, specific, and convincing justification.
3	The situation is correct with substantial and specific but not convincing justification.
2	The situation is correct with substantial but not specific and not convincing justification.
1	A situation is presented.

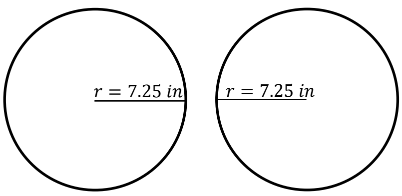


Assessment

Read and analyze each item very carefully. On your answer sheet, write the letter of the choice that corresponds to the correct answer.

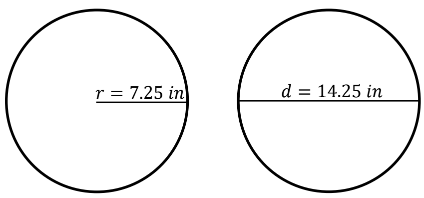
1. Which of the following illustrations do NOT show congruence?

a.



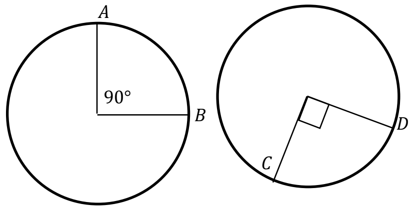
The two circles are congruent.

c.



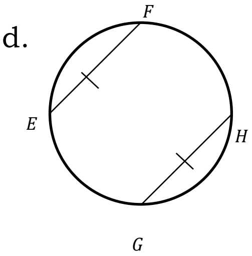
The two circles are congruent.

b.



$\widehat{AB} \cong \widehat{CD}$

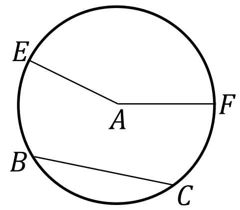
d.



$\widehat{EF} \cong \widehat{GH}$

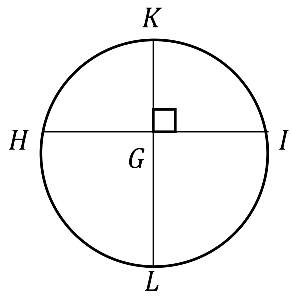
2. If an inscribed angle of a circle intercepts a semicircle, the angle is ____.
   a. acute   b. obtuse   c. right   d. straight

3. Consider $\odot A$ with $\widehat{EF} = 125°$. Which of the following statements is NOT always true?



   a. In $\odot A$, $\angle EAF = 125°$.
   b. $\widehat{BC} = 125°$ if and only if $\overline{EF} \cong \overline{BC}$.
   c. $\widehat{EF} \cong \widehat{BF}$ if and only if $\angle EAF \cong \angle BAF$.
   d. $\angle EAF \cong \angle BAF$ if and only if $\widehat{EF} = 125°$.

Refer to $\odot G$ for items 4 to 6.

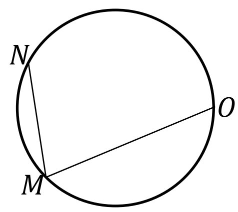


4. Which of the following best describes the illustration involving $\odot G$?
   a. $\overline{KL}$ is bisected at G.
   b. $\overline{HI}$ is bisected at G.
   c. Any two intersecting diameters are perpendicular.
   d. When diameters are perpendicular, they intersect at the center of the circle.

5. Suppose $\overline{HI} = 14.5$ and $\widehat{IKH} = 105°$, then ____.
   a. $\overline{HG} = 7.25$, $\overline{GI} = 7.25$, $\widehat{KI} = 52.5°$, and $\widehat{KH} = 52.5°$
   b. $\overline{HG} = 14.5$, $\overline{GI} = 14.5$, $\widehat{KI} = 105°$, and $\widehat{KH} = 105°$
   c. $\overline{KG} + \overline{GL} = 29$ and $\widehat{HI} + \widehat{LI} = 255°$.
   d. Insufficient data, answer cannot be determined.

6. Suppose $\widehat{HLI} = 200°$, find the measure of $\widehat{KH}$.
   a. 360°   b. 200°   c. 160°   d. 80°

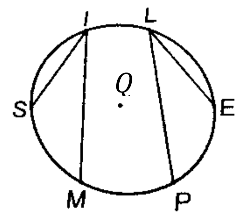
7. What is the measure of the arc intercepted by inscribed angle $\angle NMO$ if $\angle NMO = 60°$?



   a. $\widehat{NO} = 30°$ c. $\widehat{NMO}$ =120°
   b. $\widehat{NMO} = 30°$ d. $\widehat{NO} = 120°$

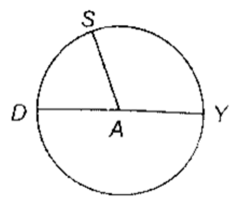
8. What phrase correctly completes the theorem, "*If two inscribed angles of a circle _____, then the angles are congruent*"?
   a. inscribe congruent arcs
   b. intercept congruent arcs
   c. inscribed congruent angles
   d. intercept congruent angles

9. In $\odot Q$, $\widehat{MS} \cong \widehat{PE}$ and $\widehat{MS} = 30°$. Which of the following statements is correct?



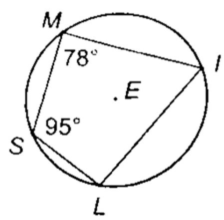
   a. $\angle SIM$ and $\angle ELP$ both measure 15°.
   b. $\angle SIM$ and $\angle ELP$ both measure 30°.
   c. $\widehat{MS}$ and $\widehat{PE}$ inscribe $\angle SIM$ and $\angle ELP$.
   d. $\widehat{MS}$ and $\widehat{PE}$ intercept $\angle SIM$ and $\angle ELP$.

10. In $\odot A$, what is the measure of $\angle SAY$ if $\widehat{DSY}$ is a semicircle and $m\angle SAD = 70$?



   a. 20°
   b. 70°
   c. 110°
   d. 150°

11. Quadrilateral $SMIL$ is inscribed in $\odot E$.
   If $m\angle SMI = 78$ and $m\angle MSL = 95$, find $m\angle MIL$.



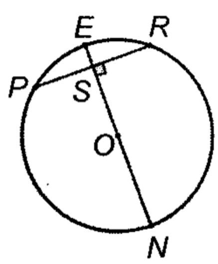
   a. 78°
   b. 85°
   c. 95°
   d. 102°

12. The opposite angles of a quadrilateral inscribed in a circle are ____.
   a. complementary b. obtuse c. right d. supplementary

13. All of the following parts from two congruent circles guarantee that two minor arcs from congruent circles are congruent except for one. Which one is it?
   a. Their corresponding congruent chords.
   b. Their corresponding central angles.
   c. Their corresponding inscribed angles.
   d. Their corresponding intercepted arcs.

Refer to ⊙O for items 14 and 15.

14. In ⊙O, what is PR if $NO = 15$ units and $ES = 6$ units?



   a. 28 units
   b. 24 units
   c. 12 units
   d. 9 units

15. In ⊙O, what is the measure of $\angle PSN$?
   a. 45°
   b. 80°
   c. 90°
   d. 180°

Additional Activity

If - then Statement

Compose three original *If-then* statements. Make sure that your statements are realistic and acceptable.

1. ____

   ____

2. ____

   ____

3. ____

   ____

Every *If-then* statement will be scored according to the rubric below.

Score	Descriptors
3	The premise is valid and the conclusion is correct/acceptable.
2	The premise is valid but the conclusion is incorrect/unacceptable.
1	The premise and the conclusion do not match.

Answer Key

What I Know

1. d	9. c
2. b	10. a
3. c	11. d
4. d	12. c
5. b	13. d
6. c	14. c
7. a	15. d
8. a	

What's In

1) ARC
2) CHORD
3) DIAMETER
4) CENTRAL ANGLE
5) INSCRIBED ANGLE

Lesson 1.

What's New

1) Yes, definition of semicircle.
2) Yes because all radii of the same circle are congruent.
3) Need to be investigated

Lesson 1. What's More

Activity 1

Statement	Reason
1. $\odot U \cong \odot K$ $\overset{\frown}{WS} \cong \overset{\frown}{LM}$	1. Given
2. In $\odot U$, m $\overset{\frown}{WS} = m\angle WUS$. In $\odot K$, m $\overset{\frown}{LM} = m\angle LKM$.	2. The degree measure of a minor arc is equal to the measure of the central angle which intercepts the arc.
3. m $\overset{\frown}{WS}$ = m $\overset{\frown}{LM}$	3. Definition of Congruent Arcs
4. $m\angle WUS = m\angle LKM$	4. Substitution Property of Equality
5. $\angle WUS \cong \angle LKM$	5. Definition of Congruent Angles

Activity 2

Statement	Reason
1. $\odot T \cong \odot N$ $\widehat{AB} \cong \widehat{OE}$	1. Given
2. $m\widehat{AB} \cong m\widehat{OE}$	2. Definition of Congruent Arcs
3. $m\widehat{AB} = m\angle ATB$ and $m\widehat{OE} = m\angle ONE$	3. The degree measure of a minor arc is equal to the measure of the central angle which intercepts it.
4. $m\angle ATB = m\angle ONE$	4. Substitution Property of Equality
5. $\angle ATB \cong \angle ONE$	5. Definition of Congruent Angles
6. $\overline{TA} \cong \overline{TB} \cong \overline{NO} \cong \overline{NE}$	6. The radii of the same circle or of congruent circles are congruent.
7. $\Delta ATB \cong \Delta ONE$	7. SAS Postulate
8. $\overline{AB} \cong \overline{OE}$	8. CPCTC

Lesson 1. What's More

Activity 3

Statement	Reasons
1. ⊙U with diameter $\overline{ES}$, $\overline{ES}$ bisects $\overline{GN}$ at I, and $\overline{ES}$ bisects $\overline{GN}$ $\widehat{GN}$	1. Given
2. $\overline{GI} \cong \overline{NI}$ $\widehat{GE} = \widehat{NE}$	2. Definition of Bisector
3. $\overline{UI} \cong \overline{UI}$	3. Reflexive/Identity Property of Equality
4. $\overline{UG} \cong \overline{UN}$	4. Radii of the same circle are congruent.
5. $\Delta GIU \cong \Delta NIU$	5. SSS Postulate
6. $\angle UIG = \angle UIN$	6. CPCTC
7. $\angle UIG$ and $\angle UIN$ are right angles.	7. Angles which form linear pair and are congruent are right angles.
8. $\overline{IU} \perp \overline{GN}$	8. Definition of Perpendicular Lines
9. $\overline{ES} \perp \overline{GN}$	9. $\overline{IU}$ is on $\overline{ES}$

Lesson 1. What I Have Learned

1. congruent
2. equal measures.
3. central angles

Lesson 1. What I can Do

The varied outputs from the students will be evaluated using the given rubrics.

Lesson 2. What's New

1) Yes because a diameter divides the circle into two equal parts called semicircles.
2) Yes because a square has four congruent vertices and these cut the circle into four congruent arcs.

Lesson 2. What 's More

Activity 1.a

Statements	Reasons
1. In the circle, $\widehat{SM}$ and $\widehat{PE}$ are the intercepted arcs of $\angle SIM$ and $\angle ELP$ respectively and $\widehat{SM} \cong \widehat{PE}$	Given
2. $m\widehat{SM} = m\widehat{PE}$	Definition of Congruent Arcs
3. $m\angle SIM = \frac{1}{2}m\widehat{SM}$ $m\angle ELP = \frac{1}{2}m\widehat{PE}$	The degree measure of an inscribed angle is one-half the degree measure of its intercepted arc.
4. $2m\angle SIM = m\widehat{SM}$ $2m\angle ELP = m\widehat{PE}$	Multiplication property of equality
5. $2m\angle SIM = 2m\angle ELP$	Substitution property of equality
6. $m\angle SIM = m\angle ELP$	Division property of equality
7. $\angle SIM \cong \angle ELP$	Congruent angles are angles with equal measures.

Lesson 2. What's More

Activity 1.b

Statements	Reasons
1. Quadrilateral DREA is inscribed in $\odot M$	1. Given
2. $m\widehat{REA} + m\widehat{RDA} = 360$	2. $\widehat{REA}$ and $\widehat{RDA}$ formed a circle.
3. $\angle RDA$ intercepts $\widehat{REA}$ and $\angle REA$ intercepts $\widehat{RDA}$	3. Definition of intercepted arc
4. $m\angle RDA = \frac{1}{2}m\widehat{REA}$ and $m\angle REA = \frac{1}{2}m\widehat{RDA}$	4. The degree measure of an inscribed angle is one-half the degree measure of its intercepted arc.
5. $2m\angle RDA = m\widehat{REA}$ and $2m\angle REA = m\widehat{RDA}$	5. Multiplication Property of Equality
6. $2m\angle RDA + 2m\angle REA = 360$	6. Substitution Property of Equality
7. $m\angle RDA + m\angle REA = 180$	7. Division Property of Equality
8. $\angle RDA$ and $\angle REA$ are supplementary	8. Definition of supplementary angles

Lesson 2.
What's More

Activity 2
a. 35
b. 55
c. 90
d. 110
e. 180

Activity 3
a. 116
b. 64
c. 116
d. 32
e. 58

Lesson 2.
What I have Learned

1. intercepted arc
2. Opposite angles

Lesson 2.
What I can Do

The varied outputs from the students will be evaluated using the given rubric.

Assessment

1. c	6. d	11. c
2. c	7. d	12. d
3. d	8. b	13. d
4. b	9. a	14. b
5. a	10. c	15. c

Additional Activity

The varied outputs from the students will be evaluated using the given rubric.

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