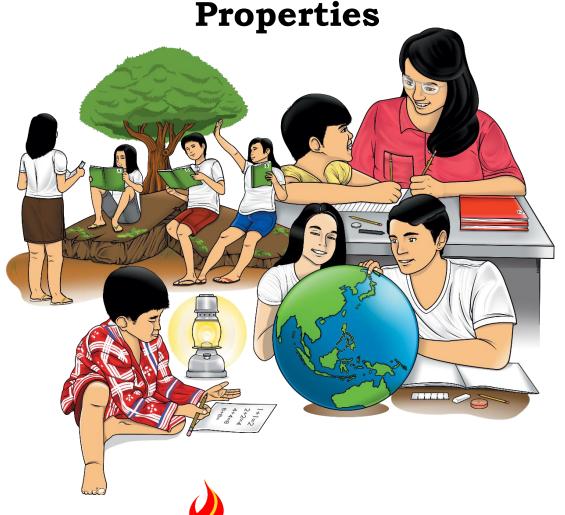




Mathematics

Quarter 2 – Module 6:
Applying the Distance Formula
to Prove Some Geometric



Mathematics – Grade 10 Alternative Delivery Mode

Quarter 2 – Module 6: Applying the Distance Formula to Prove Some Geometric Properties

First Edition, 2020

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Properties



Introductory Message

This Self-Learning Module (SLM) is prepared so that you, our dear learners, can continue your studies and learn while at home. Activities, questions, directions, exercises, and discussions are carefully stated for you to understand each lesson.

Each SLM is composed of different parts. Each part shall guide you step-bystep as you discover and understand the lesson prepared for you.

Pre-tests are provided to measure your prior knowledge on lessons in each SLM. This will tell you if you need to proceed on completing this module or if you need to ask your facilitator or your teacher's assistance for better understanding of the lesson. At the end of each module, you need to answer the post-test to self-check your learning. Answer keys are provided for each activity and test. We trust that you will be honest in using these.

In addition to the material in the main text, Notes to the Teacher are also provided to our facilitators and parents for strategies and reminders on how they can best help you on your home-based learning.

Please use this module with care. Do not put unnecessary marks on any part of this SLM. Use a separate sheet of paper in answering the exercises and tests. And read the instructions carefully before performing each task.

If you have any questions in using this SLM or any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator.

Thank you.



What I Need to Know

This module focuses generally in applying distance formula in proving properties of some geometric figures.

After going through this module, you are expected to:

- 1. find the distance between two points using the Distance Formula; and
- 2. apply Distance Formula in proving properties of some geometric figures.



A) 3a

What I Know

rect per		e letter of the co	rrect answer an	d write it on a separate	sheet of						
1.	In the Cartesian plane, what is the distance of the point (2, -3) from the origin?										
	A) $\sqrt{5}$	B) √13	C) 5	D) 13							
2.	What is the distance between point A(3, 4) and point B (10, 4)?										
	A) 4	B) 5	C) 6	D) 7							
3.	Which of the following should be the value of x so that the distance between the points $(x, -2)$ and $(12, -2)$ is 7?										
	A) 2	B) 3	C) 4	D) 5							
4.	Which of the f	ollowing equatio	n describes the	distance formula?							
	•			$\sqrt{(x_2-x_1)^2-(y_2-y_1)^2}$							
	$B) d = \sqrt{(x_2)^2}$	$(x_1 + x_1)^2 + (y_2 + y_1)^2$	$\overline{)^2}$ D) $d =$	$= \sqrt{(x_2 + x_1)^2 - (y_2 + y_1)^2}$							
5.	What is the di	stance between	two points with	coordinates (4, 3) and	(1,7)?						

A) 25 B) 16 C) 5 D) 2 6. What is the distance between point A (4a, 4a) and point C (0, a)?

B)5a

7. Both points D and U are on the fourth quadrant. If the distance between point D and U is 3 units and D is at (2,-1), which of the following are the coordinates

C) 9a

of point U? A) (2,-4) B) (1, -6) C) (4,-1)D) (2,-5)

D)12a

- 8. What is the perimeter of an equilateral triangle if two of its vertices have coordinates (0,2) and (0,5)?
 - A) 3
- B) 6
- C) 9
- D) 12
- 9. What kind of triangle is formed when its vertices (0,4), (-4,-2) and (4,-2) are plotted on the Cartesian plane?
 - A) equilateral
- B) isosceles
- C) right
- D) scalene
- 10. What type of quadrilateral is formed by the given vertices C (0,0), A (a,0), R (a,a) and E (0,a)?
 - A) kite
- B) parallelogram
- C) rectangle
- D) square
- 11. In Quadrilateral LOVE, what is the distance of point O from the origin?

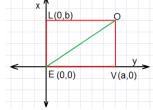


C)
$$\sqrt{a^2 + b^2}$$

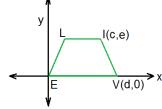
D) $\sqrt{a^2 - b^2}$

B)
$$\sqrt{a-b}$$

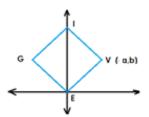
D)
$$\sqrt{a^2 - b^2}$$



- 12. What are the coordinates of point L in isosceles trapezoid LIVE?
 - A) (d-c, e)
- C) (d-c,0)
- B) (c-d,e)
- D) (e-c,d)



- 13. If GIVE is a square, what is the distance between the points V and G?
 - A) a
- C) $2\sqrt{(a + b)}$
- B) 2a
- D) $2\sqrt{(a-b)}$



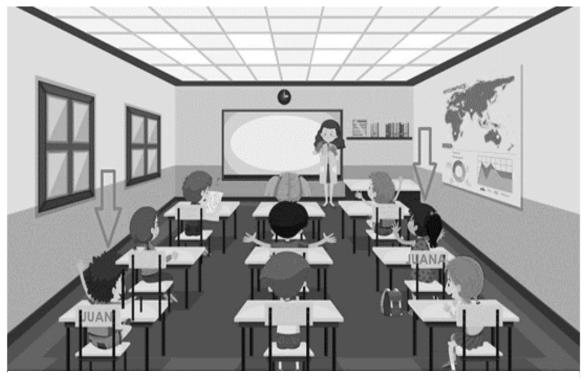
- 14. To prove that quadrilateral RICE is a rectangle, which of the following should be proven congruent?
 - A) $\overline{RI} \cong \overline{IC}$, $\overline{IC} \cong \overline{CE}$
- C) $\overline{IC} \cong \overline{CE}, \overline{RC} \cong \overline{IE}$
- B) $\overline{RE} \cong \overline{IC}, \overline{RI} \cong \overline{CE}$
- D) $\overline{RC} \cong \overline{IE}, \overline{RE} \cong \overline{IC}$
- 15. To prove that triangle ABC is a scalene triangle which of the following should be proven?
 - A) $\overline{AB} \cong \overline{BA}$
- B) $\overline{AB} \cong \overline{BC} \cong \overline{AC}$ C) $\overline{AB} \cong \overline{BC}$
- D) $\overline{AB} \ncong \overline{BC} \ncong \overline{AC}$

Lesson Distance Formula



What's In

In your previous lessons, you have learned how to plot points and name the coordinates of the points on the Cartesian plane. Examine the situation below.



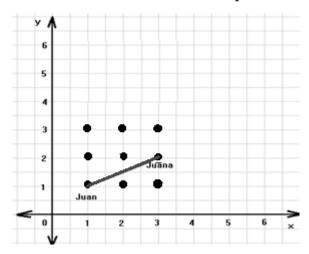
https://www.dreamstime.com/illustration/classroom-cartoon.html

The picture above depicts a classroom with 9 seats arranged in 3 rows and 3 columns. During their Mathematics class, the teacher asked Juan and Juana to describe their location.

If you were Juan and Juana,

- How would you describe your location? What mathematical concepts can you use to describe your location?
- How far are you from each other? How will you determine your distance from each other?

Now let us visualize the situation in the Cartesian plane.

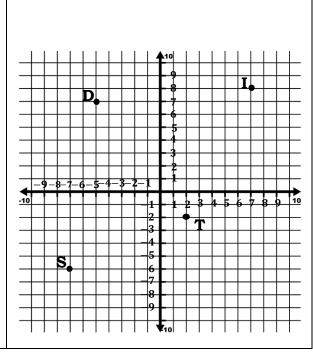


Using the illustration above, the location of Juan and Juana can be determined by naming the point of their location in the Cartesian plane. Remember that the coordinates of a point are always written as an ordered pair of the form (x,y). The first number corresponds to the x-coordinate and the second is the y-coordinate. Hence, Juan is at point (1, 1) and Juana is at point (3, 2).

Now, it's your turn!

Activity 1.

- A) Write the coordinates of each given point:
 - a) D _____
 - b) I _____
 - c) S _____
 - d) T _____
- B) Plot the following points on the Cartesian plane:
 - e) A (8, 3)
 - f) N (-6, 4)
 - g) C (-5, -7)
 - h) E (4, -9)

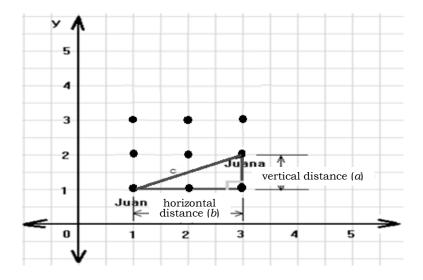


What's New



Using the situation of Juan and Juana, find the distance between them if the students in the class were seated 1 meter from each other.

The distance between two points in the coordinate plane is the length of the segment that joins the two points. Hence, to find the distance between Juan and Juana, draw a segment joining the points of their location. The figure below demonstrates that the distance between Juan and Juana can be found by forming a right triangle in which their distance is the hypotenuse, while the horizontal and the vertical segments are the legs of the right triangle.



Recall the Pythagorean Theorem: The square of the hypotenuse of a right triangle is equal to the sum of the squares of its two legs.

We will apply the Pythagorean Theorem to solve for the distance between Juan and Juana. Let c be their distance. The vertical distance and the horizontal distance which are the legs of the triangle are 1 meter and 2 meters, respectively. Hence,

$$c^{2} = 1^{2} + 2^{2}$$

$$c^{2} = 1 + 4$$

$$c^{2} = 5$$

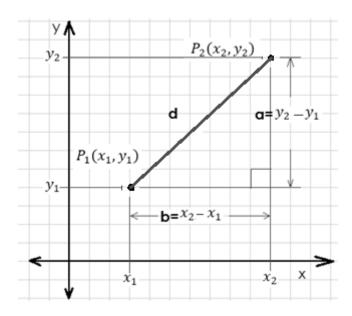
$$c = \sqrt{5} \approx 2.24$$

Therefore, the distance between Juan and Juana is $\sqrt{5}$ or approximately 2.24 meters.



What is It

Let us derive the distance formula to find the distance between two points in the coordinate plane. Consider the figure below, let d be the distance between point P_1 and point P_2 with coordinates (x_1, y_1) and (x_2, y_2) , respectively. It is also shown from the figure that the horizontal distance, b, is $x_2 - x_1$ and the vertical distance, a, is $y_2 - y_1$.



Using the Pythagorean Theorem, we have:

$$d^2 = a^2 + b^2$$

$$d^2 = (y_2 - y_1)^2 + (x_2 - x_1)^2$$

Taking the square roots of both sides of the equation:

$$\sqrt{d^2} = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$$

$$d = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$$

The Distance Formula

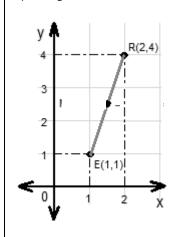
The distance, d, between points $A(x_1, y_1)$ and $B(x_2, y_2)$ may be found using the formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Example 1. Plot and solve the distance between the points E (1, 1) and R (2,4) in the coordinate plane.

Solution:

A) Graph:



B) Distance:

Let P_1 (x_1 , y_1) be equal to E (1, 1) and P_2 (x_2 , y_2) be equal to R(2,4). Hence,

$$x_1 = 1 \text{ and } y_1 = 1; \text{ and }$$

$$x_2 = 2$$
 and $y_2 = 4$

Substitute the coordinates of points E and R in the formula:

$$\mathbf{d} = \sqrt{(\mathbf{x}_2 - \mathbf{x}_1)^2 + (\mathbf{y}_2 - \mathbf{y}_1)^2}$$
$$\mathbf{d} = \sqrt{(2 - 1)^2 + (4 - 1)^2}$$

$$d = \sqrt{(2-1)^2 + (4-1)^2}$$

Simplify it further:

$$d = \sqrt{(1)^2 + (3)^2}$$

$$d = \sqrt{1+9}$$

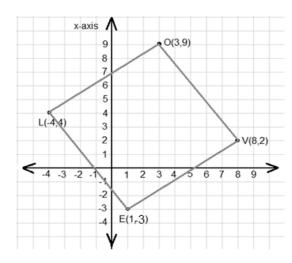
$$d = \sqrt{10}$$

Therefore the distance between points E and R is $\sqrt{10}$.

Example 2. Show that the figure formed when points L (-4, 4), O (3, 9), V (8, 2), E (1,-3) are connected consecutively is a square, then find its perimeter.

Solution:

1. Plot the points L (-4, 4), O (3, 9), V (8, 2), E (1,-3) on the coordinate plane.



- 2. To show that the figure formed is a square, we need to show that all the sides are equal in length and all angles are right angles.
 - A) Show that the lengths of \overline{LO} , \overline{OV} , \overline{VE} and \overline{EL} are congruent.
 - i) to solve the length of LO, we will use the points L (-4, 4) & O (3, 9)

$$LO = \sqrt{(3 - -4)^2 + (9 - 4)^2} = \sqrt{74}$$

ii) to solve for the length of OV, use O (3, 9) & V (8, 2)

$$OV = \sqrt{(8-3)^2 + (2-9)^2} = \sqrt{74}$$

iii) to solve for the length of VE, use V (8, 2) & E (1,-3)

$$VE = \sqrt{(1-8)^2 + (-3-2)^2} = \sqrt{74}$$

iv) to solve for the length of EL, use E (1,-3)& L (-4, 4)

$$VE = \sqrt{(-4-1)^2 + (4-3)^2} = \sqrt{74}$$

Thus, LO=OV=VE=EL.

B) Show that all angles L, O, V, & E are right angles. Note that if two lines are perpendicular, then they formed a right angle and their slopes are negative reciprocal of each other. Hence, we need to find the slope of each side.

Recall: Slope of a line

The slope of a line, m_s given two points (x_1, y_1) and (x_2, y_2) is $m = \frac{y_2 - y_1}{x_2 - x_1}$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

i) The slope of LO, use the points L (-4, 4) & O (3, 9)

$$m_{LO} = \frac{9-4}{3-(-4)} = \frac{5}{7}$$

ii) The slope of OV, use O (3, 9) & V (8, 2)

$$m_{OV} = \frac{2-9}{8-3} = -\frac{7}{5}$$

iii) The slope of VE, use V (8, 2) & E (1,-3)

$$m_{VE} = \frac{-3 - 2}{1 - 8} = \frac{5}{7}$$

iv) The slope of EL, use E (1,-3) & L (-4, 4)

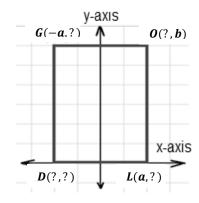
$$m_{EL} = \frac{4 - (-3)}{-4 - 1} = -\frac{7}{5}$$

- C) Based from the computed slopes of each segment, we can conclude the following:
 - The slopes of LO and OV are negative reciprocals of each other, then angle O is a right angle.
 - ii. The slopes of OV and VE are negative reciprocals of each other, then angle V is a right angle.
 - iii. The slopes of VE and EL are negative reciprocals of each other, then angle E is a right angle.
 - iv. The slopes of EL and LO are negative reciprocals of each other, then angle L is a right angle.

- 3. Since all the sides of the figure are congruent and all the angles are right angles, then quadrilateral LOVE is a square.
- 4. To solve for the perimeter, use the formula for the perimeter of square.

$$P = 4s = 4(\sqrt{74}) \approx 34.409 \text{ units}$$

Example 3. Find the coordinates (in terms of a and b) of points G, O, L, and D in the figure at the right. (Remember that the coordinate of points on the same vertical line share the same x – coordinate while points on the same horizontal line share the same y – coordinate.)



Solution:

a) finding the coordinates of G:

Since G and O lie on the same horizontal line, it implies that they have the same y – coordinate. Thus, the coordinate of G is (-a, b)

b) finding the coordinates of O:

Since O and L lie on the same vertical line, it implies that they have the same x – coordinate. Thus, the coordinate of O is (a, b).

c) finding the coordinates of L:

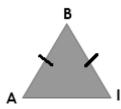
Since L lies on the x-axis, it means that its y – coordinate is 0. Thus, the coordinate of L is (a, 0).

d) finding the coordinates of D:

Since D lies on the x – axis, it means that its y- coordinate is 0. D also lies on the same vertical line with G which means that they have the same x – coordinate. Thus, the coordinates of D is(-a,0).

e) Therefore, the coordinates are G(-a,b), O(a,b), L(a,0) & D(-a,0).

Example 4. Prove that the two sides of an isosceles triangle are congruent.

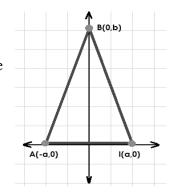


Given: $\triangle ABI$ with sides \overline{AB} , \overline{IB} , \overline{IA}

Prove: $\overline{AB} \cong \overline{IB}$

To prove:

1. Place ΔABI on the coordinate plane and label the coordinate points as shown below:



2. Find the distance between A(-a,0) and B (0,b). Substitute –a and 0 to x_2 and x_1 , respectively, and 0 and b to y_2 and y_1 , respectively.

AB =
$$\sqrt{(-a-0)^2 + (0-b)^2}$$

AB = $\sqrt{a^2 + b^2}$

3. Find the distance between I (a,0) and B (0,b). Substitute a and 0 to x_2 and x_1 , respectively, and 0 and b to y_2 and y_1 , respectively.

$$IB = \sqrt{(a-0)^2 + (0-b)^2}$$

$$IB = \sqrt{a^2 + b^2}$$

- 4. Since $AB = \sqrt{a^2 + b^2}$ and $IB = \sqrt{a^2 + b^2}$, by substitution we can say that AB = IB.
 - $\therefore \overline{AB} \cong \overline{IB}$. The two sides of an isosceles triangle are congruent.

Here are some suggestions to help you place figures for your proofs.

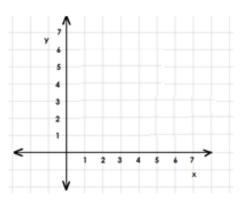
- 1. Use the origin of the coordinate plane as vertex or center of the figure.
- 2. Place at least one side of the figure on a coordinate axis, either the x or y- axis.
- 3. Keep the figure within the first quadrant if possible.
- 4. Use coordinates that make computations as simple as possible.



What's More

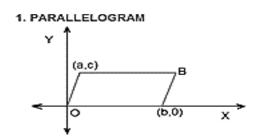
Activity 2:

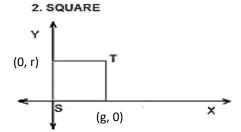
The coordinates of points C and R are (2, 5) and (7, 2), respectively. Plot these points on the coordinate plane and find their distance.



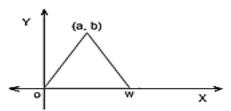
Activity 3. Use the given to answer each question.

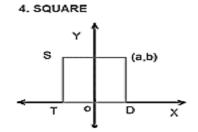
- A) Show that the figure formed when the points F (-2, 6), U (-2, -3), N (7, 6) are connected consecutively is an isosceles right triangle and find its area.
- B) Supply the missing coordinates of the points of each figure below without introducing new letters.





3. EQUILATERAL TRIANGLE





C) Prove using coordinate plane that the diagonals of an isosceles trapezoid are congruent.



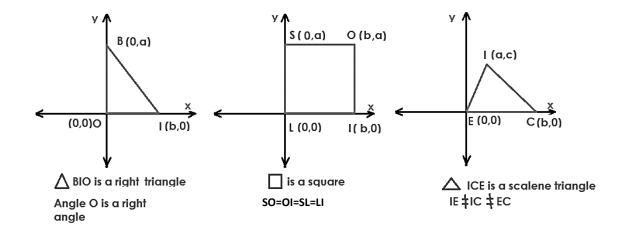
What I Have Learned

1) The Distance Formula

The distance, d, between points $A(x_1, y_1)$ and $B(x_2, y_2)$ may be found using the formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

2) With the use of the coordinate plane, distance formula is very helpful in proving properties of some geometric figures. However, the geometric figure must be placed properly on the coordinate axes so that it will be easier to prove. Here are some of the appropriate ways of placing geometric figures on the coordinate plane.





What I Can Do

Activity 4. Answer the following problems:

- 1) Lieutenant Santos orders an air strike in the battlefield targeting the enemy at a coordinate (2, 5). If he is positioned at a coordinate (-14, -12), how far is he from the target area? If the danger zone is within the 10 km radius from the strike point, is Lt. Santos safe? (Let 1 unit = 1 km)
- 2) Chester and his father stood on their newly-bought rectangular lot whose length and width are 60 and 40 meters, respectively. His father told him that the place they are standing at is one of the four boundary points of their lot. He then told Chester that they are going to put a marker on each of the four boundary points. Help Chester to locate the coordinates of the 3 boundary points using the Cartesian coordinate plane if the coordinates of their location is at (-30, 20). (Let 1 unit = 1 meter)



Assessment

Direction: Choose the letter of the correct answer. Write it on a separate sheet of paper.

1	In the	Cartesian	nlane	what is	the	distance	of the	noint i	(-5, 6)	5) from	the o	rigina
т.	m me	Cartesian	piane,	wiiat is	uic	uistance	or mic	DOME	I-J, ()) II OIII	LITE O	נווצוו:

A) 61

B) 11

D) 11

2. What is the distance between point A(-3, 1) and point B(11, 1)?

B) 12

C) 13

3. Which of the following should be the value of y so that the distance between the points (2, -2) and (2, y) is 7?

A) 2

B) 3

C) 4

D) 5

4. Which of the following describes the distance formula?

A) $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

C) $d = \sqrt{(x_2 - x_1)^2 - (y_2 - y_1)^2}$

B) $d = \sqrt{(x_2 + x_1)^2 + (y_2 + y_1)^2}$ D) $d = \sqrt{(x_2 + x_1)^2 - (y_2 + y_1)^2}$

5. What is the distance between two points whose coordinates are (4,-3) and (-4,3)?

A) 4

B) 6

C) 8

D) 10

6. What is the distance between point A(3a, 2a) and point D(-a, -6a)?

A) $2a\sqrt{10}$

B) $4a\sqrt{5}$

C)5a

D) 8a

7. What is the area of a triangle whose vertices are (0,2), (0,0) (5,0)?

A) 5 square unit

B) 8 square unit

C) 10 square

D) 12 square unit

8. What kind of triangle is formed when the vertices (-3, 5), (-3, 1) and (2, 1) are plotted on the Cartesian plane?

A) equilateral

B) isosceles

C)right

D) scalene

9. What type of quadrilateral is formed by the given vertices C(0,0), A(1,2), R(4,2) and E(3,0)?

A) kite

B) parallelogram

C) rectangle

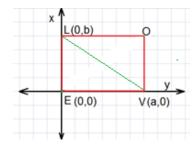
D) square

10. In Quadrilateral LOVE, what is the length of the diagonal \overline{LV} .

A) $\sqrt{a^2 + b^2}$

B) $\sqrt{a^2 - b^2}$

C) $\sqrt{a+b}$



D)
$$a^2 + b^2$$

11. What is the distance of point I

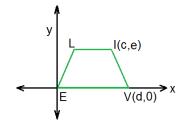
from the origin in Trapezoid LIVE?



C)
$$\sqrt{c^2 - e^2}$$

B)
$$\sqrt{c+e}$$

B)
$$\sqrt{c+e}$$
 D) $\sqrt{c^2+e^2}$



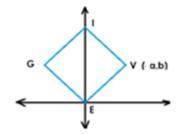
12. GIVE is a square. Find its perimeter.



B)
$$4\sqrt{a+b}$$

C)
$$4(a^2 + b^2)$$

D)
$$4\sqrt{a^2 + b^2}$$



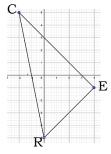
13. To prove that triangle CER is a right triangle, which of the following should be proven?

A)
$$\overline{ER} = \overline{RC} + \overline{CE}$$

B)
$$\overline{RC} = \overline{ER} + \overline{CE}$$

C)
$$\overline{ER}^2 = \overline{RC}^2 + \overline{CE}^2$$

D)
$$\overline{RC}^2 = \overline{ER}^2 + \overline{CE}^2$$



14. To prove that the diagonals of rectangle RICE are congruent which of the following should be proven?

A)
$$\overline{RI} \cong \overline{IC}$$

$$\mathrm{B})\overline{IC}\cong\overline{CE}$$

$$C)\overline{RC} \cong \overline{EC}$$

$$\mathrm{D})\overline{RC}\cong \overline{IE}$$

15. To prove that triangle ABC is an equilateral triangle which of the following should be proven?

A)
$$AB \cong BA$$

B)
$$AB \cong BC \cong AC$$

$$C)AB \cong BC$$

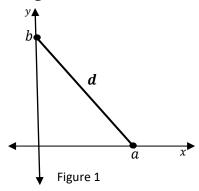
D)
$$AB \ncong BC \cong AC$$



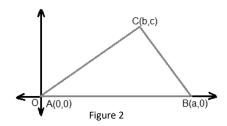
Additional Activity

Direction: Answer the following problems on a separate sheet of paper.

- 1. Draw segment MR whose endpoints are M(-1,5) and R(2, -4) on the Cartesian plane.
 - a) Find the length of segment MR.
 - b) If point S(x, -1) lies on segment MR and the length of segment SR is $\sqrt{10}$, what is x?
- 2. Refer to Figure 1. Solve the distance d in terms of a and b.

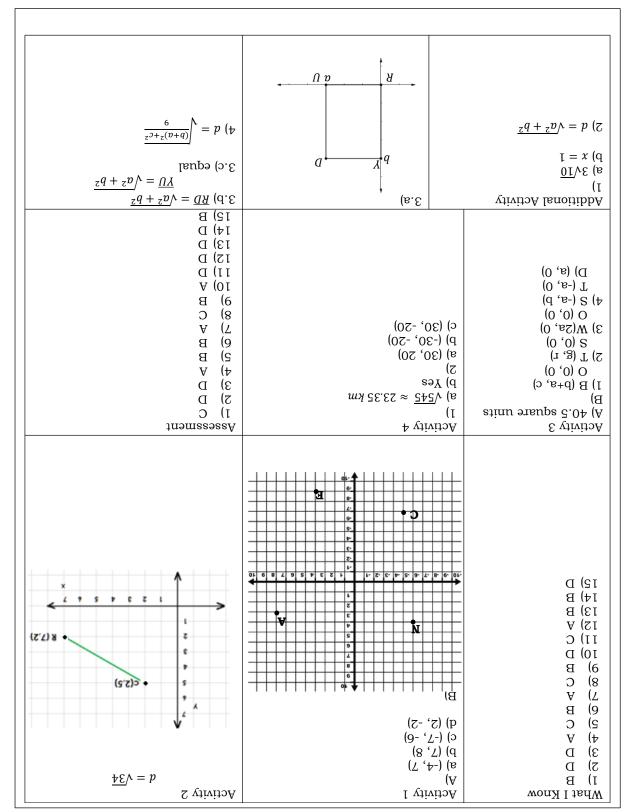


- 3) The vertices of a quadrilateral are R(0,0), U(a,0), D(a,b), Y(0,b).
 - a) Illustrate the quadrilateral in the coordinate plane.
 - b) Find the length of each diagonal.
 - c) Compare the lengths of the diagonals.
- 4) Refer to Figure 2. The *x*-coordinate of *D* is the mean of the *x*-coordinates of the vertices of triangle ABC and its *y*-coordinate is the mean of the *y*-coordinates of the vertices of triangle ABC. Find the distance between points A and *D*.





Answer Key



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