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Mathematics

Quarter 2 – Module 8:
Writing Equation of a Circle and Determining the Center and Radius of a Circle

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Alternative Delivery Mode
Quarter 2 – Module 8: Writing Equation of a Circle and Determining the Center and Radius of a Circle
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Mathematics

Quarter 2 – Module 8:
Writing Equation of a Circle and Determining the Center and Radius of a Circle

Introductory Message

This Self-Learning Module (SLM) is prepared so that you, our dear learners, can continue your studies and learn while at home. Activities, questions, directions, exercises, and discussions are carefully stated for you to understand each lesson.

Each SLM is composed of different parts. Each part shall guide you step-by-step as you discover and understand the lesson prepared for you.

Pre-tests are provided to measure your prior knowledge on lessons in each SLM. This will tell you if you need to proceed on completing this module or if you need to ask your facilitator or your teacher's assistance for better understanding of the lesson. At the end of each module, you need to answer the post-test to self-check your learning. Answer keys are provided for each activity and test. We trust that you will be honest in using these.

In addition to the material in the main text, Notes to the Teacher are also provided to our facilitators and parents for strategies and reminders on how they can best help you on your home-based learning.

Please use this module with care. Do not put unnecessary marks on any part of this SLM. Use a separate sheet of paper in answering the exercises and tests. And read the instructions carefully before performing each task.

If you have any questions in using this SLM or any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator.

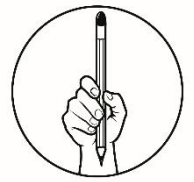
Thank you.

What I Need to Know

This module contains activities, discussions and practice exercises on how to determine the center and radius of a circle given its equation and vice versa. This particular topic is essential in Coordinate Geometry. Understanding coordinate geometry has important applications like mapping in aeronautics and navigation, finding the distance and midpoints between points and many more in other different fields. Thus, it is important for you to know how to determine the center and radius of circles given its equation and vice versa.

At the end of this module, you will be able to achieve the following objectives:

1. illustrate the center-radius form of the equation of a circle, and
2. determine the center and radius of a circle given its equation and vice versa.



What I Know

DIRECTION: Let us determine how much you already know about the equation of a circle. Read and understand each item, then choose the letter of your answer and write it on your answer sheet.

1. Which equation represents an equation of a circle in the standard form?

   A) $x + y = 7$ C) $x^2 + 4\text{x} + 2 = 0$
   B) $x + y^2 = 8$ D) $x^2 + y^2 = 25$

2. Which of the following represents an equation of a circle in the general form?

   A) $x^2 + 2x - 4y + 5 = 0$ C) $x^2 + y^2 + 4\text{x} - 4\text{y} - 28 = 0$
   B) $y^2 - 3x + y + 5 = 0$ D) none of these

3. A circle defined by $x^2 + y^2 = 9$ has a graph whose center is at _____.

   A) origin B) quadrant I C) quadrant II D) quadrant III

4. A circle with an equation defined by $(x - 5)^2 + (y + 3)^2 = 9$ has a graph whose center is located at _____.

   A) quadrant I B) quadrant II C) quadrant III D) quadrant IV

5. What is the center of the circle defined by $(x + 5)^2 + (y + 3)^2 = 36$?

   A) $(5,3)$ B) $(-5,3)$ C) $(-5,-3)$ D) $(5,-3)$

6. What is the center of the circle $x^2 + y^2 - 4x + 10y + 13 = 0$?

   A) $(2,5)$ B) $(-2,5)$ C) $(2,-5)$ D) $(-2,-5)$

7. What is the radius of the circle $x^2 + y^2 - 4x + 10y + 13 = 0$?

   A) 3 B) 4 C) 5 D) 6

8. What is the equation of the circle whose center is at $(0,5)$ and has a radius of 5?

   A) $x^2 + (y-5)^2 = 25$ C) $x^2 + (y-5)^2 = 5$
   B) $x^2 + y^2 = 25$ D) $(x-5)^2 + y^2 = 25$

9. What is the general form of the equation of a circle whose center is at $(3,0)$ and the radius is 4?

   A) $x^2 + y^2 - 4x + 10y + 13 = 0$ C) $x^2 + y^2 - 6x - 7 = 0$
   B) $x^2 + y^2 - 10x + 10y + 16 = 0$ D) $x^2 + y^2 - 6x + 7 = 0$

10. Given by the equation of the circle, $x^2 + y^2 + 4x - 4y - 28 = 0$, what is its center-radius form?

    A) $(x+2)^2 + (y-2)^2 = 36$ C) $(x-2)^2 + (y+2)^2 = 36$
    B) $(x-2)^2 + (y-2)^2 = 36$ D) $x^2 + y^2 = 36$

What's In

Your previous lessons on circles and the use of distance formula are both significantly relevant skills for you to understand the next lesson. Let's have a short recall of these topics.

- **CIRCLE**

  A circle is the set of all points, (x, y), on a plane having the same distance from a fixed point called the <u>center</u> of the circle. The distance between the center of the circle and any point on the circle is called the <u>radius</u>, r, of the circle.

- **DISTANCE FORMULA**

  Supposing the coordinates of point A is (x_1, y_1) and point B is (x_2, y_2), then the distance, d, between point A and point B is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

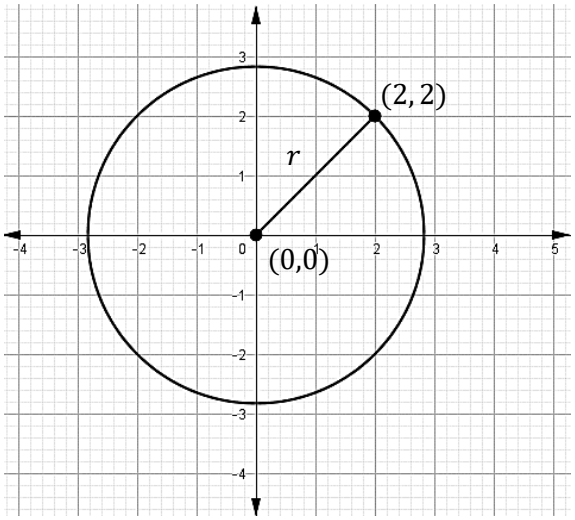
What's New

Before you proceed to the lesson proper, it is important that you know how to use the distance formula in finding the measure of the radius of a circle. The following activity will also help you understand the lesson.

Activity 1: Radius of a Circle

Direction: Given the graph of each circle, find the length of the radius r using the distance formula.

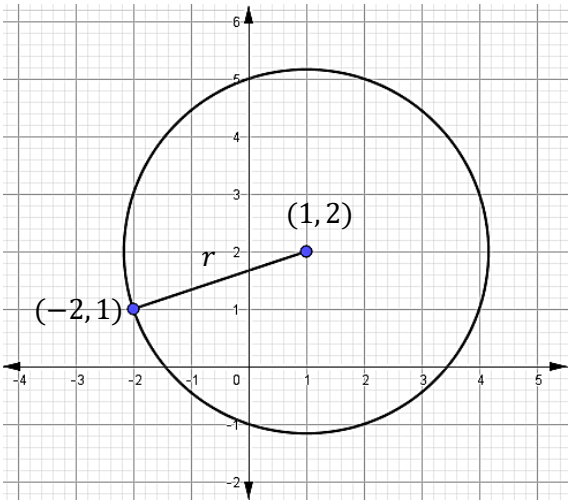
1)



Solution:

Answer: _____

2)



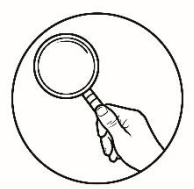
Solution:

Answer: _____

You did a good job in performing the given tasks! Now, it's your time to check your own work. If you answered both items correctly, you may proceed to the next part of this module. If not, please try again.

check your answers →

1) $2\sqrt{2}$ 2) $\sqrt{10}$



What Is It

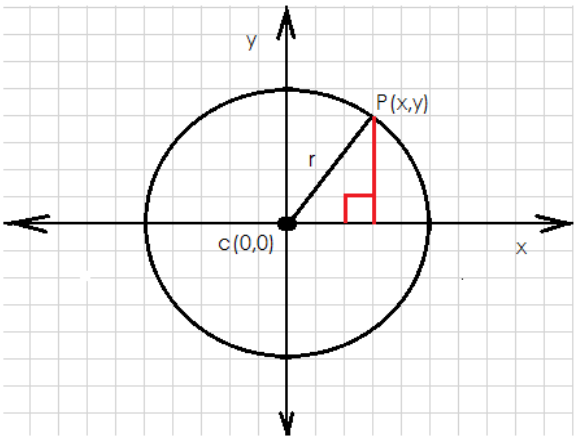
You've learned already how to find the measure / length of the radius of a circle using the distance formula when given the coordinates of the center and a point on the circle. This time you will be learning about the different forms of the equation of a circle.

A. Circle with center at the origin.

Given the circle with center at $(0,0)$. To draw its radius (r), connect the center to any point on the circle like point $P(x,y)$.

Then using the distance formula, we can find the equation of the circle.

We find the distance between the center$(0,0)$ and $P(x,y)$, which is the length of the radius(r) of the circle.



From the distance formula, let $P_1\ (0,0)$, $P_2\ (x,y)$, and r be the distance.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$r = \sqrt{(x-0)^2 + (y-0)^2}$ *Substitute P_1, P_2 and r in the formula.*

$r = \sqrt{x^2 + y^2}$ *Simplify.*

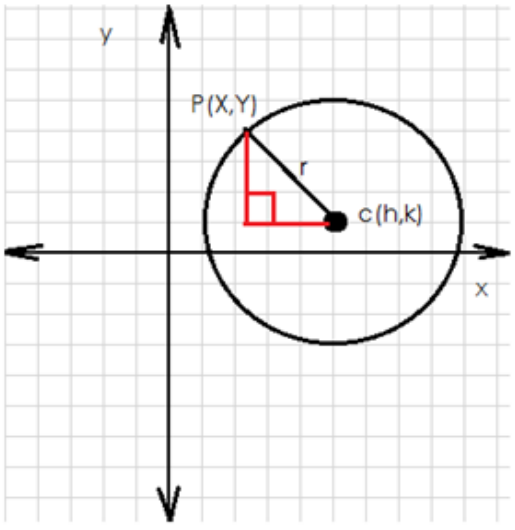
$r^2 = x^2 + y^2$ *Square both sides of the equation.*

Thus, the equation of a circle with center at the origin is $\boldsymbol{x^2 + y^2 = r^2}$.

B. Circle with center NOT at the origin.

Notice that the center of the circle is not on the origin but rather at point (h,k). At point $P(x,y)$ on the circle, the radius r is drawn connecting the two points.

We find the distance between the center (h,k) and $P(x,y)$, which is the length of the radius (r) of the circle.



From the distance formula, let $P_1\ (h,k)$, $P_2\ (x,y)$, and r be the distance.

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$r = \sqrt{(x-h)^2 + (y-k)^2}$ *Substitute P_1 and P_2 in the formula.*

$r^2 = (x-h)^2 + (y-k)^2$ *Square both sides of the equation.*

Thus, the equation of a circle with center not on the origin at any point (h,k) is $\boldsymbol{(x-h)^2 + (y-k)^2 = r^2}$.

Center-Radius Form (Standard Form) of the Equation of a Circle

Let r be the radius of a circle, the equation of a circle in center-radius form is:

(a) $\boldsymbol{x^2 + y^2 = r^2}$ with center at the origin $(0,0)$
(b) $\boldsymbol{(x-h)^2 + (y-k)^2 = r^2}$ with center not on the origin at (h,k)

Examples:

1. $x^2 + y^2 = 9$
2. $(x-3)^2 + (y+1)^2 = 4$

Expanding the standard form, we will arrive at the **general form** of the equation of a circle.

Expand: $(x-h)^2 + (y-k)^2 = r^2$

Solution: $x^2 - 2hx + h^2 + y^2 - 2ky + k^2 - r^2 = 0$

$x^2 + y^2 + (-2h)x + (-2k)y + h^2 + k^2 - r^2 = 0$

*let $C = -2h$, $D = -2k$ and $E = h^2 + k^2 - r^2$

$x^2 + y^2 + Cx + Dy + E = 0$

General Form of the Equation of a Circle

Thus, the equation of a circle in general form is $\boldsymbol{x^2 + y^2 + Cx + Dy + E = 0}$ where C, D and E are real numbers.

Examples:

1. $x^2 + y^2 - 6x - 10y + 18 = 0$
2. $x^2 + y^2 - 8x = 0$
3. $x^2 + y^2 + 2y - 3 = 0$
4. $4x^2 + 4y^2 - 16x + 24y - 36 = 0$

Note: The numerical coefficients of x^2 and y^2 are <u>both equal to 1</u>, and the value of the radius (r) is <u>positive</u>. Moreover, the numerical coefficients of x^2 and y^2 can be possibly not equal to 1 but they must be equal and cannot be equal to zero. See example 4 above.

After learning the equations of a circle in relation to its center and radius. This time, you will learn how to determine the center and radius of a circle given the equation, and to write the equation of a circle given the center and the radius.

Example 1: Find the center and radius of the circle $x^2 + y^2 = 25$.

Solution: The given equation is in the form $x^2 + y^2 = r^2$.
This means that the center of the circle is at the origin.
To find the radius, get the square root of 25, which is 5.

Note: Since $x^2 + y^2 = 25$ is in the form $x^2 + y^2 = r^2$, then $r^2 = 25$.
By extracting the square roots, $r =$ **5.**

Therefore, the center of the circle is at $(\mathbf{0}, \mathbf{0})$ and the radius is **5 units**.

Example 2: Find the center and radius of the circle $x^2 + (y - 4)^2 = 16$.

Solution: The given equation is in the form $(x - h)^2 + (y - k)^2 = r^2$.
This means that the center of the circle is at (h, k).

$x^2 + (y - 4)^2 = 16$	*Given.*
$(x - 0)^2 + (y - 4)^2 = 4^2$	*Write as* $(x - h)^2 + (y - k)^2 = r^2$.
$h = 0, k = 4, r = 4$	*Identify the values of* h, k, *and* r.

Therefore, the center of the circle is at $(\mathbf{0}, \mathbf{4})$ and the radius is **4 units**.

Example 3: Find the center and radius of the circle whose equation is represented by $x^2 + y^2 - 6x - 10y + 18 = 0$.

Solution: Since the equation is written in general form, we need to transform the equation to its standard form to determine its center and radius.

$x^2 + y^2 - 6x - 10y + 18 = 0$	*Given.*
$(x^2 - 6x) + (y^2 - 10y) + 18 = 0$	*Group terms with the same variable.*
$(x^2 - 6x) + (y^2 - 10y) = -18$	*Apply Addition Property of Equality.*
$(x^2 - 6x + _) + (y^2 - 10y + _) = -18 + _ + _$	*Apply completing the squares.*
$(x^2 - 6x + 9) + (y^2 - 10y + 25) = -18 + 9 + 25$	$\left(\frac{6}{2}\right)^2 = 9 \quad \left(\frac{10}{2}\right)^2 = 25$
$(x - 3)^2 + (y - 5)^2 = 16$	*Factoring perfect square trinomial.*
$(x - 3)^2 + (y - 5)^2 = 4^2$	*Write as* $(x - h)^2 + (y - k)^2 = r^2$.
$h = 3, k = 5, r = 4$	*Identify the values of* h, k, *and* r.

Therefore, the center of the circle is at $(\mathbf{3}, \mathbf{5})$ and the radius is **4 units**.

Example 4: Find the center and radius of the circle whose equation is represented by $4x^2 + 4y^2 - 16x + 24y - 36 = 0$.

Solution: Since the numerical coefficients of x^2 and y^2 are both not equal to 1, divide both sides of the equation by 4.

$\frac{4x^2+4y^2-16x+24y-36}{4} = \frac{0}{4}$ *Divide each term by 4.*

$x^2 + y^2 - 4x + 6y - 9 = 0$

$(x^2 - 4x) + (y^2 + 6y) - 9 = 0$

$(x^2 - 4x) + (y^2 + 6y) = 9$

$(x^2 - 4x + _) + (y^2 + 6y + _) = 9 + _ + _$

$(x^2 - 4x + 4) + (y^2 + 6y + 9) = 9 + 4 + 9$

$(x - 2)^2 + (y + 3)^2 = 22$

$(x - 2)^2 + (y + 3)^2 = \sqrt{22}$

$h = 2, k = -3, r = \sqrt{22}$

Therefore, the center of the circle is at $(\mathbf{2}, \mathbf{-3})$ and the radius is $\sqrt{\mathbf{22}}$ **units**.

Example 5. Write the equation of the circle in center-radius form whose center is at $(6,7)$ and contains the point $(2,5)$.

Solution: Step 1: Find the radius using the distance formula.

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ *Let $P_1(6,7)$ and $P_2(2,5)$*

$r = \sqrt{(2 - 6)^2 + (5 - 7)^2}$ *Substitute P_1, P_2 and r.*

$r = \sqrt{(-4)^2 + (-2)^2}$ *Perform the operations.*

$r = \sqrt{16 + 4}$ *Square each term.*

$r = \sqrt{20} = 2\sqrt{5}$ *Simplify.*

Step 2: Write the equation in center-radius form.
At $C(6, 7)$; h $= 6$, $k = 7$.

$(x - h)^2 + (y - k)^2 = r^2$ *Use the center-radius form.*

$(x - 6)^2 + (y - 7)^2 = \left(2\sqrt{5}\right)^2$ *Substitute the values of h, k, and r.*

$(x - 6)^2 + (y - 7)^2 = 20$ *Simplify.*

Therefore, the equation of the circle in center-radius form is $\mathbf{(x - 6)^2 + (y - 7)^2 = 20.}$

Example 6: Write the equation of the circle in general form with center at $(-5,-4)$ and radius of 3 units.

Solution: Given: $\mathrm{h} = -5,\ \mathrm{k} = -4,\ \mathrm{r} = 3$

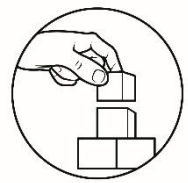
$[\mathrm{x} - (-5)]^2 + [\mathrm{y} - (-4)]^2 = 3^2$ *Substitute given in* $(x-h)^2 + (y-k)^2 = r^2$.

$(x+5)^2 + (y+4)^2 = 9$ *Simplify.*

$x^2 + 10x + 25 + y^2 + 8y + 16 = 9$ *Expand by squaring a binomial.*

$x^2 + y^2 + 10x + 8y + 32 = 0$ *Write in* $x^2 + y^2 + Ax + By + C = 0$.

Therefore, the equation of the circle in general form is $\boldsymbol{x^2 + y^2 + 10x + 8y + 32 = 0}$.



What's More

Activity 2: Pair them Up

Match the given parts of the circle with its corresponding equation. Use the letters of your answer to decode the message below.

Item	Given		Equation
1	Center $(0,5)$	$r = 6$	N. $(x-2)^2 + (y-4)^2 = 36$
2	Center $(8,-7)$	$r = 15$	R. $(x+4)^2 + (y-9)^2 = 144$
3	Center $(6,1)$	$r = 9$	A. $(x-6)^2 + (y-1)^2 = 81$
4	Center $(-4,9)$	$r = 12$	E. $(x-8)^2 + (y+7)^2 = 225$
5	Center $(2,4)$	$r = 6$	L. $x^2 + (y-5)^2 = 36$
6	Center $(-4,-4)$	$r = 4\sqrt{2}$	H. $(x-7)^2 + y^2 = 64$
7	Center $(5,5)$	$r = 3\sqrt{3}$	T. $x^2 + (y+2)^2 = 49$
8	Center $(-2,0)$	$r = 10$	M. $(x+2)^2 + y^2 = 100$
9	Center $(0,-2)$	$r = 7$	G. $(x-5)^2 + (y-5)^2 = 27$
10	Center $(7,0)$	$r = 8$	I. $(x+4)^2 + (y+4)^2 = 32$
11	Center $(-2,2)$	$r = 4\sqrt{2}$	F. $x^2 + y^2 - 6x - 10y + 18 = 0$
12	Center $(3,0)$	$r = 4$	Y. $x^2 + y^2 - 6x - 7 = 0$
13	Center $(3,5)$	$r = 4$	S. $x^2 + y^2 + 4x - 4y - 24 = 0$
14	Center $(0,0)$	$r = 5$	O. $(x-3)^2 + (y-3)^2 = 20$
15	Center $(3,3)$	$r = 2\sqrt{5}$	U. $x^2 + y^2 = 25$

15	10
6	11

1	2	3	4	5	6	5	7
	4	2	3	1	1	12	

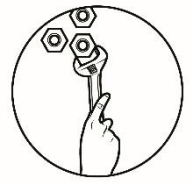
8	3	9	10
13	14	5	!

What I Have Learned

Complete the statements below:

I have learned that …

1) when the center of a circle is at the origin, the equation to be used is _____.
2) when the center of a circle is at any other point (h, k), the equation to be used is _____.
3) a circle can also be represented in its general form which is written as _____.
4) for a circle to exist, radius is _____ and coefficients of x^2 and y^2 should not be equal to _____.
5) given the coordinates of the center and any other point on the circle, the length of the radius can be computed by using the _____.
6) to write the equation of a circle, the two necessary parts must be obtained: (a) _____ and (b) _____.



What I Can Do

Word Problem:

Mang Cardo tied his carabao to a peg in the field. In the evening, dark clouds began to form and the weather turned bad. Consecutive lightning and thunder frightened the carabao causing it to run in circles straining its rope.

In the morning when the weather calmed down, Mang Cardo sent his Grade 10 son to untie the carabao. His son was surprised to see a perfectly formed circle around the carabao's peg.

If the circle formed by the carabao's foot prints is given by the equation $x^2 + y^2 - 6x + 8y - 119 = 0$, how long is the carabao's rope if the scale used is 1 meter is to 2 units?

Illustration:	Solution:
Conclusion:	

Assessment

DIRECTION: Read and understand each item, then choose the letter of your answer and write it on your answer sheet.

1. What is the center-radius form of a circle defined by $x^2 + y^2 - 2x + 4y - 1 = 0$?
   A) $(x+1)^2 + (y-2)^2 = 6$ C) $(x+1)^2 + (y-2)^2 = 8$
   B) $(x-1)^2 + (y+2)^2 = 6$ D) $(x-1)^2 + (y+2)^2 = 8$

2. Which of the following is NOT an equation of a circle in general form?
   A) $2x^2 + 2y^2 - 12x - 20y + 36 = 0$ C) $x^2 + y^2 + 10y + 9 = 0$
   B) $x^2 + y^2 - 6x - 7 = 0$ D) $x^2 + 5x - 6y + 6 = 0$

3. What is the location of the center of a circle define by $(x+6)^2 + (y+4)^2 = 9$?
   A) Quadrant I B) Quadrant II C) Quadrant III D) Quadrant IV

4. What is the equation of the circle whose center is at $(4,0)$ and the radius is 3?
   A) $x^2 + y^2 - 8x + 4y + 7 = 0$ C) $x^2 + y^2 - 8x + 7 = 0$
   B) $x^2 + y^2 + 8x - 4y - 7 = 0$ D) $x^2 + y^2 - 8y + 7 = 0$

5. What is the measure of the radius of a circle defined by $x^2 + y^2 + 8x + 8 = 0$?
   A) $2\sqrt{2}$ B) $4\sqrt{2}$ C) 4 D) 8

6. The center of a circle is at $(7,-2)$ and the radius is 5 units. What is the equation of the circle?
   A) $(x+7)^2 + (y-2)^2 = 25$ C) $(x+7)^2 + (y-2)^2 = 5$
   B) $(x-7)^2 + (y+2)^2 = 25$ D) $(x-7)^2 + (y+2)^2 = 5$

For items 7 – 8. Consider the circle defined by $3x^2 + 3y^2 - 6x - 6y + 3 = 0$.

7. What is the center-radius form of the equation of the circle?
   A) $(x-1)^2 + (y-1)^2 = 1$ C) $(x+1)^2 + (y-1)^2 = 1$
   B) $(x+1)^2 + (y+1)^2 = 1$ D) $(x-1)^2 + (y+1)^2 = 1$

8. What is the center of the circle?
   A) $(-1,1)$ B) $(-1,-1)$ C) $(1,-1)$ D) $(1,1)$

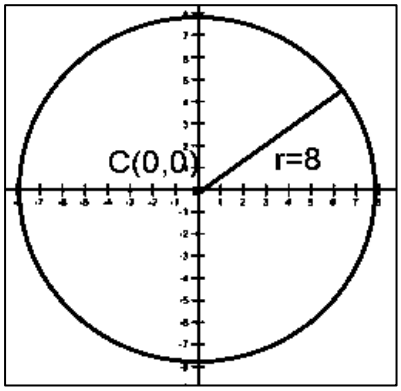
9. What is the equation of a circle whose radius is $\sqrt{13}$ units and its center is at the origin?
   A) $x^2 + y^2 = \sqrt{13}$ C) $(x+1)^2 + (y+1)^2 = 13$
   B) $x^2 + y^2 = 13$ D) $(x+1)^2 + (y+1)^2 = \sqrt{13}$

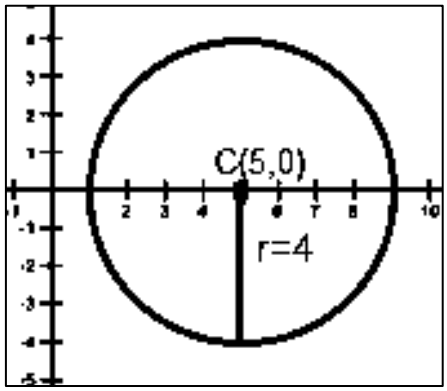
10. What is the radius of the circle given by the equation $x^2 + y^2 - 8x + 2y - 32 = 0$?
    A) 5 B) 6 C) 7 D) 8

Additional Activity

DIRECTION: Match the figure with the corresponding equation that represents it. Write the letter on each item.

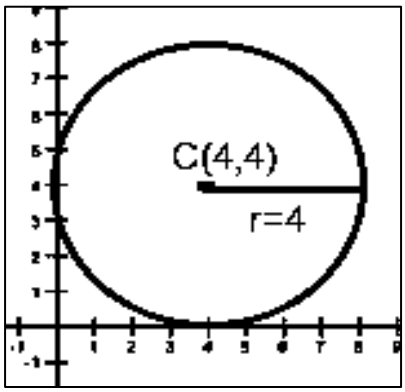
1

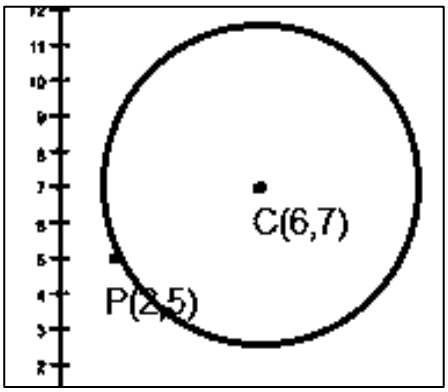




2

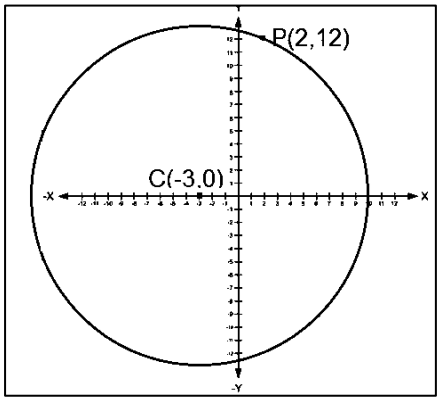
3





4

5



A) $(x - 6)^2 + (y - 7)^2 = 20$

B) $(x + 3)^2 + y^2 = 169$

C) $x^2 + y^2 = 64$

D) $(x - 4)^2 + (y - 4)^2 = 16$

E) $(x - 5)^2 + y^2 = 16$

Answer Key

What I Know

1) D	3) A	5) C	7) B	9) C
2) C	4) D	6) C	8) A	10) A

What's More (Activity 2)

1) L	4) R	7) G	10) H	13) F
2) E	5) N	8) M	11) S	14) U
3) A	6) I	9) T	12) Y	15) N

Message: OH LEARNING MATH IS REALLY FUN!

What Can I Do

The carabao's rope measures 6 meters.

ASSESSMENT

1) B	3) C	5) A	7) A	9) B
2) D	4) C	6) B	8) D	10) C

ADDITIONAL ACTIVITY

1) C	2) E	3) D	4) A	5) B

References

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