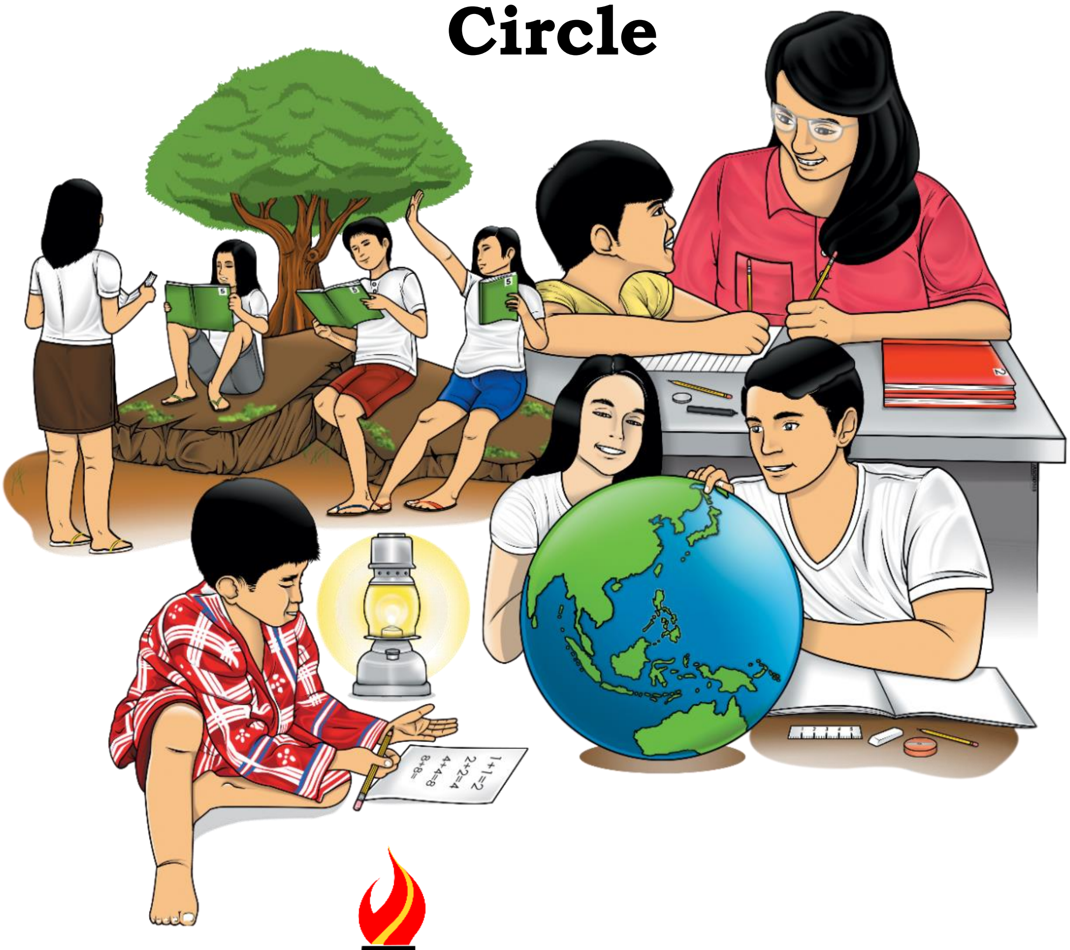
10

# Mathematics

## Quarter 2 – Module 5:
## Theorems on Secants, Tangents, Segments, and Sectors of a Circle



ALTERNATIVE DELIVERY MODE

ADM

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**Alternative Delivery Mode**
**Quarter 2 – Module 5: Theorems on Secants, Tangents, Segments, and Sectors of a Circle**
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**Development Team of the Module**

**Writer's Name:** John Denver B. Pinkihan

**Co-Writer's Name:** Heather G. Banagui

**Reviewer's Name:** Bryan A. Hidalgo

**Management Team:**

May B. Eclar
Benedicta B. Gamatero
Carmel F. Meris
Ethielyn E. Taqued
Edgar H. Madlaing
Marciana M. Aydinan
Lydia I. Belingon

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Office Address: Wangal, La Trinidad, Benguet
Telefax: (074) 422-4074
E-mail Address: car@deped.gov.ph

# 10

# Mathematics

## Quarter 2 – Module 5:
## Theorems on Secants, Tangents, Segments, and Sectors of a Circle

DEPARTMENT OF EDUCATION

# Introductory Message

This Self-Learning Module (SLM) is prepared so that you, our dear learners, can continue your studies and learn while at home. Activities, questions, directions, exercises, and discussions are carefully stated for you to understand each lesson.

Each SLM is composed of different parts. Each part shall guide you step-by-step as you discover and understand the lesson prepared for you.

Pre-tests are provided to measure your prior knowledge on lessons in each SLM. This will tell you if you need to proceed on completing this module or if you need to ask your facilitator or your teacher's assistance for better understanding of the lesson. At the end of each module, you need to answer the post-test to self-check your learning. Answer keys are provided for each activity and test. We trust that you will be honest in using these.

In addition to the material in the main text, Notes to the Teacher are also provided to our facilitators and parents for strategies and reminders on how they can best help you on your home-based learning.

Please use this module with care. Do not put unnecessary marks on any part of this SLM. Use a separate sheet of paper in answering the exercises and tests. And read the instructions carefully before performing each task.

If you have any questions in using this SLM or any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator.

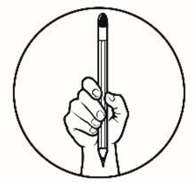
Thank you.

## What I Need to Know

This module contains activities, discussions and practice exercises on secants, tangents, segments and sectors of a circle.

After going through this module, you are expected to:

- Illustrate secants, tangents, segments, and sectors of a circle,
- Identify secant, tangents, segments, and sectors given an illustration,
- Proves theorems on secants, tangents and segments, and
- Find an unknown measurement of an angle and segments formed when secants and tangents intersect in a point inside, on and outside a circle.



## What I Know

<u>Direction:</u> Read each item carefully and write the letter of the correct answer on a separate sheet of paper.

1. Which of the following line segments intersects a circle at exactly two points?
   A. line  B. ray  C. secant  D. tangent

2. Which of the following is the region bounded by two radii of the circle and their intercepted arc?
   A. area  B. circumference  C. sector  D. semicircle

3. How many pairs of common tangents can be drawn from the circles in *Figure 1*?

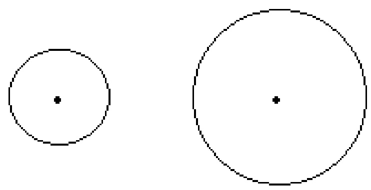


*Figure 1*

   A. 1  B. 2  C. 3  D. 4

**For items 4-8**. In *Figure 2*, $\overline{AB}$, $\overline{AD}$, and $\overline{FG}$ are tangent segments of ⊙C

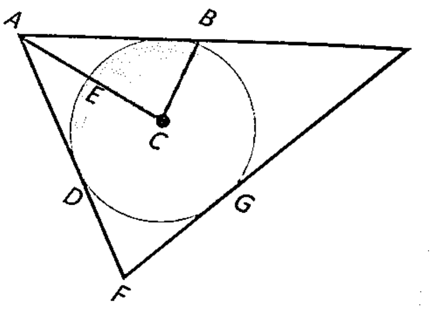



*Figure 2*

4. Which of the following are the intercepted arcs of $\angle DFG$?

A. $\overset{\frown}{DG}$ and $\overset{\frown}{DBG}$ C. $\overset{\frown}{BG}$ and $\overset{\frown}{EB}$

B. $\overset{\frown}{EB}$ and $\overset{\frown}{DG}$ D. $\overset{\frown}{DG}$ and $\overset{\frown}{BG}$

5. If $m\angle DAB$=84, then what is the $m\angle DAC$=_______.
 A. 16° B. 31° C. 42° D. 168°

6. In the given figure, if measure of $\overline{AB}$ is 12. Which of the following is the measurement of $\overline{AD}$?
A. 3 B. 6 C. 8 D. 12

7. Find the measurement of ∠F given that m$\overset{\frown}{DG}$ =78.
 a. 39° b. 102° c. 141° d. 156°

8. Which of the following is the m$\angle ABC$?
 a. 100° b. 90° c. 60° d. 45°

**For items 9-13**. Refer to *Figure* 3.

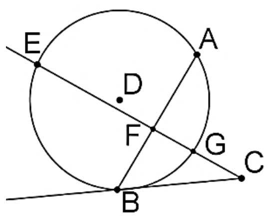



*Figure 3*

9. If $\overline{BC}$ = 12 and $\overline{GC}$ = 6, then which of the following is the length of $\overline{EG}$?
 a. 12 b. 18 c. 24 d. 72

10. If $\overline{AF}$ = 8, $\overline{FB}$ = 6, and $\overline{FG}$= 2, then which of the following is $\overline{EF}$?
 a. 10 b. 12 c. 16 d. 24

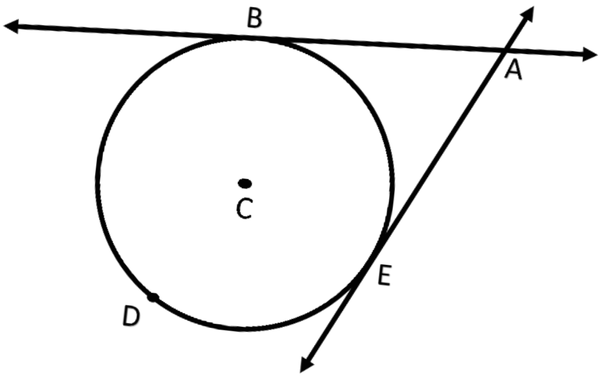
11. If m$\angle AFG$ = 70 and m $\overset{\frown}{AG}$ = 80, then which of the following is m $\overset{\frown}{BE}$ ?
 a. 60° b. 70° c. 80° d. 220°

12. If m$\overset{\frown}{AB}$ = 110 , m $\overset{\frown}{AG}$ = 60 , and m $\overset{\frown}{BE}$ = 100, then m$\angle C$=____.
   a. 100°   b. 75°   c. 50°   d. 25°

13. If m$\overset{\frown}{AB}$ = 110, then which of the following is the $m\angle ABC$?
   a. 55°   b. 90°   c. 110°   d. 220°

14. The following statement is a theorem on two tangents intersecting outside a circle.

If two tangents intersect in the exterior of a circle, then the measure of the angle formed is one-half the difference of the measures of the intercepted arcs.




Which of the following is needed to show to prove the theorem?
   a. ∠BAE is an angle formed by $\overleftrightarrow{AB}$ and $\overleftrightarrow{AE}$ outside the circle.
   b. ∠BAE intercepts $\overset{\frown}{BE}$ and $\overset{\frown}{EDB}$.
   c. m$\overset{\frown}{BE}$ < m $\overset{\frown}{EDB}$
   d. ∠BAE = $\frac{1}{2}$ (m $\overset{\frown}{EDB}$- m$\overset{\frown}{BE}$)

15. Which of the following describes a major sector of a circle?
   a. A region bounded by two radii and an arc of the circle.
   b. A region bounded by two radii and an arc which measures 90°
   c. A region bounded by two radii and an arc which measures 180°
   d. A region bounded by two radii and an arc which measure 200°.

# What's In

Before we proceed to our lesson, we need to review related concepts discussed in your Math 8 and concepts discussed in the previous modules. Activity 1 is a review on the Properties of Equality and Congruence, and Activity 2 is a review on chords, arcs, central angles, and inscribed angles.

**Activity 1: Match Me**
Match the mathematical statement in Column A by drawing an arrow to the corresponding property of equality or congruence in Column B. You can match more than one mathematical statement to one property.

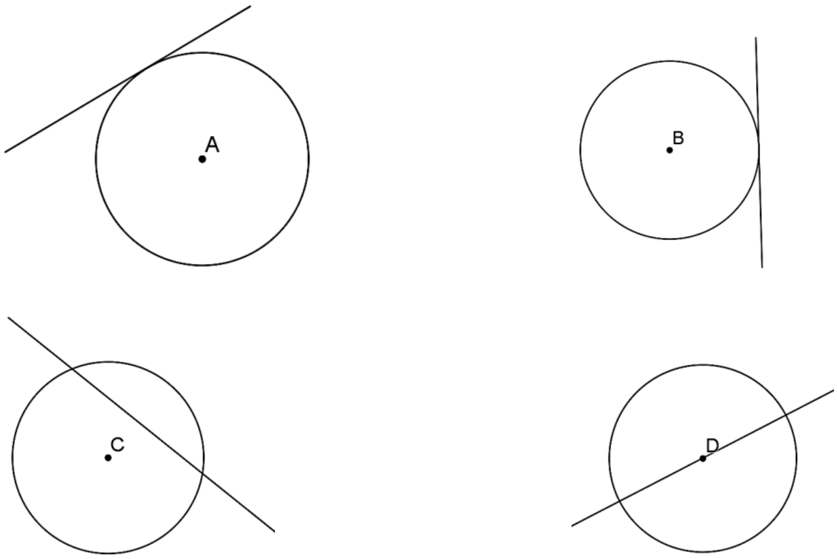
| Mathematical Statements | Property of Equality or Congruence |
|---|---|
| 1. $m\overline{AB} = m\overline{AB}$ | a. Addition Property of Equality |
| 2. $\overline{AB} \cong \overline{AB}$ | b. Subtraction Property of Equality |
| 3. If $\overline{AB} \cong \overline{CD}$, then $\overline{AB} + \overline{EF} \cong \overline{CD} + \overline{EF}$ | c. Transitive Property |
| | d. Substitution Property |
| 4. If $\overline{AB} \cong \overline{CD}$, then $\overline{AB} - \overline{EF} \cong \overline{CD} - \overline{EF}$ | e. Symmetric Property |
| 5. If $\angle A \cong \angle B$ and $\angle B \cong \angle C$ then $\angle A \cong \angle C$ | f. Reflexive Property |
| 6. If $m\angle A = m\angle B$ and $m\angle B = m\angle C$ then $m\angle A = m\angle C$ | |
| 7. If $m\overline{AB} = m\overline{DC} + m\overline{EF}$ and $m\overline{EF} = m\overline{MN}$ then $m\overline{AB} = m\overline{DC} + m\overline{MN}$ | |

**Activity 2**: **Can you illustrate me?**
In each item, draw a circle illustrating the following parts. Label and thicken the indicated part. You can use a colored pencil, crayon or marker.

1. Central angle A
2. Inscribed angle B
3. Points C and D on a circle
4. Points E and F outside a circle
5. Intercepted arc by a central angle G
6. Intercepted arc by an inscribed angle H

# *What's New*

**Activity 3: Touch or Divide?**
Do you think tangent lines and secant lines are the same? To answer this question, consider the following illustrations of tangent lines and secant lines.




In the figures, ⊙A and ⊙B are intersected by tangent lines while ⊙C and ⊙D are intersected by secant lines. Do you think there are similarities on tangent and secant lines? How about their differences?

**Tangent Line**
A **tangent** to a circle is a line coplanar with the circle and intersects it in one and only one point. The point of intersection of the line and the circle is called the **point of tangency**.

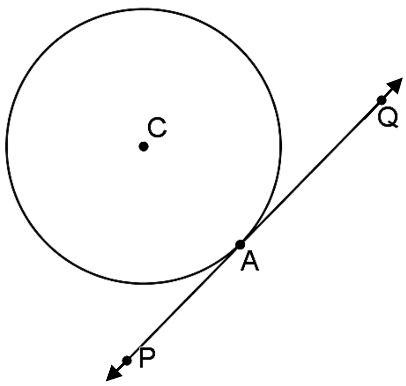



*Figure 4*

Example: In *Figure 4*, $\overleftrightarrow{PQ}$ intersects $\odot C$ at A only. Based on the definition of tangent line, $\overleftrightarrow{PQ}$ is a tangent line and A is the point of tangency.

**Secant**

A **secant** is a line or segment or ray that intersects a circle at exactly two points. A secant contains chord of a circle.

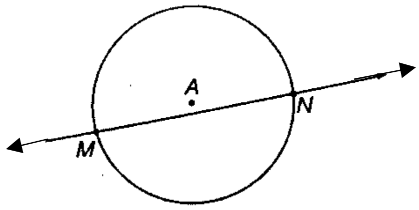



*Figure 5*

Example: In Figure 5, $\overleftrightarrow{MN}$ is a **secant** line of $\odot A$. It intersects the circle at two points.

## *What Is It*

The following is the summary of what to be discussed in this section:

1. Definition of Terms
2. Postulate on Tangent Line
3. Theorems on Tangent Line
4. Theorems on Angles formed by Tangent Lines and Secant Lines
5. Theorems on Segments formed by Tangent Segments and Secant Segments

**Common Tangent**

A ***common tangent*** is a line or segment or ray that is tangent to two circles in the same plane.

a. ***Common internal tangents*** intersect the segment joining the centers of the two circles.

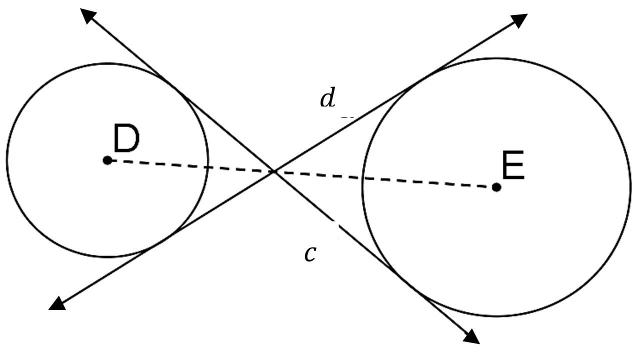



*Figure 6*

Example: In *Figure 6*, *c* and *d* are both common internal tangents of $\odot D$ and $\odot E$. Notice that they intersect the line connecting the center of $\odot D$ and $\odot E$.

b**. *Common external tangents*** are lines or rays or segment that do not intersect the segment joining the centers of the two circles.

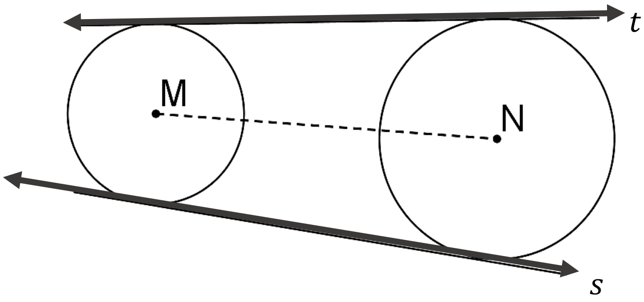



*Figure 7*

Example: In *Figure 7*,*t* and *s* are common external tangents. Note that they do not intersect the line connecting the centers of ⊙M and ⊙N.

**External Secant Segment**

An **external secant segment** is the part of a secant segment that is outside of a circle.

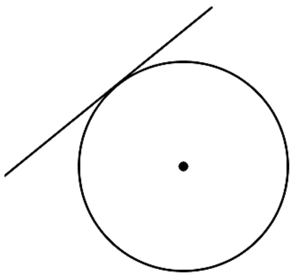

*Figure 8*

Example: In *Figure 8*, secant segments are illustrated outside ⊙C and ⊙D. In ⊙C. $\overline{AB}$ is a segment of secant line *h* and it is outside the circle. In ⊙D, $\overline{EC}$ and $\overline{FC}$ are segments of secant line *f* and secant line *k* respectively outside the circle. For these reasons, $\overline{AB}$, $\overline{EC}$ and $\overline{FC}$ are external secant segments.

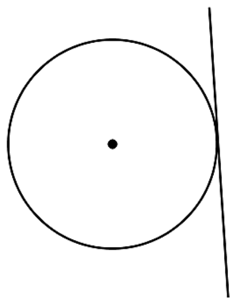
**Activity 4: Tangent or Secant**

To check your understanding on tangent and secant lines, supply what is being asked in the problems that follows.

1. In each figure, identify if the figure illustrates a t**angent line**, **secant line**, or **neither**.

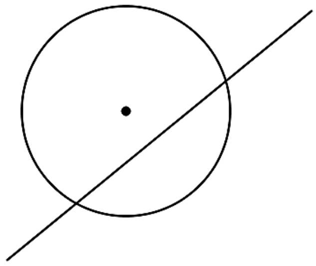


a.

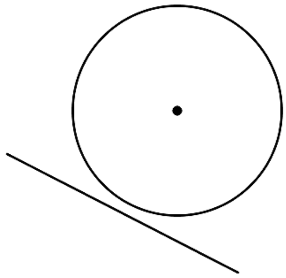


d.

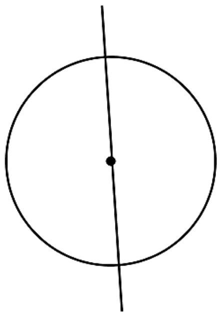
b.



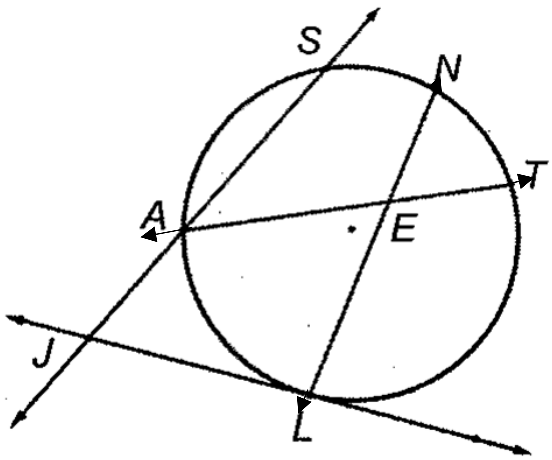
e.



c.

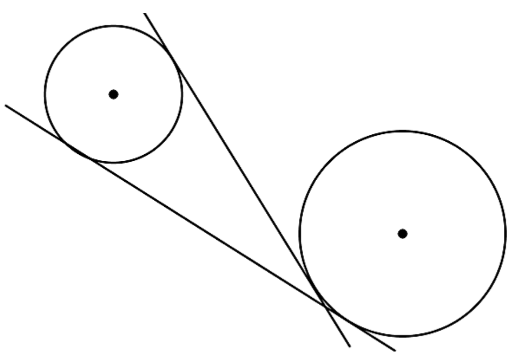


2. In the circle, name which lines are **tangent** and which lines are **secant**.

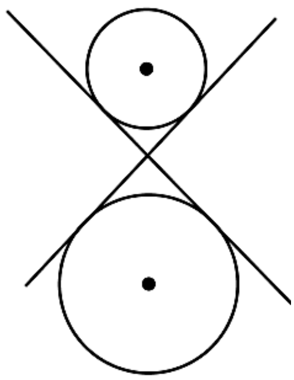


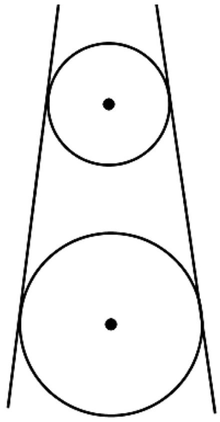

3. In each figure, identify if **common internal tangents**, **common external tangents** or **neither** are being illustrated.

a.

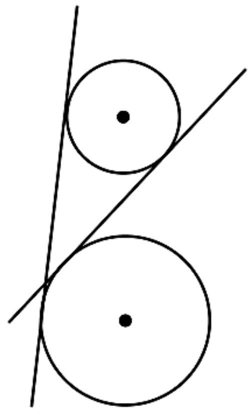


c.



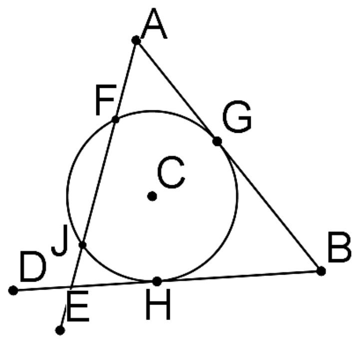


b.

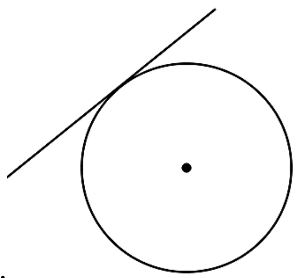


d.

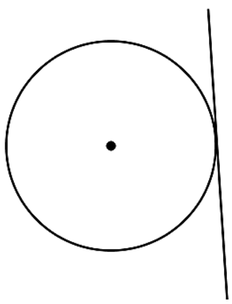
4. In ʘC, identify which segments are **external secant segment**.




In item **number 1**,
lines in letters **a and d** are **tangent lines** since they intersect or touch a circle at only one point

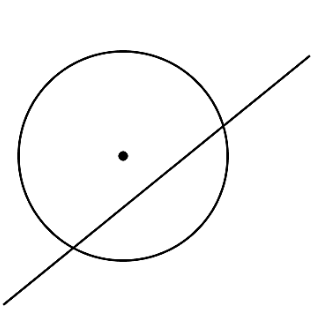


a.

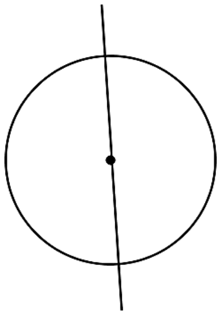


d.

Lines in letters **b and c** are **secant lines** since each intersects a circle at two points.

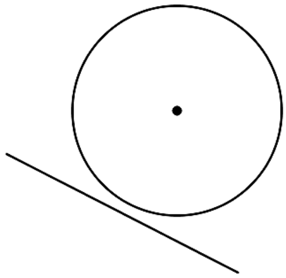


b.



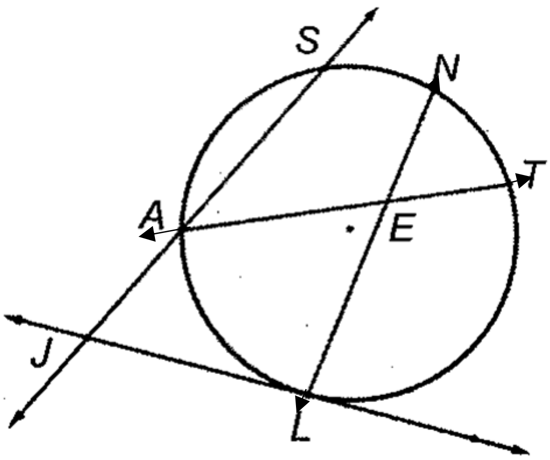
c.

Line in letter e is neither a tangent line nor a secant line since it does not touch any point on a circle.

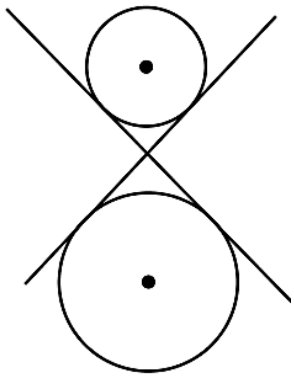


e.

In item **number 2,** the only **tangent line** is $\overleftrightarrow{JL}$ while the **secant lines** are $\overleftrightarrow{SJ}$, $\overleftrightarrow{NL}$ and $\overleftrightarrow{AT}$.

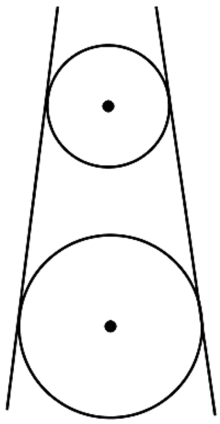



In item **number 3**, letter **d** illustrates lines that are **common internal tangents**,



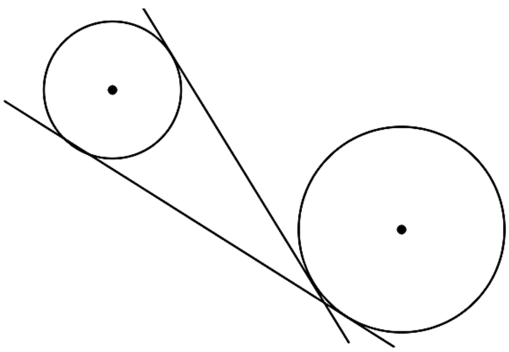
c.

letter **b** illustrates lines that are **common external tangents**,

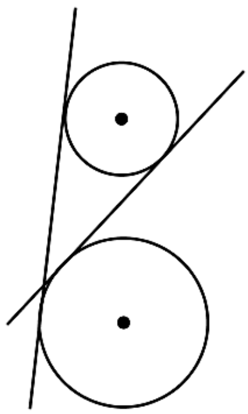


b.

and letter **a** and letter **e** both **neither** illustrate common internal tangents nor common external tangents.

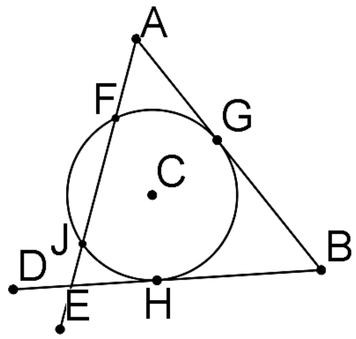


a.



d.

In item **4**, $\overline{AE}$ is only the external secant line. **$\overline{AF}$, and $\overline{JE}$** are segments outside $\odot C$. Thus, the e**xternal segments of a secant line are $\overline{AF}$, and $\overline{JE}$**.




**Postulate on Tangent Line**

At a given point on a circle, **one and only one** line can be drawn that is tangent to the circle.

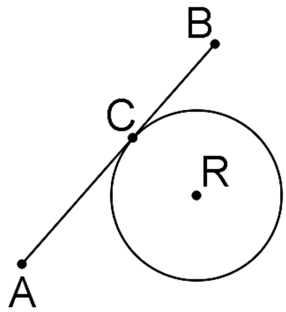



*Figure 9*

To illustrate, consider ⊙R in *Figure 9*. If C is a point on the circle, then $\overline{AB}$ is the only line that can be drawn through C that is tangent to ʘR.

**Theorems on Tangent Line**

1. If a line is tangent to a circle, then it is perpendicular to the radius drawn to the point of tangency.

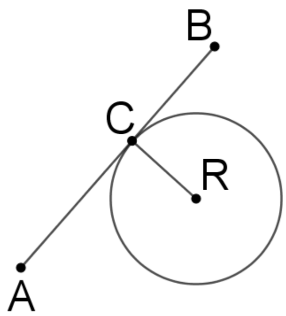



*Figure 10*

In *Figure 10*, $\overline{AB}$ is tangent to ʘR, at C. If we connect C to R, then $\overline{CR} \perp \overline{AB}$

2. If a line is perpendicular to the radius of a circle at its endpoint on the circle, then the line is tangent to the circle.

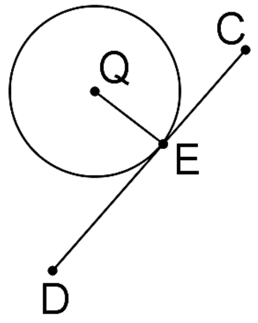



*Figure 11*

ʘQ in *Figure 11,* $\overline{DC} \perp \overline{QE}$ and $\overline{QE}$ is a radius of circle Q. Therefore, $\overline{DC}$ is tangent to ʘQ.

3. If two segments from the same exterior point are tangent to a circle, then the two segments are congruent.

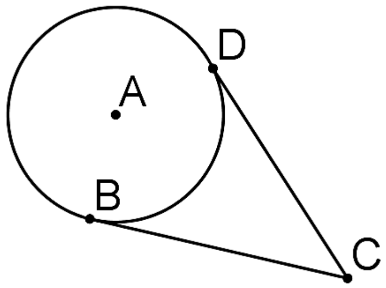



*Figure 12*

In *Figure 12,* $\overline{DC}$ and $\overline{BC}$ are tangent to circle A at D and B, respectively, from the common external point, C. From the theorem, $\overline{BC} \cong \overline{DC}$.

**Theorems on Angles Formed by Tangents and Secants**

1. If two tangents intersect in the exterior of a circle, then the measure of the angle formed is one-half the difference of the measures of the intercepted arcs.

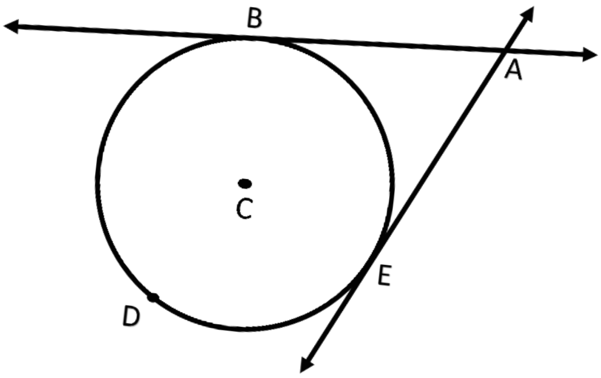



*Figure 13*

In *Figure 13*, tangent $\overleftrightarrow{BA}$ and tangent $\overleftrightarrow{AE}$ intersect outside ʘC at point A. $\overset{\frown}{BE}$ and $\overset{\frown}{EDB}$ are the two intercepted arcs of ∠BAE. The theorem states that:

$$m\angle BAE = \frac{1}{2}(m\overset{\frown}{EDB} - m\overset{\frown}{BE})$$

2. If two secants intersect in the exterior of a circle, then the measure of the angle formed is one-half the positive difference of the measures of the intercepted arcs.

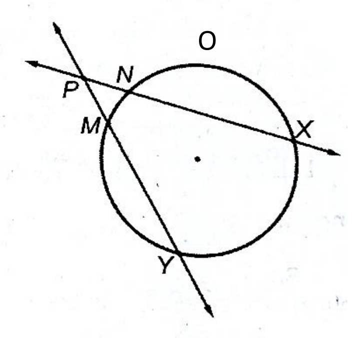



*Figure 14*

In *Figure 14,* $\overleftrightarrow{NX}$ and $\overleftrightarrow{MY}$ are two secants intersecting outside the circle at point P. $\overset{\frown}{XY}$ and $\overset{\frown}{MN}$ are the two intercepted arcs of $\angle XPY$. The theorem states that:

$$m\angle XPY = \frac{1}{2}(m\overset{\frown}{XY} - m\overset{\frown}{MN})$$

3. If a secant and a tangent intersect in the exterior of a circle, then the measure of the angle formed is one-half the difference of the measures of the intercepted arcs.

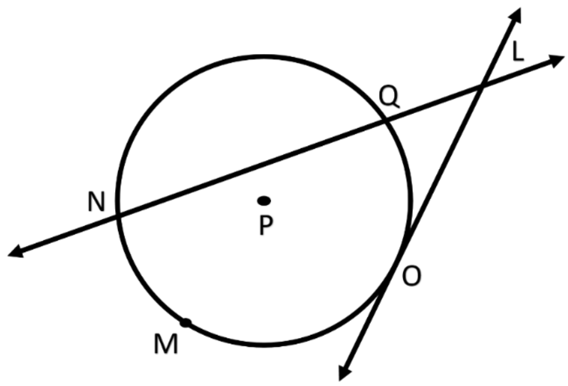



*Figure 15*

In *Figure 15*, secant $\overleftrightarrow{NL}$ and tangent $\overleftrightarrow{LO}$ intersect outside the circle at point L. $\overset{\frown}{OMN}$ and $\overset{\frown}{QO}$ are the two intercepted arcs of $\angle NLO$. The theorem states that:

$$m\angle NLO = \frac{1}{2}(m\overset{\frown}{OMN} - m\overset{\frown}{QO})$$

4. If two secants intersect in the interior of a circle, then the measure of an angle formed is one-half the sum of the measures of the arcs intercepted by the angle and its vertical angle.

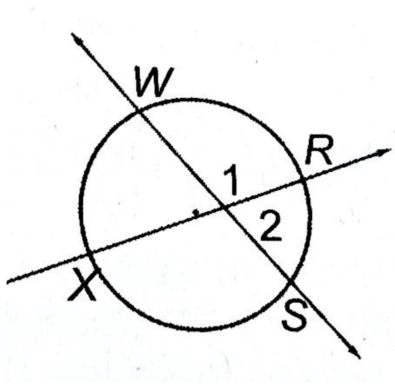



*Figure 16*

In *Figure 16*, $\overleftrightarrow{WS}$ and $\overleftrightarrow{RX}$ are two secants intersecting inside the circle. $\overset{\frown}{WR}$ and $\overset{\frown}{XS}$ are the two intercepted arcs of ∠1 while $\overset{\frown}{XW}$ and $\overset{\frown}{RS}$ are the two intercepted arcs of ∠2. The theorem states that:

$m\angle 1=\frac{1}{2}(m\overset{\frown}{WR}+m\overset{\frown}{XS})$ $\qquad$ $m\angle 2=\frac{1}{2}(m\overset{\frown}{XW}+m\overset{\frown}{RS})$

5. If a secant and a tangent intersect at the point of tangency, then the measure of each angle formed is one-half the measure of its intercepted arc.

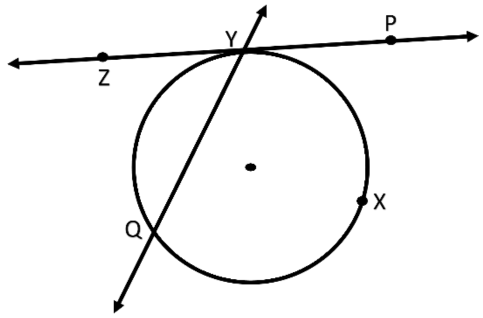



*Figure 17*

In *Figure 17*, $\overleftrightarrow{ZP}$ is tangent at the circle at Y. Secant $\overleftrightarrow{QY}$ and tangent $\overleftrightarrow{ZP}$ intersectat Y. $\overset{\frown}{QY}$ is the intercepted arc of ∠ZYQ while $\overset{\frown}{YXQ}$ is the intercepted arc of ∠QYP. The theorem states that:

$m\angle ZYQ=\frac{1}{2}m\overset{\frown}{QY}$ $\qquad$ $m\angle QYP=\frac{1}{2}m\overset{\frown}{YXQ}$

**Theorems on Secant Segments, Tangents Segments, and External Segments**

1. If two secant segments are drawn to a circle from an exterior point, then the product of the lengths of one secant segment and its external secant segment is equal to the product of the lengths of the other secant segment and its external secant segment.

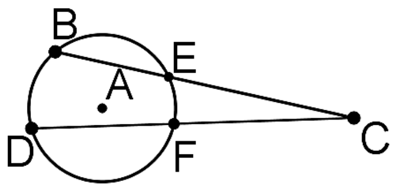



*Figure 18*

In ʘA of *Figure 18*, $\overline{BC}$ and $\overline{DC}$ are secant segments from external point C. $\overline{EC}$ is the external secant segment of $\overline{BC}$ and $\overline{FC}$ is the external secant segment of $\overline{DC}$. The theorem states that:

$$(\overline{BC})\bullet(\overline{EC}) = (\overline{DC})\bullet(\overline{FC})$$

2. If a tangent segment and a secant segment are drawn to a circle from an exterior point, then the square of the length of the tangent segment is equal to the product of the lengths of the secant segment and its external secant segment.

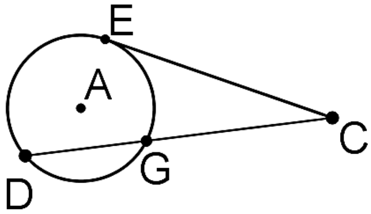



*Figure 19*

In ʘA of *Figure 19*, $\overline{EC}$ and $\overline{DC}$ are tangent segment and secant segment respectively from external point C. $\overline{GC}$ is the external secant segment of $\overline{DC}$. The theorem states that:

$$(EC)^2 = (\overline{DC})\bullet(\overline{GC})$$

3. If two secant lines intersect inside the circle, the product of the lengths of segments formed inside the circle are equal.

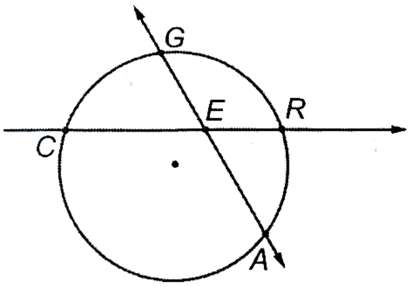



*Figure 20*

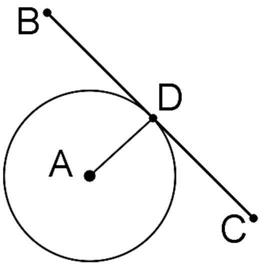
In *Figure 20,* secant lines $\overleftrightarrow{GA}$ and $\overleftrightarrow{CR}$ intersect inside the circle. $\overline{GE}$, $\overline{EA}$, $\overline{RE}$ and $\overline{EC}$ are the segments formed inside the circle. Thus,

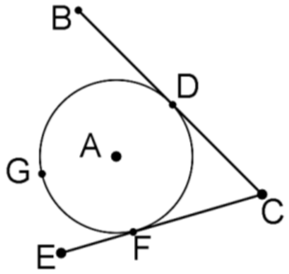
$$(\overline{GE})\bullet(\overline{EA}) = (\overline{RE})\bullet(\overline{EC})$$

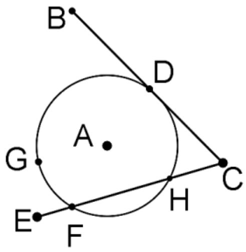
**Activity 5: Match Me**

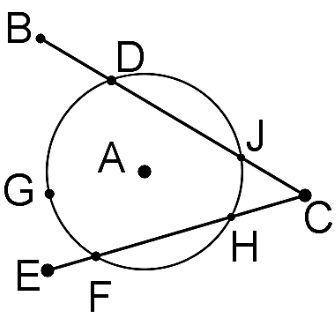
Match illustrations in column A with the relationship about its angles formed, tangent segment, and secant segments in column B. An illustration in column A can be matched to more than one relationship in column B. Or more than one illustration in column A can be match to one relationship in column B. In addition, any line segment that looks like a tangent line is tangent, any line segment that looks like secant line is secant and a point that looks like a point of tangency is really the point of tangency.

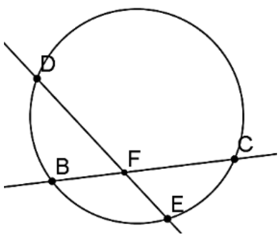
| Column A | Column B |
|---|---|
| | a. $(\overline{DC})^2 = (\overline{FC})(\overline{HC})$ |

1. 


2. 


3. 


4. 


5. 


b. $(\overline{DC})(\overline{JC}) = (\overline{FC})(\overline{HC})$

c. $\overline{DC} \cong \overline{FC}$

d. If $\overline{BC}$ is tangent to ⊙A at D, then $\overline{BC} \perp \overline{AD}$

e. In ⊙A, if $\overline{BC} \perp \overline{AD}$, then $\overline{BC}$ is tangent to the circle at D

f. $m\angle BCE = \frac{1}{2}(m\overset{\frown}{FGD} - m\overset{\frown}{DF})$

g. $m\angle BCE = \frac{1}{2}(m\overset{\frown}{FGD} - m\overset{\frown}{JH})$

h. $m\angle BCE = \frac{1}{2}(m\overset{\frown}{FGD} - m\overset{\frown}{DH})$

i. $(\overline{DF}) \bullet (\overline{FE}) = (\overline{BF}) \bullet (\overline{FC})$

Did you answer the Match Me activity correctly?

After illustrating the theorems, we consider the proof of some in the next discussions.

**Proof**

1. If two secants intersect in the exterior of a circle, then the measure of the angle formed is one-half the positive difference of the measures of the intercepted arcs.
*Step 1:To prove the theorem, it would be easier to transform to an equivalent if-then statement using Figure 21.*

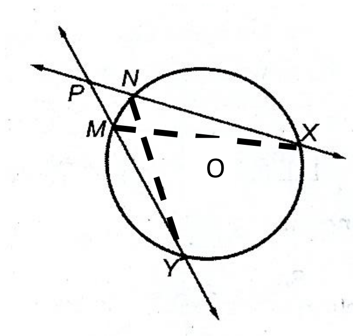



*Figure 21*

Step 1: If-then: If $\overleftrightarrow{NX}$ and $\overleftrightarrow{MY}$ are two secants intersecting outside the circle at point P, $\overset{\frown}{XY}$ and $\overset{\frown}{MN}$ are the two intercepted arcs of $\angle XPY$ then $m\angle XPY=\frac{1}{2}(m\overset{\frown}{XY}-m\overset{\frown}{MN})$

*Step 2: Identify the given and what to prove. In the if- then statement, the if statement is the given and the then statement is the conclusion.*

Step 2:
Given:
$\overleftrightarrow{NX}$ and $\overleftrightarrow{MY}$ are two secants intersecting outside the circle at point P,
$\overset{\frown}{XY}$ and $MN$ are the two intercepted arcs of $\angle XPY$

Need to Show: $m\angle XPY=\frac{1}{2}(m\overset{\frown}{XY}-m\overset{\frown}{MN})$

*Step 3: Prove now the theorem*

| Statement | Reason |
|---|---|
| 1. Connect N and Y, and M and X | By construction |
| 2. ∠XMY is an exterior angle of ΔPXM. | Definition of an exterior angle of a triangle. |
| 3. m∠XMY=m∠XPY+m∠NXM | The measure of the exterior angle of a triangle is equal to the sum of the measures of its remote interior angles. |
| 4. $\overset{\frown}{MN}$ is intercepted by inscribed ∠NXM and $\overset{\frown}{XY}$ is intercepted by inscribed ∠XMY | Definition of intercepted arc |
| 5. $m\angle NXM = \frac{1}{2}m\overset{\frown}{MN}$ <br> $m\angle XMY = \frac{1}{2}m\overset{\frown}{XY}$ | The measure of an inscribed angle is one half the measure of its intercepted arc. |
| 6. $\frac{1}{2}m\overset{\frown}{XY} = m\angle XPY + \frac{1}{2}m\overset{\frown}{MN}$ | Substitution from statements 3 to 5 |
| 7. $\frac{1}{2}mXY - \frac{1}{2}m\overset{\frown}{MN} = m\angle \overset{\frown}{XPY}$ | Subtraction Property |

| | |
|---|---|
| 8.$m\angle XPY = \frac{1}{2}(m\overset{\frown}{XY} - m\overset{\frown}{MN})$ | By factoring |

■

2. If a secant and a tangent intersect at the point of tangency, then the measure of each angle formed is one-half the measure of its intercepted arc.

   *Following the steps given in example 1,*

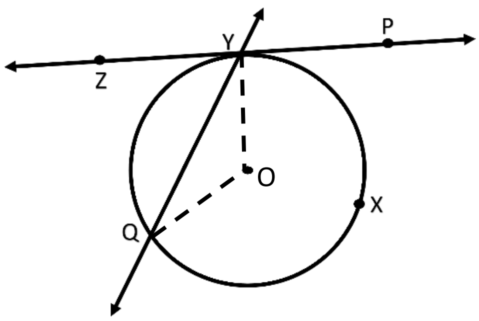



*Figure 22*

Step 1: If,

- $\overleftrightarrow{ZP}$ is tangent to ⊙O at Y, and
- Secant $\overleftrightarrow{QY}$ and tangent $\overleftrightarrow{ZP}$ intersectat Y then

$$m\angle ZYQ = \frac{1}{2}m\overset{\frown}{QY} \quad \text{and} \quad m\angle QYP = \frac{1}{2}m\overset{\frown}{YXQ}$$

Step 2:
Given:

$\overleftrightarrow{ZP}$ is tangent to ⊙O at Y

Secant $\overleftrightarrow{QY}$ and tangent $\overleftrightarrow{ZP}$ intersect at Y

Need to Show:

$$m\angle ZYQ = \frac{1}{2}m\overset{\frown}{QY} \quad \text{and} \quad m\angle QYP = \frac{1}{2}m\overset{\frown}{YXQ}$$

Proof: Since we need to show two relationships, the proof of the theorem shall be divided into two. The first part of the proof is to show that $m\angle ZYQ = \frac{1}{2}m\overset{\frown}{QY}$. Consider the proof that follows

| Statement | Reasons |
|---|---|
| 1.Connect O to Q and O to Y forming $\overline{OQ}$ and $\overline{OY}$ | By construction |
| 2. YQ is intercepted by ∠YOQ | Definition of intercepted arc. |
| 3. $m\angle YOQ = m\overset{\frown}{YQ}$ | The measure of a central angle is equal to the measure of its intercepted arc. |
| 4.$\overleftrightarrow{ZP}$ is tangent to the circle at Y | Given |
| 5.$\overline{OY} \perp \overleftrightarrow{ZP}$ | If a line is tangent to a circle, then it is perpendicular to the radius drawn to the point of tangency. |
| 6.$m\angle ZYO = 90$ | Perpendicular lines form a 90° angle. |
| 7. $\angle ZYQ + \angle QYO = \angle ZYO$ | Whole part postulate |
| 8.$m\angle ZYQ = m\angle ZYO - m\angle QYO$ | Subtraction Property |
| 9.$m\angle ZYQ = 90 - m\angle QYO$ | Substitution from statements 6 to 8 |

| | |
|---|---|
| 10. $m\angle QYO = 90 - m\angle ZYQ$ | Addition property of equality |
| 11. $\overline{YO}$ and $\overline{OQ}$ are radii of $\odot O$ | Definition of radius of a circle |
| 12. $\overline{YO} = \overline{OQ}$ | Radii of a circle are equal |
| 13. $\Delta YOQ$ is an isosceles triangle | At least two sides of an isosceles triangle are equal |
| 14. $m\angle QYO = m\angle OQY$ | Angles opposite the equal sides of an isosceles triangles are equal. |
| 15. $m\angle QYO + m\angle OQY + m\angle YOQ = 180$ | The sum of the interior angles of a triangle is 180 |
| 16. $m\angle QYO + m\angle QYO + m\angle YOQ = 180$ | Substitution from statements 14 to 15 |
| 17. $2m\angle QYO + m\angle YOQ = 180$ | Simplification |
| 18. $2(90 - m\angle ZYQ) + m\angle YOQ = 180$ | Substitution from statements 10 to 17 |
| 19. $180 - 2m\angle ZYQ + m\angle YOQ = 180$ | Distributive Property |
| 20. $-2m\angle ZYQ + m\angle YOQ = 0$ | Subtraction Property |
| 21. $m\angle YOQ = 2m\angle ZYQ$ | Addition Property |
| 22. $\frac{1}{2}m\angle YOQ = m\angle ZYQ$ | Division Property |
| 23. $m\angle YOQ = mYQ$ | A central angle is equal to its intercepted arc. |
| 24. $\frac{1}{2}m\widehat{YQ} = m\angle \widehat{ZYQ}$ | Substitution from statements 23 to 22 |

The first part is done for you, prove the second part of the theorem.

**Activity 6: Can You Complete by Proof?**
The other theorems will be left for you to prove.

The last concept we need to discuss is about sectors of a circle.

**Sector of a Circle**

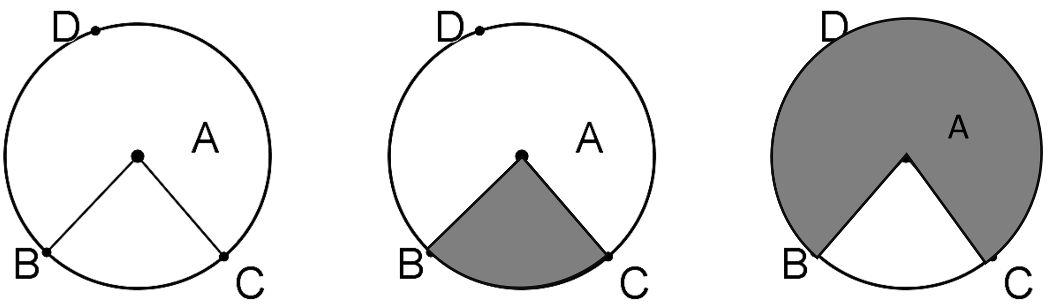
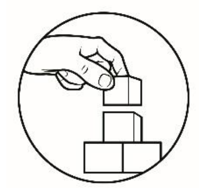



*Figure 23*

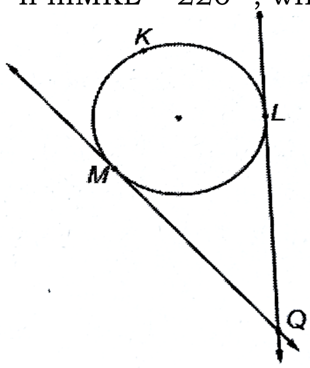
The region of the circle bounded by radii and an arc are called **sectors**, as illustrated *Figure 23. T*he shaded region in the center circle is a minor sector since it is bounded by a minor arc and radii. While the shaded region in the right most figure is a major sector since it is bounded by two radii and a major arc.



## *What's More*

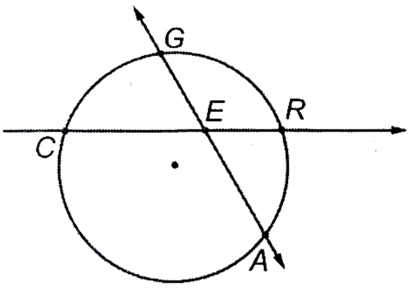
**Activity 7.** Apply the different theorems discussed to solve for the unknown in the following exercises.

1. If $m\overset{\frown}{MKL} = 220°$ , what is $m\angle MQL$?

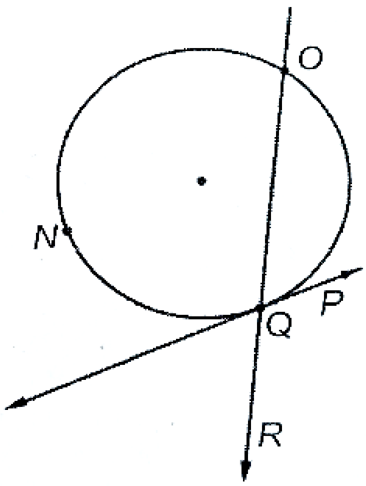



2. Suppose $m\overset{\frown}{CG} = 6x + 5$, $m\overset{\frown}{RA} = 4x + 15$, and $m\angle AEC = 120$, Find

a. x b. $m\overset{\frown}{CG}$ c. $m\overset{\frown}{RA}$




3. If $m\overset{\frown}{QNO} = 238°$, what is $m\angle PQO$? $m\angle PQR$?




## *What I Have Learned*

*In the previous activities that you have done, were you able to apply the theorems you have learned? In the next activity you are to summarize now your understanding about the theorems.*

**I think I do!**
Answer the following questions.

1. What is the relationship of two secants intersecting in the exterior of a circle to the measures of its intercepted arcs?

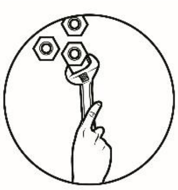
2. What is the relationship of a secant and a tangent intersecting in the exterior of a circle to its intercepted arcs?

3. What is the relationship of two secants intersecting in the interior of a circle to the measures of the intercepted arcs and its vertical angles?

4. What is the relationship among the segments formed inside a circle when two secant lines intersect in the interior of a circle?

5. How are the two secants segments drawn from the exterior point to the circle related to their external secant segments?

6. How about if a tangent segment and a secant segment are drawn to the circle from an exterior point, what will be its relationship to its segments formed?



## *What I Can Do*

Did you know that the tire of a car touches the road at a point when running? Or in other words, a tire is tangent on the road. What do you think is the reason?

Can you cite 5 other situations where tangent line or secant line is applied around us?

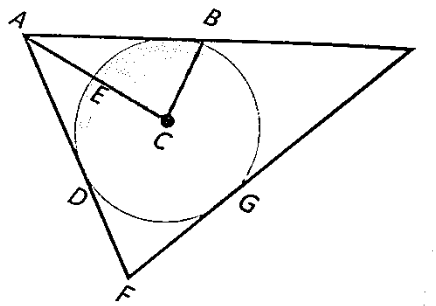
## Assessment

Directions: Read carefully each item and write the letter of the correct answer on a separates sheet of paper

1. Which of the following line segments intersects a circle at exactly two points?
   A. line  B. ray  C. secant  D. tangent

2. Which of the following is the region bounded by two radii of the circle and their intercepted arc?
   A. area  B. circumference  C. semicircle  D. sector

**For items 3-7**. Refer to the figure at the right.




$\overline{AB}$, $\overline{AD}$, and $\overline{FG}$ are tangent segments of ⊙C.

3. Which of the following are the intercepted arcs of $\angle AFG$?
A. $\overset{\frown}{DG}$ and $\overset{\frown}{DBG}$  C. $\overset{\frown}{BG}$ and $\overset{\frown}{EB}$
B. $\overset{\frown}{EB}$ and $\overset{\frown}{DG}$  D. $\overset{\frown}{DG}$ and $\overset{\frown}{BG}$

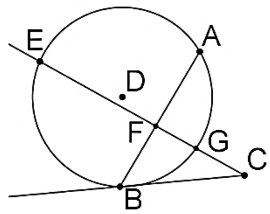
4. If $m\angle DAB$=84, then $m\angle DAC$=_______.
   A. 16°  B. 42°  C. 51°  D. 168°

5. In the figure given that $\overline{AB}$ is 12. Which of the following is the measurement of $\overline{AD}$?
   A. 3  B. 6  C. 12  D. 24

6. Find the measurement of ∠F given m $\overset{\frown}{DG}$ =78.
   A. 39°  B. 102°  C. 141°  D. 156°

7. Which of the following is the m$\angle ABC$?
   A. 100°  B. 90°  C. 60°  D. 45°

**For items 8-15**. Refer to the figure at the right.




8. If $\overline{BC}$ = 12 and $\overline{GC}$ = 6, then which of the following is the length of $\overline{EG}$?
A. 12  B. 18  C. 24  D. 72

9. If $\overline{AF}$ = 8, $\overline{FB}$ = 6, and $\overline{FG}$= 2, then which of the following is $\overline{EF}$?
   A. 10  B. 12  C. 16  D. 24

10. If m$\angle AFG$ = 70 and m$\overset{\frown}{AG}$ = 80, then which of the following is the m$\overset{\frown}{BE}$ ?
   A. 60°  B. 70°  C. 80°  D. 220°

11. If m$\overset{\frown}{AB}$ = 110 , m$\overset{\frown}{AG}$ = 60 , and m$\overset{\frown}{BE}$ = 100, then m$\angle C$=____.
   A. 100°   B. 75°   C. 50°   D. 25°

12. If m$\overset{\frown}{AB}$ = 110, then which of the following is the $m\angle ABC$?_.
A. 55°   B. 90°   C. 110°   D. 220°

13. Which of the following arcs is an intercepted arc of $\angle EFA$?
A. $\overset{\frown}{EA}$   B. $\overset{\frown}{AGB}$   C. $\overset{\frown}{BG}$   D. $\overset{\frown}{BEA}$

14. In the figure which of the following represents a tangent?
A. $\overline{BC}$   B. $\overline{EC}$   C. $\overline{BA}$   D. $\overline{BF}$

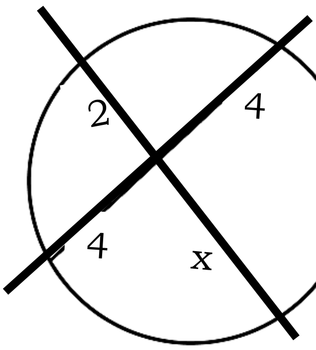
15. In the figure which of the following represents a secant?
A. $\overline{BC}$   B. $\overline{EC}$   C. $\overline{BA}$   D. $\overline{BF}$

## *Additional Activity*

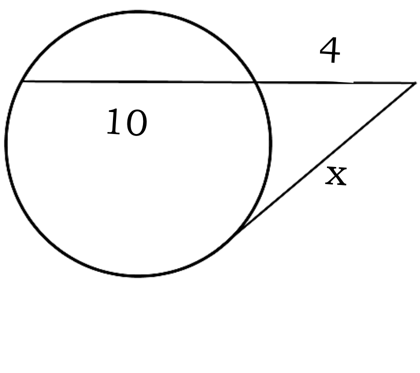
*Mathematical Reasoning*

Investigate and explain what is wrong with the student's solution below.

1. Solution: $\frac{x}{2} = \frac{4}{4}$
   $4x = 4(2)$
   $x = 2$




2. Solution: $x^2 = 4(10)$
   $x^2 = 40$
   $x = \sqrt{40}$
   $x = 2\sqrt{10}$



# Answer Key

**What I Know**
1. C
2. C
3. B
4. A
5. C
6. D
7. B
8. B
9. B
10. D
11. A
12. D
13. A
14. D
15. D

**What's In**
**Activity 1**
1. f
2. f
3. a
4. b
5. c
6. c
7. d

**What's It :**
Activity 5
1. d,e
2. c,f
3. a, h
4. b,g
5. i

**What's More**
**Activity 7**

1. $m\angle MQL = \frac{1}{2}(\text{MKL} - \text{ML})$
   $= \frac{1}{2}(220-140)$
   $= 40$
2. $\angle$ AEC and $\angle$CEG form a Linear Pair. It is given that m $\angle$AEC is 120 then m$\angle$CEG is 60. In addition,
   $m\angle CEG = \frac{1}{2}(m \text{ arc CG} + \text{m arc RA})$.
   Substituting the given and m$\angle$CEG, then $x=10$, $CG = 65$ and $AR = 55$.

3. m$\angle$PQO =61°

$\angle$PQR= 119°

**ASSESSMENT**
1. C
2. D
3. A
4. B
5. C
6. B
7. B
8. B
9. D
10. A
11. D
12. A
13. A
14. A
15. B

**ADDITIONAL ACTIVITY**
1. From the theorem when two secants intersect inside the circle,
   2(x) = 4 (4) thus
   x = 8
2. From the theorem, when two secants intersect outside the circle,
   $x^2=4(16)$ thus
   x= 8

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**For inquiries or feedback, please write or call:**

Department of Education - Bureau of Learning Resources (DepEd-BLR)

Ground Floor, Bonifacio Bldg., DepEd Complex
Meralco Avenue, Pasig City, Philippines 1600

Telefax: (632) 8634-1072; 8634-1054; 8631-4985

Email Address: blr.lrqad@deped.gov.ph * blr.lrpd@deped.gov.ph