Case 1 (2024): Migration & Default Risk (3 questions)

Assume the following one period migration & default transition matrix holds

Rating	AAA	AA	A	BBB	BB	B	CCC	D (default)
AAA	91.115%	8.179%	0.607%	0.072%	0.024%	0.003%	0.000%	0.000%
AA	0.844%	89.626%	8.954%	0.437%	0.064%	0.036%	0.018%	0.021%
A	0.055%	2.595%	91.138%	5.509%	0.499%	0.107%	0.045%	0.052%
BBB	0.031%	0.147%	4.289%	90.584%	3.898%	0.708%	0.175%	0.168%
BB	0.007%	0.044%	0.446%	6.741%	83.274%	7.667%	0.895%	0.926%
B	0.008%	0.031%	0.150%	0.490%	5.373%	82.531%	7.894%	3.523%
CCC	0.000%	0.015%	0.023%	0.091%	0.388%	7.630%	83.035%	8.818%

Table 1: One Period Migration & Default Transition Matrix

Furthermore, assume that a one factor Merton model holds which drives rating transition, i.e.:

$$X_i = \sqrt{\rho} \cdot Y + \sqrt{1-\rho} \cdot \epsilon_i$$

Where Y denotes the common factor and ϵ_i the idiosyncratic factor (both standard normal).

Portfolio

We are interested in analyzing the one-period credit (migration & default) risk, for a portfolio of bonds.

The market values of the (zero-coupon) bonds are provided below:

i) for the base case (market value <u>before</u> migration event)
ii) for the shocked case (market value <u>after</u> migration event);

Rating	Base Value	Shocked Value
AAA	99.40	99.50
AA	98.39	98.51
A	97.22	97.53
BBB	92.79	92.77
BB	90.11	90.48
B	86.60	88.25
CCC	77.16	77.88
D (default)		60.00

Table 2: 5-year zero-coupon bond values in the base and after shock

Assume the following two portfolios (with total market value of 1,500 mln EUR) with the following allocation of the market value over the different ratings:

- <u>Portfolio I (Investment Grade):</u> 60% AAA, 30% AA, 10% BBB.
- <u>Portfolio II (Junk):</u> 60% BB, 35% B, 5% CCC.

In this exercise we would like to analyze the credit risk (migration & default) given the above setup.

Exercise

Question 1 – A concentrated investment

Assume **a single issuer** per rating class in the investment portfolio, with the market value allocated pro rata over the rating classes (so for portfolio 1, in total 60%*1,500mln EUR market value is invested in the AAA class, 30%*1,500mln EUR in the AA class and 10%*1,500mln EUR in the BBB class).

For $\rho \in \{0,\ 33\%, 66\%, 100\%\}$:

- Compute the expected (average) portfolio value.
- Compute the 90% and 99.5% Value-at-Risks and Expected Shortfalls

Single Issuer per rating	Rho	Expected Value	90% VaR	99.5% VaR	90% ES	99.5% ES
Portfolio I	0%					
Portfolio I	33%					
Portfolio I	66%					
Portfolio I	100%					
Portfolio II	0%					
Portfolio II	33%					
Portfolio II	66%					
Portfolio II	100%					

Question 2 (diversified investments)

Assume now a more diversified portfolio where the total invested amount is allocated pro rata over the rating classes (so for portfolio 1, in total 60%*1,500mln EUR market value is invested in the AAA class, 30%*1,500mln EUR in the AA class and 10%*1,500mln EUR in the BBB class).

Only now assume **100 issuers** per rating class, where per rating class the issuer has an equal market value for that class. So for portfolio I, 60%*1,500mln EUR=900mln EUR is invested in the AAA, with a 9 mln EUR (= 900 mln EUR / 100 issuers) investment per issuer in this AAA class (and a similar logic for the ratings and for portfolio II).

For $\rho \in \{0, 33\%, 66\%, 100\%\}$:

- Compute the expected (average) portfolio value.
- Compute the 90% and 99.5% Value-at-Risks and Expected Shortfalls

100 Issuers per rating	Rho	Expected Value	90% VaR	99.5% VaR	90% ES	99.5% ES
Portfolio I	0%					
Portfolio I	33%					
Portfolio I	66%					
Portfolio I	100%					
Portfolio II	0%					
Portfolio II	33%					
Portfolio II	66%					
Portfolio II	100%					

Question 3

Please comment on the results of the analyses, in particular on:

a) Differences in the results between portfolio I and II (Motivate your answer)
b) Differences in the results for increasing levels of ρ (Motivate your answer)

c) Explain how the one factor model could be extended to accommodate for a heavier tail dependence between defaults (i.e., increased clustering of defaults), whilst keeping the migration probabilities unchanged.