Confidence intervals

Some of you asked how to compute a confidence interval for Monte Carlo estimations, please below how to compute these (differently) for expected values and for a correlation coefficient. Though it is a bit of a detailed topic, it might be useful to know in practice when you need to assess the accuracy of any Monte Carlo (risk management) simulations you are performing.

Often we perform simulations are we interested in how accurate our Monte Carlo estimate is, or whether more simulations would be require to obtain a stable estimate. 9 times out of 10, we estimating an Expected Value (e.g. the expected discounted forward price, or expected discounted call option pay-off). In the cases where we are dealing with an Expectation, we can use the central limit theorem and the fact that our Monte Carlo estimates are i.i.d: "For a large number of independent identically distributed random variables X_1, ... , X_n, with finite variance, the average approximately has a normal distribution, no matter what the distribution of the X_i is, with the approximation roughly improving in proportion to sqrt(n)".

This implies for an expected value, 95% confidence interval and n simulations:
a) **Confidence interval for an expected value:** Can be computed as [Sample Mean + N^-1(2.5%)*Sample St Dev / Sqrt(n) , Sample Mean + N^-1(97.5%)*Sample St Dev / Sqrt(n)]
b) **In practice:** You will need 4 times more scenario's to obtain a twice as accurate estimate, and you will need 100 times more scenarios for a 10 times as accurate Monte Carlo estimate. Runs up quickly, **and generally some uncertainty around Monte Carlo estimates is accepted (within materiality thresholds).**

General Rule (expected value): The above applies to any expected value estimated via Monte Carlo simulation (e.g. for the discounted forward price and call option price). For more details, please see: https://en.wikipedia.org/wiki/Confidence_interval.Links to an external site. Generally we are dealing with this situation, and we have to compute a confidence interval around an expected value.

Exceptions (e.g. correlation, value-at-risk): In some cases, however. you need to compute a confidence intervals for non-expected values, such as a correlation coefficient (or a value-at-risk estimate). In these cases, confidence intervals are computed differently. How to compute a confidence intervals for a correlation efficient is explained here: https://shandou.medium.com/how-to-compute-confidence-interval-for-pearsons-r-a-brief-guide-951445b9cb2dLinks to an external site.