# Case 2 (2024): CDS Stripping

The following CDS spreads apply for counterparty "C", assuming annually compounded interest rates of 3% and a LGD (Loss Given Default) of 40%;

| Maturity | CDS Rate (in bps) | Formula |
|---|---|---|
| 1Y | 100 | $R(1)$ |
| 3Y | 110 | $R(3)$ |
| 5Y | 120 | $R(5)$ |
| 7Y | 120 | $R(7)$ |
| 10Y | 125 | $R(10)$ |

## Exercise

<u>Question 1 (Simplified CDS stripping):</u>

Derive the CDS curve, where you assume <u>continuous premium payments</u>, and piece-wise constant hazard rates. To this end:

1. Calculate the Average Hazard rates using $\lambda_{Average}(T) = \frac{R(T)}{LGD}$.
2. From this calculate the cumulative default probability and forward hazard rates.

Please fill in the following table;

| Maturity | CDS Rate (in bps) | (Average) Hazard Rate (see above "simple" definition) | Forward Hazard Rate (between $T_{i-1}$ and $T_i$) | Default Probability (between $T_{i-1}$ and $T_i$) |
|---|---|---|---|---|
| 1Y | 100 | | | |
| 3Y | 110 | | | |
| 5Y | 120 | | | |
| 7Y | 120 | | | |
| 10Y | 125 | | | |

**Note:**

- The simplified setup with continuous premium payments is a different procedure as the iterative stripping procedure which you are asked to apply in question 2.
- The main objective is to use the resulting hazard rates and default probabilities from this question and benchmark these outcomes against the results obtained from question 2.
- In Question 3 you are then asked to compare the results between question 1 and 2.

Question 2:

Strip the CDS curve assuming quarterly premiums including accrued premium at default, and piece-wise constant hazard rates. This entails iteratively solving for the Forward Hazard Rates using the following formulas:

- $\lambda_1$: $CDS(0,1,R(1),\mathrm{LGD};\{\lambda_1\}) = 0$
- $\lambda_2$: $CDS(0,3,R(3),\mathrm{LGD};\{\lambda_1,\lambda_2\}) = 0$
- $\lambda_3$: $CDS(0,5,R(5),\mathrm{LGD};\{\lambda_1,\lambda_2,\lambda_3\}) = 0$
- $\lambda_4$: $CDS(0,7,R(7),\mathrm{LGD};\{\lambda_1,\lambda_2,\lambda_3,\lambda_4\}) = 0$
- $\lambda_5$: $CDS(0,10,R(10),\mathrm{LGD};\{\lambda_1,\lambda_2,\lambda_3,\lambda_4,\lambda_5\}) = 0$

Where: $CDS\big(0,T,R(T),\mathrm{LGD};\bar{\boldsymbol{\lambda}}\big) =$

$$R(T)\cdot\left\{\sum_{i=1}^{N} e^{-r\cdot T_i}\cdot(T_i-T_{i-1})\cdot Q(\tau>T_i) + \sum_{i=1}^{N} e^{-r\cdot T_i^{Mid}}\cdot\big(Q(\tau>T_{i-1})-Q(\tau>T_i)\big)\cdot\frac{(T_i-T_{i-1})}{2}\right\}$$

$$-LGD\cdot\sum_{i=1}^{N} e^{-r\cdot T_i^{Mid}}\cdot\big(Q(\tau>T_{i-1})-Q(\tau>T_i)\big)$$

Where $T_i^{Mid} = \frac{(T_i+T_{i-1})}{2}$ and where in which the (forward) hazard rate is computed as:

$$\lambda(u) = \begin{cases} \lambda_1 \; if\; 0\le u<1 \\ \lambda_2 \; if 1\le u<3 \\ \lambda_3 \; if 3\le u<5 \\ \lambda_4 \; if 5\le u<7 \\ \lambda_5 \; if 7\le u<10 \end{cases}$$

**Hint:** to solve for the forward rates, you can use root finding procedures, which exists in many software packages (e.g. using SciPy in Python)

**Please fill in the following table;**

| Maturity | CDS Rate (in bps) | (Average) Hazard Rate (defined as: $\frac{\int_0^{T_i}\lambda(u)du}{T_i}$) | Forward Hazard Rate (between $T_{i-1}$ and $T_i$) | Forward Default Probability (between $T_{i-1}$ and $T_i$) |
|---|---|---|---|---|
| 1Y | 100 | | | |
| 3Y | 110 | | | |
| 5Y | 120 | | | |
| 7Y | 120 | | | |
| 10Y | 125 | | | |

**Note: It is not allowed to use full third party CDS software packages (such as QuantLib) for solving the above CDS spreads, i.e. you are expected to implement your own CDS stripping procedure and submit your code together with your answer.**

Question 3:

Compare and comment on the hazard rate / default probability differences between question 1 and 2.

i. For which maturities and/or economic situations are the results close to each other?
ii. Under which circumstances, or changes the input parameters, could potential differences be observed?

Please comment on your findings.