Case 3 (2024): CVA Equity Derivatives

You can work in groups of 2 students. Hand-in your group solution together with your code (in the same pdf). In case you feel some information is missing or incorrect, please specify which information you need and continue the exercise based on that. Please answer the below 5 questions and comment on your findings in a clear and concise manner: Briefly comment on your approach, calculation steps & results.

Please upload your report, as well as your code in one or two pdf files to canvas!

Good Luck!

Introduction

Goal of this exercise is to price the (unilateral) CVA charge for an equity portfolio from the bank's stand point towards the counterparty "C".

Assumptions

1. The following forward hazard rates apply for counterparty "C", and assuming an LGD of 40%;

T_{i-1}	T_i	Forward Hazard rate λ between T_{i-1} and T_i
0	1	2.00%
1	3	2.15%
3	5	2.20%

2. Current risk-free interest rate term structure equal to $r = 3\%$ continuously compounding rate

3. Consider two underlying equity process:
   a. STOXX50 (SX5E): $S_{SX5E}(t+\Delta) = S_{SX5E}(t)\exp\left[\left(r - q - \frac{1}{2}\sigma_{SX5E}^2\right)\cdot\Delta + \sigma_{SX5E}\cdot Z_{SX5E}\cdot\sqrt{\Delta}\right]$
   b. AEX (AEX): $S_{AEX}(t+\Delta) = S_{AEX}(t)\exp\left[\left(r - q - \frac{1}{2}\sigma_{AEX}^2\right)\cdot\Delta + \sigma_{AEX}\cdot Z_{AEX}\cdot\sqrt{\Delta}\right]$

   With dividend yield $q = 2\%$, Δ the time step between time t and $t+\Delta$, volatilities $\sigma_{AEX} = 15\%$ and $\sigma_{SX5E} = 15\%$, and with Z_{SX5E} and Z_{AEX} two standard normal distributions with correlation $\rho = 80\%$.

4. Consider an exposure from the bank towards the counterparty "C" consisting of the following equity derivatives portfolio:

Instrument	Type	Underlying	# Contracts	Base Level	Strike	Vol	Maturity
1	Forward	SX5E	10,000	4,235	4,235		5 Years
2	Forward	AEX	55,000	770	770		5 Years
3	Put Option	SX5E	10,000	4,235	3,388	15%	5 Years
4	Put Option	AEX	55,000	770	616	15%	5 Years

Part 1 - CVA Calculations

Please answer below questions 1-3 1, note that there also two questions for part 2 of the case study. Briefly outline your approach and comments on your results;

Question 1: Equity Model Simulation

Verify the correctness of the Equity Simulation model implementations:

a) Verify that the expected discounted payoff for the equity forwards aligns with the t=0 valuation for the two forwards.
b) Verify that the expected discounted payoff for the put options aligns with the t=0 analytical Black-Scholes valuation formula for the two options.
c) Verify that the correlation between the log-returns of AEX and $SX5E$ equals 80%.

In all above questions:

- Confirm a correct Monte Carlo model implementation by verifying your average simulated value is "close" to the theoretical values.
- In all cases provide 95% confidence intervals for your simulated outcomes.
- Explain your answers.

Question 2: Exposures & CVA

Calculate and plot the **monthly expected (positive) exposure** profiles for:

a) The netted portfolio of contracts.

Calculate the following **CVA charges**:

b) Each underlying contract separately.
c) The portfolio of contracts assuming the contracts are not in a netting agreement.
d) The portfolio of contracts assuming the contracts are in a netting agreement.

Question 3: Impact of Model Parameters

Take the portfolio of contracts, assuming the contracts are in a netting agreement.

Calculate the following **CVA charges** for the portfolio of contracts assuming:

a) All Equity volatilities are equal to 30% instead of 15%
b) Correlation between the log-returns is equal to 40% instead of 80%.

c) Comment on the outcomes of your results in (a) and (b).

Part 2 – Credit Risk Management

Please answer below questions 4&5. Briefly outline your approach and comments on your results;

Question 4: Collateral Impact on CVA

Take the portfolio of contracts, assuming the contracts are in a netting agreement.

Assume collateral is posted, equal to the size of the exposure, but with different frequencies;

That is, assume collateral is posted every y months, with $y = \{1, 2, 3, \dots, 12, 24, 36, 48, 60\}$.

a) Plot the impact of the collateral determination frequency on the CVA charge.

   Comment on your results and compare with the "no-collateral" CVA charge.

For question (b) assume that no collateral is posted, but instead an initial margin (independent amount) must be posted.

b) Analyze the impact of posting an initial margin amount equal to 1mln, 10mln and 100mln EUR.

   Comment on your results and compare with the "no-initial margin" CVA charge.

Question 5: Credit Risk Exposure Hedging

Take the portfolio of contracts, assuming the contracts are in a netting agreement, but without collateral. The bank would like to construct a hedge portfolio to neutralize (first order) changes of the underlying CDS rates for counterparty "C" to offset the corresponding CVA charge movements.

a) Calculate impacts on the CVA charge for a 10 bps increase in each of the underlying forward hazard rates. This should hence result in the following table;

CVA Charge	CVA Charge Price Change
Increase Forward Hazard Rate [0,1]	
Increase Forward Hazard Rate [1,3]	
Increase Forward Hazard Rate [3,5]	

b) Calculate the impact on CDS contract values for a 10 bps increase of forward hazard rates.

   Note 1: A change of the [0,1] forward hazard rate impacts the 1Y, 3Y and 5Y CDS Price.
   Note 2: You are allowed to use either the simple or advanced CDS pricing formula (with accrued premium), but do align with a time-dependent hazard rate function as from assumption (1).

CDS Price Changes	1Y CDS	3Y CDS	5Y CDS
Increase Forward Hazard Rate [0,1]			
Increase Forward Hazard Rate [1,3]	0		
Increase Forward Hazard Rate [3,5]	0	0	

c) Explain how CDS contracts can be used to hedge (neutralize) the CVA charge movements with respect to changes in the underlying forward hazard rates for counterparty “C”.

d) Calculate the notionals for the CDS contracts with 1Y, 3Y and 5Y maturity, in order to construct a first order (delta) hedge of the CVA charge, with respect to movements in the underlying forward hazard rates for counterparty “C”.

   That is, start with the 5Y CDS contract and backwards solve for the CDS notionals which neutralize the CVA charge movement as function of forward hazard rates changes.

e) If we move forward in time and when market circumstances change, explain why the delta hedge should be adjusted, and when this would be needed.