10 Sample Exercises for CCSR (2024)

1. Merton's Model

In Merton's model, let the debt have maturity T and the debt to be paid at maturity be F. Assume the firm value evolves like a geometric Brownian motion with

$$dV(t) = (r - k)dt + \sigma V(t)dW(t)$$

Hence default probability is given by: $\Psi(d), d = \frac{\log\frac{F}{V(0)} - \left((r-k) - 0.5 \cdot \sigma^2\right) \cdot T}{\sigma\sqrt{T}}$

a) Compute the probability of default in case $r = 8\%$, $k = 4\%$, $\sigma = 20\%$ and for $T = 3$, $V(0) = 100$ and $F = 85$.

b) Is the default probability increasing or decreasing in the debt level ?
Is the default probability increasing or decreasing in the volatility ?
Is the default probability increasing or decreasing in the return r.
Motivate.

c) The instantaneous default probability goes to zero in Merton's model (for t goes to 0), how does this compare with the instantaneous probability of default in an intensity model ?

d) How can Merton's model be improved to get a more realistic behavior for short term spreads.

2. CDS

Assume a CDS quoted spread is 400 bps and the recovery is estimated to be 20%. Assume that the premium leg of the CDS pays continuously and the hazard is constant.

a) Compute the hazard rate.

b) Compute the probability of default between year 3 and 5.

c) What is probability of surviving an infinite time ?

d) If recovery increases, but the spread remains the same, does the probability of default increase or decrease? Motivate

e) Is it true that if the CDS spread doubles and the recovery remains the same, then the probability of default in 5 years also doubles?

3. Hazard Rates

Assume a piecewise constant hazard rate where

$$\gamma(t) = 2\% \text{ for } 0 \leq t \leq 1$$
$$\gamma(t) = 4\% \text{ for } 1 \leq t \leq 2$$
$$\gamma(t) = 2\% \text{ for } 2 \leq t \leq 3$$

Compute:

a) The probability of defaulting in one year, two year and three years.

b) The probability of surviving more than two years.

c) Assume a recovery of 50%, zero interest rates and the CDS sells protection from today to three years. Compute the price of the default leg / protection leg of the CDS.

4. Default Dependency

Consider a portfolio of 3 companies and assume a one factor model. Assume that the probability of default of company k, conditional on a realization of the common factor y, is given by $p_k(y)$ with k =1,2 or 3. Model the default of company k by a latent variable V(k) which has a standard normal distribution. Assume that the common factor Y has a standard normal distribution.

a) Describe the one-factor model.
b) Give the probability of one default conditional on Y=y.
c) Give a formula for the probability of one default.
d) What happens to the price of a 1st-to-default swap if the asset correlations are increased?
e) How can we extend the model to accommodate heavier tail dependence between defaults?

5. CDO

An investor sells protection of the 9-12% tranche of the iTraxx pool. We assume a recovery of 40% for all names. Each single name in the portfolio has a credit position in the index of 1/125 = 0.8% and participates to the aggregate loss in terms of $0.8\% \cdot LGD = 0.8\% \cdot 0.6 = 0.48\%$. This means that each default corresponds to a loss of 0.48% in the global portfolio.

a) How many defaults are needed before the tranche starts experiencing some losses.
b) How many defaults are needed before the tranche is wiped out completely.
c) Assume we want to compute the periodic spread of the equity tranche 0-3% tranche. Assume a one-factor Gaussian copula for the pricing ? Will the tranche price increase or decrease in case we increase the correlation ?
d) Same as c) only now for the 30-60% tranche.
e) Same as c) only now for the 60-90% tranche.

6. Trading strategy

Consider the following trading strategy. Sell CDS protection on ten names where each name has a CDS premium (spread) of 170bp and simultaneously buy protection through a First to Default on this basket at a spread of 1500bp. At the moment of the first default you unwind the remaining 9 CDS. This is a profitable strategy since it gives 200bp per annum as long as there is no default and no cashflows after the first default.

a) Explain where this argument can go wrong.

How does this depend on the default correlation ?

7. First to Default Basket / Copula

We are given three CDS on names 1,2,3. Assume each single name default is modeled with a constant hazard rate and the CDS premium legs pay continuously, so that the hazard rate formulas are:

$$\lambda_i = \frac{R_{CDS,i}}{1 - rec_i}, i = 1,2,3$$

Assume a one year maturity and all three names having a recovery of 40% and spreads respectively of 50, 100 and 200 bps.

a) If the defaults are connected through an independence copula and interest rates are zero, compute the default leg price of a third to default leg.
b) If the defaults are connected through a maximum dependence copula, compute the default leg price of a third to default leg.
c) In which of the two cases a) and b) is above protection more expensive? Motivate

8. Third to Default Basket / Copula

We are given three CDS on names 1,2,3. Assume a one year maturity and all three names having a recovery of 60% and **hazard rates** respectively of 300, 400 and 500 bps.

a) If the defaults are connected through an independence copula and interest rates are zero, compute the default leg price of a third to default leg.
b) If the defaults are connected through a maximum dependence copula, compute the default leg price of a third to default leg.
c) In which of the two cases a) and b) is above protection more expensive? Motivate

9. Complexity – Open Questions

a) How can regulators create a negative feedback loop in real estate markets via mortgage regulations?
b) In what cases can "the subjective, non-scientific learning process of people" lead to booms and busts in macro economics?

10. Complexity – Multiple Choice Questions

i) Networks with multiples nodes and edges give results that usually cannot be found with closed form mathematical models because

A. There are more edges than nodes
B. Nodes grow non-linear
C. A shock perturbates through a network, often causing many feedback loops
D. A shock perturbates through a network, always with negative feedback loops

ii) When we are confronted with increased uncertainty:

A. Our Cortisol level increases and we become more risk averse
B. Our Testosteron level increases and we become more risk averse
C. Our Cortisol level increases and we get a higher risk appetite
D. Our Testosteron level increases and we get a higher risk appetite

iii) Regulators that want to create a negative feedback loop in financial markets when their participants become euphoric can for example

A. Allow more lending of banks when markets do well
B. Demand more capital from banks when volatility (good measure for risk) goes up in financial markets
C. Put a limit on loan-to-income for mortgages instead of a limit on loan-to-value
D. Allow for more risk taking when volatility goes down in financial markets

iv) A long term scenario can be probable (reasonably likely), plausible (easy to imagine, even though likelihood not high) and possible (challenging to our imagination but not “armagedon”like). When drafting scenario’s, the process should focus on

A. Probable because it needs to get attached to a probability
B. Plausible but not Possible because otherwise people will lose faith in the process
C. Possible and Plausible because we need to stretch our imagination
D. Probable and Plausible but not Possible because that is too unrealistic

Exercise Set - Answers (2024)

Exercise 1

a) $\Psi(d) = 26\%$, where $d = \frac{\log\frac{F}{V(0)} - \left((r-k)-0.5\cdot\sigma^2\right)\cdot T}{\sigma\sqrt{T}} = -0.6420$,
b Increasing in F, decreasing in r, increasing in σ
c) Instantaneous probability of default goes to zero in Merton's model, whereas it is non-zero in intensity models.
d) By introducing a jump diffusion process.

Exercise 2

a) Can use simple formula (see assumptions): $\lambda = \frac{R}{LGD} = \frac{400bps}{1-0.2} = 500\ bps$
b) Use intensity model formulas:

$$Q(3 < \tau < 5) = Q(\tau > 3) - Q(\tau > 5) = \exp(-\lambda \cdot 3) - \exp(-\lambda \cdot 5) = 8.1907\%$$

c) Use limit formula with $\lambda > 0$:

$$\lim_{u\to\infty} Q(t \geq u) = \exp(-\lambda \cdot u) = 0$$

d) We have $R \approx \lambda \cdot LGD$, so when the recovery ($REC = 1 - LGD$) increases, the loss given default LGD goes down. With an equal spread, the default probability hence has to increase.

   Or alternatively: if you have the same spread, but the loss upon a default is smaller (e.g. 1% vs 20%), than the default probability automatically has to be higher.
e) This is not the case (note: $\lambda_{double} = \frac{2\cdot R}{LGD} = 2\cdot\lambda$) due to non-linearity of the exponent:

$$1 - \exp(-2\lambda \cdot 10) \neq 2 \cdot [1 - \exp(-\lambda \cdot 10)]$$

Exercise 3

General formula: $Q(t \leq T) = 1 - exp\left(-\int_0^T \lambda(u)du\right)$
Time-dependent intensities: compute integrals using time-dependent intensity.

a) This implies using intensity model formulas with $\lambda_1 = 0.02, \lambda_2 = 0.04, \lambda_3 = 0.02$:
   i. $Q(t \leq 1) = 1 - \exp\left(-\lambda_1 \cdot 1\right)$
   ii. $Q(t \leq 2) = 1 - \exp\left(-[\lambda_1 \cdot 1 + \lambda_2 \cdot 1]\right)$
   iii. $Q(t \leq 3) = 1 - \exp\left(-[\lambda_1 \cdot 1 + \lambda_2 \cdot 1 + \lambda_3 \cdot 1]\right)$
b) $Q(t > 2) = 1 - Q(t \leq 2) = \exp\left(-[\lambda_1 \cdot 1 + \lambda_2 \cdot 1]\right)$
c) CDS payoff is equal to the loss given default when default happens before 3 years.
   Take risk-neutral expectations and use the special fact (!) that interest rates are zero:

$$E\left[D(0,\tau) \cdot LGD \cdot 1_{\{\tau<3y\}}\right] \stackrel{!}{=} LGD \cdot E\left[1_{\{\tau<3y\}}\right] = LGD \cdot Q(t < 3)$$

Exercise 4

a) For each firm i the value at time T is modeled as:

$$V_i(T) = \sqrt{\rho_i}Y + \sqrt{1-\rho_i}\epsilon_i$$

Here Y is a standard normally distributed common factor and the ϵ_i's are independent.

b) We have that conditional on Y, the probability of default of company i is $p_i(y) = p_i$. So, conditional on Y , we have that the probability of precisely one default is

$$P(X=1|Y=y) = p_1(1-p_2)(1-p_3) + p_2(1-p_1)(1-p_3) + p_3(1-p_1)(1-p_2)$$

c) Integrate over all possible outcomes of the common factor:

$$\int_{-\infty}^{\infty} P(X=1|Y=y)\phi(y)dy$$

Where $\phi(y)$ denotes a standard normally distributed factor.

d) An increasing correlation means that the probability of all three company's surviving increases. Hence the price of a first-to-default CDS decreases.
e) Increase likelihood of simultaneous default, e.g. by using a copula with more tail dependence, e.g. the student-t copula.

Exercise 5

a) 9%/0.48%=18.75, so after 19 defaults.
b) 12%/0.48=25, so after 25 defaults.
c) Equity tranche will be less expensive with higher correlation as probability of no defaults increases.
d) Super senior tranche will be more expensive with higher correlation as probability of many defaults increases.
e) Nothing changes, since even if all 125 firms default, there is only a $125 * 0.48 = 60\%$ loss. Hence the 60%-90% tranche is safe under a 40% recovery assumption.

Exercise 6

a) Suppose there is correlation between defaults. Then at the moment of the first default, the remaining companies are probable to a higher default rate and you cannot unwind at a favorable price. If the correlation is positive, the CDS price is likely to have moved in the wrong. So you cannot make a riskless profit here.

b) A high correlation means that at the moment of the first default, probabilities of other defaults are higher as well. This means that CDS spreads for the remaining firms are high, so that it is costly to unwind. With a low correlation this effect is less likely.

Exercise 7

a) $LGD \cdot P(\tau_3 < T) = LGD \cdot P(\tau_1 < T) \cdot P(\tau_2 < T) \cdot P(\tau_3 < T) = 0.6 \cdot \left(1 - e^{-\lambda_1 \cdot T}\right) \cdot \left(1 - e^{-\lambda_2 \cdot T}\right) \cdot \left(1 - e^{-\lambda_3 \cdot T}\right) = 0.0002698\%$

b) The name defaulting as third will be the one with the smallest hazard rate (default time for that name will be the latest), being the first firm. The price of default leg will then be:

$$LGD \cdot P\left(\tau^{(3)} < T\right) = LGD \cdot P(\tau_1 < T) = 0.6 \cdot \left(1 - e^{-\lambda_1 \cdot T}\right) = 0.4979\%$$

c) Protection is more expensive under case b (perfect dependence), as in this case when the first name with defaults, then automatically all other firms will default as well, whereas in independent case, even if this first name defaults, the others can still survive as those defaults are fully independent.

Exercise 8

a) Due to Independence: we can factor out the default events and write: $LGD \cdot Q(\tau_1 < T \text{ and } \tau_2 < T \text{ and } \tau_3 < T) = 40\% \cdot \left(1 - e^{-3\% \cdot 1}\right) \cdot \left(1 - e^{-4\% \cdot 1}\right) \cdot \left(1 - e^{-5\% \cdot 1}\right) = 0.00226\%$

b) Due to perfect dependence: the name defaulting as third will be the one with the smallest hazard rate (default time for that name will be the latest), being the firm with hazard rate of 300 bps.
The price of default leg will then be:

$$LGD \cdot Q(\tau_1 < T \text{ and } \tau_2 < T \text{ and } \tau_3 < T) = LGD = 40\% \cdot \left(1 - e^{-3\% \cdot 1}\right) = 1.182\%$$

c) Protection is more expensive under case ii (perfect dependence), as in this case when the first name with 300bps defaults, then automatically all other firms will default as well, whereas in independent case, even if this first name defaults, the others can still survive as those defaults are fully independent. (This can also be seen in the prices of the default legs under both assumptions)

Exercise 9

a) Regulators can create a negative feedback loop by putting a limit on loan-to-income for mortgages instead of a limit on loan-to-value.
b) Learning is not in line with reality. When markets rise, they switch from fundamental analysis to extrapolative rules This self-enforces the feedbackloop but they ignore the fact that the more people join, the more “saturated” this strategy will get and it becomes economically unsustainable. But at the same time more people will follow the rule until it tips

Exercise 10

Answers:

- i) C
- ii) A
- iii) C
- iv) C