Quantitative Risk Management: The Challenges & The Basics

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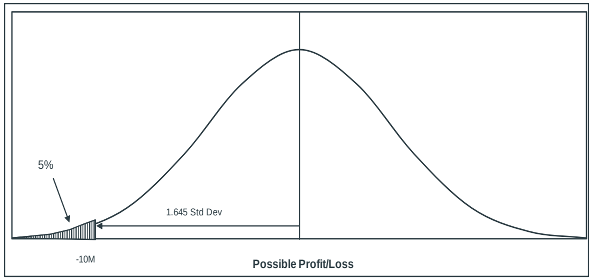


Definition of risk

What is Risk?

"hazard, a chance of bad consequences, loss or exposure to mischance" [OED]

"any event or action that may adversely affect an organization's ability to achieve its objectives and execute its strategies"

"the quantifiable likelihood of loss or less-than-expected returns"

Types of financial risks

- Market risk
- Credit risk
- Liquidity risk
- Model risk
- Operational risk
- Cyber risk
- Business risk
- Legal risk
- Climate risk
- etc.

Where do these risks come from?

- market risk — the risk of a change in the value of a financial position due to changes in the value of the underlying components on which that position depends, such as stock and bond prices, exchange rates, commodity prices, etc.

- credit risk — the risk of not receiving promised repayments on outstanding investments such as loans and bonds, because of the "default" of the borrower.

- operational risk — the risk of losses resulting from inadequate or failed internal processes, people and systems, or external events.

- Liquidity risk: risk of not being able to roll over debt, or risk associated with not being able to close existing positions due to lack of liquidity on markets

- Model risk: risk associated with models used for pricing, risk management etc being “wrong”: inappropriate model, wrong parameters, wrong calibration, wrong use, …

Our first focus: MARKET RISK

- Risk (unfavorable changes, i.e., losses of portfolio value) due to movements in
  - Stocks and stock indices
  - Exchange rates
  - Interest Rates
  - Other asset prices: commodities, real estate funds, etc

Our second risk: credit and counterparty risk

- Risk of individual loans (corporate, mortgages, personal loans) not being repaid: default risk
- Modelling of PoD but also of LGD
- Risk of corporate loans: downgrades in ratings of counterparties
- Related risk (falls under interest rate risk): early prepayments of loans/mortgages, withdrawals from NMD

Our third risk: model risk

- Mitigation of model risk: MODEL VALIDATION

QRM: what are challenges?

- Mathematical rigor and precision in current practices
- Going beyond current practices
- Dealing with:
  - Dependence of risks and multivariate nature of problem
  - Extreme values and heavy tails
  - Concentration risk
  - Scale of portfolios and liquidity risk
  - Model risk
  - Data issues
  - Understanding current regulation

Interdependence

The multivariate nature of risk presents an important challenge. Whether we look at market risk or credit risk, or overall enterprise-wide risk, we are generally interested in some form of aggregate risk that depends on high-dimensional vectors of underlying risk factors such as individual asset values in market risk, or credit spreads and counterparty default indicators in credit risk.

A particular concern in our multivariate modelling is the phenomenon of dependence between extreme outcomes, when many risk factors move against us simultaneously.

> "Extreme, synchronized rises and falls in financial markets occur infrequently but they do occur. The problem with the models is that they did not assign a high enough chance of occurrence to the scenario in which many things go wrong at the same time—the perfect storm scenario."

Extremes and heavy tails matter

“From the point of view of the risk manager, inappropriate use of the normal distribution can lead to an understatement of risk, which must be balanced against the significant advantage of simplification. From the central bank's corner, the consequences are even more serious because we often need to concentrate on the left tail of the distribution in formulating lender-of-last-resort policies. Improving the characterization of the distribution of extreme values is of paramount importance.”

“With globalisation increasing, you'll see more crises. Our whole focus is on the extremes now - what's the worst that can happen to you in any situation - because we never want to go through that [LTCM] again. “

Concentration risk

In a perfect storm scenario the risk manager discovers that the diversification he thought he had is illusory; practitioners describe this also as a concentration of risk.

> "Over the last number of years, regulators have encouraged financial entities to use portfolio theory to produce dynamic measures of risk. VaR, the product of portfolio theory, is used for short-run, day-to-day profit and loss exposures. Now is the time to encourage the BIS and other regulatory bodies to support studies on stress test and concentration methodologies. Planning for crises is more important than VaR analysis. And such new methodologies are the correct response to recent crises in the financial industry."

Problem of scale

A further challenge in QRM is the typical scale of our portfolios, which at their most general may represent the entire position in risky assets of a financialinstitution.

Calibration of detailed multivariate models for all risk factors is an almost impossible task and hence any sensible strategy involves dimension reduction, that is to say the identification of key risk drivers and a concentration on modelling the main features of the overall risk landscape with a fairly broad brush approach.

Interdisciplinarity: finance, maths, data, implementation

The quantitative risk manager of the future should have a combined skillset that includes concepts, techniques and tools from many fields:

- mathematical finance;
- statistics and financial econometrics;
- actuarial mathematics;
- non-quantitative skills, especially communication skills;
- humility — QRM is a small piece of a bigger picture!

Current regulation
Main goals for banks: liquidity and capital adequacy

- Basel III
- FRTB
- TRIM
- IRRBB
- IFRS 9, 17
- Solvency II
- IORP
- GDPR
- MIFID

Well-known (to you) and new stuff coming up next:

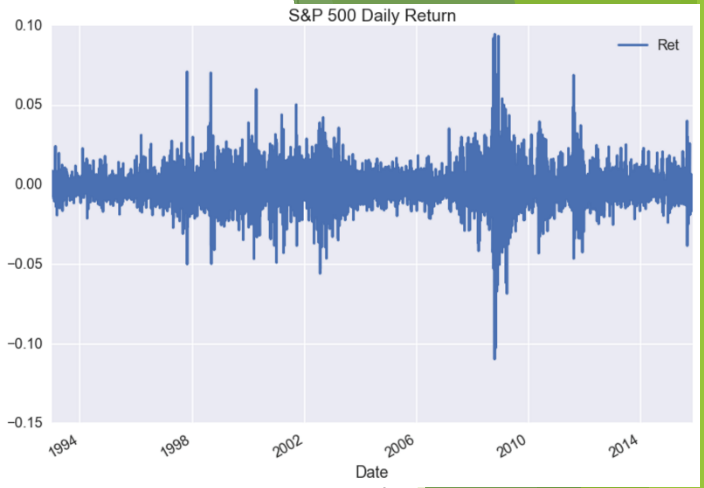
- Volatility
- Portfolio variance and volatility
- Main risk measures: VaR and ES
- Basic RM methods: historical, Variance-Covariance, *N* and *t* distributions
- Time-dependent volatility: EWMA, ARCH and GARCH

- New stuff:

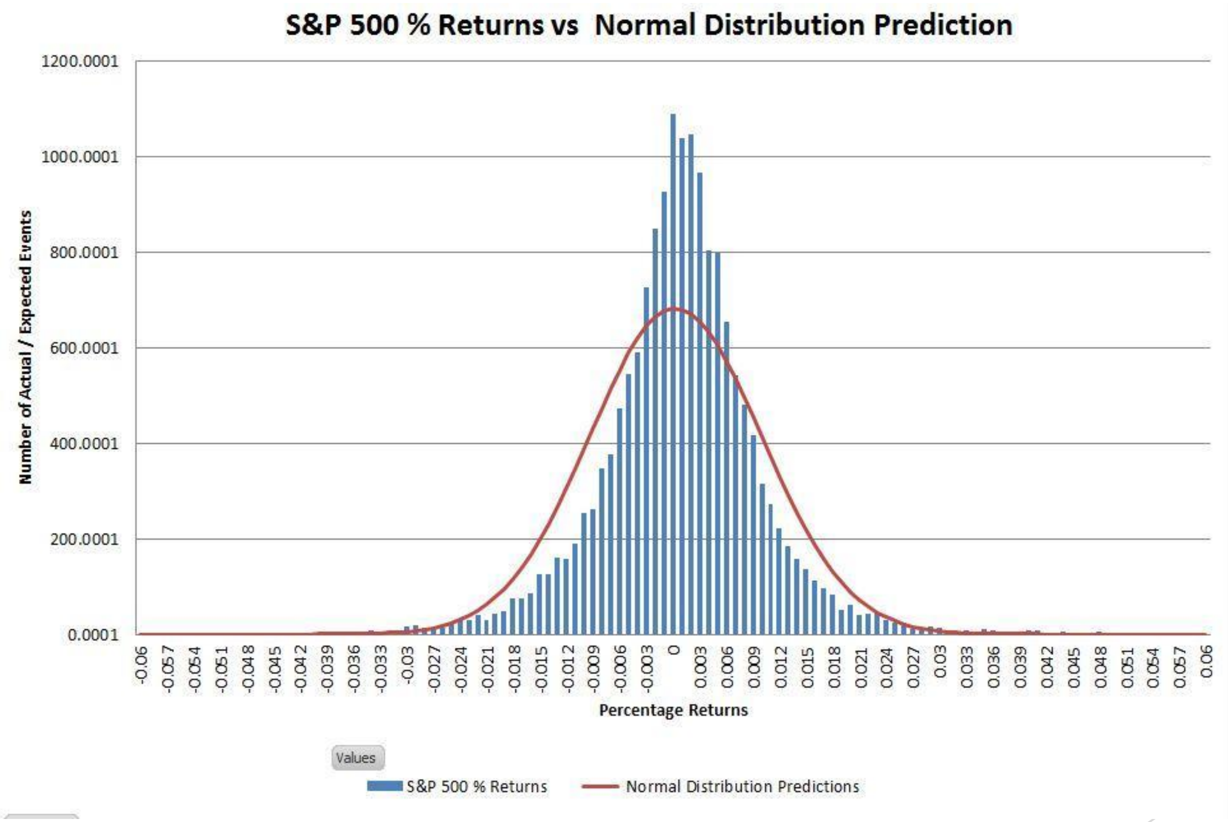
Filtered Historical Simulation

Backtesting of RM models

Variance and volatility



- Financial returns: random variables
- Horizon: day, week, month, year → depends on financial institution/its department, risk type, etc
- We model random variables by their probability distribution (full model) or their moments
- First moment: average, or expected return
- Second (central) moment: variance (squared volatility)
- Third (central) moment: related to skewness
- Fourth (central) moment: related to kurtosis



S&P 500 % Returns vs Normal Distribution Prediction
1200.0001
1000.0001
800.0001
600.0001
400.0001
200.0001
0.0001
Number of Actual / Expected Events
-0.06
-0.057
-0.054
-0.051
-0.048
-0.045
-0.042
-0.039
-0.036
-0.033
-0.03
-0.027
-0.024
-0.021
-0.018
-0.015
-0.012
-0.009
-0.006
-0.003
0
0.003
0.006
0.009
0.012
0.015
0.018
0.021
0.024
0.027
0.03
0.033
0.036
0.039
0.042
0.045
0.048
0.051
0.054
0.057
0.06
Percentage Returns
Values
S&P 500 % Returns
Normal Distribution Predictions

Standard Approach to Estimating Volatility

- Define σ_n the volatility on day n
- Define asset price S_i at end of day i
- Define log-return $u_i = \ln(S_i/S_{i-1})$

$$\sigma_n^2 = \frac{1}{m-1}\sum_{i=1}^{m}(u_{n-i} - \bar{u})^2$$

$$\bar{u} = \frac{1}{m}\sum_{i=1}^{m} u_{n-i}$$

- Simplifications:
- Define u_i as $(S_i - S_{i-1})/S_{i-1}$
- Assume that the mean value of u_i is zero
- Replace $m-1$ by m

$$\sigma_n^2 = \frac{1}{m}\sum_{i=1}^{m} u_{n-i}^2$$

Daily vs Yearly Volatilities

- Volatility is usually measured as % per year
- In RM/VaR applications, we often measure daily volatility:

$$\sigma_{day} = \frac{\sigma_{year}}{\sqrt{252}}$$

σ_{day} is the standard deviation of the return in one day

Similar square root of time scaling rule holds for all time horizons, i.e., to go from yearly to weekly to monthly volatility and back

EXAMPLE

The Variance and the Volatility of an *N*-asset Portfolio

- Variance of *N*-asset portfolio:

$$Var(r_p) = \sum_{i=1}^{N} w_i^2 \operatorname{var}(r_i) + 2\sum_{i=1}^{N} \sum_{j=i+1}^{N} w_i w_j \operatorname{cov}(r_i, r_j)$$

- The most important thing when calculating portfolio variance/volatility is the variance-covariance matrix or correlation matrix of the returns

- The above formula follows from properties of variance: check! Do it first for portfolio of two assets and then roll it out for *N* assets

Equally weighted portfolio

- Assume *n* assets, equal correlations, weights and volatilities
- Then the portfolio variance is:

$$Var(r_p) = \frac{\sigma^2}{n} + \frac{n-1}{n}\rho\sigma^2 \rightarrow \rho\sigma^2 = Cov(i, j)$$

- Prove this
- What happens if correlations and volatilities are different? Prove that:

$$\begin{aligned} Var(R_P) \; &= \; \frac{1}{n}(\text{Average Variance of the Individual Stocks}) \\ &\quad + \left(1 - \frac{1}{n}\right)(\text{Average Covariance between the Stocks}) \end{aligned}$$

Risk-Free Securities

- We put a fraction *x* of the money in a risky portfolio and the rest in a risk-free instrument (e.g., cash).
  - The expected return is:

$$\begin{aligned} E[R_{xP}] &= (1-x)r_f + xE[R_P] \\ &= r_f + x(E[R_P] - r_f) \end{aligned}$$

  - The volatility is:

$$\begin{aligned} SD[R_{xP}] &= \sqrt{(1-x)^2 Var(r_f) + x^2 Var(R_P) + 2(1-x)xCov(r_f, R_P)} \\ &= \sqrt{x^2 Var(R_P)} \\ &= xSD(R_P) \end{aligned}$$

Leverage

- Leverage:
  - Borrowing money to invest in risky assets.
  - Or shorting some risky assets to invest in other risky assets.
  - Such portfolios are called *leveraged portfolios*.
  - **These are more risky than unleveraged positions.**

  - **Example**: you have 1000 euros and you borrow another 1000 (against interest of e.g. 5%). You invest all 2000 in a stock (or stock index) with volatility of 20% and expected return of 10%.
  - Your resulting portfolio has expected return of 15% = 2*10%-5%, but the volatility of 2*20%=40%!

Worked out example of leverage

Expected Return and Volatility with a Short Sale

Problem

Suppose you have \$20,000 in cash to invest. You decide to short sell \$10,000 worth of Coca-Cola stock and invest the proceeds from your short sale, plus your \$20,000, in Intel. What is the expected return and volatility of your portfolio?

- Vols are resp. 0.25 and 0.5, assume that the correlation is zero (of course it is not, but this will be then a lower bound) and expected returns are 6% and 26%.
- Note: how do calculations change if correlation is 0.2?

Solution

We can think of our short sale as a negative investment of $-\$10{,}000$ in Coca-Cola stock. In addition, we invested $+\$30{,}000$ in Intel stock, for a total net investment of $\$30{,}000 - \$10{,}000 = \$20{,}000$ cash. The corresponding portfolio weights are

$$x_I = \frac{\text{Value of investment in Intel}}{\text{Total value of portfolio}} = \frac{30{,}000}{20{,}000} = 150\%$$

$$x_C = \frac{\text{Value of investment in Coca-Cola}}{\text{Total value of portfolio}} = \frac{-10{,}000}{20{,}000} = -50\%$$

Note that the portfolio weights still add up to 100%. Using these portfolio weights, we can calculate the expected return and volatility of the portfolio using Eq. 11.3 and Eq. 11.8 as before:

$$E[R_P] = x_I E[R_I] + x_C E[R_C] = 1.50 \times 26\% + (-0.50) \times 6\% = 36\%$$

$$SD(R_P) = \sqrt{Var(R_P)} = \sqrt{x_1^2 Var(R_I) + x_C^2 Var(R_C) + 2x_I x_C Cov(R_I, R_C)}$$

$$= \sqrt{1.5^2 \times 0.50^2 + (-0.5)^2 \times 0.25^2 + 2(1.5)(-0.5)(0)} = 76.0\%$$

Note that in this case, short selling increases the expected return of your portfolio, but also its volatility, above those of the individual stocks.

Risk measures: VaR and ES

Value at Risk (VaR)

- Given the time horizon *h*, confidence level (probability) *a*,

what is the loss that will not be exceeded with probability a, when holding a portfolio for h days?

- Formal definition:

VaR(h,a) = inf { l: P(Loss > l) <= 1-a }

In other words, in 1-*a* % of cases, losses are *VaR* dollars (euros,…) **or more**

- Another formal definition: a quantile of the distribution of portfolio losses:

$$\mathrm{VaR}_\alpha = q_\alpha(F_L) = F_L^{\leftarrow}(\alpha) \, ,$$

i.e. the α-quantile of F_L.

Another risk measure: Expected Shortfall (ES)

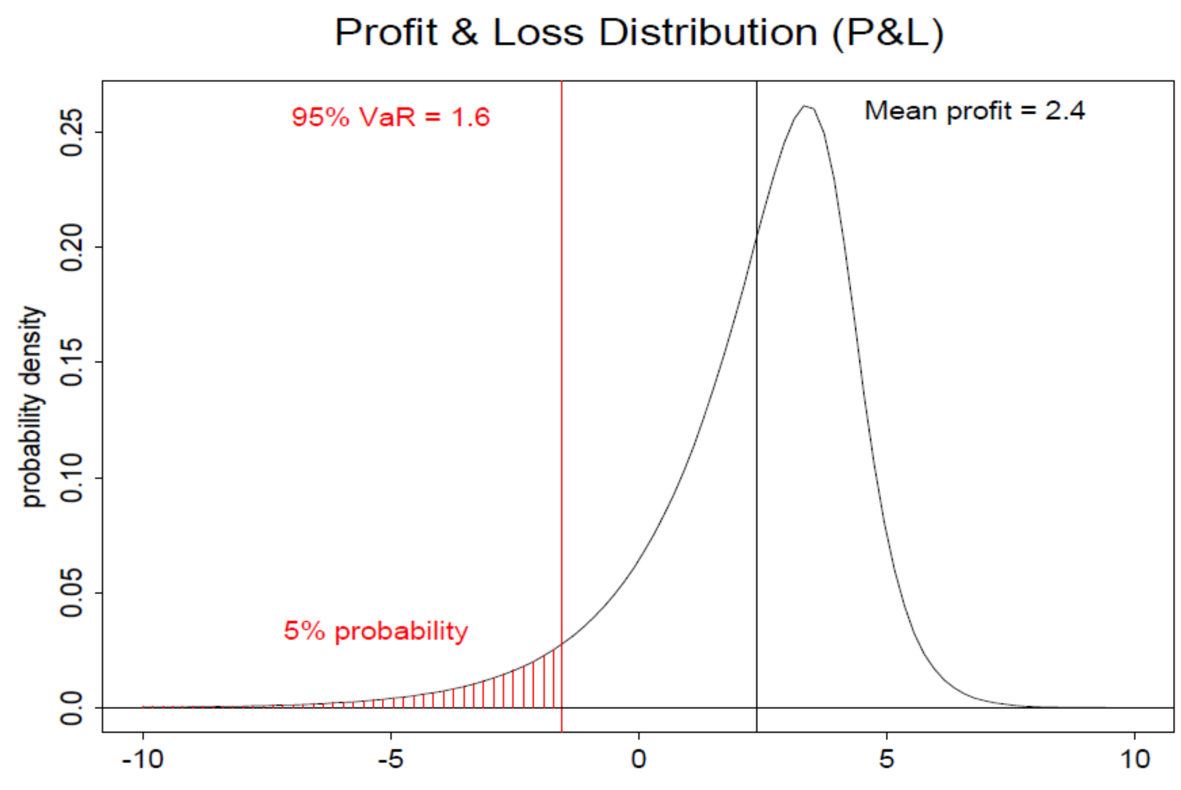
- Expected shortfall is the *average loss when VaR is exceeded:*

$$\mathsf{ES}_\alpha = E\left(L \mid L > \mathrm{VaR}_\alpha\right) \ ,$$

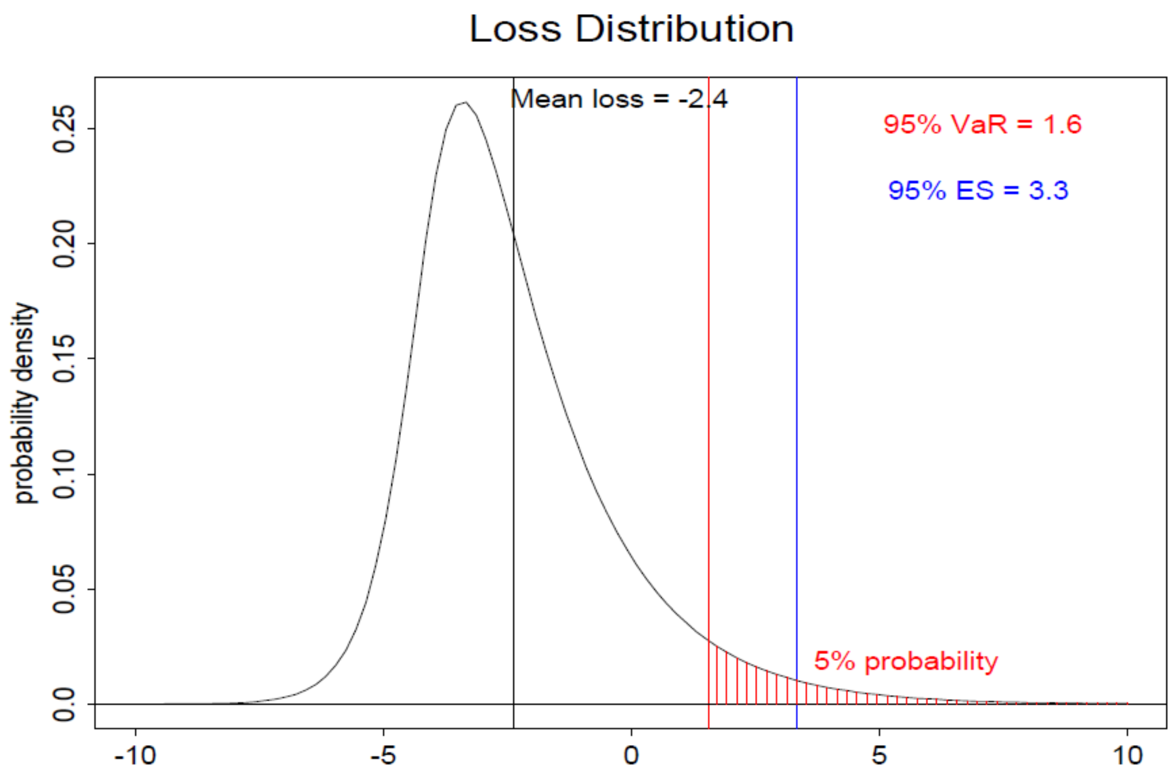
- So, given the loss exceeded VaR, how much money I will lose on average?

- ES gives information about frequency and size of large losses, while VaR gives information about frequency of large losses but says not much about potential size.

VaR in Visual Terms



Losses and Profits



Loss distribution vs P/L distribution

- In QRM course (and in market risk management in general), we always work with LOSS distribution and not P/L distribution

- So losses are positive, and gains are negative

- This is because we are focused on losses and at the same time, we like working with positive random variables

Advantages of VaR

- It captures an important aspect of risk

  in a single number
- It is easy to understand
- It asks the simple question: “How bad can things get?”

- Disadvantage: it gives you a **minimum** loss which occurs with a certain probability
- ES overcomes this disadvantage

VaR vs. ES

- VaR is the loss level that will not be exceeded with a specified probability
- Expected Shortfall is the expected loss given that the loss is greater than the VaR level
- ES is theoretically more appealing. It also contains more information about large losses than VaR.

Problem with VaR (more about this later)

- VaR is not a “coherent” risk measure, i.e. does not always reflect benefits of diversification:

  VaR(P1+P2) is not always < *VaR(P1)* + *VaR(P2)*

- However, ES does have this property

VaR, ES and Regulatory Capital

- Regulators used to base the capital they require banks to keep on VaR

- The market-risk capital was k times the 10-day 99% VaR where k is at least 3

- FRTB: in 2015, BIS changed its capital requirements calculations. It used to be based on VaR(10 days, 99%). Now it is based on ES(10 days, 97.5%). Otherwise the formula stays the same.

- Why change in confidence level?

Read the following chapters/sections of the book:

- Chapters 1 and 3
- Chapter 2: after the next lecture
- Refresh your knowledge about all discussed concepts: distributions, moments, quantiles of a distribution, portfolio variance etc.
- Do questions asked in slides: leveraged portfolio example, portfolio variance formula derivation.
- Do Exercises 1 t/m 12 in HWS1
- Start on Assignment 1: form groups, define portfolio, gather data