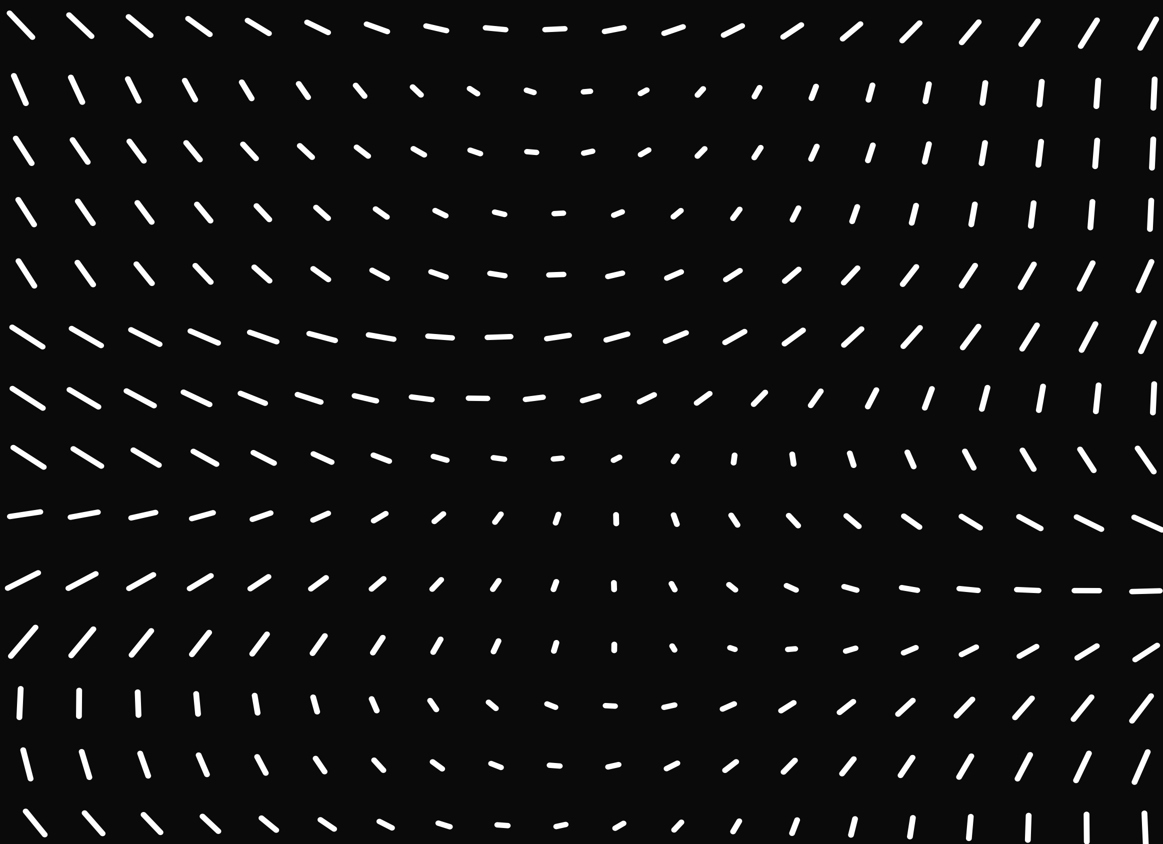

§2.1.

#1. 2.1 F

2.2.

(a) F

(b) T

(c) F

(d) T

(e) F

2.4. T

2.7. T.

#2 #109 2.1 (i) Indep.

2.1 (f) Indep

#3. pg 110 2.3 (d) $\left(\begin{array}{cccc|c} 1 & 0 & -2 & 8 & -5 \\ 0 & 1 & 1 & 1 & 4 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right) \Rightarrow$ Pivot Columns are $\begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix}$ and $\begin{pmatrix} 3 \\ 0 \\ -1 \end{pmatrix}$

2.8 (a) 3 pivots. $\Rightarrow$ apply Gauss Jordan Elimination.

(b) Restricting down to the 3x3 Matrix, we should obtain Identity Matrix.

§2.2

2.14 T This would force the matrix to be the zero matrix.

2.15 F If x_i's are scalar multiples of eachother. this is simply not true.

2.16 F If $\{x_1, x_2, x_3, x_4\}$ Contains the dependent vector, its span is 3-dimensional.

2.17 T

2.18 T

#2. 2.32

(c) Apply row reduction to find 3 pivot Columns.

Consider $\left\{\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}\right\}$ as another Basis.

(d) Same as above, take $\{e_1, e_2, e_3, e_4\}$ as a Basis.

#3

pg 124.

2.38 (a) $\left(\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$ Rank 1.

$$\begin{aligned} x_1 + 2x_2 + x_3 &= 0 \\ 0 + x_2 + 0 &= x_2 \\ 0 + 0 + x_3 &= x_3 \end{aligned} \rightarrow \begin{aligned} x_1 + 0 &= -2x_2 - x_3 \\ 0 + x_2 + 0 &= x_2 \\ 0 + 0 + x_3 &= x_3 \end{aligned}$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix} x_2 + \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} x_3.$$

Basis for null space.

(c) Basis for Null space, $\left\{ \begin{pmatrix} 5 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} -3 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -4 \\ 0 \\ 0 \\ 1 \end{pmatrix} \right\}$

Rank 1

pg 125.

2.42. $\dim \leq 2$, W, Z are dependent.

2.46. $\begin{pmatrix} 2 & 3 \\ 3 & -5 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ So yes they are indep.

§2.3

#143. 2.20

(a) T. $A: \mathbb{R}^5 \to \mathbb{R}^3$, with at $\dim(\ker A) \geq 3 \Rightarrow \dim(\mathrm{Rank}(A)) \leq 2$

(b) F.

(c) F

2.21 T

2.22. T

2.23 F

2.25 T The rank is at least 3.
But the # of pivots possible is 3, so $\exists$ exactly 3 pivots i.e. Rank = 3.

2.27 F Rank + Nullity = 4 $\Rightarrow$ A is $n \times 4$ matrix, nothing more can be said.

$A^t_{n \times m}$

rank $A^t = m$

$\Rightarrow$ Ran $A = m$

$A_{m \times n}$

$\Rightarrow$ So $Ax = b$ is always solvable.

pg. 143

2.64 B is, C is not.

2.65 (a) $\begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

(b) $(10001)^t, (01005)^t, (001-2)^t$

(c) $A_1 = 2R_1 + 2R_2 + 3R_3$

$A_2 = R_1 + R_2 + 3R_3$

$A_3 = 0R_1 + R_2 + 2R_3$

$A_4 = 3R_1 + 3R_2 + 8R_3$

d) Nope, Need 4 pivots.

(e) Verify Theorem

2.66.

(b) Column space = $\left\{ \begin{pmatrix} -1 \\ 4 \\ 3 \end{pmatrix}, \begin{pmatrix} 4 \\ 4 \\ 0 \end{pmatrix}, \begin{pmatrix} -2 \\ 2 \\ -3 \end{pmatrix} \right\}$

Row space = $\{ e_1^t, e_2^t, e_3^t \}$

(d) Column Space = $\left\{ \begin{pmatrix} 1 \\ 2 \\ 3 \\ -2 \end{pmatrix} \right\}$

Row Space = $\left\{ \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} \right\}$

2.72. (a) A is 4×5, $\dim(\operatorname{Rank} A) + \dim(\operatorname{Null} A) = 5$

$2 + \dim(\operatorname{Null} A) = 5$

$\dim(\operatorname{Null} A) = 3$

It is enough to prove that $\{X_1, X_2, X_3\}$ are lin. independent. This would prove it is a basis for Null A $\Leftrightarrow$ Span Solution space for $A\vec{x} = \vec{0}$

via Computation $\Rightarrow$ $(X_1 | X_2 | X_3) = \begin{pmatrix} 17 & 1 & 5 \\ 6 & 2 & 4 \\ -13 & 3 & 3 \\ 1 & 4 & 2 \\ 2 & 5 & 1 \end{pmatrix} \approx \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

$\Rightarrow$ Columns are lin dep.

(b) $\exists ?\ B_{4\times5}$, $\dim(\text{Rank } B) = 3$, $\dim(\text{Null } A) = 2$

- If there were such a B, its Null Space would have dim 3 which is not possible.