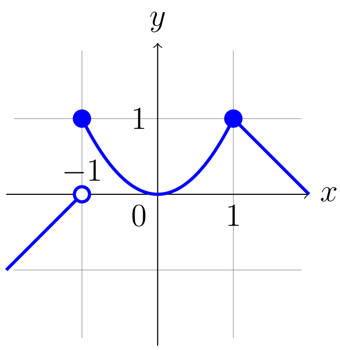
Homework 2.2

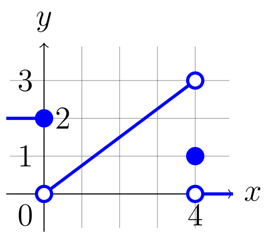
7. (a) $\lim_{t\to 0^-} g(t) = -1$. (b) $\lim_{t\to 0^+} g(t) = -2$. (c) $\lim_{t\to 0} g(t)$ does not exist.
(d) $\lim_{t\to 2^-} g(t) = 2$. (e) $\lim_{t\to 2^+} g(t) = 0$. (f) $\lim_{t\to 2} g(t)$ does not exist.
(g) $g(2) = 1$. (h) $\lim_{t\to 4} g(t) = 3$.

8. (a) $\lim_{x\to -3} A(x) = \infty$. (b) $\lim_{x\to 2^-} A(x) = -\infty$.
(c) $\lim_{x\to 2^+} A(x) = \infty$. (d) $\lim_{x\to -1} A(x) = -\infty$.
(e) V.A.: $x = -3$, $x = -1$, $x = 2$.

11. $a \neq -1$. [Hint: $\lim_{x\to -1^-} f(x) = 0 \neq 1 = \lim_{x\to -1^+} f(x)$.]



18.



40. $\lim_{x\to 2^-} \dfrac{x^2 - 2x}{x^2 - 4x + 4}[= \lim_{x\to 2^-} \dfrac{x\cancel{(x-2)}}{(x-2)^{\cancel{2}}} : \dfrac{(+)}{(-)}] = -\infty$.

42. $\lim_{x\to 0^+} (\dfrac{1}{x} - \ln x^2)[: (+) - (-)] = \infty$.

Homework 2.3

2. (a) $\lim_{x\to 2}[f(x)+g(x)][=-1+2]=1$.

(b) $\lim_{x\to 0}[f(x)-g(x)]$ does not exist. $[\lim_{x\to 0^-}(f-g)=5\neq 3=\lim_{x\to 0^+}(f-g).]$

(c) $\lim_{x\to -1}[f(x)g(x)][=1\cdot 2]=2$.

(d) $\lim_{x\to 3}\dfrac{f(x)}{g(x)}$ does not exist. $[\lim_{x\to 3} f/g=\infty,\ \lim_{x\to 3^+} f/g=-\infty.]$

(e) $\lim_{x\to 2} x^2 f(x)[=4\cdot(-1)]=-4$.

(f) $f(-1)+\lim_{x\to -1} g(x)[=3+2]=-1$.

16. $\lim_{x\to -1}\dfrac{2x^2+3x+1}{x^2-2x-3}=\dfrac{1}{4}$. [Hint: Reduce $x+1$.]

22. $\lim_{u\to 2}\dfrac{\sqrt{4u+1}-3}{u-2}=\dfrac{2}{3}$. [Hint: Multiply $\sqrt{4u+1}+3$ then reduce $u-2$.]

24. $\lim_{h\to 0}\dfrac{(3+h)^{-1}-3^{-1}}{h}=-\dfrac{1}{9}$. [Hint: Combine then reduce h.]

31. $\lim_{h\to 0}\dfrac{(x+h)^3-x^3}{h}=3x^2$. [Hint: Expand then reduce h.]

39. $\lim_{x\to 0} x^4\cos\dfrac{2}{x}=0$. [Hint: $-1\leq\cos\dfrac{2}{x}\leq 1$, $-x^4\leq x^4\cos\dfrac{2}{x}\leq x^4$.]

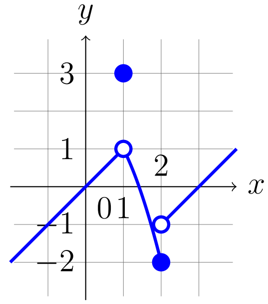
44. $\lim_{x\to -2}\dfrac{2-|x|}{2+x}=1$.

51. $c=7$. [Hint: Solve $\sqrt{2+c}=\lim_{t\to 2^+}B(t)=\lim_{t\to 2^-}B(t)=3$.]

52. (a) (i) $\lim_{x\to 1^-} g(t)=1$. (ii) $\lim_{x\to 1} g(t)=1$. (iii) $g(1)=3$.

(iv) $\lim_{x\to 2^-} g(t)=-2$. (v) $\lim_{x\to 2^+} g(t)=-1$. (vi) $\lim_{x\to 2} g(t)$ does not exist.

(b)



53. (a) (i) $\lim_{x\to -2^+} [[x]] = -2$. (ii) $\lim_{x\to -2} [[x]]$ does not exist.
[Hint: $\lim_{x\to -2^-} [[x]]] = -3$.] (iii) $\lim_{x\to -2.4} [[x]] = -3$.
(b) (i) $\lim_{x\to n^-} [[x]] = n-1$. (ii) $\lim_{x\to n^+} [[x]] = n$.
(c) $a \notin \mathbb{Z}$.

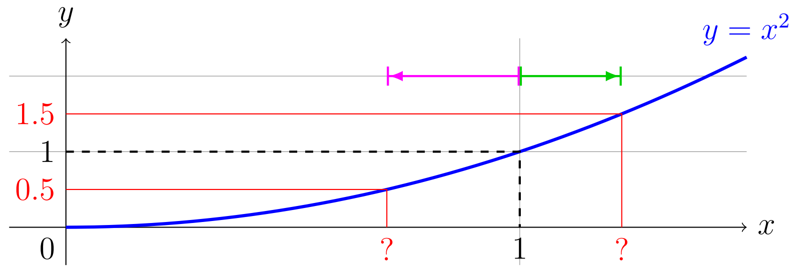
55. $\lim_{x\to 2} f(x) = \lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x) = -1 \neq 0 = f(2)$.

60. (a) $\lim_{x\to 0} f(x)[= \lim_{x\to 0} \frac{f(x)}{x^2} \lim_{x\to 0} x^2 = 5 \cdot 0] = 0$.
(b) $\lim_{x\to 0} \frac{f(x)}{x}[= \lim_{x\to 0} \frac{f(x)}{x^2} \lim_{x\to 0} x = 5 \cdot 0] = 0$.

65. $a = 15$, $\lim_{x\to -2} \frac{3x^2 + ax + a + 3}{x^2 + x - 2} = -1$.
[Hint: $\lim_{x\to -2} (x^2 + x - 2) = 0 \implies \lim_{x\to -2} (3x^2 + ax + a + 3) = 0$.]

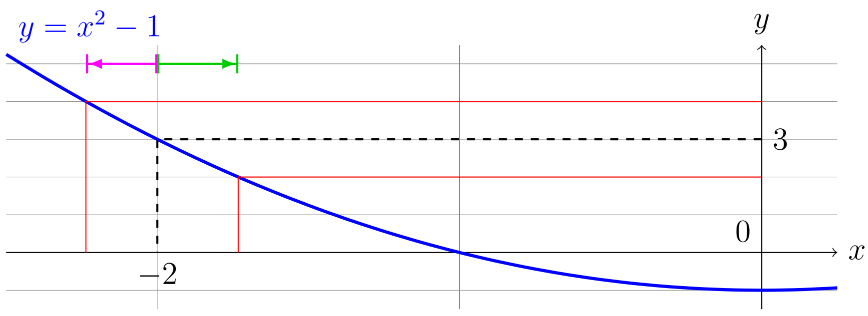
Homework 2.4

4. $\delta \le [\min\{\sqrt{1.5}-1, 1-\sqrt{0.5}\} =]\sqrt{1.5}-1 (\approx 0.2247)$.
[Hint: $|x^2-1| < \frac{1}{2}$, $-0.5 < x^2-1 < 0.5$, $0.5 < x^2 < 1.5$,
$\sqrt{0.5} < x < \sqrt{1.5}$, $-(1-\sqrt{0.5}) < x-1 < \sqrt{1.5}-1$.]



22. $\dfrac{x^2+x-6}{x-2} = \dfrac{(x+3)(x-2)}{x-2} = x+3$ when $x \neq 2$.
$\left|\dfrac{x^2+x-6}{x-2} - 5\right| = |x+3-5| = |x-2| < \varepsilon$.
$\forall\, \varepsilon > 0$, choose $\delta = \varepsilon$.
If $0 < |x-2| < \delta$, then $|f(x)-L| = \left|\dfrac{x^2+x-6}{x-2} - 5\right| = |x-2| < \delta = \varepsilon$.

31. If $|x+2| < 1$ then $|x-2| < 5$, $|(x^2-1)-3| = |x+2||x-2| < 5|x+2| < \varepsilon$.
$\forall\, \varepsilon > 0$, choose $\delta = \min\left\{1, \dfrac{\varepsilon}{5}\right\}$.
If $0 < |x-(-2)| < \delta$, then $|x+2| < 1 \implies |x-2| < 5$ and $|x+2| < \dfrac{\varepsilon}{5}$,
and hence $|f(x)-L| = |(x^2-1)-3| = |x+2||x-2| < \dfrac{\varepsilon}{5} \cdot 5 = \varepsilon$.
♦ : $\max \delta = \sqrt{4+\varepsilon} - 2$. ($\because 2-\sqrt{4-\varepsilon} > \sqrt{4+\varepsilon}-2$ for $\varepsilon < 4$.)



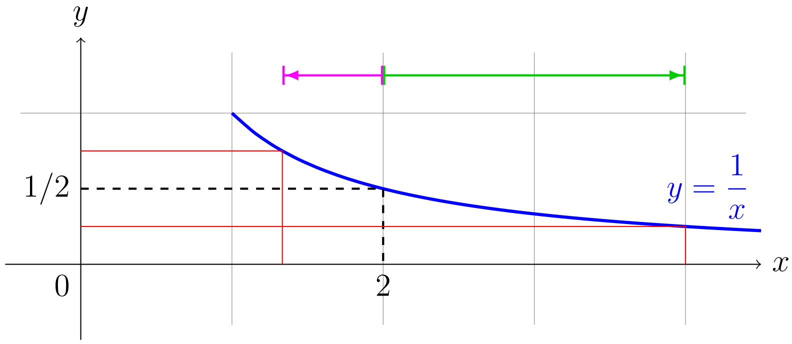
36. $\left|\frac{1}{x}-\frac{1}{2}\right| = \frac{|x-2|}{2|x|}$, if $|x-2|<1$, then $|x|>1$, $\frac{|x-2|}{2|x|} < \frac{|x-2|}{2\cdot 1} < \varepsilon$.

$\forall\ \varepsilon > 0$, choose $\delta = \min\{1, 2\varepsilon\}$.

If $0 < |x-2| < \delta$, then $|x-2| < 1 \implies |x| > 1$, and $|x-2| < 2\varepsilon$,

and hence $|f(x)-L| = \left|\frac{1}{x}-\frac{1}{2}\right| = \frac{|x-2|}{2|x|} < \frac{2\varepsilon}{2\cdot 1} = \varepsilon$.

♦ : $\max\delta = \frac{4\varepsilon}{2\varepsilon+1}$.$(\because \frac{4\varepsilon}{1-2\varepsilon} > \frac{4\varepsilon}{2\varepsilon+1}$ for $\varepsilon < \frac{1}{2}$.)



42. $\frac{1}{(x+3)^4} > M$, $|x+3| < \frac{1}{\sqrt[4]{M}}$.

$\forall\ M > 0$, choose $\delta = \min\left\{\frac{1}{\sqrt[4]{M}}\right\}$.

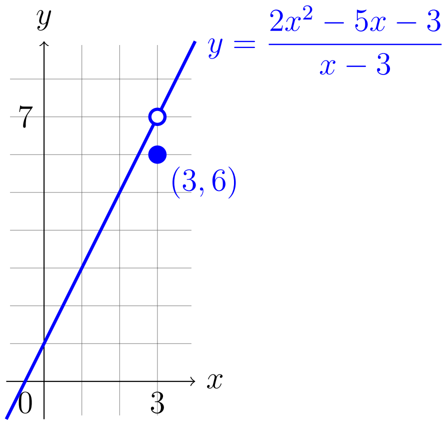
If $0 < |x-(-3)| < \delta$, then $f(x) = \frac{1}{(x+3)^4} > \frac{1}{\delta^4} = \frac{1}{(1/\sqrt[4]{M})^4} = M$.

Homework 2.5

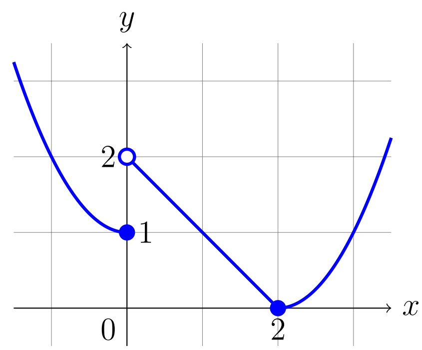
4. $[-3, 2], (-2, -1), (-1, 0], (0, 1), (1, 3]$.

11. $\lim_{x \to 4} f(x) = 16 + \sqrt{3} = f(4)$.

22. $\lim_{x \to 3} f(x) = 7 \neq 6 = f(3)$.



43. discontinuous: $x = 0$;
continuous from left: $x = 0$.



45. $\frac{2}{3}$. [Hint: Solve $4c + 4 = \lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x) = 8 - 2c$.]

48. (a) $f \circ g = \dfrac{1}{1/x^2} = x^2$, $x \neq 0$. (b) $g(0)$ is undefined.

52. If $f(3) < 6$, then by the Intermediate Value Theorem, there exists $c \in (2, 3) \ni f(c) = 6$, a solution to $f(x) = 6$ other than 1 and 4.

53. $f(1) = -1 < 0 < 15 = f(2)$.

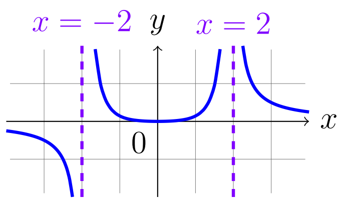
67. Nowhere, no value. [Hint: f has no limit everywhere.]

71. f is continuous at $a \neq 0$. $\lim_{x \to 0} f(x) = 0 = f(0)$ by the Squeeze Theorem.
[Hint: $-x^4 \leq x^4 \sin(1/x) \leq x^4$.]

Homework 2.6

3. (a) $\lim_{x\to\infty} f(x) = -2$. (b) $\lim_{x\to-\infty} f(x) = 2$.
(c) $\lim_{x\to 1} f(x) = \infty$. (d) $\lim_{x\to 3} f(x) = -\infty$.
(e) V.A.: $x = 1$, $x = 3$; H.A.: $y = -2$, $y = 2$.

6.



20. $\lim_{t\to\infty} \dfrac{t - t\sqrt{t}}{2t^{3/2} + 3t - 5} = -\dfrac{1}{2}$. [Hint: $\div t^{3/2}$.]

28. $\lim_{x\to-\infty} (\sqrt{4x^2 + 3x} + 2x) = -\dfrac{3}{4}$.
[Hint: Multiply $\sqrt{4x^2+3x} - 2x$ then $\div x (< 0)$, $\dfrac{\sqrt{4x^2+3x}}{x} = -\sqrt{4 + \dfrac{3}{x}}$.]

32. $\lim_{x\to\infty} (e^{-x} + 2\cos 3x)$ does not exist.

34. $\lim_{x\to-\infty} \dfrac{1 + x^6}{x^4 + 1} = \infty$. [Hint: $\div x^4$.]

42. $\lim_{x\to\infty} [\ln(2+x) - \ln(1+x)] = 0$. [Hint: Combine and continuity.]

50. V.A.: $x = 0$, $x = 1$, $x = -1$; H.A.: $y = -1$. [Hint: $y = \dfrac{1 + x^4}{x^2(1-x)(1+x)}$.]

55. (a) 0. (b) $\pm\infty$.

67. $\lim_{x\to\infty} f(x) = 5$.
[Hint: $\lim_{x\to\infty} \dfrac{10e^x - 21}{2e^x} = \lim_{x\to\infty} \dfrac{5\sqrt{x}}{\sqrt{x-1}} = 5$ and the Squeeze Theorem.]

76. (a) $x > 10^8$.
(b) $\forall\ \varepsilon > 0$, choose $N = \dfrac{1}{\varepsilon^2}$.
If $x > N$, then $|f(x) - 0| = \dfrac{1}{\sqrt{x}} < \dfrac{1}{\delta} = \dfrac{1}{\sqrt{1/\varepsilon^2}} = \varepsilon$.

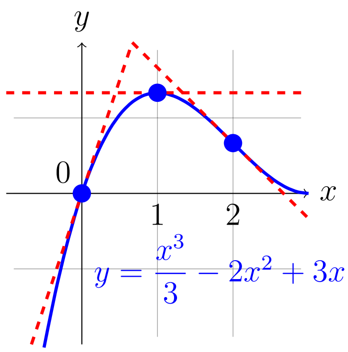
Homework 2.7

6. $y = 9(x-2) + 3 = 9x - 15.$

13. -9.6 m/s.

22. $f(4) = 3,\ f'(4) = \dfrac{1}{4}.$

23.



28. $g'(1) = 4,\ y = 4(x-1) - 1 = 4x - 5.$

35. $f'(a) = \dfrac{-1}{\sqrt{1-2a}}$. [Hint: By definition $\lim\limits_{x\to a} \dfrac{\sqrt{1-2x} - \sqrt{1-2a}}{x-a}$.]

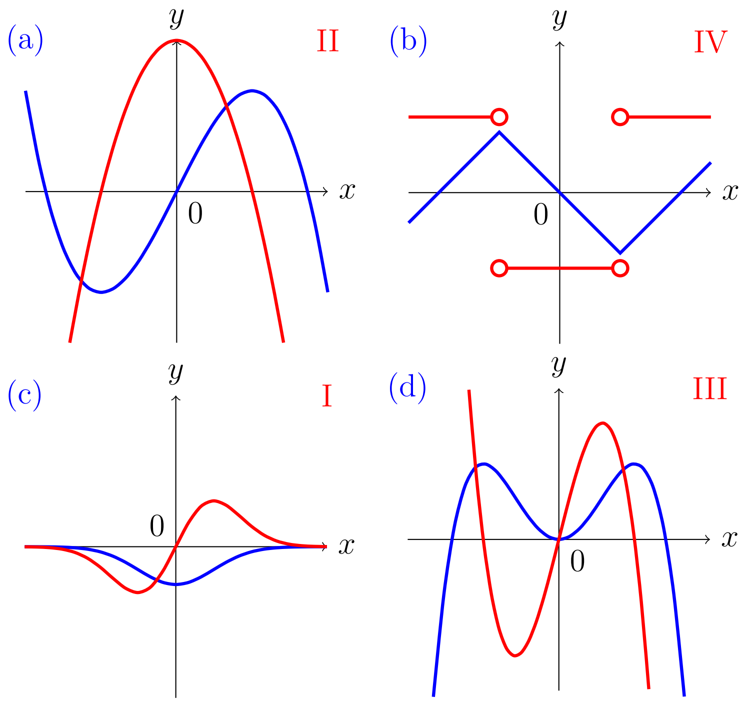
53. (a) The rate of change of the production cost with respect to the number of kilograms of produced gold.
(b) To produce the 50th kilogram of gold costs about 36 dollars.
(c) Decreases in short term and increases in long term. Producing the next gold in the same gold mine is cheaper than the previous one, but finding the new gold mine costs a lot.

60. $f'(0) = 0.$

[Hint: By definition $\lim\limits_{x\to 0} \dfrac{x^2 \sin\frac{1}{x} - 0}{x - 0} = \lim\limits_{x\to 0} x \sin\dfrac{1}{x} = 0$ by the Squeeze Theorem.]

Homework 2.8

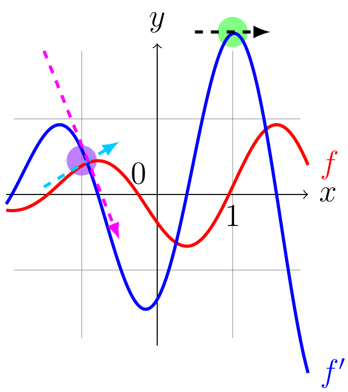
3. (a)'=II, (b)'=IV, (c)'=I, (d)'=III.



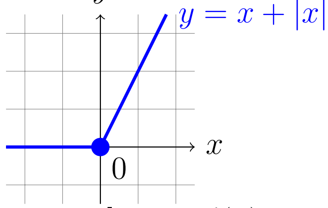
28. $f'(x) = \dfrac{2x^2 - 6x + 2}{(2x-3)^2}$. Both $(-\infty, \frac{3}{2}) \cup (\frac{3}{2}, \infty)$.

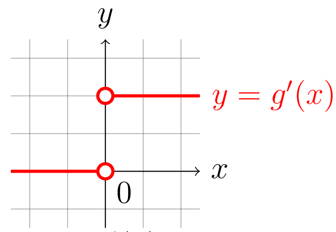
42. $x = -1$ (discontinuous), $x = 2$ (corner).

48. $f'(-1) > f''(1)$. [Hint: Slopes of blue curve at -1 is negative, and of red is positive, so f is red and f' is blue. Slope of f' at 1 is near 0.]



62. (a)



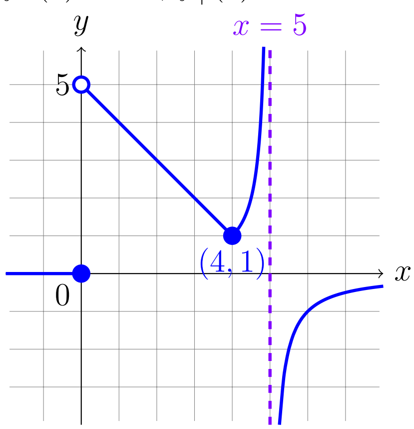


(b) $x \neq 0$. [Hint: $g'(x) = 2$ or $x > 0$ and $g'(x) = 0$ for $x < 0$.]

(c) $g'(x) = 1 + \dfrac{|x|}{x}$.

64. (a) $f'_-(4) = -1$, $f'_+(4) = 1$.

(b)



(c) $x = 0$ (discontinuous), $x = 5$ (infinite discontinuous).

(d) $x = 0$, $x = 4$ (corner), $x = 5$.