Homework 3.1

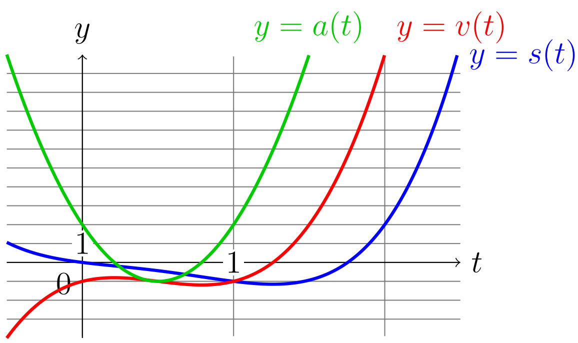
7. $f'(t) = 6t^2 - 6t - 4$.

29. $f'(v) = -\frac{2}{3}v^{-5/3} - 2e^v$.

35. tangent line: $y = \frac{1}{2}(x-2) + 3 = \frac{1}{2}x + 2$.

38. tangent line: $y = \frac{3}{2}(x-1) + 1 = \frac{3}{2}x - \frac{1}{2}$,
normal line: $y = -\frac{2}{3}(x-1) + 1 = -\frac{2}{3}x + \frac{5}{3}$.

50. (a) $v(t) = 4t^3 - 6t^2 + 2t - 1$, $a(t) = 12t^2 - 12t + 2$.
(b) $a(1) = 2$ m/s^2.
(c)



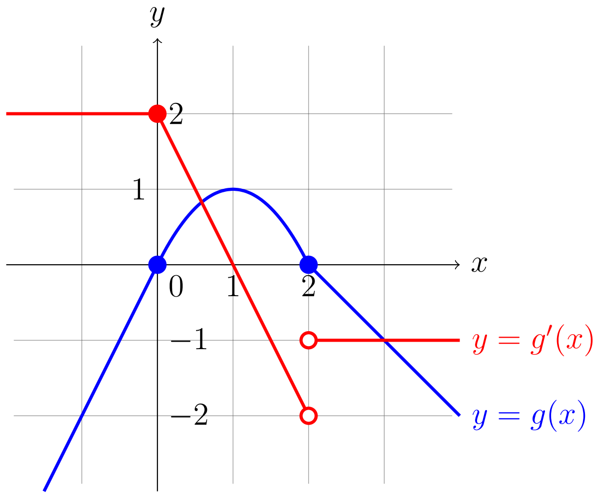
56. $f'(x) = e^x - 2 = 0$ when $x = \ln 2$. (tangent line: $y = 2 - 2\ln 2$.)

61. $y' = \frac{1}{2\sqrt{x}} = \frac{1}{2}$ when $x = 1$, normal line: $y = -2(x-1)+1$, $2x + y = 3$.

68. $A = B = -\frac{1}{2}$, $C = -\frac{3}{4}$.
[Hint: $y' = 2Ax + B$, $y'' = 2A$, $(2A) + (2Ax + B) - 2(Ax^2 + Bx + C) = (-2A)x^2 + (2A - 2B)x + (2A + B - 2C) = x^2$, $-2A = 1$, $2A - 2B = 0$, $2A + B - 2C = 0$.]

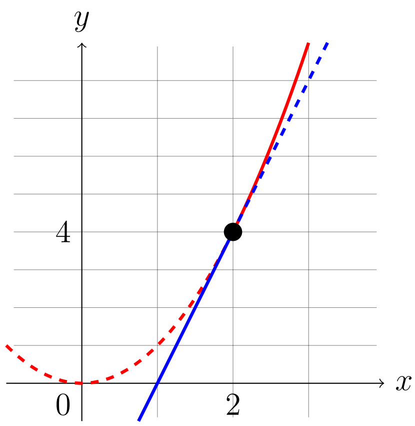
72. $x \neq 2$, $g'(x) = \begin{cases} 2 & \text{if } x \leq 0 \\ 2-2x & \text{if } 0 < x < 2 \\ -1 & \text{if } x \boxed{>} 2 \end{cases}$.

[Hint: $\lim\limits_{x\to 0^-} g'(x) = 2 = \lim\limits_{x\to 0^+} g'(x)$, $\lim\limits_{x\to 2^-} g'(x) = -2 \neq -1 = \lim\limits_{x\to 2^+} g'(x)$.]



81. $m = 4$, $b = -4$.

[Hint: Solve $\lim\limits_{x\to 2^-} f(x) = \lim\limits_{x\to 2^+} f(x)$ & $\lim\limits_{x\to 2^-} f'(x) = \lim\limits_{x\to 2^+} f'(x)$.]



83. $\lim\limits_{x\to 1} \dfrac{x^{1000}-1}{x-1} = 1000$. [Hint: $f(x) = x^{1000}$, find $f'(1)$.]

Homework 3.2

2. $f'(x) = \left\langle \begin{array}{ll} \dfrac{(x^4 - 5x^3 + \sqrt{x})'(x^2) - (x^4 - 5x^3 + \sqrt{x})(x^2)'}{(x^2)^2} & \text{(Quotient Rule)} \\ (x^2 - 5x + x^{-3/2})' & \text{(Power Rule)} \end{array} \right\rangle$

$= 2x - 5 - \dfrac{3}{2}x^{-5/2}.$

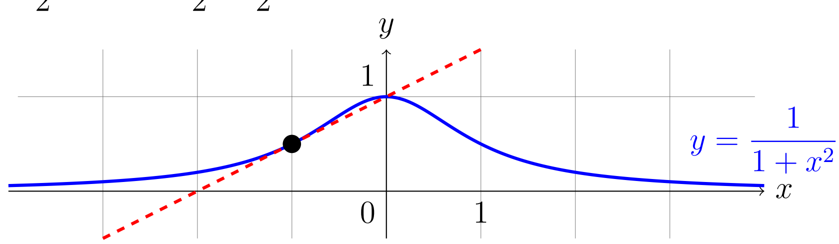
10. $J'(v) = 1 + v^{-2} + 6v^{-4}.$

19. $y' = -s^{-2} + \dfrac{3}{2}s^{-5/2}.$

28. $f'(x) = \dfrac{e^x}{2\sqrt{x}} + \sqrt{x}e^x$, $f''(x) = -\dfrac{e^x}{4x\sqrt{x}} + \dfrac{e^x}{\sqrt{x}} + \sqrt{x}e^x.$

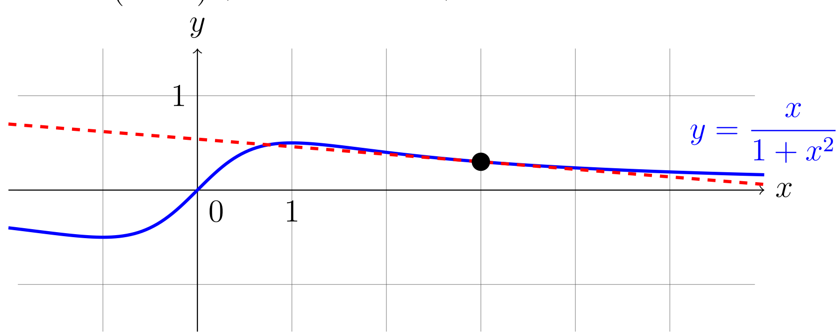
35. (a) $y = \dfrac{1}{2}(x+1) + \dfrac{1}{2} = \dfrac{1}{2}x + 1.$

♦ (b)



36. (a) $y = -0.08(x-3) + 0.3 = -0.08x + 0.54.$

♦ (b)



44. (a) $h'(4) [= 3f'(4) + 8g'(4)] = -6.$

(b) $h'(4) [= f'(4)g(4) + f(4)g'(4)] = 24.$

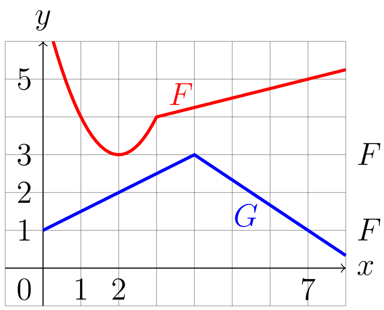
(c) $h'(4) [= \dfrac{f'(4)g(4) - f(4)g'(4)}{[g(4)]^2}] = \dfrac{36}{25}.$

(d) $h'(4) [= \dfrac{g'(4)[f(4) + g(4)] - g(4)[f'(4) + g'(4)]}{[f(4) + g(4)]^2}] = -\dfrac{36}{49}.$

45. $f'(0)[= e^0 g(0) + e^0 g'(0)] = 7.$

50. (a) $P'(2)[= F'(2)G(2) + F(2)G'(2) = 0 \cdot 2 + 3 \cdot \frac{1}{2}] = \frac{3}{2}.$

(b) $Q'(7)[= \dfrac{F'(7)G(7) - F(7)G'(7)}{[G(7)]^2} = \dfrac{\frac{1}{4} \cdot 1 - 5 \cdot \frac{-2}{3}}{1^2}] = \dfrac{43}{12}.$



$F(2) = 3,\ F'(2) = 0,\ G(2) = 2,\ G'(1) = \frac{1}{2}.$

$F(7) = 5,\ f'(7) = -\frac{1}{3},\ G(7) = 1,\ G'(7) = -\frac{1}{4}.$

56. $Q'(0) = 4$. [Hint: $Q = \frac{F}{G}$, $Q' = \frac{F'G - FG'}{G^2}$.]

Homework 3.3

1. $f'(x) = 2x\sin x + x^2\cos x$.

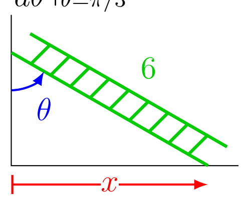
9. $y' = \dfrac{2 - \tan x + x\sec^2 x}{(2-\tan x)^2}$.

16. $f'(t) = e^t\cot t + te^t\cot t - te^t\csc t$.

22. tangent line: $y = x + 1$.

35. (a) $v(t) = 8\cos t$, $a(t) = -8\sin t$.
(b) $x(\frac{2\pi}{3}) = 4\sqrt{3}$, $v(\frac{2\pi}{3}) = -4$, $a(\frac{2\pi}{3}) = -4\sqrt{3}$, left $(v(\frac{2\pi}{3}) < 0)$.

37. $\left.\dfrac{dx}{d\theta}\right|_{\theta=\pi/3} = 3$ m/rad. [Hint: $x(\theta) = 6\sin\theta$.]



39. $\displaystyle\lim_{x\to 0}\frac{\sin 5x}{3x} = \frac{5}{3}$.

47. $\displaystyle\lim_{\theta\to 0}\frac{\cos\theta - 1}{2\theta^2} = -\frac{1}{4}$. [Hint: $\cos\theta = 1 - 2\sin^2\dfrac{\theta}{2}$ or $\times\dfrac{\cos\theta+1}{\cos\theta+1}$.]

51. $\dfrac{d^{99}}{dx^{99}}(\sin x) = -\cos x$.

57. $\displaystyle\lim_{\theta\to 0^+}\frac{d}{s} = 1$. [Hint: $s = r\theta$, $d = 2r\sin\dfrac{\theta}{2}$.]

Homework 3.4

5. $\dfrac{dy}{dx} = \dfrac{e^{\sqrt{x}}}{2\sqrt{x}}$.

9. $f'(x) = \dfrac{5}{2\sqrt{5x+1}}$.

16. $g'(x) = (2x-1)e^{x^2-x}$.

19. $h'(t) = \dfrac{2}{3}(t+1)^{-1/3}(2t^2-1)^3 + (t+1)^{2/3}12t(2t^2-1)^2$.

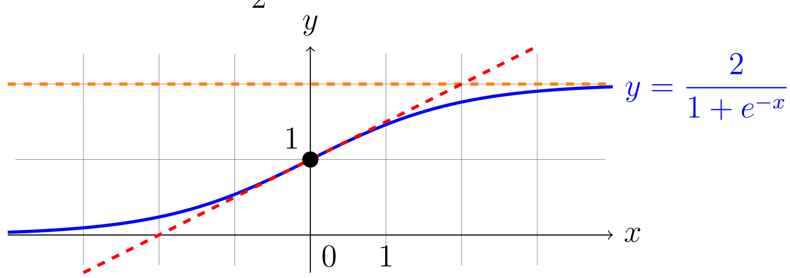
22. $y' = 5\left(x + \dfrac{1}{x}\right)^4\left(1 - \dfrac{1}{x^2}\right)$.

31. $F'(t) = e^{t\sin 2t}(\sin 2t + 2t\cos 2t)$.

41. $f'(t) = e^t \sec^2(e^t) + e^{\tan t}\sec^2 t$.

45. $y' = \dfrac{-\pi\cos(\tan\pi x)\sec^2(\pi x)\sin\sqrt{\sin(\tan\pi x)}}{2\sqrt{\sin(\tan\pi x)}}$.

55. (a) tangent line: $y = \dfrac{1}{2}x + 1$.
(b)



59. $\left((2n+\frac{1}{2})\pi, 3\right)$, $\left((2n+\frac{3}{2})\pi, -1\right)$, $n \in \mathbb{Z}$.
[Hint: Solve $f'(x) = 2\cos x + 2\sin x\cos x = 2\cos x\,(1+\sin x) = 0$.]

63. (a) $h'(1) = f'(g(1))g'(1) = 5 \cdot 6 = 30$.
(b) $H'(1) = g'(f(1))f'(1) = 9 \cdot 4 = 36$.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	3	2	4	6
2	1	8	5	7
3	7	2	7	9

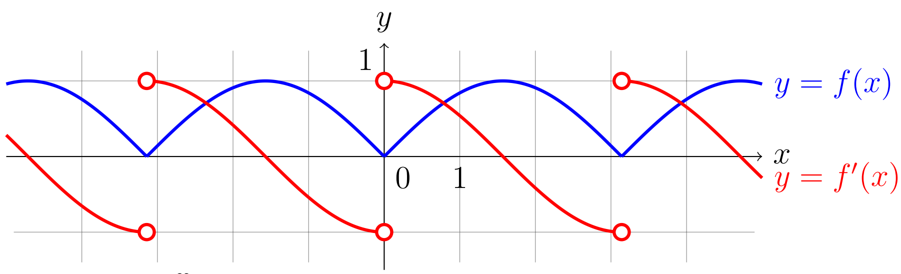
69. (a) $F'(x) = f'(e^x)e^x$. (b) $G'(x) = e^{f(x)}f'(x)$.

73. $F'(0) = f'(3f(4f(0)))3f'(4f(0))4f'(0) = 96$.

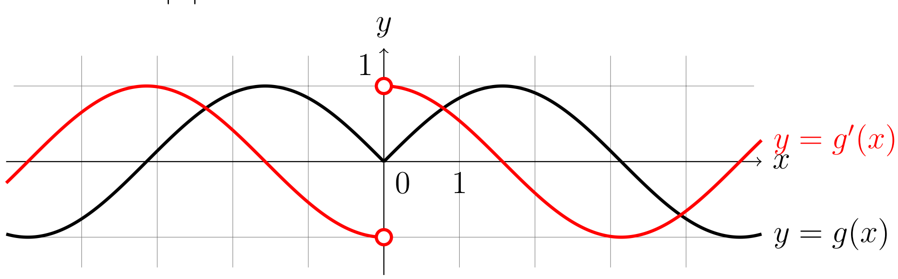
77. $y^{(50)} = -2^{50}\cos 2x$.

98. (a) $\dfrac{d}{dx}|x| = \dfrac{d}{dx}\sqrt{x^2} = \dfrac{2x}{2\sqrt{x^2}} = \dfrac{x}{|x|}$, $x \neq 0$.

(b) $f'(x) = \dfrac{\sin x \cos x}{|\sin x|}$, $f(x)$ is not differentiable when $x = n\pi$, $n \in \mathbb{Z}$.



(c) $g'(x) = \dfrac{x}{|x|}\cos|x|$, $g(x)$ is not differentiable when $x = 0$ (at 0).



Homework 3.5

3. (a) $y' = -\dfrac{\sqrt{y}}{\sqrt{x}}$.

(b) $y' = 1 - \dfrac{1}{\sqrt{x}}$. [Hint: $y = (1-\sqrt{x})^2$.]

(c) $-\dfrac{\sqrt{y}}{\sqrt{x}} = -\dfrac{1-\sqrt{x}}{\sqrt{x}} = 1 - \dfrac{1}{\sqrt{x}}$.

11. $\dfrac{dy}{dx} = -\dfrac{2xy^2 + \sin y}{2x^2y + x\cos y}$.

15. $\dfrac{dy}{dx} = \dfrac{y^2 - ye^{x/y}}{y^2 - xe^{x/y}}$.

22. $g'(0) = 0$.
[Hint: $g(0) + 0 \sin g(0) = 0^2$, $g(0) = 0$,
$g'(x) + \sin g(x) + xg'(x)\cos g(x) = 2x$,
$g'(0) + \sin 0 + (0)g'(0)\cos 0 = 2(0)$.]

30. tangent line: $y = \dfrac{1}{\sqrt{3}}(x + 3\sqrt{3}) + 1 = \dfrac{1}{\sqrt{3}}x + 4$.
[Hint: $\dfrac{2}{3}\dfrac{1}{\sqrt[3]{x}} + \dfrac{2}{3}\dfrac{y'}{\sqrt[3]{y}} = 0$, $\dfrac{2}{3}\dfrac{1}{\sqrt[3]{-3\sqrt{3}}} + \dfrac{2}{3}\dfrac{y'}{\sqrt[3]{1}} = 0$, $y' = \dfrac{1}{\sqrt{3}}$.]

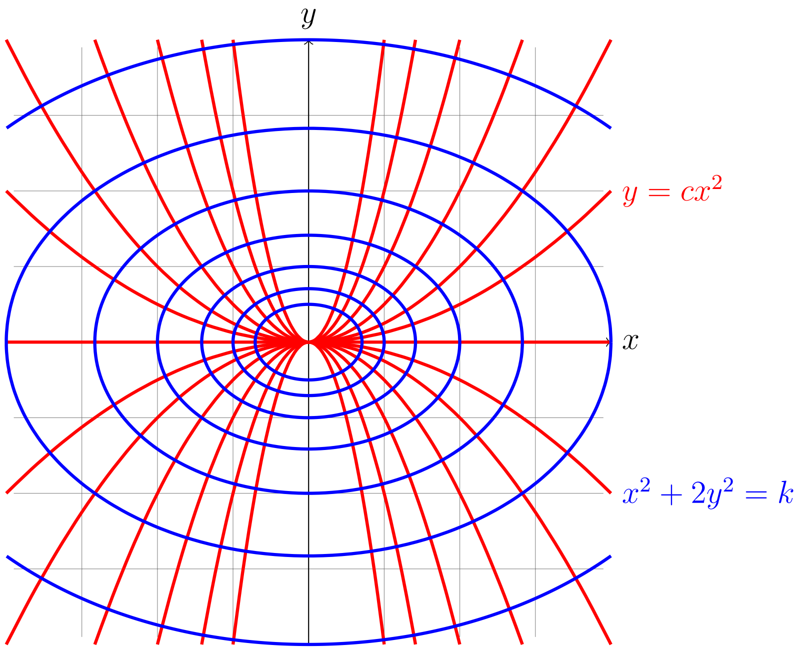
39. $y''(0) = \dfrac{1}{e^2}$.
[Hint: $xy + e^y = e$, $0y + e^y = e \implies y = 1$,
$y + xy' + e^y y' = 0$, $1 + 0y' + e^1 y' = 0 \implies y'(0) = -e^{-1} = \dfrac{-1}{e}$,
$2y' + xy'' + e^y y'^2 + e^y y'' = 0$, $2(-e^{-1}) + 0y'' + e^1(-e^{-1})^2 + e^1 y'' = 0$.]

44. $\dfrac{x_0}{a^2}x + \dfrac{y_0}{b^2}y = 1$. [Hint: $\dfrac{2x}{a^2} + \dfrac{2yy'}{b^2} = 0$, $\dfrac{2x_0}{a^2} + \dfrac{2y_0y'}{b^2} = 0$.]

49. $y' = \dfrac{1}{2(1+x)\sqrt{x}}$.

57. $y' = \sin^{-1} x$.

67.



$y = cx^2 \implies y' = 2cx$, $x^2 + 2y^2 = k \implies y' = -\dfrac{x}{2y} = -\dfrac{x}{2cx^2} = \dfrac{-1}{2cx}$.
When $c \neq 0$, curves are orthogonal; when $c = 0$, horizontal line $y = cx^2 = 0$ intersects $x^2 + 2y^2 = k$ orthogonally at $(\pm\sqrt{k}, 0)$ since ellipse has vertical tangent at them.

77. (a) $f(f^{-1}(x)) = x$, $f'(f^{-1}(x))(f^{-1})'(x) = 1$, $(f^{-1})'(x) = \dfrac{1}{f'(f^{-1}(x))}$.

(b) $(f^{-1})'(5) = \dfrac{1}{f'(f^{-1}(5))} = \dfrac{1}{f'(4)} = \dfrac{1}{2/3} = \dfrac{3}{2}$.

Homework 3.6

5. $f'(x) = \dfrac{1}{5}(\ln x)^{-4/5}\dfrac{1}{x}.$

11. $F'(t) = 2\dfrac{\ln t}{t}\sin t + (\ln t)^2 \cos t.$

15. $F'(s) = \dfrac{1}{s \ln s}.$

22. $y' = \dfrac{1 + \ln x}{x \ln x \ln 2}.$

25. $y' = \tan x,\ y'' = \sec^2 x,$

30. $f'(x) = \dfrac{1}{x \ln x \ln \ln x},\ \{x \colon x > e\} = (e, \infty).$

34. tangent line: $y = x - 1.$

45. $y' = x^{\sin x}\left(\dfrac{\sin x}{x} + \ln x \cos x\right).$

49. $y' = (\tan x)^{1/x}\left(\dfrac{\sec^2 x}{x \tan x} - \dfrac{\ln \tan x}{x^2}\right).$

52. $y' = \dfrac{\ln y - y/x}{\ln x - x/y} = \dfrac{\ln y^{xy} - y^2}{\ln x^{xy} - x^2}.$

55. $\lim\limits_{x \to 0} \dfrac{\ln(1+x)}{x} = \lim\limits_{x \to 0} \dfrac{\ln(1+x) - 0}{x - 0} = \dfrac{d}{dx}\ln(1+x)\Big|_{x=0} = \dfrac{1}{1+0} = 1.$

56. $\because x > 0,\ n \to \infty \iff x/n \to 0^+,\ \lim\limits_{n\to\infty}\left(1 + \dfrac{x}{n}\right)^n = \lim\limits_{n\to\infty} e^{\ln(1+x/n)^n}$

$= e^{\lim_{n\to\infty} n \ln(1+x/n)} = (e^x)^{\lim_{x/n\to 0^+} \frac{\ln(1+x/n)}{x/n}} \overset{\mathbf{55.}}{=} e^{x \cdot 1} = e^x.$

[Another sol] $\because x > 0,\ n \to \infty \iff n/x \to \infty,\ \lim\limits_{n\to\infty}\left(1 + \dfrac{x}{n}\right)^n$

$= \lim\limits_{n/x\to\infty}\left[\left(1 + \dfrac{1}{n/x}\right)^{n/x}\right]^x = \left[\lim\limits_{n/x\to\infty}\left(1 + \dfrac{1}{n/x}\right)^{n/x}\right]^x = e^x.$

Homework 3.9

1. $\dfrac{dV}{dt} = 3x^2\dfrac{dx}{dt}$. [Hint: $V = x^3$.]

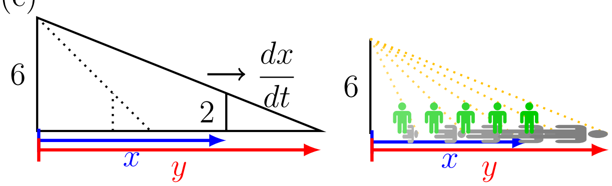
7. $\dfrac{dS}{dt} = 128\pi$ cm^2/min. [Hint: $S = 4\pi r^2$.]

11. $\dfrac{dz}{dt} = -18$.

15. (a) A 2 m man walk away a light on 6 m pole, t for time (in seconds), x for the distance from the pole to the man, $\dfrac{dx}{dt} = 1.5$ m/s.

(b) distance y m from the pole to the tip of man's shadow, when $x = 10$ m, $\dfrac{dy}{dt} =$?

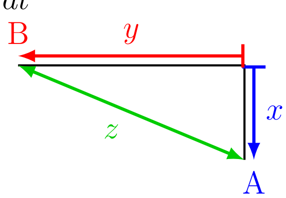
(c)



(d) $\dfrac{y-x}{y} = \dfrac{2}{6}$, $y = \dfrac{3}{2}x$.

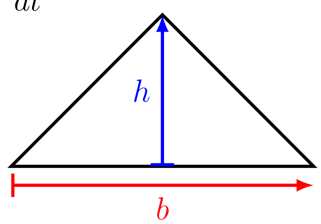
(e) $\dfrac{dy}{dt} = \dfrac{d}{dt}\dfrac{3}{2}x = \dfrac{3}{2}\dfrac{dx}{dt} = \dfrac{9}{4}$ m/s.

17. $\dfrac{dz}{dt} = 78$ km/h.



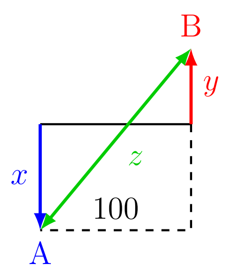
[Hint: $z^2 = x^2 + y^2$, $\dfrac{dx}{dt} = 30$, $\dfrac{dy}{dt} = 72$, $x = 2\dfrac{dx}{dt} = 60$, $y = 2\dfrac{dy}{dt} = 144$, $\dfrac{dz}{dt} =$?]

21. $\dfrac{db}{dt} = -1.6$ cm/min.



[Hint: $A = \dfrac{1}{2}bh$, $\dfrac{dh}{dt} = 1$, $\dfrac{dA}{dt} = 2$, $h = 10$, $A = 100$, $\dfrac{db}{dt} = ?$]

23. $\dfrac{720}{13} \approx 55.3846$ km/h.



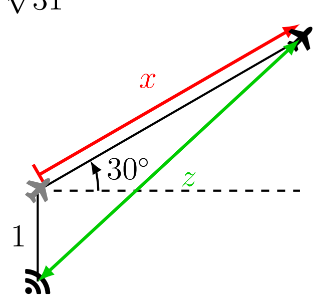
[Hint: A move south x km in $\dfrac{dx}{dt} = 35$ km/h, B move north y km in $\dfrac{dy}{dt} = 25$ km/h, of distance z km, $(x+y)^2 + 100^2 = z^2$, $x = 4\dfrac{dx}{dt}$, $y = 4\dfrac{dy}{dt}$, $\dfrac{dz}{dt} = ?$]

29. $\dfrac{dh}{dt} = \dfrac{4}{3\pi}$ m/min. [Hint: $V = \dfrac{1}{3}\pi r^2 h = \dfrac{1}{12}\pi h^3$, $\dfrac{dV}{dt} = 3$, $h = 3$, $\dfrac{dh}{dt} = ?$]

31. $\dfrac{dA}{dt} = 150\sqrt{3}$ cm^2/min. [Hint: $A = \dfrac{\sqrt{3}}{4}a^2$, $\dfrac{da}{dt} = 10$, $a = 30$, $\dfrac{dA}{dt} = ?$]

39. $\dfrac{dR}{dt} = \dfrac{107}{810} \approx 0.1321$ Ω/s.

47. $\dfrac{1650}{\sqrt{31}} \approx 296.3487$ km/s.



[Hint: flight distance x km, distance z km, $z^2 = (x\cos 30°)^2 + (x \sin 30° + 1)^2$ or apply the Cosine Law: $z^2 = 1^2 + x^2 - 2x\cos 120° = x^2 + x + 1$, $\dfrac{dx}{dt} = 300$, $x = \dfrac{1}{60}\dfrac{dx}{dt}$, $\dfrac{dz}{dt} = ?$]

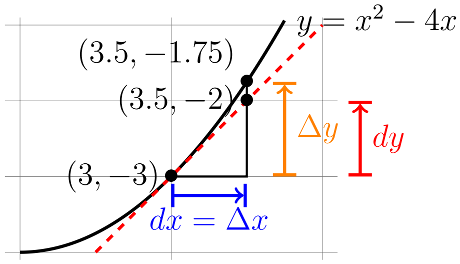
Homework 3.10

4. $L(x) = \ln 2 \cdot x + 1 = (\ln 2)x + 1 = x \ln 2 + 1.$
(**Attention:** $\ln 2x = \ln(2x) \neq \ln 2^x = x \ln 2.$)

11. (a) $dy = (1 - 4x)e^{-4x}\ dx.$ (b) $dy = \dfrac{-2t^3}{\sqrt{1-t^4}}\ dt.$

17. (a) $dy = \dfrac{x}{\sqrt{3+x^2}}\ dx.$ (b) $dy = \dfrac{1}{\sqrt{3+1^2}}(-0.1) = -0.05.$

19. $dy = 1$, $\Delta y = 1.25.$



[Hint: $dy = (2x - 4)\ dx$.]

25. $\sqrt[3]{1001} \approx 10 + \dfrac{1}{300} = 10.00\bar{3} \approx 10.0033.$
[Hint: $f(x) = \sqrt[3]{x}$ at $x = 10$ and $dx = 1$.]

28. $\cos 29^\circ \approx \dfrac{\sqrt{3}}{2} + \dfrac{\pi}{360} \approx 0.8748.$
[Hint: $f(x) = \cos x$ at $x = \dfrac{\pi}{6}$ and $dx = -1^\circ = -\dfrac{\pi}{180}$.]

35. (a) $\dfrac{42}{\pi} \approx 27\ \text{cm}^2$, $\dfrac{1}{84} \approx 0.0119.$
[Hint: $C = 2\pi r = 64$, $A = 4\pi r^2 = \dfrac{1}{\pi}C^2$, $dA = \dfrac{2}{\pi}C\ dC$, $\dfrac{dA}{A} = 2\dfrac{dC}{C}$.]
(b) $\dfrac{1764}{\pi^2} \approx 179\ \text{cm}^3$, $\dfrac{1}{56} \approx 0.0179$
[Hint: $V = \dfrac{4}{3}\pi r^3 = \dfrac{1}{6\pi^2}C^3$, $dV = \dfrac{1}{2\pi^2}C^2\ dC$, $\dfrac{dV}{V} = 3\dfrac{dC}{C}$.]

39. $I = \dfrac{V}{R}$, $dI = -\dfrac{V}{R^2}\ dR$, $\dfrac{dI}{I} = \dfrac{-V\ dR/R^2}{V/R} = -\dfrac{dR}{R}.$
[Hint: the same in magnitude(大小相同).]