Macro Problem Set 3

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1 Value function iteration

1.1 Elementary iteration

Question 1.1 In order to analyze the results in the vectorization part, I report the outcome obtained with the elementary iteration procedure. I used a grid of 2000 points, evenly spaced between $0.95k_{ss}$ and $1.05k_{ss}$ and 0.01 as threshold. Convergence was reached after 151 minutes and 184 iterations. Figure 1 depicts the results. Consumption is almost linear (the steps are because of the number of grid points - with a higher number they would be less pronounced but it would take a lot for the code to be run). The dashed line in the picture regarding capital is the 45° line and the dot in the figure regarding capital is the steady state (also in the pictures below).

1.2 Vectorization

Question 1.2 I used a grid of 2000 points, evenly spaced between $0.95k_{ss}$ and $1.05k_{ss}$ and 0.01 as threshold. Convergence was reached after 42 seconds and 184 iterations. This algorithm was much faster than the previous one. Figure 2 plots the results. The ball in the subplot regarding capital is the steady state.

Question 1.3 I used a grid of 2000 points, evenly spaced between $\frac{1}{2}k_{ss}$ and $\frac{3}{2}k_{ss}$ and 0.01 as threshold. Convergence was reached after 46 seconds and 180 iterations. Figure 3 plots the results. Notice that before the value function appeared almost linear, because we were considering a small interval around the steady state for computing the grid. Here, as the interval is larger it starts to be curved. The result are similar to the shooting algorithm, as consumption converges just below 2.

Question 1.4

Figure 1: Elementary iteration, grid between $0.95k_{ss}$ and $1.05k_{ss}$

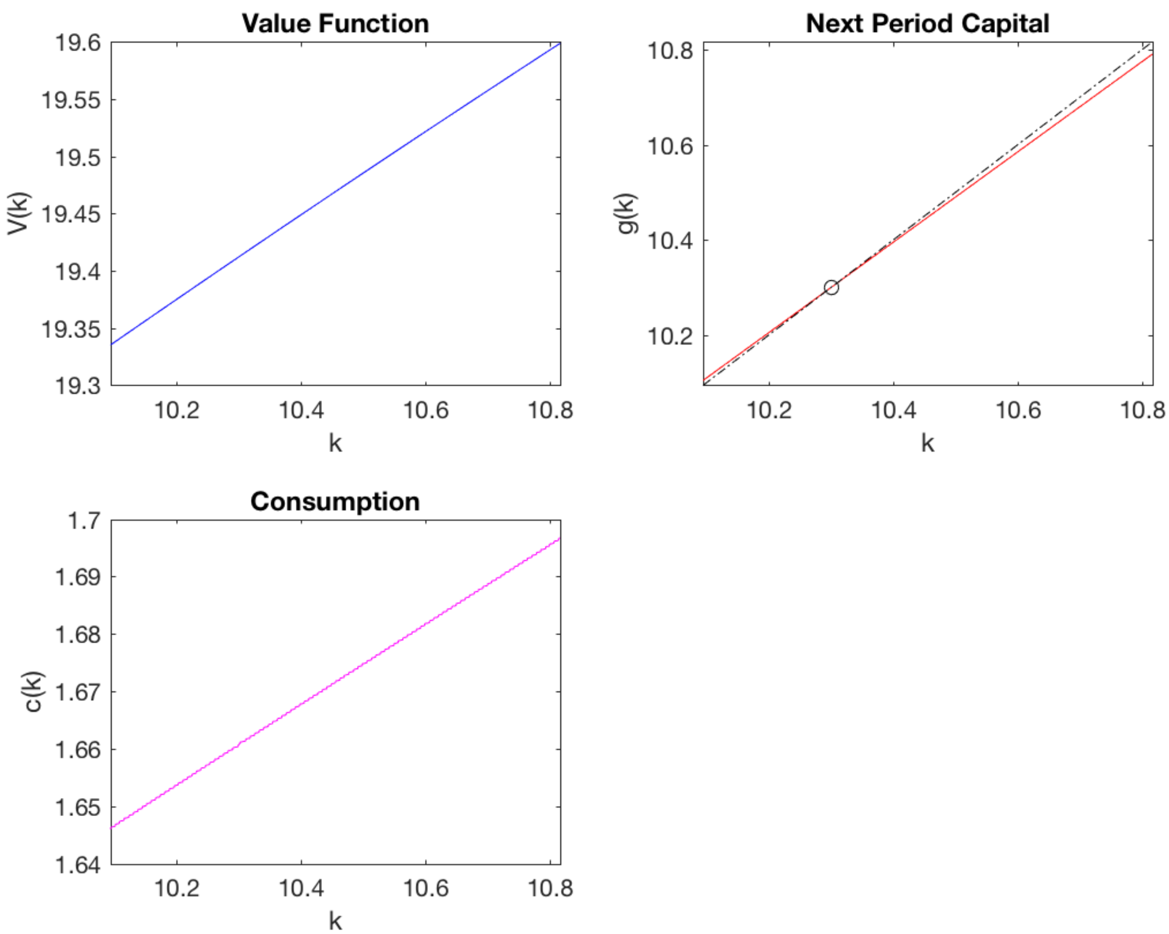


Figure 2: Vectorization, grid between $0.95k_{ss}$ and $1.05k_{ss}$

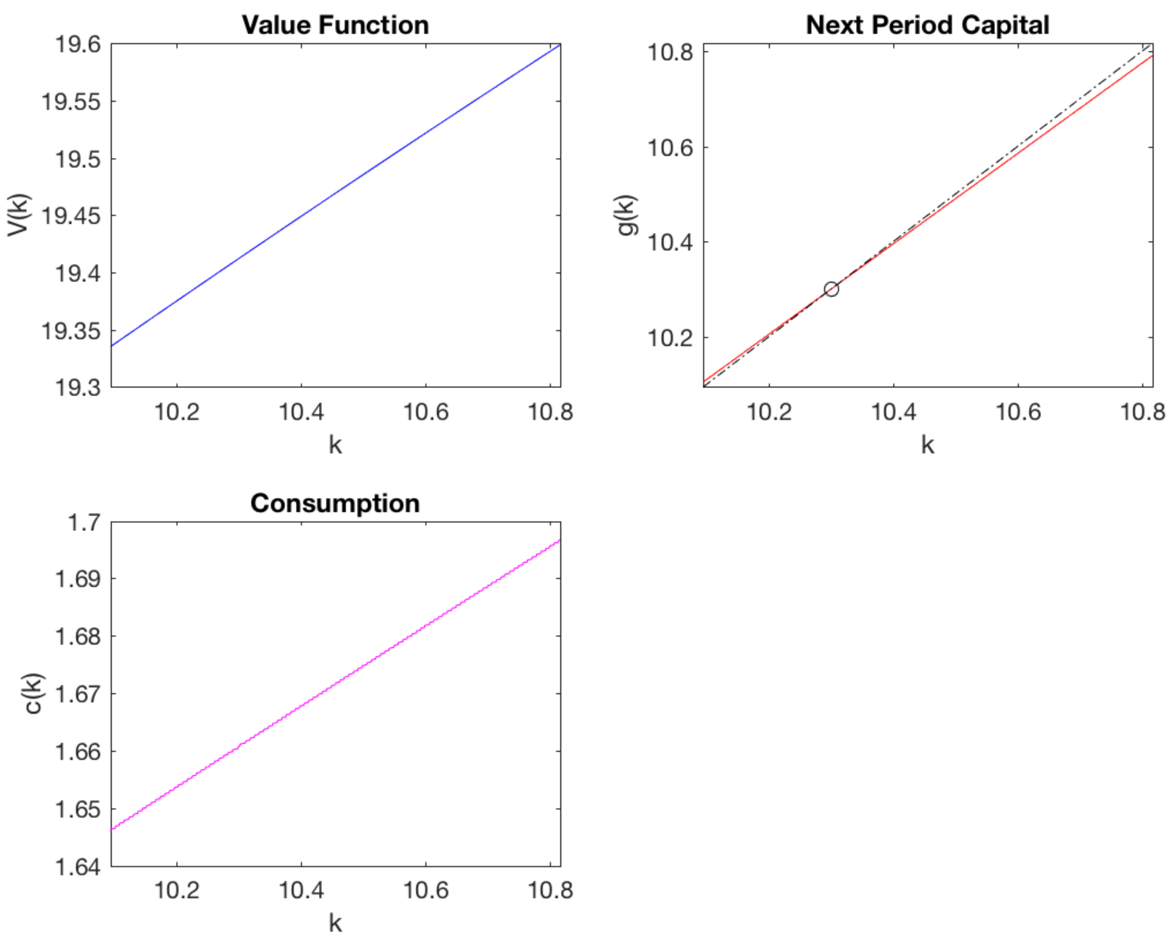


Figure 3: Vectorization, grid between $\frac{1}{2}k_{ss}$ and $\frac{3}{2}k_{ss}$

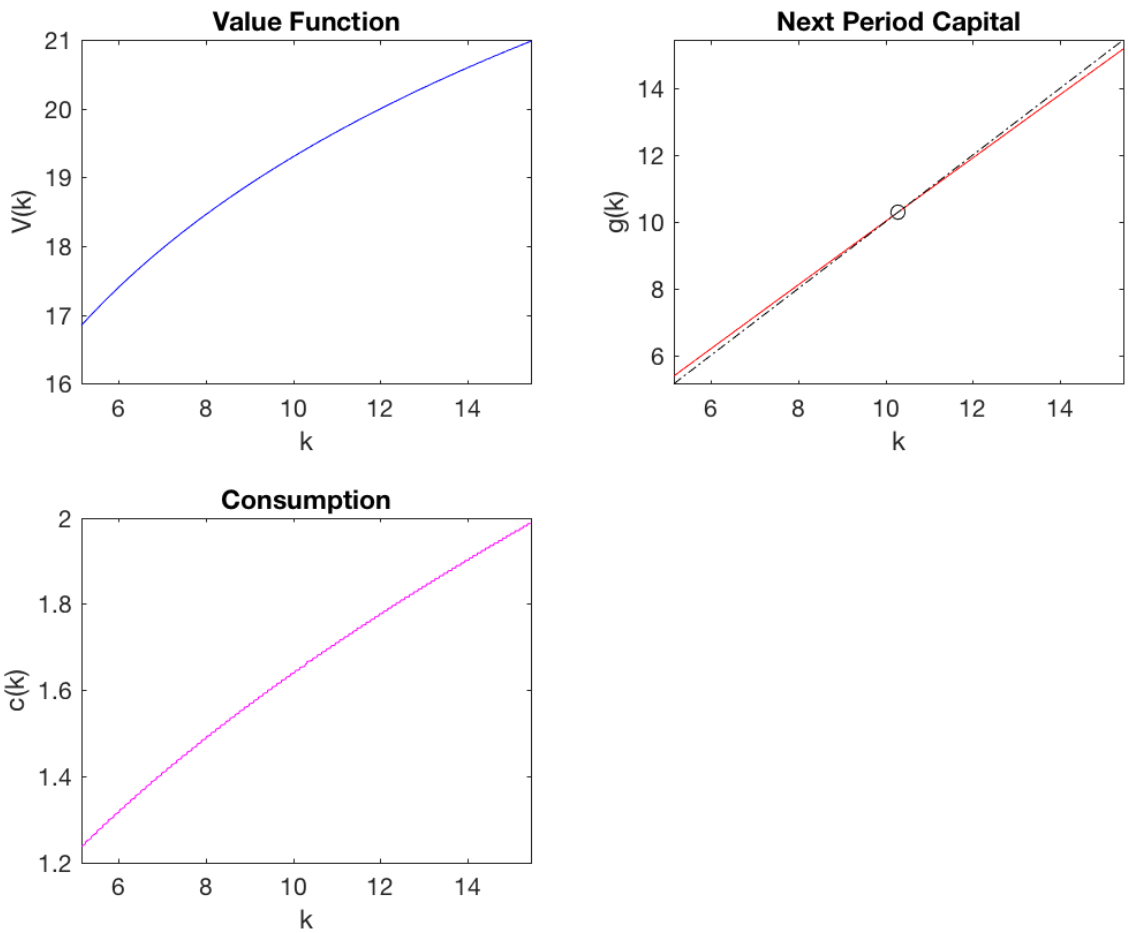
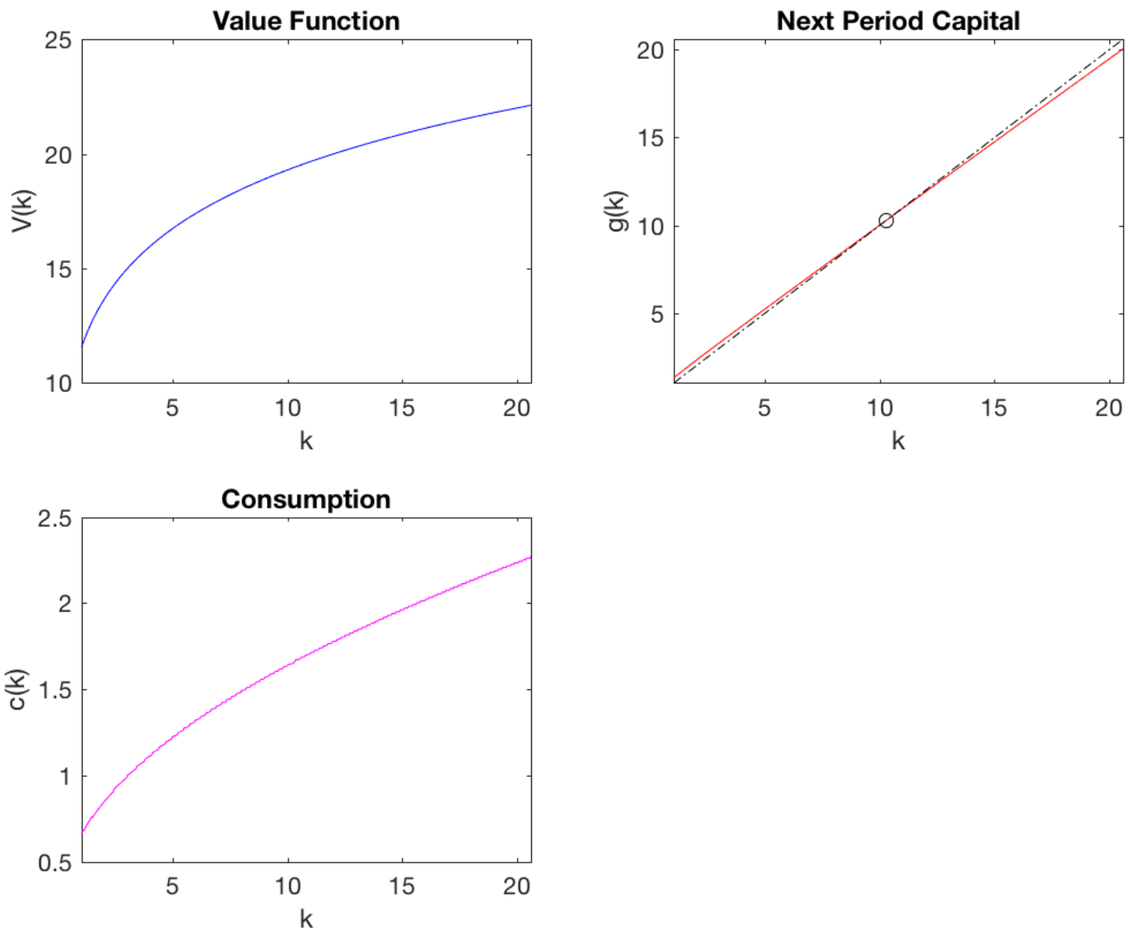


Figure 4: Vectorization, grid between $0.1k$ and $2k_{ss}$



Baseline, grid between $0.1k$ **and** $2k_{ss}$ I used a grid of 2000 points, evenly spaced between $0.1k_{ss}$ and $2k_{ss}$ and 0.01 as threshold. Convergence was reached after 87 seconds. Figure 4 plots the results. The value function is more curved as the interval considered is larger.

Increased depreciation, $\delta = 0.1$ As shown in Figure 5, when the depreciation rate increases, capital depreciates more rapidly, and indeed the dashed line is below the baseline. At the same time, remember that consumption depends negatively on k' (i.e. if capital tomorrow is bigger then consumption today is lower). Hence, given that k' is lower with a higher depreciation rate for each k, consumption increases for each k: indeed, the dashed line is above the baseline in the picture regarding consumption.

Figure 5: Baseline vs Increased depreciation ($\delta = 0.1$)

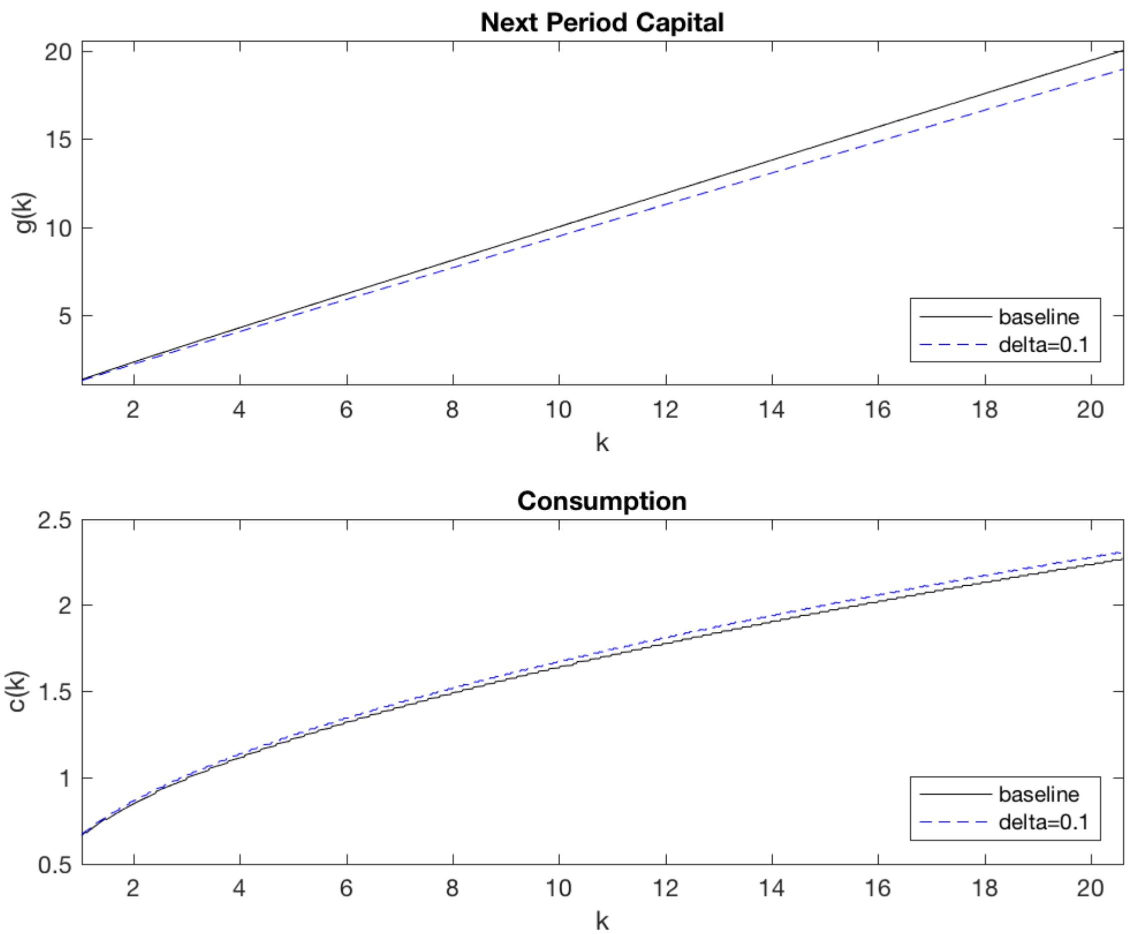
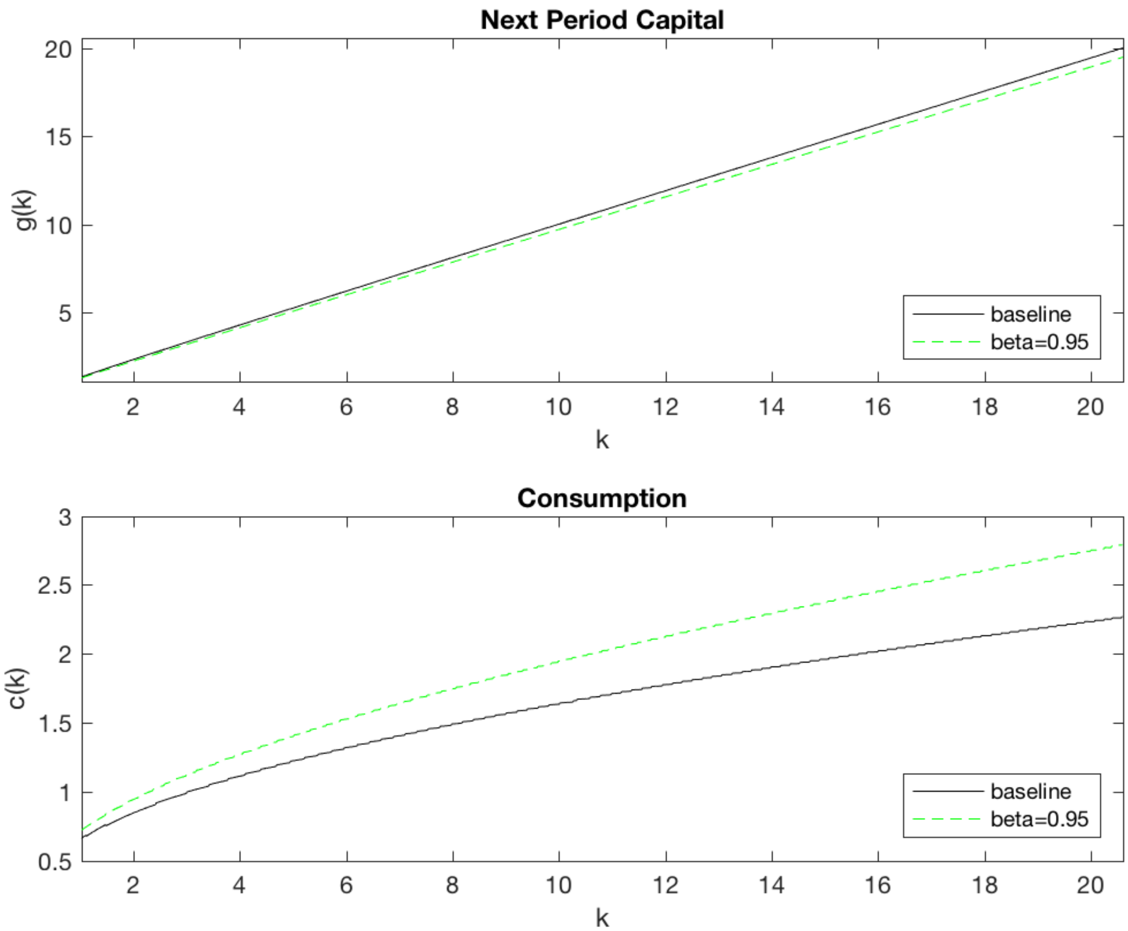


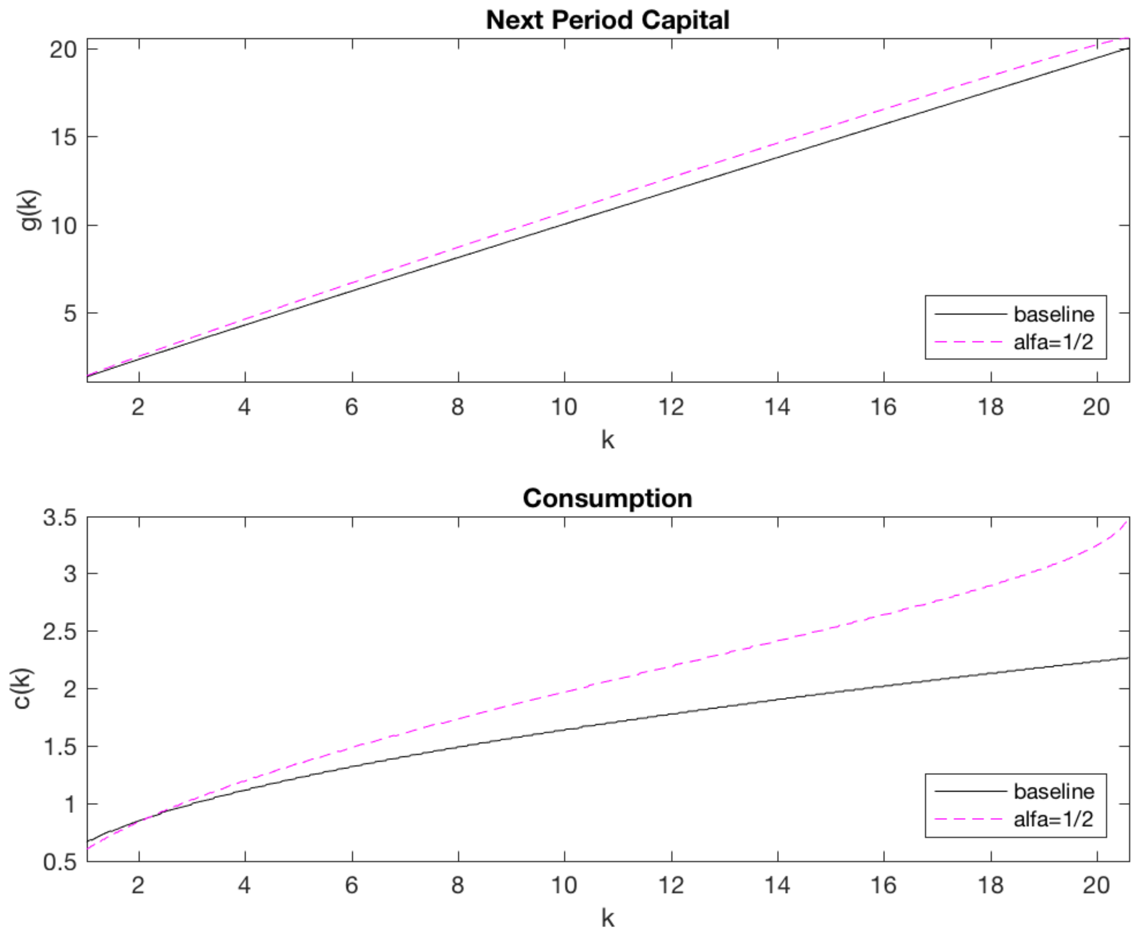
Figure 6: Baseline vs More impatience ($\beta = 0.95$)



More impatience, $\beta = 0.95$ As shown in Figure 6, increasing β implies more impatience in consumption, meaning that future consumption is valued relatively less than today's. This implies that less capital will be accumulated in the next period in order to finance higher consumptio today. Indeed, the picture clearly shows that k' decreases and c increases after this shift.

Higher capital share, $\alpha = 1/2$ If α increases, then the marginal productivity of capital increases, meaning that a unit of capital produces more output than before. Given that output is used to finance consumption and capital, they both increase: the dashed line is above the baseline in both graphs. Figure 7 plots the results. An interesting thing to notice is the fact that for lower values of capital consumption actually decreases when alpha is increased. This is because, given that capital is more productive, it is better to consume less today in order to accumulate more

Figure 7: Baseline vs Higher capital share ($\alpha = 1/2$)



capital and get higher consumption in the future.

Lower intertemporal elasticity of substitution, $\gamma = 3$ If γ increases, it means that the consumer is less prone to consume tomorrow rather than today, because the increment in utility between two consecutive periods is lower than before. Indeed, consumption at the beginning is higher and then becomes lower: the dashed line is higher than before and then goes below the baseline. Capital is not affected because the overall amount of consumption does not change, it only changes its inter-temporal allocation.

Figure 8: Baseline vs Lower intertemporal elasticity of substitution ($\gamma = 3$)

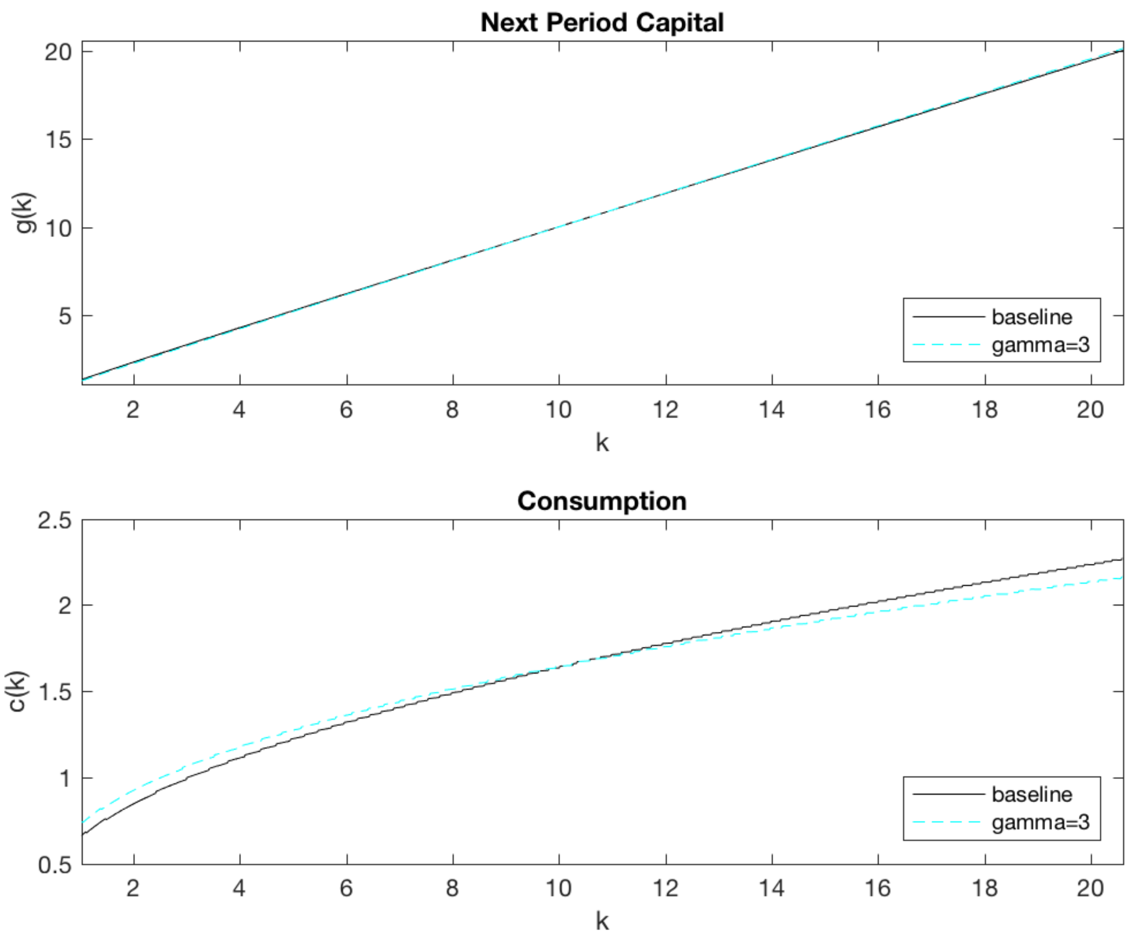
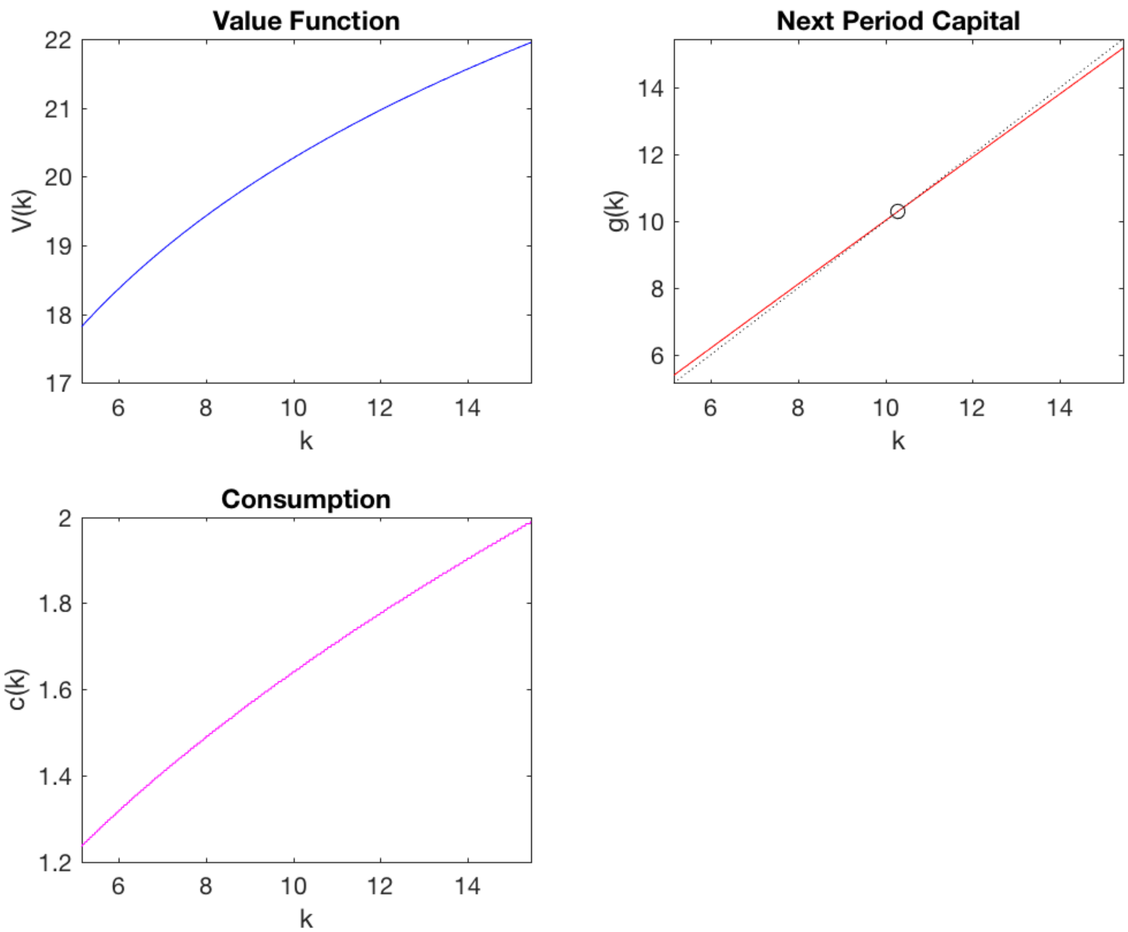


Figure 9: Interpolation



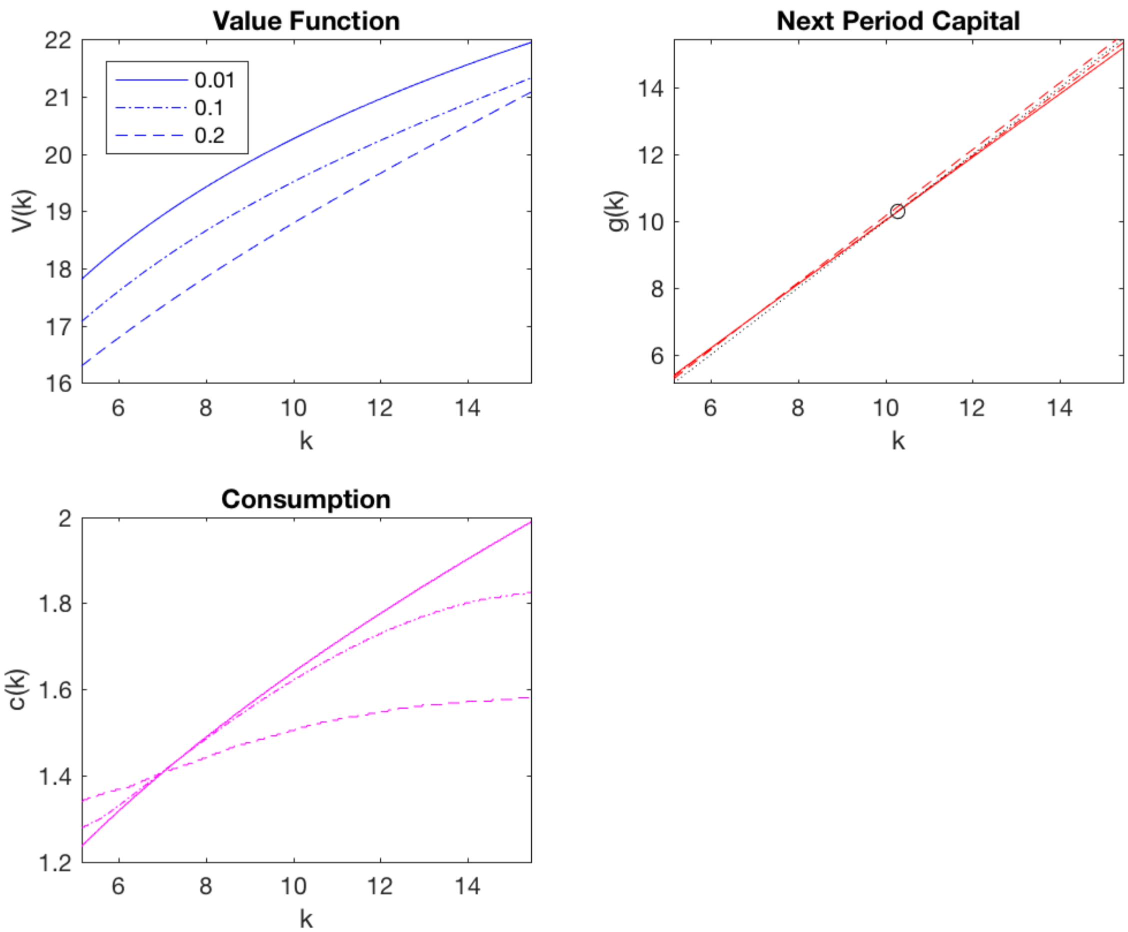
1.3 Grid interpolation

I used a grid of 700 points both for capital and consumption, evenly spaced between $\frac{1}{2}k_{ss}$ and $\frac{3}{2}k_{ss}$ for capital and between 1 and 5 for consumption. 0.01 was mantained as threshold. Convergence was reached after 13 seconds and 251 iterations. Figure 9 plots the results. Notice that here a considerably lower number of grid points for capital delivers a similar result to before. Here, indeed, only 700 points are needed while before 2000 were needed.

1.4 Capital quality shock

I used a grid of 1000 points both for capital and consumption, evenly spaced between $\frac{1}{2}k_{ss}$ and $\frac{3}{2}k_{ss}$ for capital and between 1 and 5 for consumption. 0.01 was mantained as threshold. I chose three

Figure 10: Interpolation with capital quality shocks



different values for ε: 0.01, 0.10, 0.20. Results are reported in Figure 10. Clearly, as ε increases, the value function goes down. Next period capital remains almost the same, while consumption is a bit higher for low values of capital and then is lower. Intuitively, as ε increases, there is more uncertainty regarding the value of capital tomorrow (and therefore also of consumption). Because of risk aversion, she prefers to smooth consumption more as the magnitude of the shock increases. Capital remains almost the same because what changes is only the inter-temporal amount of consuption, not the total. Being the value function dependent on the reward function, it is lower when consumption is more smooth.

2 Markov chains

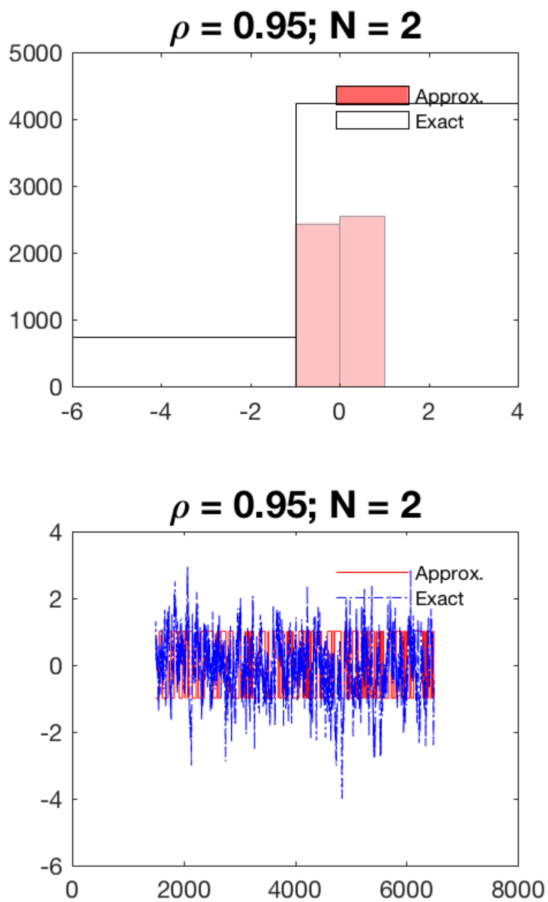
Attached is the code I used to solve exercise 2.3, part (b) of the book.

```
%exercise2.3 part b Sargent book
pi0=[0.5 0.5]'; P1=[1 0;0 1]; P2=[0.5 0.5;0.5 0.5];
gamma=2.5;beta=0.95;
c=[1^(1-gamma)/(1-gamma) 5^(1-gamma)/(1-gamma)]';
I=eye(2,2); C=zeros(2,2);
C1=I-beta*P1; C2=I-beta*P2;
D1=inv(C1); D2=inv(C2);
E1=D1*c; E2=D2*c;
V1=E1'*pi0; V2=E2'*pi0;
```

3 Approximation of AR(1) process using Markov chains

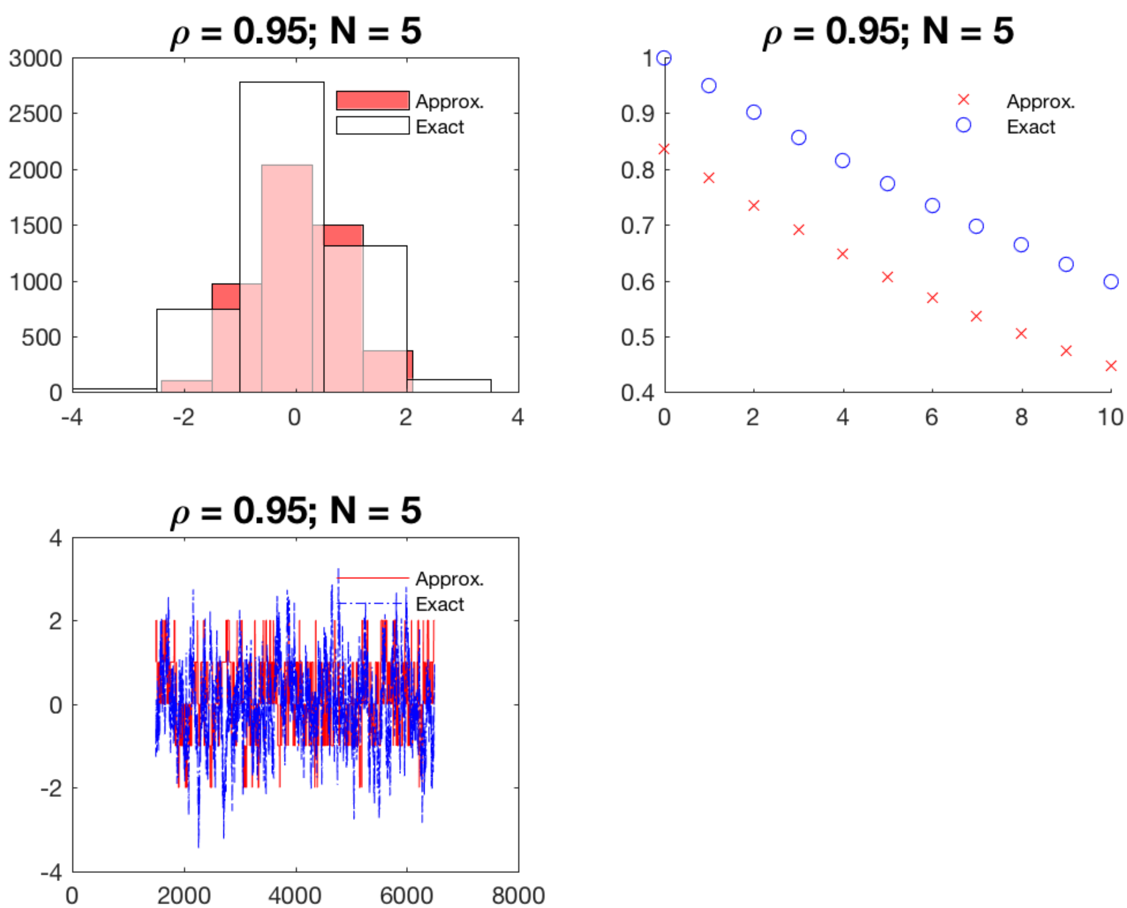
The figures below plot the histograms and the autocovariance functions of the exact $AR(1)$ process and those obtained with the approximation process. For the approximated process, I used 6500 observations and I truncated the first 1500 in order to get results that do not depend on the initial conditions. The figures below show how changes in N and ρ affect the graphs. Clearly, as N increases, the approximated process matches better the real one because it can obscillate between more states. However, as ρincreases and the process becomes more persistent, the approximation loses forecasting power. This is because as ρ gets closer to 1, the $AR(1)$ process becomes non-invertible and this induces numerical instability on the approximation method.

Figure 11:



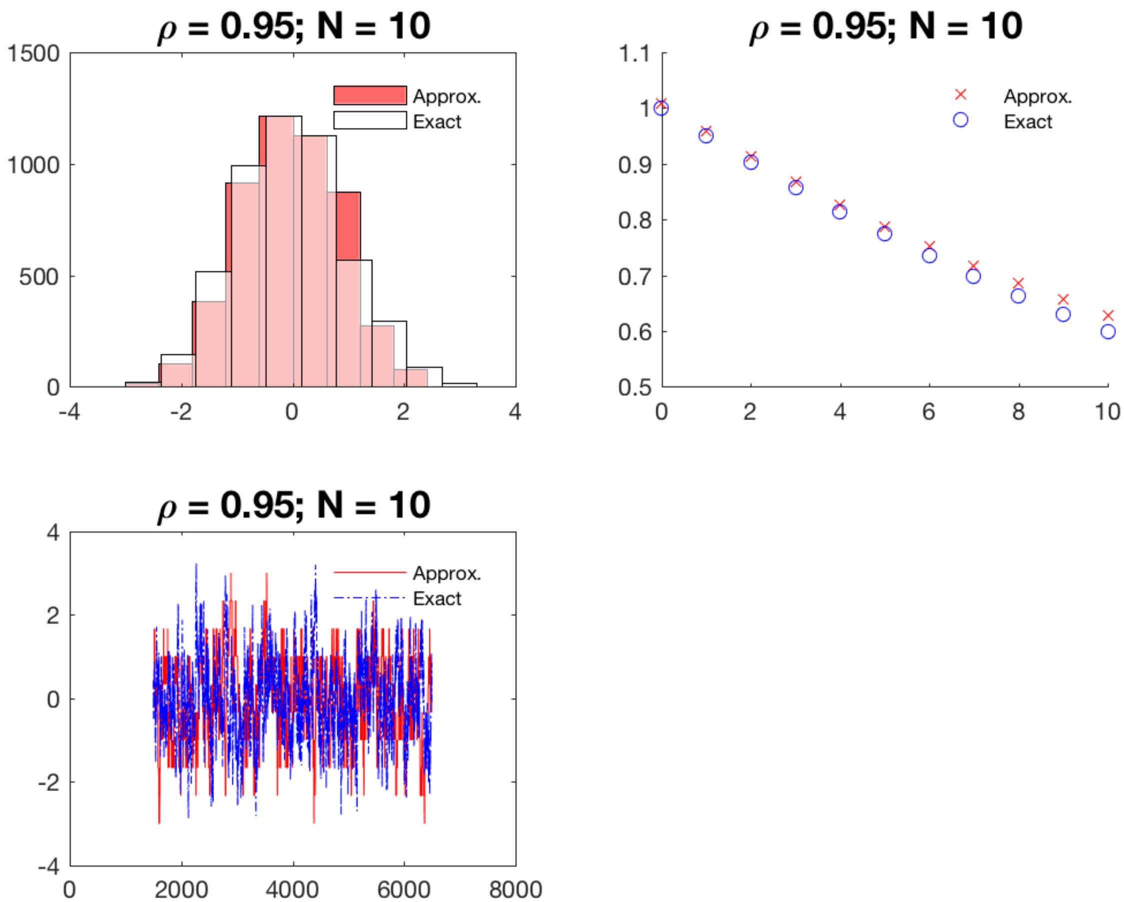
ρ = 0.95; N = 2
Approx.
Exact
ρ = 0.95; N = 2
Approx.
Exact
ρ = 0.95; N = 2
Approx.
Exact

Figure 12:



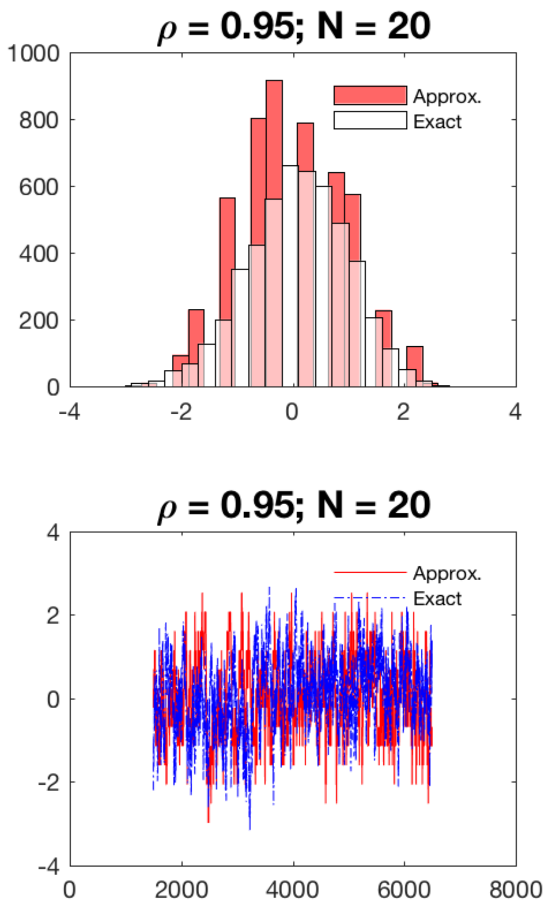
ρ = 0.95; N = 5
Approx.
Exact
ρ = 0.95; N = 5
Approx.
Exact
ρ = 0.95; N = 5
Approx.
Exact

Figure 13:



ρ = 0.95; N = 10
Approx.
Exact
1500
1000
500
0
-4
-2
0
2
4
ρ = 0.95; N = 10
Approx.
Exact
1.1
1
0.9
0.8
0.7
0.6
0.5
0
2
4
6
8
10
ρ = 0.95; N = 10
Approx.
Exact
4
2
0
-2
-4
0
2000
4000
6000
8000

Figure 14:



ρ = 0.95; N = 20
1000
800
600
400
200
0
-4
-2
0
2
4
Approx.
Exact
ρ = 0.95; N = 20
1
0.9
0.8
0.7
0.6
0.5
0
2
4
6
8
10
Approx.
Exact
ρ = 0.95; N = 20
4
2
0
-2
-4
0
2000
4000
6000
8000
Approx.
Exact

Figure 15:

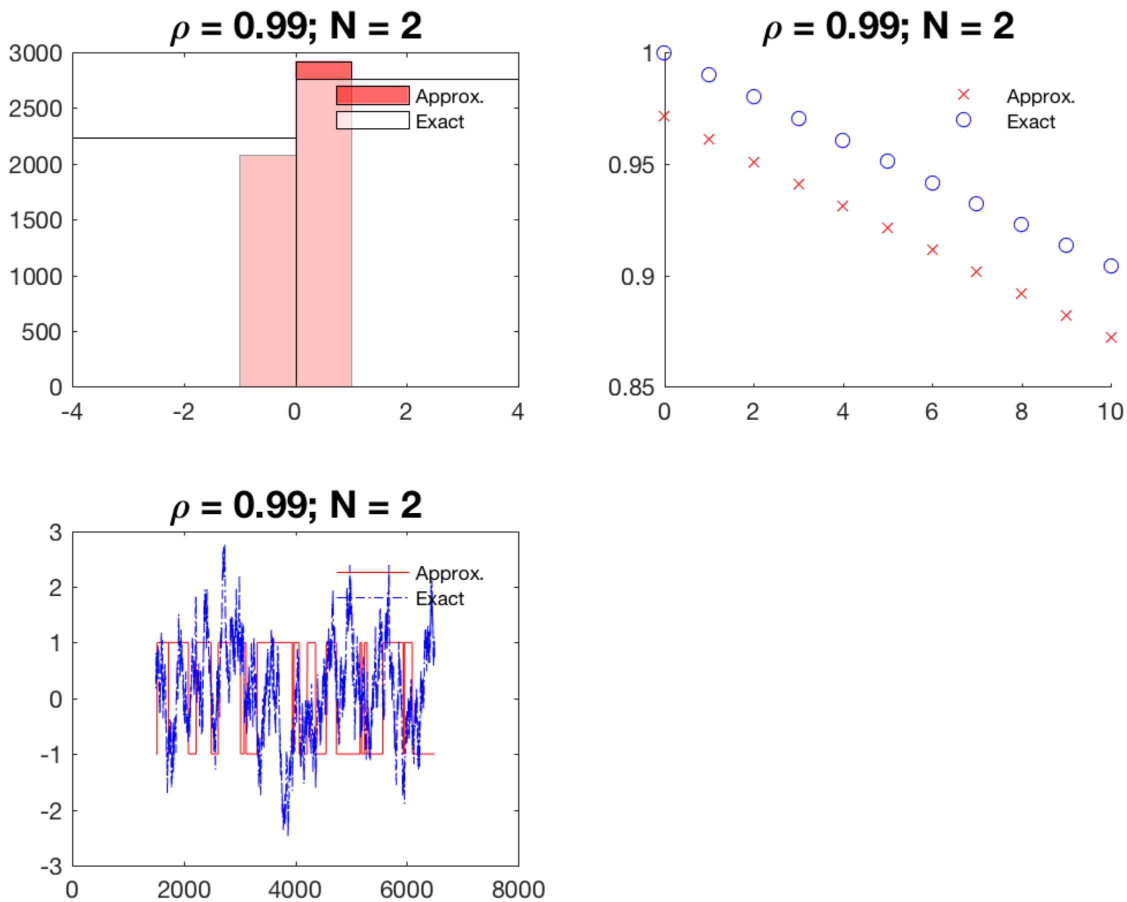
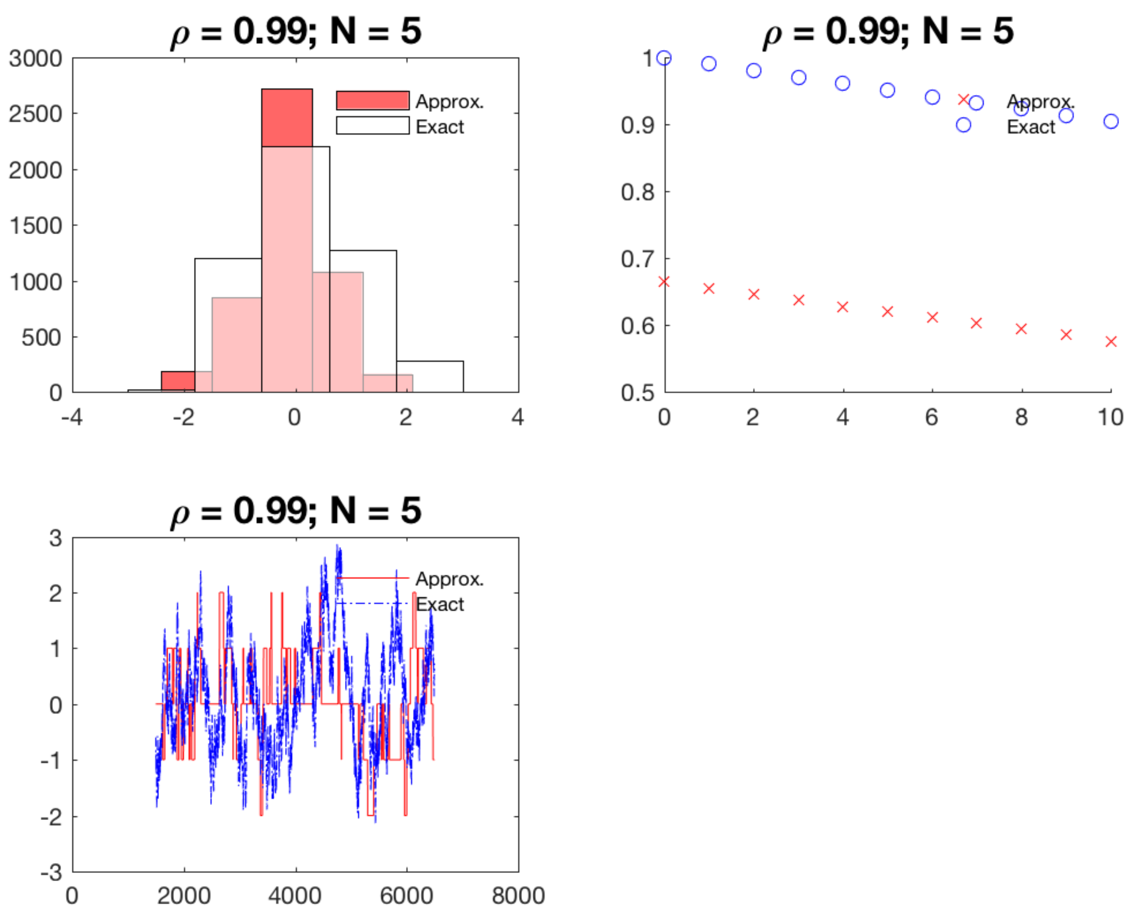

ρ = 0.99; N = 2
Approx.
Exact
ρ = 0.99; N = 2
Approx.
Exact
ρ = 0.99; N = 2
Approx.
Exact

Figure 16:



ρ = 0.99; N = 5
Approx.
Exact
ρ = 0.99; N = 5
Approx.
Exact
ρ = 0.99; N = 5
Approx.
Exact

Figure 17:

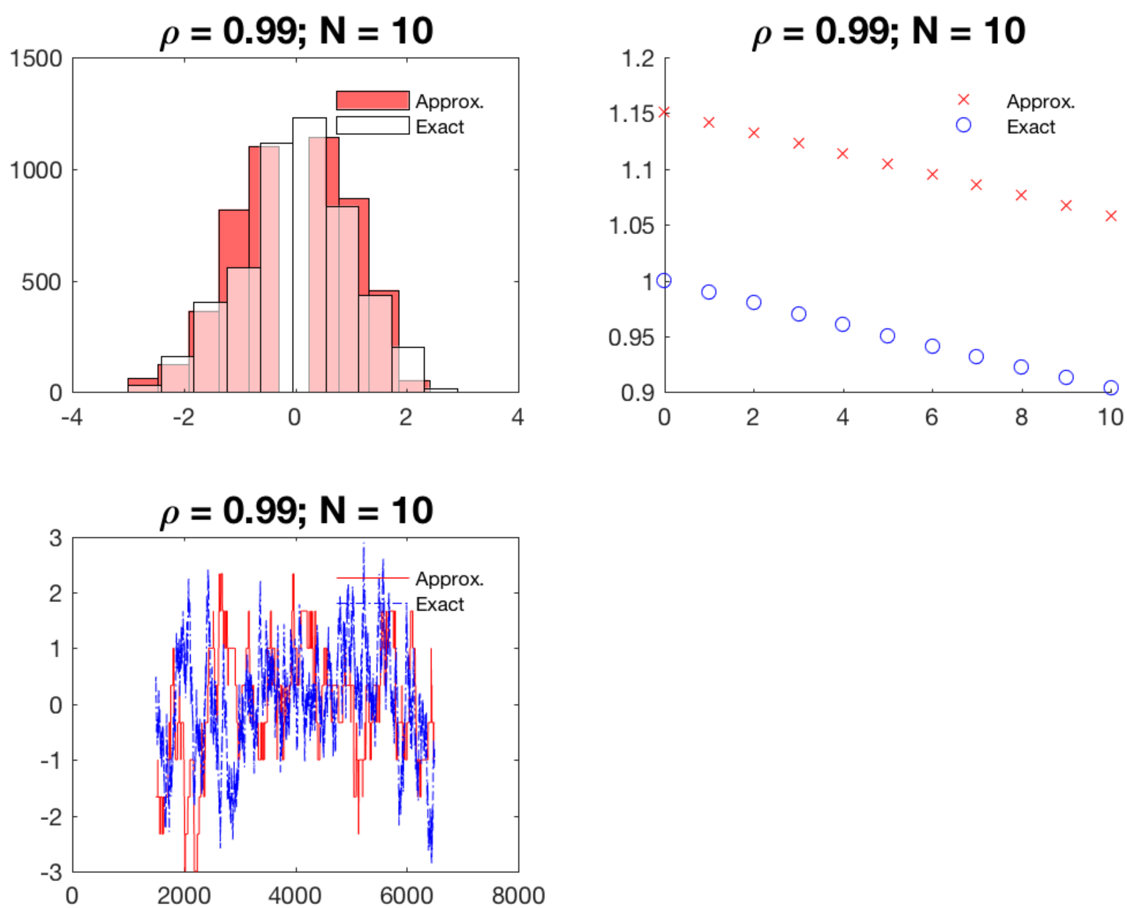
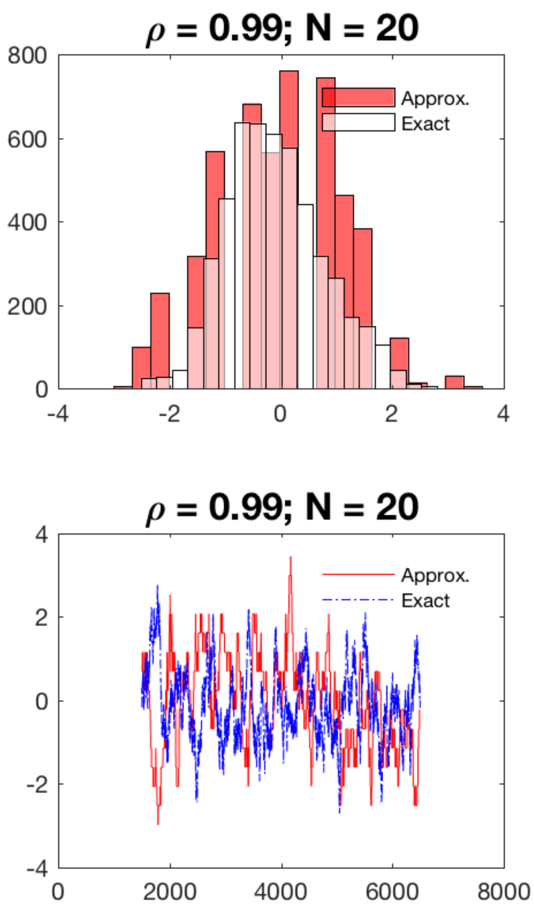

ρ = 0.99; N = 10
Approx.
Exact
1500
1000
500
0
-4
-2
0
2
4
ρ = 0.99; N = 10
Approx.
Exact
1.2
1.15
1.1
1.05
1
0.95
0.9
0
2
4
6
8
10
ρ = 0.99; N = 10
Approx.
Exact
3
2
1
0
-1
-2
-3
0
2000
4000
6000
8000

Figure 18:



ρ = 0.99; N = 20
Approx.
Exact
800
600
400
200
0
-4
-2
0
2
4
ρ = 0.99; N = 20
Approx.
Exact
1.4
1.3
1.2
1.1
1
0.9
0
2
4
6
8
10
ρ = 0.99; N = 20
Approx.
Exact
4
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-2
-4
0
2000
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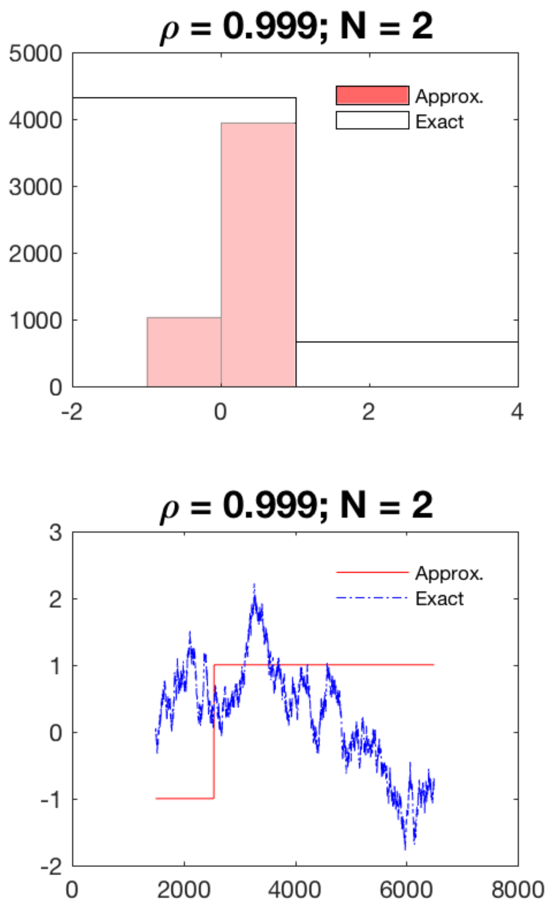
6000
8000

Figure 19:



ρ = 0.999; N = 2
Approx.
Exact
ρ = 0.999; N = 2
Approx.
Exact
ρ = 0.999; N = 2
Approx.
Exact

Figure 20:

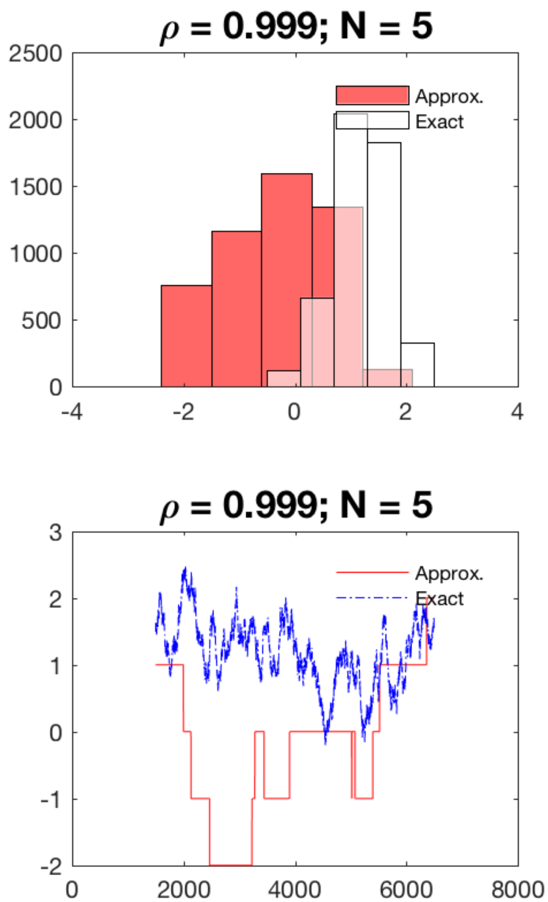
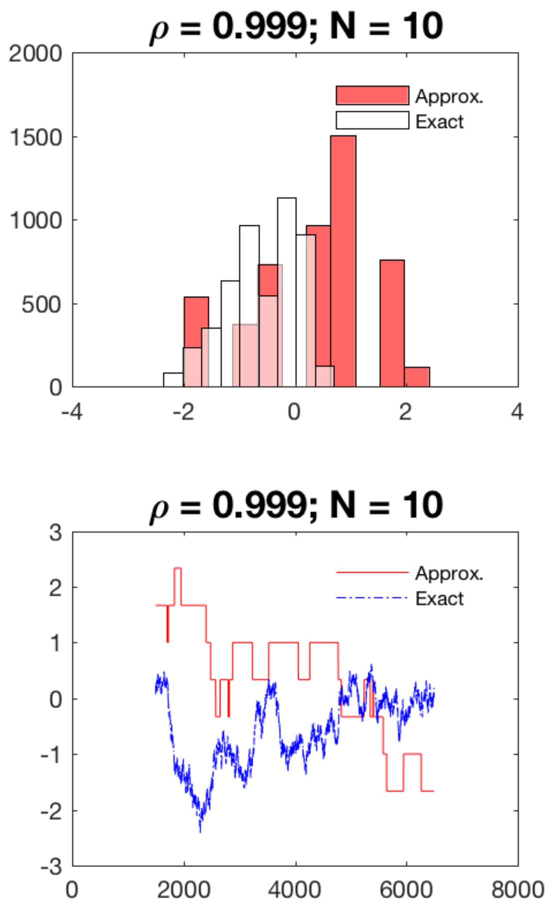
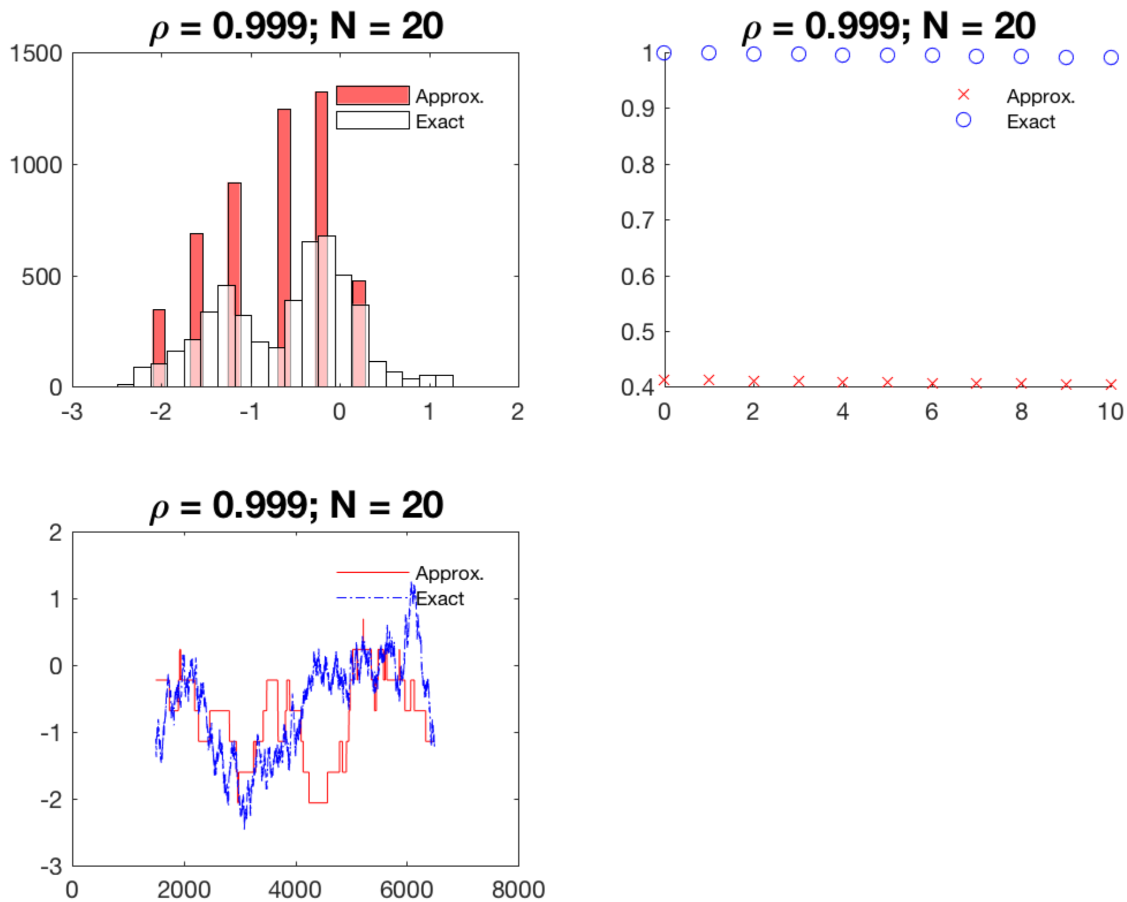

ρ = 0.999; N = 5
Approx.
Exact
ρ = 0.999; N = 5
Approx.
Exact
ρ = 0.999; N = 5
Approx.
Exact

Figure 21:



ρ = 0.999; N = 10
Approx.
Exact
2000
1500
1000
500
0
-4
-2
0
2
4
ρ = 0.999; N = 10
1.15
1.1
1.05
1
0.95
Approx.
Exact
0
2
4
6
8
10
ρ = 0.999; N = 10
Approx.
Exact
3
2
1
0
-1
-2
-3
0
2000
4000
6000
8000

Figure 22:



ρ = 0.999; N = 20
Approx.
Exact
1500
1000
500
0
-3
-2
-1
0
1
2
ρ = 0.999; N = 20
Approx.
Exact
1
0.9
0.8
0.7
0.6
0.5
0.4
0
2
4
6
8
10
ρ = 0.999; N = 20
Approx.
Exact
2
1
0
-1
-2
-3
0
2000
4000
6000
8000