Computational Exercise 1

Part a, b, c, d See MATLAB code.

Part e Figure 1 presents the results of the simulations for $\hat{\beta}_0$ (Unrestricted, $\mathcal{A}$, $\mathcal{B}$) and $\hat{\beta}_2$ (Unrestricted, $\mathcal{B}$) for the three values of ρ (0 - first row, 0.5 - second row, 0.9 - third row). All distributions look normal and are centered at the true parameter, as expected. Looking at the first column, it is clear that the unconstrained estimator has the highest variance and it is the less efficient (the distributions of the estimators obtained from restrictions $\mathcal{A}$, $\mathcal{B}$ are more centered on the true parameter and less dispersed). This is in line with expectations because restrictions $\mathcal{A}$, $\mathcal{B}$ "match" the values for betas imposed in the model. Notice also, that as the correlation increases, the distribution of the beta obtained from restriction $\mathcal{A}$ is more centered and less dispersed around 1 than that of the estimator obtained from $\mathcal{B}$. This is because $\mathcal{B}$ has more "degrees of freedom" than $\mathcal{A}$: while the latter imposes the true value for each parameter as restriction, the former imposes only that the sum is 2, which is true in the case in which the coefficients are both one but it can be true also in other cases (e.g one is zero and the other is two). Hence, the estimator obtained from $\mathcal{B}$ is less precise and this is more evident when correlation is high. Indeed, when correlation increases, is more difficult to estimate β_1 and β_2 separately as their regressors become more and more correlated (multicollinearity), so that the OLS is less precise because it has to invert a matrix that is almost singular. Looking at the second column, notice first that it is not useful to plot the case $\mathcal{A}$ because we would get a histogram with (almost) all observations at 1. Then, the same comments as above can be made: the unrestricted estimator is less precise for the same reason as before and as correlation increases the estimate becomes more dispersed (note that the range of the x-axis in the last picture increases) because it becomes more difficult to disentangle the effects among the two coefficients. Overall, it is useful to notice that restricted estimators are more efficient if the restrictions imposed are correct and they are more efficient the "tighter" the restriction is (as explained above when comparing the efficiency in $\mathcal{A}$ and $\mathcal{B}$).

Figure 1: Results for $\hat{\beta}_0$ and $\hat{\beta}_2$