Exercise 4

Part (a)

I estimate the following VAR(2):

$$z_t = F_1 z_{t-1} + F_2 z_{t-2} + \varepsilon_t \quad ; \; z_t = \begin{bmatrix} \triangle \log y_t \\ Zic_t \end{bmatrix}$$

where $F_1 = \begin{bmatrix} \beta_1^1 & \beta_2^1 \\ \beta_3^1 & \beta_4^1 \end{bmatrix}$ and $F_2 = \begin{bmatrix} \beta_1^2 & \beta_2^2 \\ \beta_3^2 & \beta_4^2 \end{bmatrix}$ are 2x2 matrices. Note that in this case SUR estimation is equivalent to OLS equation by equation as the set of regressors is the same.

Below the estimated matrices:

$$\hat{F}_1 = \begin{bmatrix} 0.5722 & -0.0516 \\ 0.9008 & 0.8915 \end{bmatrix}; \quad \hat{F}_2 = \begin{bmatrix} 0.2554 & 0.0313 \\ 0.0842 & -0.0188 \end{bmatrix}$$

The code that performs the above is reported here:

```
%% Exercise 4
load ps2.mat;
% Estimate 2nd order VAR - equivalent to OLS eq by eq in this case %
beta_dlny = regress(dlny(3:end),[dlny(2:end-1) dlny(1:end-2) ...
                                 Zic(2:end-1) Zic(1:end-2)]);
beta_Zic  = regress(Zic(3:end),[dlny(2:end-1) dlny(1:end-2) ...
                                 Zic(2:end-1) Zic(1:end-2)]);
% Collect the estimated companion matrices
F1        = [beta_dlny(1,1) beta_dlny(3,1);beta_Zic(1,1) beta_Zic(3,1)];
F2        = [beta_dlny(2,1) beta_dlny(4,1);beta_Zic(2,1) beta_Zic(4,1)];
```

Part (b)

We have to compute the Beveridge-Nelson gap:

$$\log y_t - \tau_t = -\sum_{h=1}^{\infty} \mathbb{E}_t(\triangle \log y_t - \mu_{\triangle \log y})$$

where $\mu_{\triangle \log y}$ is the unconditional mean of GDP growth. In order to get the summation term, first I have to estimate:

$$\tilde{z}_t = F_1^{'} \tilde{z_{t-1}} + F_2^{'} \tilde{z_{t-2}} + \eta_t \quad ; \; \tilde{z}_t = \begin{bmatrix} \triangle \log y_t - \mu_{\triangle \log y} \\ Zic_t \end{bmatrix}$$

Now, I define the following:

$$e' = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}; \quad \underset{4x4}{A} = \begin{bmatrix} F_1' & F_2' \\ \underset{2x2}{I} & \underset{2x2}{0} \end{bmatrix}; \quad \underset{4x1}{Z_t} = \begin{bmatrix} \tilde{z}_t \\ z_{\tilde{t-1}} \end{bmatrix}$$

to rewrite the above VAR (2) as:

$$Z_t = AZ_{t-1} + \upsilon_t$$

Then the h-step ahead forecast of $\triangle \log y_t - \mu_{\triangle \log y}$ as:

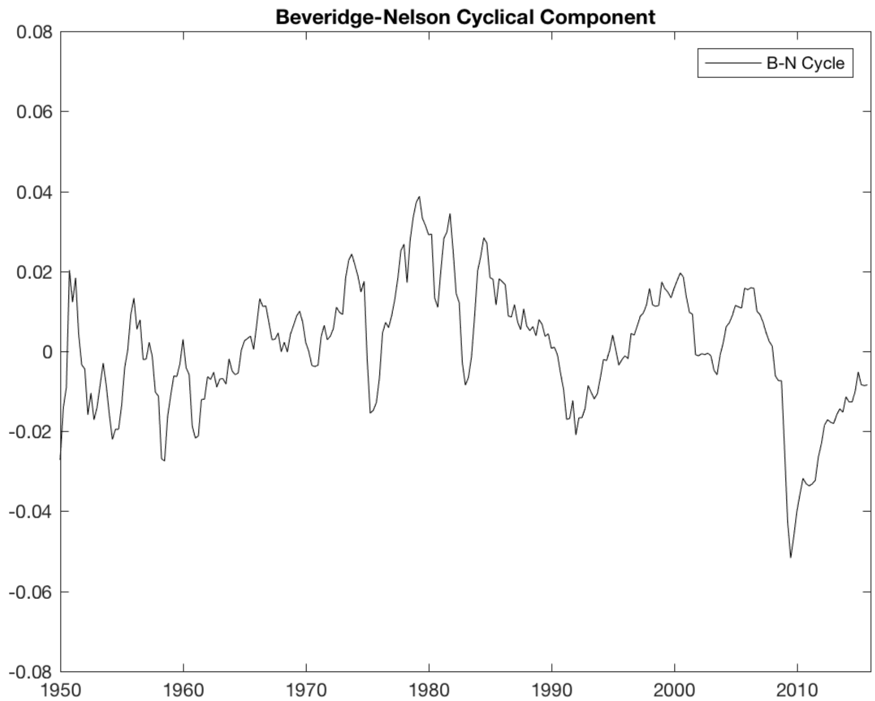
$$\sum_{h=1}^{\infty} \mathbb{E}_t(\triangle \log y_{t+h} - \mu_{\triangle \log y}) = \sum_{h=1}^{\infty} e' A^h Z_t = e' \sum_{h=1}^{\infty} A^h Z_t = e'(I-A)^{-1} A Z_t$$

since the eigenvalues of A are inside the unit circle. Using the estimated matrices in the above formulas we can get the output gap. The code below performs what described above.

```
% Beveridge—Nelson output gap %
betaBN_dlny = regress(dlny(3:end),[dlny(2:end—1)—mean(dlny) dlny(1:end—2)—mean(dlny) ...
            Zic(2:end—1) Zic(1:end—2)]);
betaBN_Zic  = regress(Zic(3:end),[dlny(2:end—1)—mean(dlny) dlny(1:end—2)—mean(dlny) ...
            Zic(2:end—1) Zic(1:end—2)]);
% Collect the estimated companion matrices
F1_BN      = [betaBN_dlny(1,1) betaBN_dlny(3,1);betaBN_Zic(1,1) betaBN_Zic(3,1)];
F2_BN      = [betaBN_dlny(2,1) betaBN_dlny(4,1);betaBN_Zic(2,1) betaBN_Zic(4,1)];
A          = [F1_BN F2_BN;eye(2,2) zeros(2,2)];
e          = [1 0 0 0];
Zt  = [(dlny(3:end)—mean(dlny))';(Zic(3:end))';...
      (dlny(2:end—1)—mean(dlny))';(Zic(2:end—1))'];
for i=1:size(Zt,2)
        cycle(i,1) = — e*inv(eye(4,4)—A)*A*Zt(:,i);
end
```

Figure 1 plots the output gap. From the picture we can see that at the end of the sample (2007-2015) the economy was almost -5% below the trend and then slowly recovered to around -1% at the end of the sample.

Figure 1: B-N Output Gap



Part (c)

We have from above that:

$$\log y_t - \tau_t = -e'(I-A)^{-1}AZ_t = B(\theta)Z_t$$

Hence by the delta method:

$$\sqrt{T}(B(\hat{\theta}) - B(\theta)) \sim N\left[0; \left.\frac{\partial B}{\partial \theta}\right|_{\theta=\hat{\theta}} V_{\hat{\theta}} \left.\frac{\partial B'}{\partial \theta}\right|_{\theta=\hat{\theta}}\right]$$

Given the value of the asymptotic variance obtained above, I can get the asymptotic standard error σ and get the ± 1 st.dev. of the estimates and use them to compute confidence intervals.

The following part of the code does what described above and Figure 2 plots the results.

```
% Asymptotics SE report range of end-of-sample in +-1 std
B       = e*inv(eye(4,4)-A)*A;  B_prime = gradient(B);
Zt_1    = [(dlny(2:end-1)-mean(dlny))';(Zic(2:end-1))';...
                     (dlny(1:end-2)-mean(dlny))';(Zic(1:end-2))'];
eps     = Zt-A*Zt_1;
V       = length(eps)^(-1)*eps*eps';
AVar    = B_prime*V*B_prime';
Asd     = AVar.^(1/2);
```

```
for i=1:size(Zt,2)
        cycle_l(i,1) = -e*inv(eye(4,4)-A)*A*Zt(:,i)-1.65*Asd;
        cycle_h(i,1) = -e*inv(eye(4,4)-A)*A*Zt(:,i)+1.65*Asd;
end
```

Figure 2: B-N output gap within ±1 St.Dev.