Exerises 2.1.5 and 2.1.6 - Linear Approximation

I simulated in MATLAB the evolution of k starting from two different points, $\frac{1}{2}k_{ss}$ and $\frac{3}{2}k_{ss}$. Figure 1 depicts the results. It is clear that both paths converge to the steady state after around 100 periods, but the path starting from $\frac{1}{2}k_{ss}$ reaches the steady state more rapidly. Indeed, given the decreasing nature of the path and the fact that $\frac{1}{2}k_{ss}$ is nearer to the steady state, it converges fastly.

Figure 2, instead, plots the corrisponding evolution of consumption for these two paths. Both paths converge around 1.66, the steady state value for consumption. Again, starting from $\frac{1}{2}k_{ss}$ convergence is faster.

Figure 3 depicts the phase diagram for both the two cases ($\frac{1}{2}k_{ss}$ case in red and from $\frac{3}{2}k_{ss}$ case in blue). They overlap and have an almost linear shape (the range of the y axis is small) and they both converge to the steady state.

Exercise 2.2 - Shooting algorithm

After having expressed k_{t+2} as function ok k_{t+1} and k_t (calculations can be found at the end of page 3 of the manuscript), I follow the steps of the shooting algorithm starting from the guess $\frac{1}{2}k_{ss}$. The algorithm works in this way: if the difference between the last value obtained of capital is positive, I subtract $\frac{1}{j}$ with j being the repetition's number. Viceversa if the difference is negative. Indeed, if the path undershoots the steady state, it means that the initial guess was too low and hence it must be raised; viceversa if the path overshoots the steady state. Figure 4 plots some path for some different guesses. Some paths overshoot the steady state, others become zero (they are excluded because capital cannot be negative). The violet line represents the evolution of capital that converges to the steady state (after 304 repetitions), with tolerance 0.001.

Figure 5 plots the same in the $\frac{3}{2}k_{ss}$ case. Again, some paths overshoot the steady state and the violet line represents the evolution of capital that converges to the steady state (after 314 repetitions), with tolerance 0.001.

Figure 6 plots the consumption paths associated with the capital paths plotted above in the $\frac{1}{2}k_{ss}$ case. Again, some paths explode and other becomes zero because they correspond to non feasible paths of capital. The violet line is the one associated with the convergent path and, hence, makes consumption converge to its steady state value. Figure 7 does the same for the $\frac{3}{2}k_{ss}$ case.

Figure 8 plots the phase diagram for the convergent path in the $\frac{1}{2}k_{ss}$ case (green) and in the $\frac{3}{2}k_{ss}$ case (magenta). Notice that both converge to the steady state, one from above and the other from below.

Figure 9 plots the phase diagram of some non convergent paths for both the two cases.

Finally, Figure 10 compares the phase diagrams obtained with the shooting algorithm and with the lineas approximation for the $\frac{1}{2}k_{ss}$ case. Figure 11 does the same for the $\frac{3}{2}k_{ss}$ case. In the $\frac{1}{2}k_{ss}$ case, the linear approximation approaches the steady state from above while the shooting algorithm from below. Clearly, the difference in path around the initial conditions is relevant but as the paths reach the steady state, the points of the two sequences become closer. In the $\frac{3}{2}k_{ss}$ case, instead, they both approach the steady state from above and the difference around the initial conditions is smaller but still relevant. Again, the difference vanishes as the two paths converge to the steady state.

Figure 1: Evolution of k_t starting from $\frac{1}{2}k_{ss}$ (red) and from $\frac{3}{2}k_{ss}$ (blue), (k_{ss} in green)

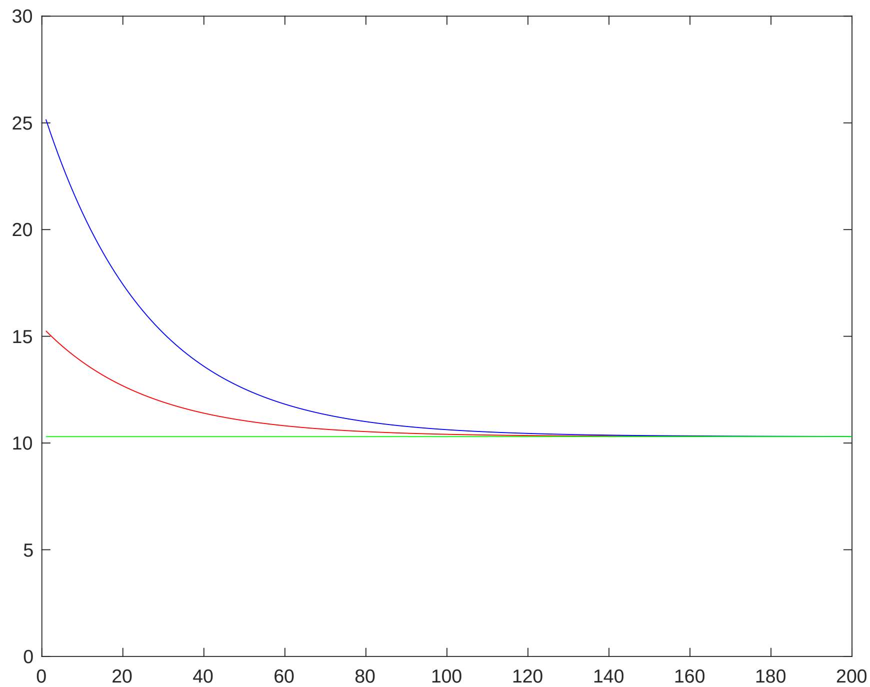

Figure 2: Evolution of c_t starting from $\frac{1}{2}k_{ss}$ (magenta) and from $\frac{3}{2}k_{ss}$ (cyan)

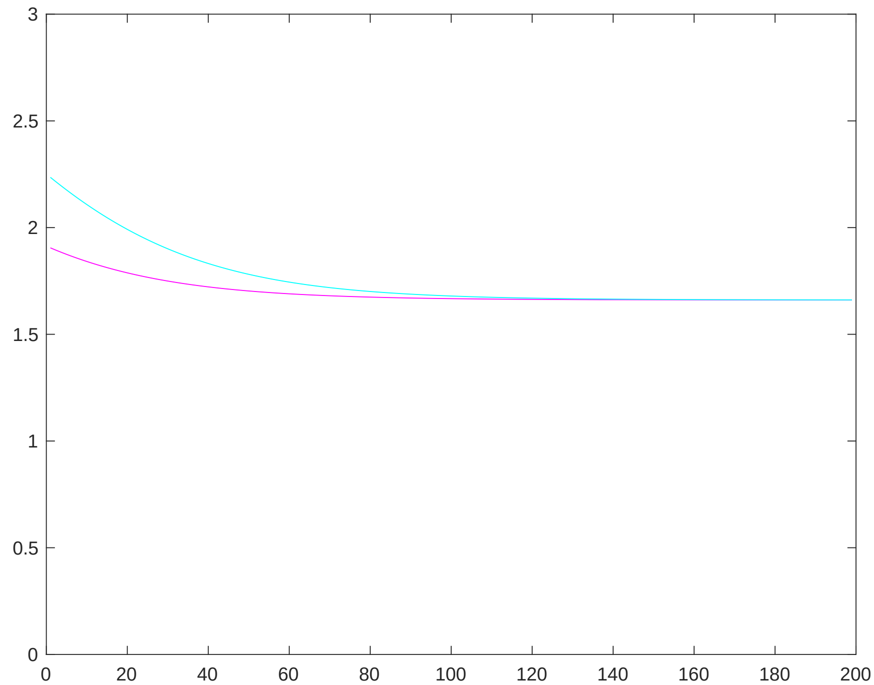

Figure 3: Phase diagram, $\frac{1}{2}k_{ss}$ case in red and from $\frac{3}{2}k_{ss}$ case in blue

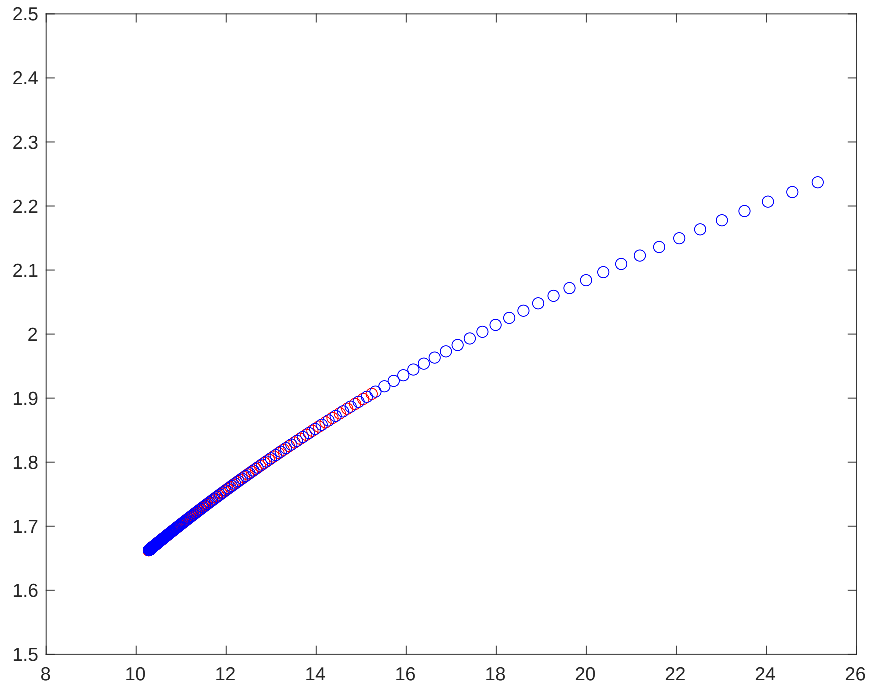

Figure 4: Evolution of capital ($\frac{1}{2}k_{ss}$ case)

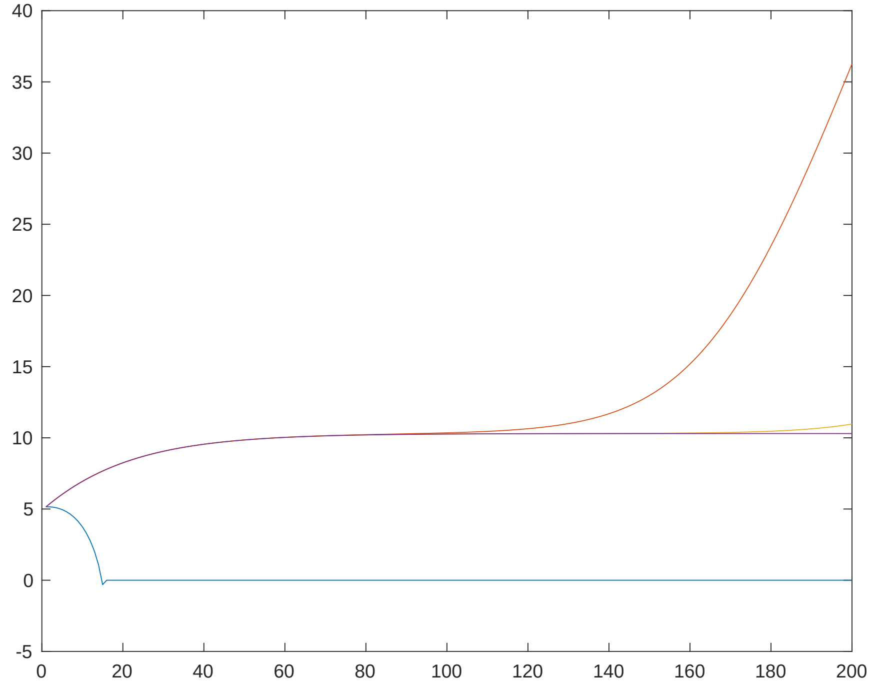

Figure 5: Evolution of capital ($\frac{3}{2}k_{ss}$ case)

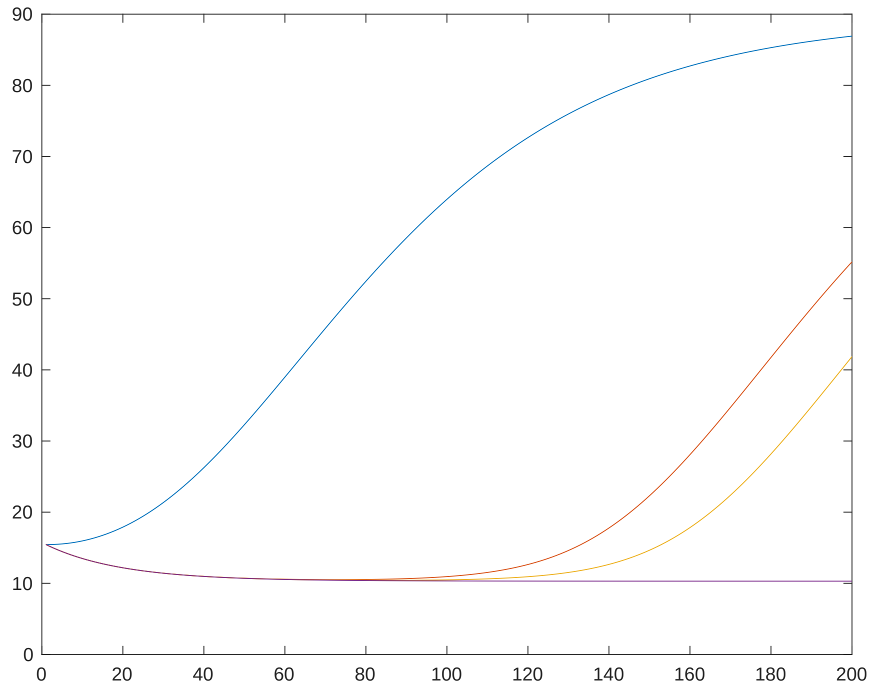

Figure 6: Evolution of consumption ($\frac{1}{2}k_{ss}$ case)

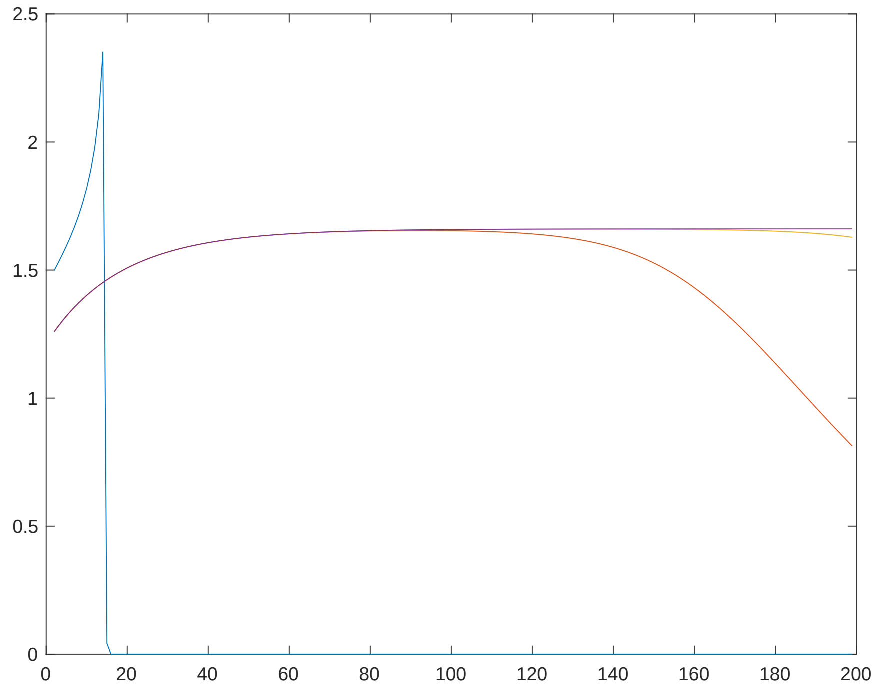

Figure 7: Evolution of consumption ($\frac{3}{2}k_{ss}$ case)

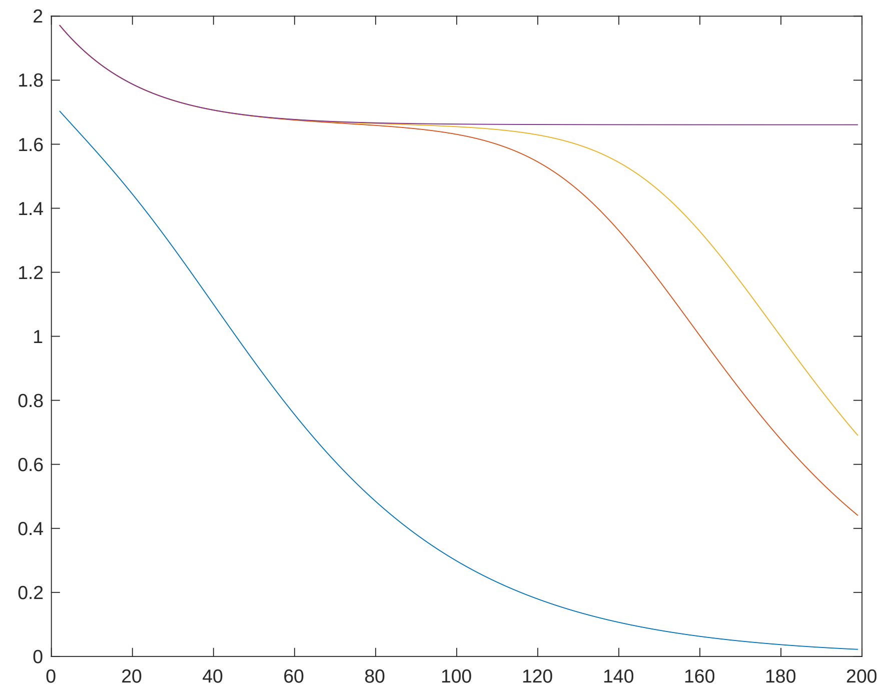

Figure 8: Phase diagram for the two convergent paths, $\frac{1}{2}k_{ss}$ case (green) and $\frac{3}{2}k_{ss}$ case (magenta)

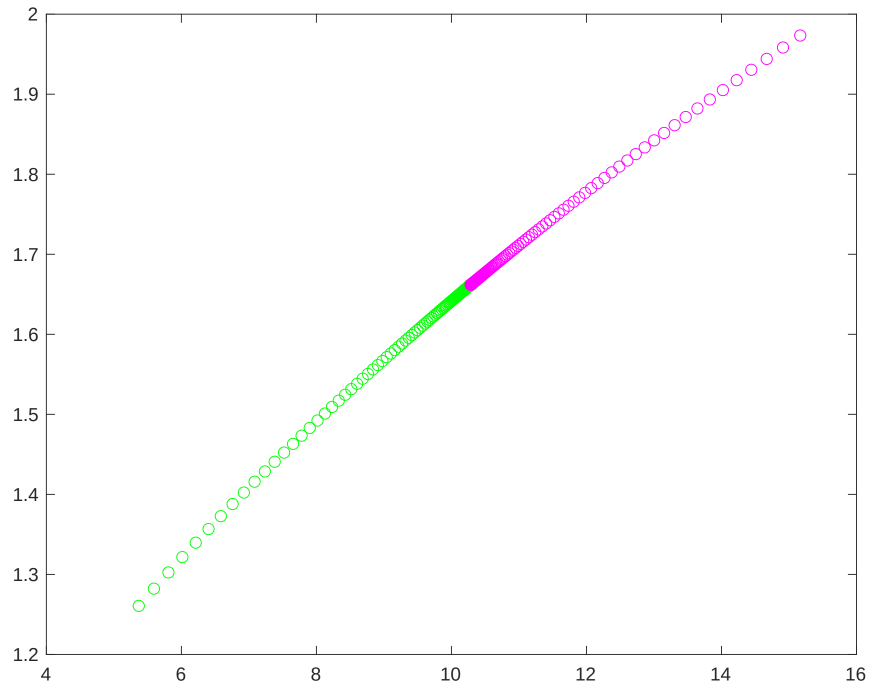

Figure 9: Phase diagrams of non-convergent paths

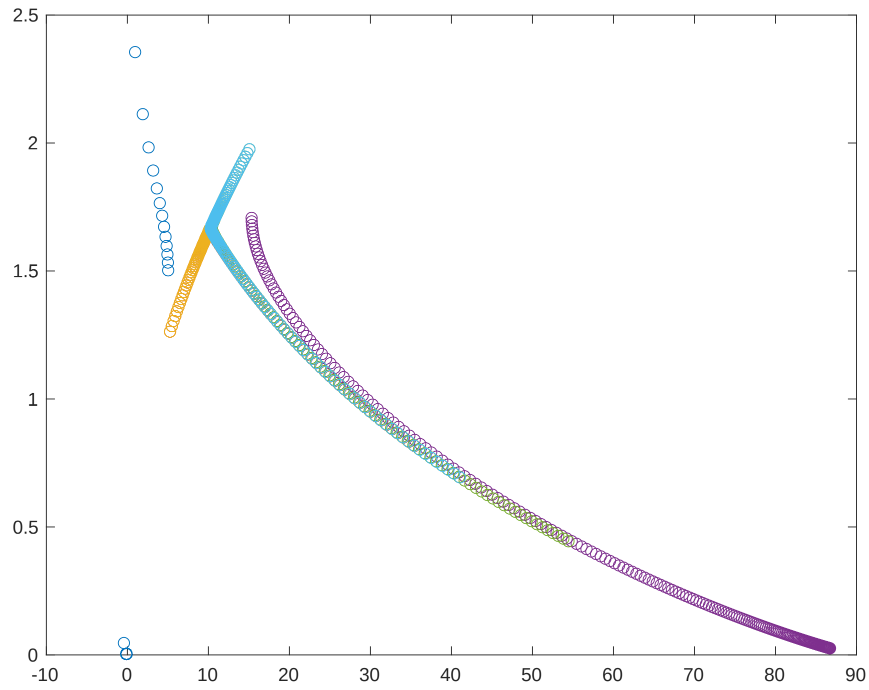

Figure 10: Comparison of phase diagrams (shooting-blue, approximation-red), $\frac{1}{2}k_{ss}$ case

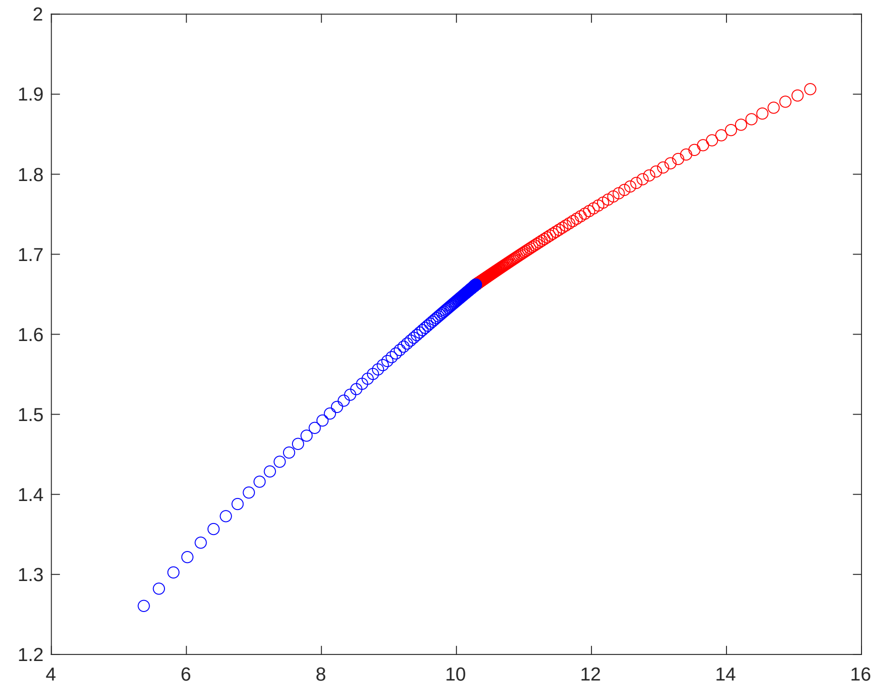

Figure 11: Comparison of phase diagrams (shooting-blue, approximation-red), $\frac{3}{2}k_{ss}$ case