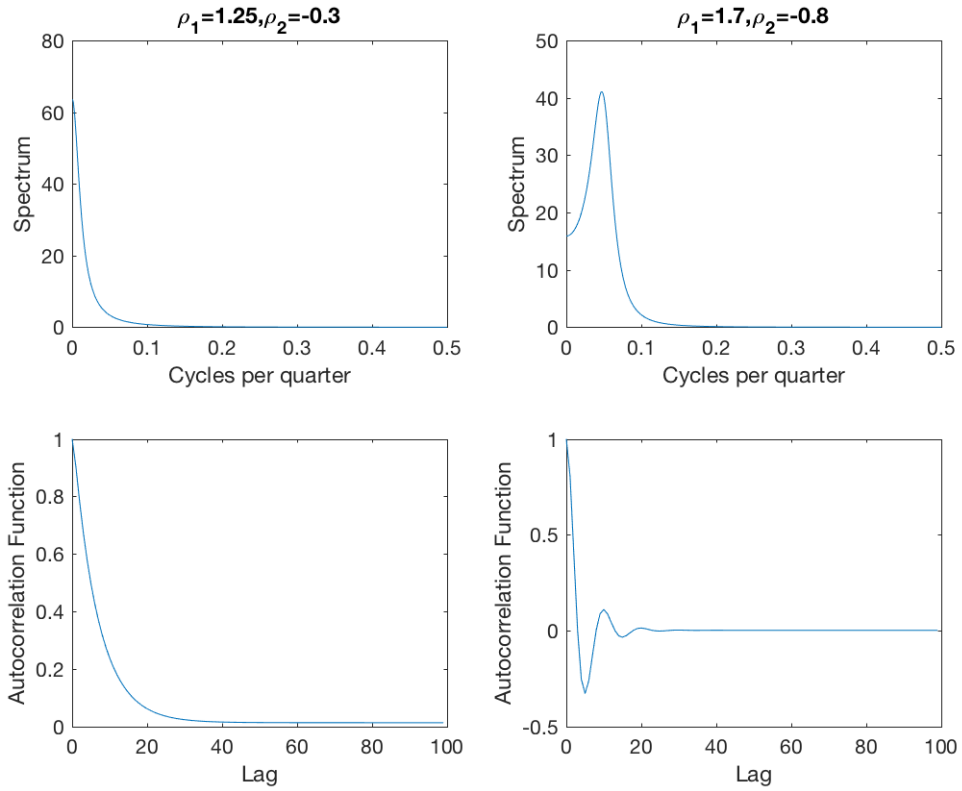
Exercise 4

For the description see the handwritten part.

Figure 1: Spectrum and Autocorrelation functions under the two parametrizations



Code

```
%% Econometrics II - Part II %%
%  Problem Set 1
%  Gualtiero Azzalini
addpath('/Users/Gualtiero/Dropbox/A NYU/Metrics/cogley/psets/1')
addpath('/Users/Gualtiero/Documents/MATLAB/Add-Ons/mfe_toolbox/utility')
clear all;

%% Exercise 4
rho1    = [1.25; 1.7];
rho2    = [-0.3; -0.8];
dt      = 1000;
w       = linspace(0,pi,dt)';
i       = sqrt(-1);
```

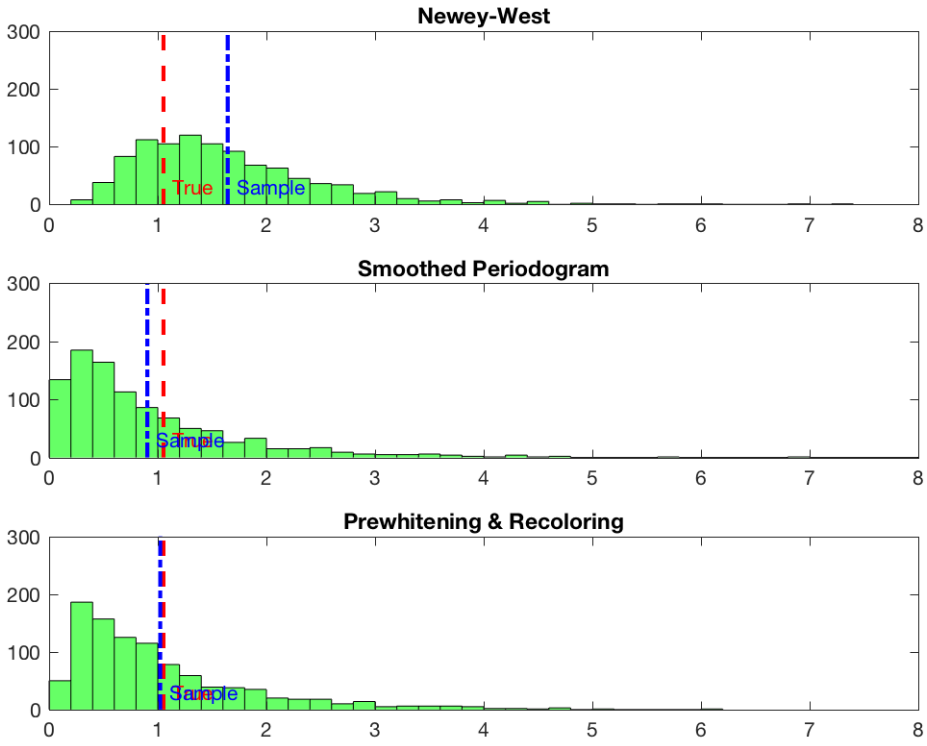
```
for j=1:2
for p = 1:length(w)
fxx(p,1)    = 1/(2*pi*(1+rho1(j,1)^(2)+rho2(j,1)^(2)+...
                        2*(rho1(j,1)*rho2(j,1)-rho1(j,1))*cos(w(p,1))-...
                        2*rho2(j,1)*cos(2*w(p,1))));
end
gamma = ifft(fxx);
acf = gamma/gamma(1,1);
eval(['gamma_',num2str(j),'=gamma']);
eval(['fxx_',num2str(j),'=fxx']);
eval(['acf_',num2str(j),'=acf']);
end

figure(1)
subplot(2,2,1);
plot(w/(2*pi),fxx_1);title('\rho_{1}=1.25,\rho_{2}=-0.3');
ylabel('Spectrum');xlabel('Cycles per quarter');
subplot(2,2,3); plot(0:1:99,acf_1(1:100));
ylabel('Autocorrelation Function');xlabel('Lag');
subplot(2,2,2); plot(w/(2*pi),fxx_2);title('\rho_{1}=1.7,\rho_{2}=-0.8');
ylabel('Spectrum');xlabel('Cycles per quarter');
subplot(2,2,4); plot(0:1:99,acf_2(1:100));
ylabel('Autocorrelation Function');xlabel('Lag');
saveas(figure(1),'exercise4.png')
```

Exercise 5

Figure 2 plots the results of the estimation of the asymptotic variance under the three different specifications. From the picture it is clear that the most accurate method is the Prewhitening and Recoloring method is the one that gets closer to the true variance and that the Newey-West method (with 4 lags) has the estimate that is the furthest among the three, probably because of the small sample size. The Smoothed Periodogram, instead is in the middle between the other two methods. This result is consistent with what we have seen in class, when we stated that it is best practice to use Prewhitening and Recoloring as the chi-squared distributions that are employed in Smoothed Periodogram works better in finite samples (while here we have 1000 simulations).

Figure 2: Asymptotic Variance under different methods



Code

```
%% Exercise 5
clear
rho   = 0.9; sigma = 1; T     = 100; N     = 1000; nlag  = 4;
% True Variance
[autocorr, gamma0] = acf(rho,0,99);
acv                = autocorr.*gamma0;
fxx                = abs(fft(acv/(pi)));
TrueVar            = 2*pi*fxx(1)/T;
rng(12345);
epsilon = randn(T,N);
for n=1:N
x(1,n) = epsilon(1,n);
for t=2:T
x(t,n) = rho*x(t-1,n)+epsilon(t,n);
end
end
xhat = mean(x);
```

```
% Newey–West
for n=1:N
xhat_m          = x(:,n)-xhat(:,n);
nw      = (xhat_m'*xhat_m)/T;
for pp=1:nlag
w       = (nlag+1-pp)./(nlag+1);
gamma = (T^(-1))*((xhat_m(1+pp:T))'*(xhat_m(1:T-pp)));
gamma_p   = gamma+gamma';
nw = nw+w*gamma_p;
end
AsyNW(n,1)=nw/sqrt(T);

% Smoothed Periodogram
width       = 5;
dZ          = fft(x(:,n),2*T);
I           = inv(pi*T)*abs(dZ(1:T,:)).^2;
I_smooth    = conv2(I,ones(width,1)/width,'same');
AsySP(:,n)= 2*pi*I_smooth(1)/T;

% Prewhitening & Recoloring
width         = 51;
r             = x(1:end-1,n) \ x(2:end,n);
u             = x(2:end,n)- r*x(1:end-1,n);
dZ1           = fft(u,2*T);
I1            = inv(pi*T)*abs(dZ1(1:T,:)).^2;
I1_smooth     = conv2(I1,ones(width,1)/width,'same');
fxx_PW        = abs((1 - r*exp(-1i*(1:100)'*(pi/100))).^(-1).*I1_smooth ...
.*(1 - r*exp(1i*(1:100)'*(pi/100))).^(-1));
AsyPW(:,n)   = 2*pi*fxx_PW(1)/T;
end
figure(2)
subplot(3,1,1)
histogram(AsyNW,'BinWidth',0.2,'FaceColor','green');
title('Newey–West');xlim([0 8]), ylim([0 300])
set(vline([TrueVar, mean(AsyNW)],{'r--','b-.'},{'True','Sample'}),'LineWidth',2)
subplot(3,1,2)
histogram(AsySP,'BinWidth',0.2,'FaceColor','green');
title('Smoothed Periodogram');xlim([0 8]), ylim([0 300])
set(vline([TrueVar, mean(AsySP)],{'r--','b-.'},{'True','Sample'}),'LineWidth',2)
subplot(3,1,3)
histogram(AsyPW,'BinWidth',0.2,'FaceColor','green');
```

```
title('Prewhitening & Recoloring');xlim([0 8]), ylim([0 300])
set(vline([TrueVar, mean(AsyPW)],{'r--','b-.'},{'True','Sample'}),'LineWidth',2)
saveas(figure(2),'exercise5.png')
```