Convolutional layer-Convolution

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1 Convolution

Let off the concept of convolution first. For simplicity, consider an image of 5 x 5 and a 3 x 3 convolution kernel. There are 9 parameters in the convolution kernel here. In this case, the convolution kernel actually has 9 neurons, and their output forms a 3 x 3 matrix, called the feature graph. The first neuron is connected to the first 3 x 3 part of the image, and the second neuron is connected to second part. As shown in the following figure 1.

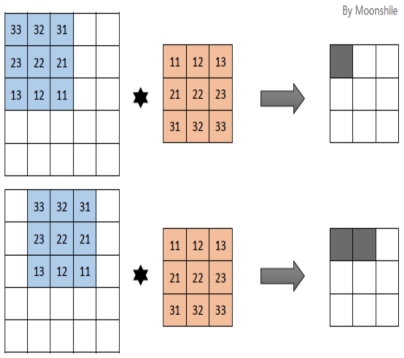

Figure 1: Convolution Process

Above the graph is the output of the first neuron, and below is the output of the second neurons. The formula of each neuron is

$$f(x) = act(\sum_{i,j}^{n} \theta_{(n-i)(n-j)} x_{ij} + b) \tag{1}$$

Now let's recollect the discrete convolution operation. Assuming that there are two dimensional discrete functions f (x, y) and g (x, y), their convolution is defined as:

$$f(m,n) * g(m,n) = \sum_{u}^{\infty} \sum_{v}^{\infty} f(u,v) g(m-u, n-v) \tag{2}$$

The 9 neurons in the above example are actually equivalent to the convolution operation of the image and convolution kernel after the output of the neurons is completed. [1]

References

[1] Cheng Guan. Convolutional - convolution. *CSDN*, 135(6):269–284, 2018.