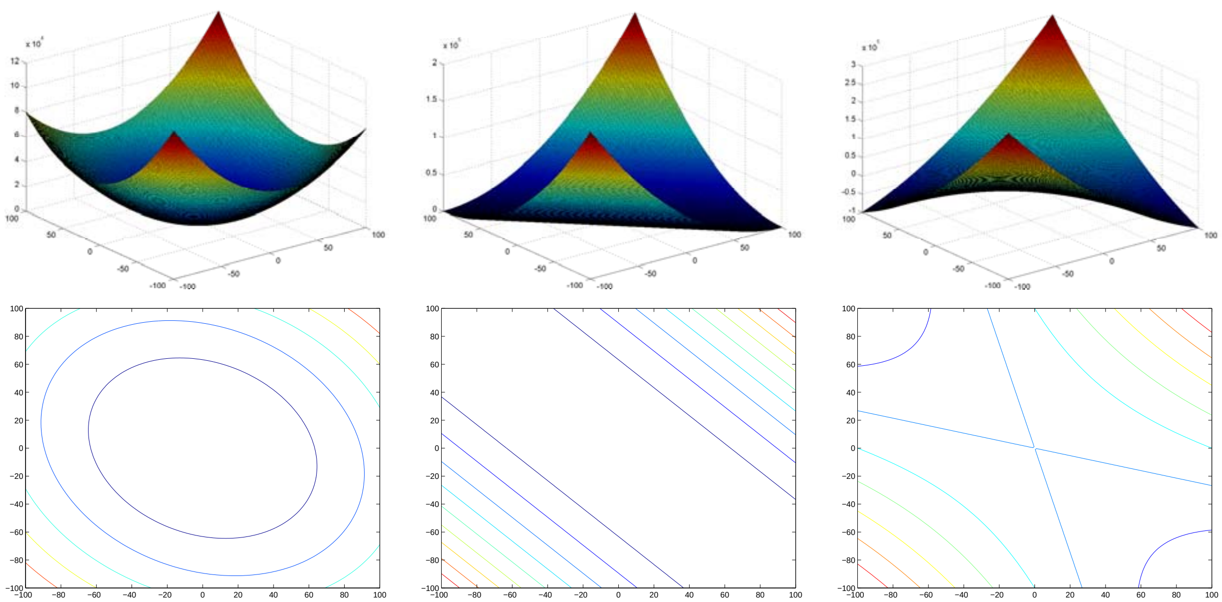
Solutions to Homework Set Two

ECE 175
Electrical and Computer Engineering
University of California San Diego

Nuno Vasconcelos

1. a)



$b = 1$ $\qquad$ $b = 5$ $\qquad$ $b = 10$

b) The eigenvalues are:

$b = 1$	$b = 5$	$b = 10$
$(6, 4)$	$(10, 0)$	$(15, -5)$

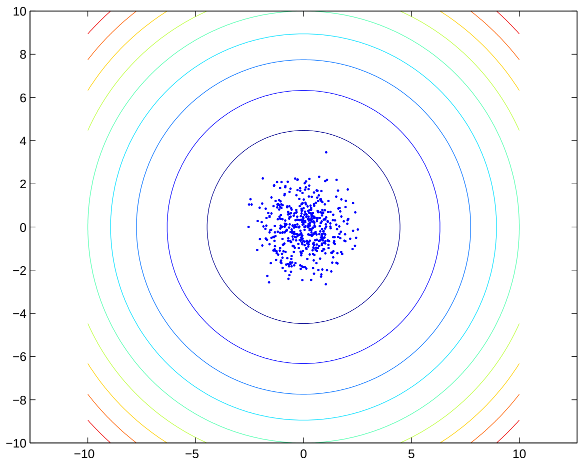
c) one of the criteria for positive-definiteness is that the matrix can only have positive eigenvalues. Also, semi-definite positive matrices only have non-negative eigenvalues. Hence

$b = 1$	$b = 5$	$b = 10$
positive definite	semi-positive definite	neither

d) When both eigenvalues are positive, the function has positive curvature along the two eigenvector directions. Hence it is a bowl. A zero eigenvector means no curvature along the associated eigenvector. Hence the function is flat, or 1D. A negative eigenvalue means negative curvature along one of the eigenvectors. Hence the function is "cup-like" in one dimension, and "cap-like" in the second. This means that the function is a saddle. Note that, in this example, the eigenvectors were $(1, 1)$ and $(1, -1)$ for the three cases.

e) When $b = 10$, $f(\mathbf{x})$ is negative for some $\mathbf{x}$. So, it cannot be a norm. When $b = 5$, $f(\mathbf{x}) = 0$ for some $\mathbf{x} \neq \mathbf{0}$. So, it cannot be a norm. The only function that satisfies all the necessary conditions of a norm is $b = 1$. It is, therefore, not surprising that this is the only case where we have a positive definite function.

2. a)



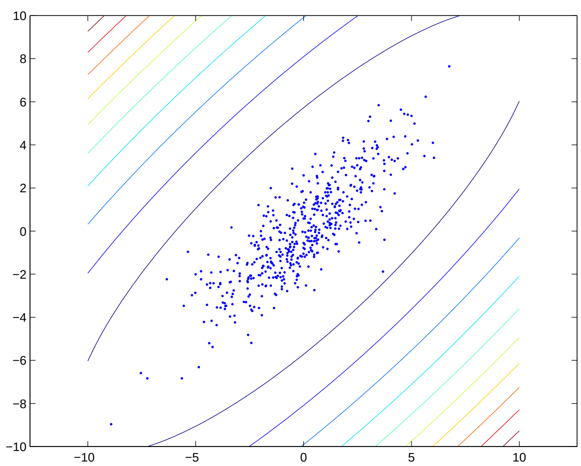
b) According to MATLAB, the new covariance is

$$\begin{bmatrix} 5.6773 & 4.5470 \\ 4.5470 & 5.2784 \end{bmatrix}.$$

We know that the covariance is

$$\mathbf{C} = \mathbf{T}\mathbf{T}^T = \begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix},$$

which is close enough. The Mahalanobis distance has matrix $(\mathbf{T}\mathbf{T}^T)^{-1}$ instead of the identity. The data looks like the following.



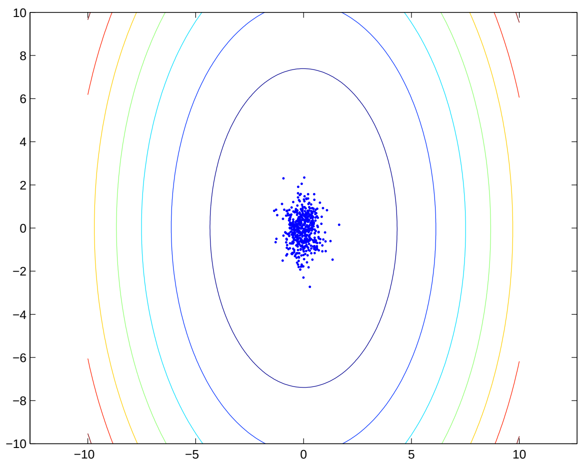
c) We now need

$$\mathbf{C} = \mathbf{T}\mathbf{T}^T = \begin{bmatrix} 1/5 & 0 \\ 0 & 1/2 \end{bmatrix},$$

or

$$\mathbf{T} = \begin{bmatrix} 1/\sqrt{5} & 0 \\ 0 & 1/\sqrt{2} \end{bmatrix}.$$

In this case, the data looks like



3. The error rates for each class, i.e. P(Error|Class = i) for all $i = 0 \ldots 9$ are given in the following table.

Table 1: Probability of error for each class

0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
0.00	**0.00**	**0.13**	**0.16**	**0.13**	**0.06**	**0.05**	**0.02**	**0.25**	**0.19**

The total error rate is P(error) = 0.094. The following figure shows some of the misclassified images. We can see from these images that the NN does a good job of classification based on the exact visual qualities of the images. In the misclassified images too, the test images were not clear visually and hence matched 'better' with other classes on a visual level. Note that this is the case as we used the image intensities as our feature space to start with. If we are able to construct another feature space that has the structure of the digits, we might be able to do better.

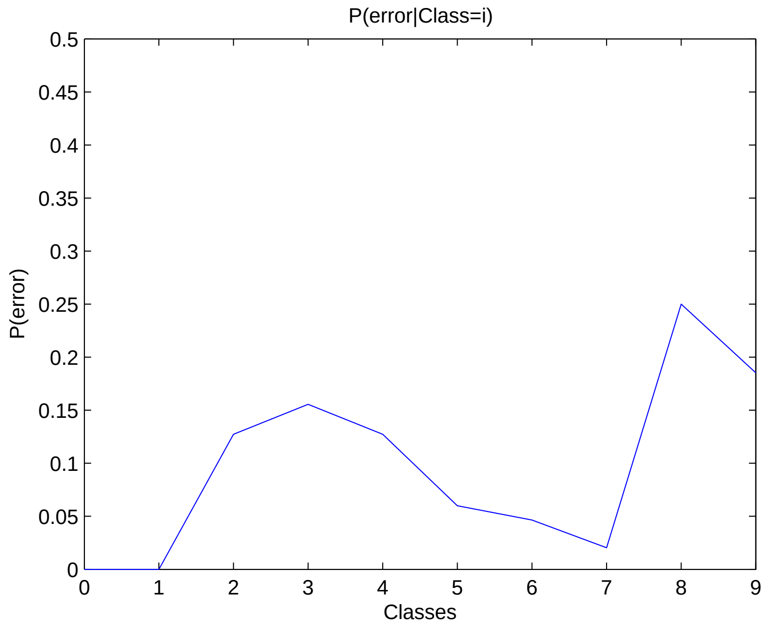


Figure 1: Probability of error for each class

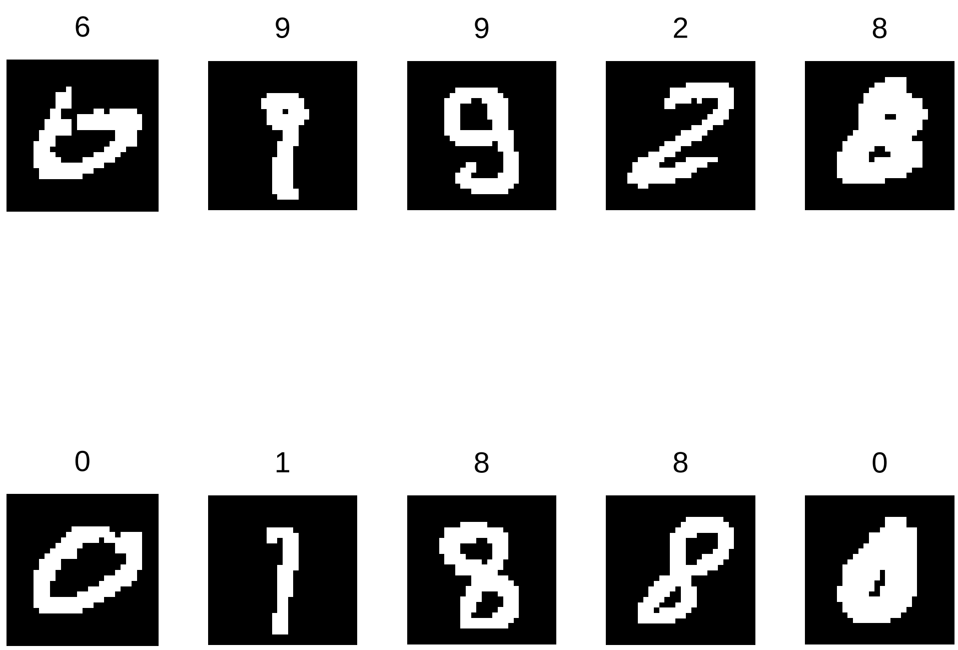


Figure 2: Misclassification, First row are the test images and the second row is the corresponding nearest image in the training database