Homework 6

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1 PCA

1.1

(a) For any i, we have:

$$\frac{\partial J}{\partial p_{i2}} = 2e_{i2}^T(x_i - p_{i1}e_1 - p_{i2}e_2)$$
$$= 2(e_{i2}^T x_i - p_{i1}e_{i2}^T e_1 - p_{i2}e_{i2}^T e_2)$$
$$= 2(e_{i2}^T - p_{i2}) = 0$$
$$p_{i2} = e_2^T x_i$$

(b)

$$\frac{\partial \tilde{J}}{\partial e_2} = -2Se_2 + 2\lambda_2 e_2 + \lambda_{12}e_1 = 0$$
$$\tilde{e_2} = \frac{1}{2}(S - I\lambda)^{-1}\lambda_{12}e_1$$
$$\frac{\partial \tilde{J}}{\partial \lambda_2} = e_2^T e_2 - 1 = 0$$
$$e_2^T e_2 = 1$$
$$\frac{\partial \tilde{J}}{\partial \lambda_{12}} = e_2^T e_1 - 0 = 0$$
$$e_2^T e_1 = 0$$

so e_2 is the second largest eigenvalues of S

1.2

(a)
eigenvalue : 1626.52644399, 7.09745924, 28.98609676
corresponding eigenvector:

$$(0.21793758, 0.41449518, 0.88357057)^T$$
$$(0.94428286, -0.31834854, -0.0835709)^T$$
$$(-0.24664366, -0.85255378, 0.46078081)^T$$

(b)
eigenvector $(0.94428286, -0.31834854, -0.0835709)^T$ with eigenvalue 7.09745924, because it has minimum eigenvalue, which means it has lowest information (minimum variance)
(c)
eigenvector $(0.21793758, 0.41449518, 0.88357057)^T$ with eigenvalue 1626.52644399 contains the most of information(maximum variance)

2 HMM

(a)

$$\alpha_1(1) = b_{1A}\pi_1 = 0.28$$
$$\alpha_1(2) = b_{2A}\pi_2 = 0.06$$
$$\alpha_2(1) = b_{1G}(\alpha_1(1)a_{11} + \alpha_1(2)a_{21}) = 0.0992$$
$$\alpha_2(2) = b_{2G}(\alpha_1(1)a_{12} + \alpha_1(2)a_{22}) = 0.0184$$
$$\alpha_3(1) = b_{1C}(\alpha_2(1)a_{11} + \alpha_2(2)a_{21}) = 0.008672$$
$$\alpha_3(2) = b_{2C}(\alpha_2(1)a_{11} + \alpha_2(2)a_{22}) = 0.009264$$
$$\alpha_4(1) = b_{1G}(\alpha_3(1)a_{11} + \alpha_3(2)a_{21}) = 0.00425728$$
$$\alpha_4(2) = b_{2G}(\alpha_3(1)a_{12} + \alpha_3(2)a_{22}) = 0.00145856$$
$$\alpha_5(1) = b_{1T}(\alpha_4(1)a_{11} + \alpha_4(2)a_{21}) = 0.0003989248$$
$$\alpha_5(2) = b_{2T}(\alpha_4(1)a_{12} + \alpha_4(2)a_{22}) = 0.0005179776$$
$$\alpha_6(1) = b_{1A}(\alpha_5(1)a_{11} + \alpha_5(2)a_{21}) = 0.000210532352$$
$$\alpha_6(2) = b_{2A}(\alpha_5(1)a_{12} + \alpha_5(2)a_{22}) = 0.00007811$$
$$P(O;\Theta) = \alpha_6(1) + \alpha_6(2) = 0.000288642352$$

(b)

$$\beta_6(1) = 1$$
$$\beta_6(2) = 1$$
$$\beta_5(1) = (\beta_{1A}a_{11}\beta_6(1) + \beta_{2A}a_{12}\beta_6(2)) = 0.36$$
$$\beta_5(2) = (\beta_{1A}a_{21}\beta_6(1) + \beta_{2A}a_{22}\beta_6(2)) = 0.28$$
$$\beta_4(1) = (\beta_{1T}a_{11}\beta_5(1) + \beta_{2T}a_{12}\beta_5(2)) = 0.0456$$
$$\beta_4(2) = (\beta_{1T}a_{21}\beta_5(1) + \beta_{2T}a_{22}\beta_5(2)) = 0.0648$$
$$\beta_3(1) = (\beta_{1G}a_{11}\beta_4(1) + \beta_{2G}a_{12}\beta_4(2)) = 0.017184$$
$$\beta_3(2) = (\beta_{1G}a_{21}\beta_4(1) + \beta_{2G}a_{22}\beta_4(2)) = 0.015072$$
$$\beta_2(1) = (\beta_{1C}a_{11}\beta_3(1) + \beta_{2C}a_{12}\beta_3(2)) = 0.00227904$$
$$\beta_2(2) = (\beta_{1C}a_{21}\beta_3(1) + \beta_{2C}a_{22}\beta_3(2)) = 0.00340032$$
$$\beta_1(1) = (\beta_{1G}a_{11}\beta_2(1) + \beta_{2G}a_{12}\beta_2(2)) = 0.0008643056$$
$$\beta_1(2) = (\beta_{1G}a_{21}\beta_2(1) + \beta_{2G}a_{22}\beta_2(2)) = 0.0007726848$$
$$P(X_6 = S_1|O;\theta) = \frac{\alpha_6(1)\beta_6(1)}{\alpha_6(1)\beta_6(1) + \alpha_6(2)\beta_6(2)} = 0.72938$$
$$P(X_6 = S_2|O;\theta) = 1 - P(X_6 = S_1|O;\theta) = 0.27062$$

(c)

$$P(X_4 = S_1|O;\theta) = \frac{\alpha_4(1)\beta_4(1)}{\alpha_4(1)\beta_4(1) + \alpha_4(2)\beta_4(2)} = 0.672559$$
$$P(X_4 = S_2|O;\theta) = 1 - P(X_4 = S_1|O;\theta) = 0.327441$$

(d)

$$P(X_1 = S_1|O;\theta) = \frac{\alpha_1(1)\beta_1(1)}{\alpha_1(1)\beta_1(1) + \alpha_1(2)\beta_1(2)} = 0.83938$$
$$P(X_1 = S_2|O;\theta) = 1 - P(X_1 = S_1|O;\theta) = 0.16062$$

max is S_1

$$P(X_2 = S_1|O;\theta) = \frac{\alpha_2(1)\beta_2(1)}{\alpha_2(1)\beta_2(1) + \alpha_2(2)\beta_2(2)} = 0.78324$$
$$P(X_2 = S_2|O;\theta) = 1 - P(X_2 = S_1|O;\theta) = 0.21676$$

max is S_1

$$P(X_3 = S_1|O;\theta) = \frac{\alpha_3(1)\beta_3(1)}{\alpha_3(1)\beta_3(1) + \alpha_3(2)\beta_3(2)} = 0.51627$$
$$P(X_3 = S_2|O;\theta) = 1 - P(X_3 = S_1|O;\theta) = 0.48373$$

max is S_1

$$P(X_5 = S_1|O;\theta) = \frac{\alpha_5(1)\beta_5(1)}{\alpha_5(1)\beta_5(1) + \alpha_5(2)\beta_5(2)} = 0.49753$$
$$P(X_5 = S_2|O;\theta) = 1 - P(X_5 = S_1|O;\theta) = 0.50247$$

max is S_2
Most likely explanation is $S_1S_1S_1S_1S_2S_1$
(e)

A:

$$P(O_7 = A|O;\theta) = P(X_6 = S_1|O;\theta)b_{1A} + P(X_6 = S_2|O;\theta)b_{2A} = 0.345876$$

C:

$$P(O_7 = C|O;\theta) = P(X_6 = S_1|O;\theta)b_{1C} + P(X_6 = S_2|O;\theta)b_{2C} = 0.154124$$

G:

$$P(O_7 = G|O;\theta) = P(X_6 = S_1|O;\theta)b_{1G} + P(X_6 = S_2|O;\theta)b_{2G} = 0.345876$$

T:

$$P(O_7 = T|O;\theta) = P(X_6 = S_1|O;\theta)b_{1T} + P(X_6 = S_2|O;\theta)b_{2T} = 0.154124$$

A,G is most likely after $o_{1:6}$