Reinforcement Learning

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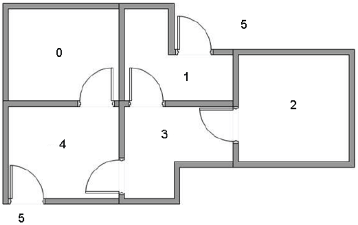
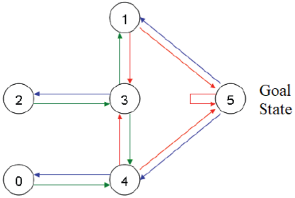
Reinforcement Learning

Example: Intelligent Navigation



Building Map



Graph

Table of Contents

Reinforcement Learning

Reinforcement Learning

- Agent and environment
- Action a, reward r
- *Policy* (π): State (s) and action $a \triangleq \pi(s)$
- *Samples*: $(s_1, a_1, r_1, \cdots, s_t)$, $t = 1, 2, \cdots$
- *Reinforcement learning*: $\max(r)$,
  s.t. $(s_1, a_1, r_1, \cdots, s_t) \to \pi$

MDP: Markov Decision Process

- A state s_t is *Markov* if $P(s_{t+1}|s_t) = P(s_{t+1}|s_1, \cdots, s_t)$.
- *Value function*: $v(s) \triangleq \mathbb{E}(G_t|s_t)$
- *Return*: $G_t \triangleq \sum_{k=0}^{\infty} \lambda^k \cdot r_{t+k+1}$, λ is a discount factor.

M. Littman (2015) Reinforcement learning improves behaviour from evaluative feedback. Nature, 521: 445-451.

Reinforcement Learning

Bellman Equation

$\because$ *Value function:* $v(s) = \mathbb{E}(G_t|s_t) = \mathbb{E}(\sum_{k=0}^{\infty} \lambda^k \cdot r_{t+k+1}|s_t)$
$\therefore v(s) = \mathbb{E}(r_{t+1} + \lambda \cdot G_{t+1}|s_t)$; $v(s) = \mathbb{E}(r_{t+1} + \lambda \cdot v(s_{t+1})|s_t)$
$\therefore$ *Action-value function:*

$$Q^{\pi}(s,a) \triangleq \mathbb{E}_{s'}(r + \lambda \cdot Q^{\pi}(s',a')|s,a)$$

Optimal action-value function:

$$Q^{*}(s,a) = \mathbb{E}_{s'}(r + \lambda \cdot \max_{a'} Q^{*}(s',a')|s,a)$$

Iteratively,

$$Q_{i+1}(s,a) = \mathbb{E}_{s'}(r + \lambda \cdot \max_{a'} Q_i(s',a')|s,a) \rightarrow Q^{*}, \text{ if } i \rightarrow \infty$$

MDP: Markov Decision Process

- A state s_t is Markov if $P(s_{t+1}|s_t) = P(s_{t+1}|s_1, \cdots, s_t)$.
- Value function: $v(s) \triangleq \mathbb{E}(G_t|s_t)$
- Return: $G_t \triangleq \sum_{k=0}^{\infty} \lambda^k \cdot r_{t+k+1}$, λ is discount factor.

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Reinforcement Learning

Bellman Equation

$\because$ Value function: $v(s) = \mathbb{E}(G_t|s_t) = \mathbb{E}(\sum_{k=0}^{\infty} \lambda^k \cdot r_{t+k+1}|s_t)$
$\therefore$ $v(s) = \mathbb{E}(r_{t+1} + \lambda \cdot G_{t+1}|s_t)$; $v(s) = \mathbb{E}(r_{t+1} + \lambda \cdot v(s_{t+1})|s_t)$
$\therefore$ Action-value function:

$$Q^\pi(s,a) \triangleq \mathbb{E}_{s'}(r + \lambda \cdot Q^\pi(s',a')|s,a)$$

The optimal action-value function:

$$Q^*(s,a) = \mathbb{E}_{s'}(r + \lambda \cdot \max_{a'} Q^*(s',a')|s,a)$$

Iteratively,

$$Q_{i+1}(s,a) = \mathbb{E}_{s'}(r + \lambda \cdot \max_{a'} Q_i(s',a')|s,a) \rightarrow Q^*, \text{ if } i \rightarrow \infty$$

Q-Learning

$$Q(s_t,a_t) \leftarrow Q(s_t,a_t) + \alpha(r_{t+1} + \lambda \cdot \max_a Q(s_{t+1},a) - Q(s_t,a_t))$$

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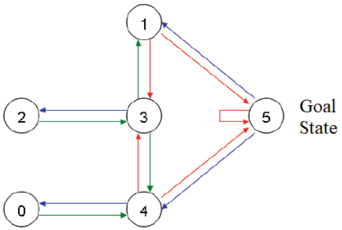
Reinforcement Learning

Deep Q-Learning



Q-Learning

$$Q(s_t, a_t) \leftarrow Q(s_t, a_t) + \alpha(r_{t+1} + \lambda \cdot \max_a Q(s_{t+1}, a) - Q(s_t, a_t))$$

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Deep Q-Learning

- For a deep network *w.r.t.* Q and weight w,

$$Q(s, a, w) = Q^{\pi}(s, a).$$

- The loss/objective function is,

$$L(w) \triangleq \mathbb{E}([r + \gamma \cdot \max_{a'} Q(s', a', w) - Q(s, a, w)]^2)$$

- The gradient is,

$$\frac{\partial L(w)}{\partial w} = \mathbb{E}([r + \gamma \cdot \max_{a'} Q(s', a', w) - Q(s, a, w)]) \cdot \frac{\partial Q(s, a, w)}{\partial w}$$

V.Mnih, et al. (2015) Human-level control through deep reinforcement learning. Nature, 518: 529-533.

Questions?

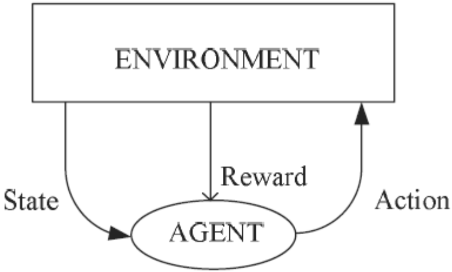
Policy and Value Iterations

- Reinforcement learning is learning what to do - how to map situations to actions — so as to *maximize a numerical reward.*
- Reinforcement learning receives *reward/penalty* or *trial/error* for its actions solve a problem.
- Reinforcement learning could *learn the best policy* and *maximize the total reward.*
- The *sequence of actions* has the maximum cumulative reward.
- For each policy π, there is a reward $v^{\pi}(s_t)$, we hope to find the *optimal policy*,

$$v^{*}(s_t) = \max_{\pi}(v^{\pi}(s_t)), \forall s_t$$

E. Alpaydn (2010) Introduction to Machine Learning (2nd Eition). The MIT Press.

Policy and Value Iterations



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Policy and Value Iterations

In a simple case, action $a(t) \triangleq \pi(s_t)$, $Q(a_t) \triangleq r(a_t) > 0$. If $r(a)$ is the reward,

$$Q(a_{t+1}) \leftarrow Q_t(a) + \eta \cdot [r(a_{t+1}) - Q(a_t)]$$

In a *full* reinforcement learning, the value of state s_t satisfies,

$$v(s_t) = \max_{a_t} Q(s_t, a_t);$$

where

$$a_t^* = \arg\max_{a_t} Q(s_t, a_t); \pi^*(s_t^*) = a_t^*;$$

E. Alpaydn (2010) Introduction to Machine Learning (2nd Eition). The MIT Press.

Policy and Value Iterations

Value iteration:

$$v(s_t) \leftarrow v(s_t) + \eta \cdot [r_{t+1} + \gamma \cdot v(s_{t+1}) - v(s_t)]$$

Termination condition:

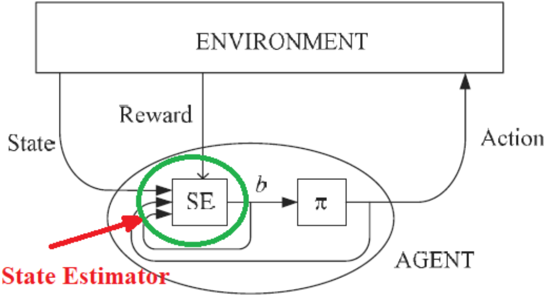
$$|v^{(l+1)}(s) - v^{(l)}(s)| < \delta; \delta > 0; l = 1, 2, 3, \cdots$$

Policy iteration:

$$\pi \leftarrow \pi' = \arg\max_{\pi}(v^{\pi}(s')); v^{\pi}(s) \leftarrow v^{\pi}(s');$$

V. Mnih, et al. (2016) Asynchronous Methods for Deep Reinforcement Learning. International Conference on Machine Learning, pp.1928-1937.

Policy and Value Iterations



E. Alpaydn (2010) Introduction to Machine Learning (2nd Eition). The MIT Press.

Monte-Carlo Method

- **Episode**: $\exists T$, $(s_1, a_1, r_2, \cdots, s_T) \to \pi$
- **Monte-Carlo Method**: Using empirical mean to replace Bellman equation instead of expected return, i.e.,

$$v_\pi(s) = \frac{1}{T}\sum_{t=1}^{T}(G_t | s_t = s)$$

where $G_t = \sum_{k=1}^{T-t} \lambda^{k-1} r_{t+k}$. Hence,

$$v(s_t) \leftarrow v(s_t) - \alpha \cdot (G_t - v(s_t))$$

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TD: Temporal Difference Method

- **Episode**: $\exists T$, $(s_1, a_1, r_2, \cdots, s_T) \to \pi$
- **Monte-Carlo Method**:

$$v(s_t) \leftarrow v(s_t) - \alpha \cdot (G_t - v(s_t))$$

- **Temporal Difference (TD)**:

$$v(s_t) \leftarrow v(s_t) - \alpha \cdot (r_{t+1} + \gamma \cdot v(s_{t+1}) - v(s_t))$$

  where $\delta = r_{t+1} + \gamma \cdot v(s_{t+1}) - v(s_t)$ is named as the *TD error*, $r_{t+1} + \gamma \cdot v(s_{t+1})$ is called the *TD target*.
- **Dynamic Programming Method**:

$$v(s_t) = \mathbb{E}_\pi(r_{t+1} + \gamma \cdot v(s_{t+1}))$$

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Double Q-Learning

- **Episode**: $\exists T,\ (s_1, a_1, r_2, \cdots, s_T) \to \pi$
- **SARSA (State-Action-Reward-State-Action)** :
  $Q(s,a) \leftarrow Q(s,a) + \alpha \cdot [r + \gamma \cdot Q(s', a') - Q(s,a)]$
  $s \leftarrow s'$, $a \leftarrow a'$.
- **Q-Learning** (an *off-policy* TD control algorithm):
  $Q(s,a) \leftarrow Q(s,a) + \alpha \cdot [r + \gamma \cdot \max_a Q(s', a) - Q(s,a)]$,
  $s \leftarrow s'$, $a \leftarrow a'$.
- **Double Q-Learning:**
  $Q_1(s,a) \leftarrow$
  $Q_1(s,a) + \alpha \cdot [r + \gamma \cdot Q_2(s', \arg\max_a Q_1(s', a)) - Q_1(s,a)]$,
  $Q_2(s,a) \leftarrow$
  $Q_2(s,a) + \alpha \cdot [r + \gamma \cdot Q_1(s', \arg\max_a Q_2(s', a)) - Q_2(s,a)]$

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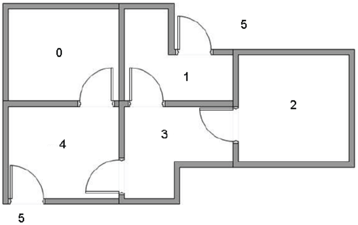
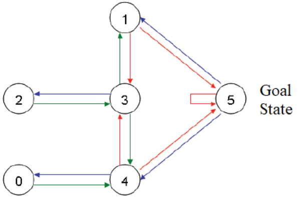
Questions?

Example: Intelligent Navigation



Building Map



Graph

Reinforcement Learning

Example: Intelligent Navigation

$$
R = \begin{array}{c c}
 & \text{Action} \\
\text{State} & \begin{array}{cccccc} 0 & 1 & 2 & 3 & 4 & 5 \end{array} \\
\begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} &
\begin{bmatrix}
-1 & -1 & -1 & -1 & 0 & -1 \\
-1 & -1 & -1 & 0 & -1 & 100 \\
-1 & -1 & -1 & 0 & -1 & -1 \\
-1 & 0 & 0 & -1 & 0 & -1 \\
0 & -1 & -1 & 0 & -1 & 100 \\
-1 & 0 & -1 & -1 & 0 & 100
\end{bmatrix}
\end{array}
\qquad
Q = \begin{array}{c c}
 & \begin{array}{cccccc} 0 & 1 & 2 & 3 & 4 & 5 \end{array} \\
\begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} &
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\end{array}
$$

Step 0

Example: Intelligent Navigation

$$Q(s,a) = r(s,a) + \lambda \cdot \max_{a'} Q(s',a')$$

$$R = \begin{array}{c} \\ \text{State} \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \text{Action} \\ \begin{array}{cccccc} 0 & 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} -1 & -1 & -1 & -1 & 0 & -1 \\ -1 & -1 & -1 & 0 & -1 & 100 \\ -1 & -1 & -1 & 0 & -1 & -1 \\ -1 & 0 & 0 & -1 & 0 & -1 \\ 0 & -1 & -1 & 0 & -1 & 100 \\ -1 & 0 & -1 & -1 & 0 & 100 \end{bmatrix} \end{array}$$

$$Q = \begin{array}{c|cccccc} & 0 & 1 & 2 & 3 & 4 & 5 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 4 & 0 & 0 & 0 & 0 & 0 & 0 \\ 5 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}$$

Step 0

Reinforcement Learning

Example: Intelligent Navigation

$$Q(1,5) = R(1,5) + \lambda \cdot \max(Q(5,5), Q(5,4), Q(5,1)) = 100.$$

$$Q(3,1) = R(3,1) + \lambda \cdot \max(Q(1,3), Q(1,5)) = 80, \lambda = 0.8.$$

$$Q = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{cccccc} 0 & 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 100 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Step 1

$$Q = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{cccccc} 0 & 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 100 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 80 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Step 2

Example: Intelligent Navigation

$$Q = \begin{array}{c|c} & \begin{array}{cccccc} 0 & 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 0 & 0 & 0 & 0 & 400 & 0 \\ 0 & 0 & 0 & 320 & 0 & 500 \\ 0 & 0 & 0 & 320 & 0 & 0 \\ 0 & 400 & 256 & 0 & 400 & 0 \\ 320 & 0 & 0 & 320 & 0 & 500 \\ 0 & 400 & 0 & 0 & 400 & 500 \end{bmatrix} \end{array}$$

Step 6x6

$$Q = \begin{array}{c|c} & \begin{array}{cccccc} 0 & 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 0 & 0 & 0 & 0 & 80 & 0 \\ 0 & 0 & 0 & 64 & 0 & 100 \\ 0 & 0 & 0 & 64 & 0 & 0 \\ 0 & 80 & 51 & 0 & 80 & 0 \\ 64 & 0 & 0 & 64 & 0 & 100 \\ 0 & 80 & 0 & 0 & 80 & 100 \end{bmatrix} \end{array}$$

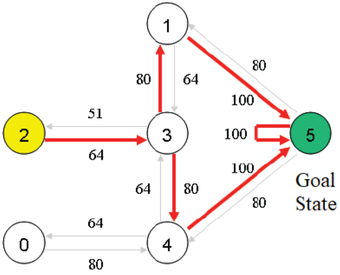
Step 6x6+1

Reinforcement Learning

Example: Intelligent Navigation

$$
Q = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array}
\begin{array}{c}
\begin{array}{cccccc} 0 & 1 & 2 & 3 & 4 & 5 \end{array} \\
\begin{bmatrix}
0 & 0 & 0 & 0 & 80 & 0 \\
0 & 0 & 0 & 64 & 0 & 100 \\
0 & 0 & 0 & 64 & 0 & 0 \\
0 & 80 & 51 & 0 & 80 & 0 \\
64 & 0 & 0 & 64 & 0 & 100 \\
0 & 80 & 0 & 0 & 80 & 100
\end{bmatrix}
\end{array}
$$

Step 36+1

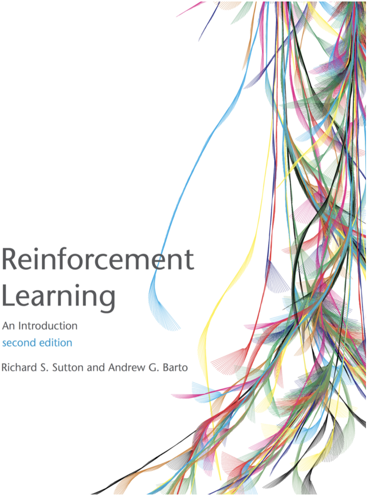


Deep Q-Learning: Implementation

```python
        start_room = index

        print("##################################",

        current_state = start_room

        step = 0

        target_state = 5

        while current_state != target_state:
            out_result = self.session.run(self.q_eval, fee
                self.q_eval_input: self.state_list[current_s

            next_state = np.argmax(out_result[0])

            print("Agent currernt state:", current_state, "

            current_state = next_state

            step += 1

        print("Agent started from room:", start_room, "

        print("##################################",


if __name__ == "__main__":
    q_network = DeepQNetwork()
    q_network.pay()
```

```
 [-1.  0. -1. -1.  0. 100.]]
################################## Agent in room:  0 starts moving ########
Agent currernt state: 0  to next state: 4
Agent currernt state: 4  to next state: 5
Agent started from room: 0  after:  2 steps arrive room 5
################################## Agent at room: 5 Agent is at Room 5 ###
####
################################## Agent in room:  1 starts moving ########
Agent currernt state: 1  to next state: 5
Agent started from room: 1  after:  1 steps arrive room 5
################################## Agent at room: 5 Agent is at Room 5 ###
####
################################## Agent in room:  2 starts moving ########
Agent currernt state: 2  to next state: 3
Agent currernt state: 3  to next state: 4
Agent currernt state: 4  to next state: 5
Agent started from room: 2  after:  3 steps arrive room 5
################################## Agent at room: 5 Agent is at Room 5 ###
####
################################## Agent in room:  3 starts moving ########
Agent currernt state: 3  to next state: 4
Agent currernt state: 4  to next state: 5
Agent started from room: 3  after:  2 steps arrive room 5
################################## Agent at room: 5 Agent is at Room 5 ###
####
################################## Agent in room:  4 starts moving ########
Agent currernt state: 4  to next state: 5
Agent started from room: 4  after:  1 steps arrive room 5
################################## Agent at room: 5 Agent is at Room 5 ###
####
>>>
```

Questions?


Reinforcement
Learning
An Introduction
second edition
Richard S. Sutton and Andrew G. Barto