

# COMP809 Data Mining and Machine Learning

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Classification Basic Concept- Decision Tree (1)



## Weekly Learning Outcomes

- Classification Basic Concept
- Classification Algorithms
- Decision Tree

# Prediction Problems

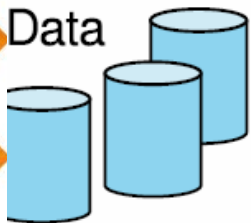
## ■ Classification

- *predicts categorical class labels (discrete or nominal)*
- *classifies data (constructs a model) based on the training set and the values (class labels) in a classifying attribute and uses it in classifying new data*

## ■ Numeric Prediction

- *models continuous-valued functions, i.e., predicts unknown or missing values*

Data



Pre-processing  
Normalization  
Feature selection

- Handling missing data
- Outliers
- PCA
- Signal-to-noise ratio



Modelling  
Learning

- Naive Bayes
- kNN, wkNN
- Decision Trees
- Linear Regression
- Neural Networks
- k-means clustering
- Association Rules

Optimization

- GA
- PSO
- D/E
- EDA
- ES



Evaluation and  
performance  
metrics

- Cross-validation
- Metrics in classification
- Metrics in regression
- Significance tests



# Supervised vs. Unsupervised Learning

## ■ Supervised learning (classification)

- *Supervision: The training data (observations, measurements, etc.) are accompanied by **labels** indicating the class of the observations*
- *New data is classified based on the training set*

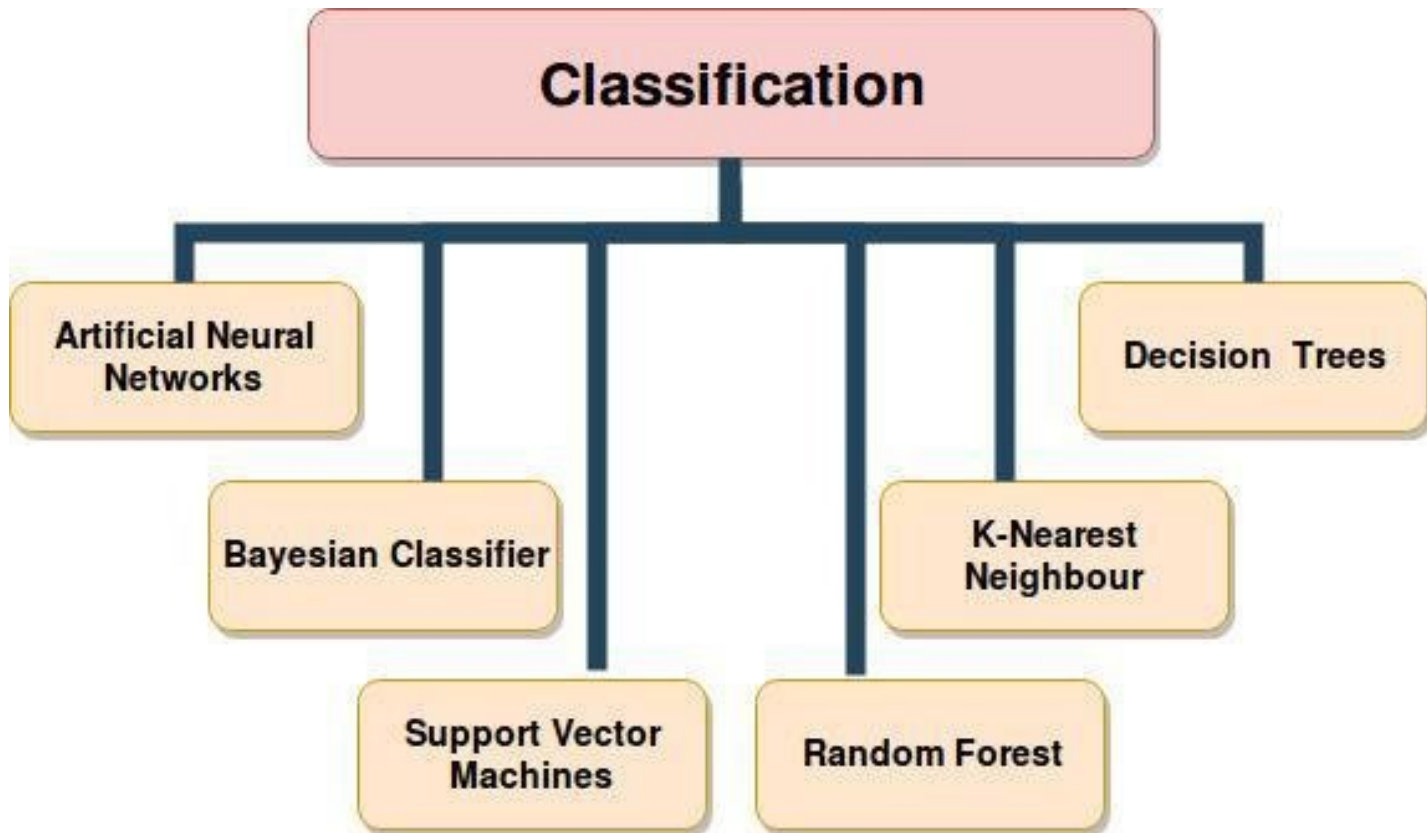
## ■ Unsupervised learning (clustering)

- *The class labels of training data is unknown*
- *Given a set of measurements, observations, etc. with the aim of establishing the existence of classes or clusters in the data*

# Classification: Definition

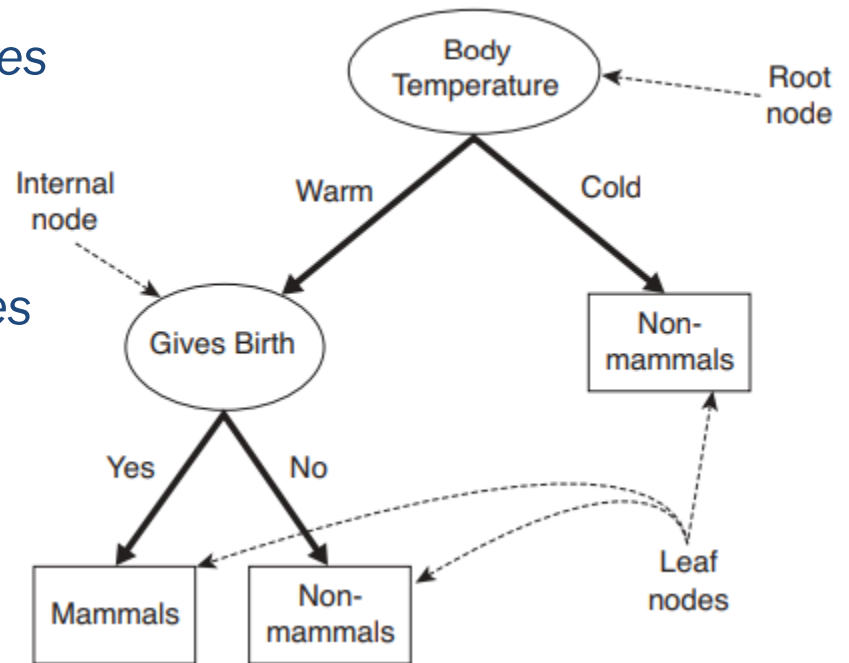
- Given a collection of records (*training set*)
  - *Each record contains a set of **attributes**, one of the attributes is the **class**.*
- Find a *model* for class attribute as a function of the values of other attributes.
- Goal: previously unseen records should be assigned a class as accurately as possible.
  - *A **test set** is used to determine the accuracy of the model.*
  - *Usually, the given data set is divided into training and test sets, with training set used to build the model and test set used to validate it.*

# Classification Algorithms



# Mammals Classification

- Root node
  - No incoming edges
  - Zero or more outgoing edges
- Internal Node
  - One incoming edge
  - Two or more outgoing edges
- Leaf/Terminal Node
  - One incoming edges
  - No outgoing edges





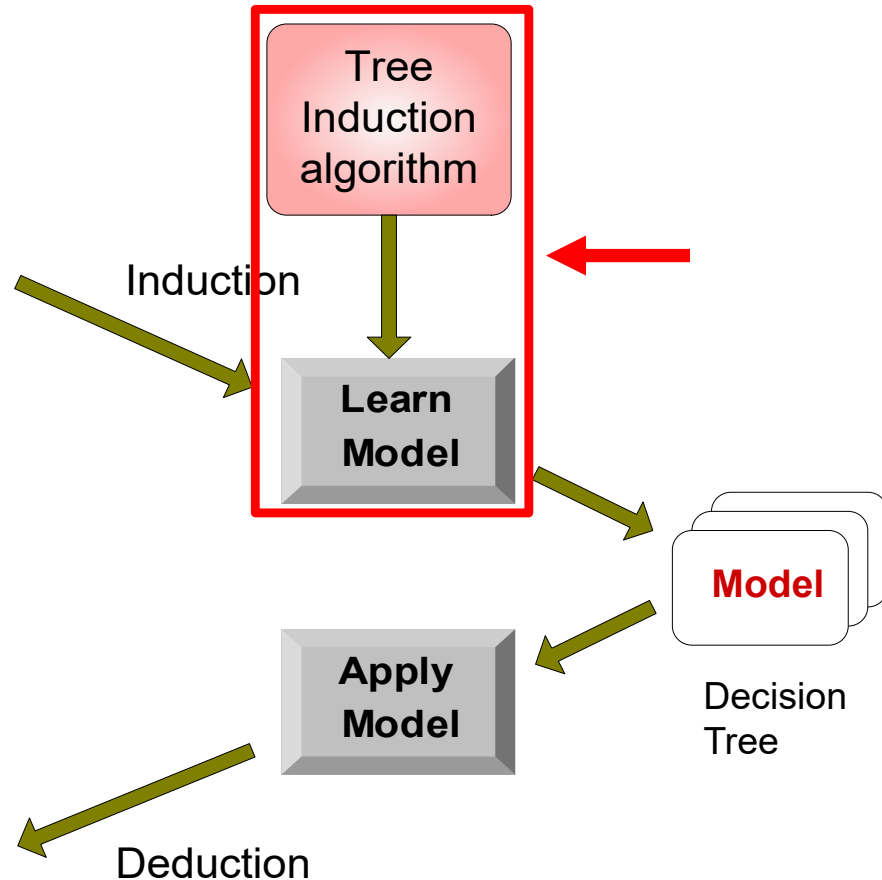
# Decision Tree Classification Task

Tid	Attrib1	Attrib2	Attrib3	Class
1	Yes	Large	125K	No
2	No	Medium	100K	No
3	No	Small	70K	No
4	Yes	Medium	120K	No
5	No	Large	95K	Yes
6	No	Medium	60K	No
7	Yes	Large	220K	No
8	No	Small	85K	Yes
9	No	Medium	75K	No
10	No	Small	90K	Yes

Training Set

Tid	Attrib1	Attrib2	Attrib3	Class
11	No	Small	55K	?
12	Yes	Medium	80K	?
13	Yes	Large	110K	?
14	No	Small	95K	?
15	No	Large	67K	?

Test Set

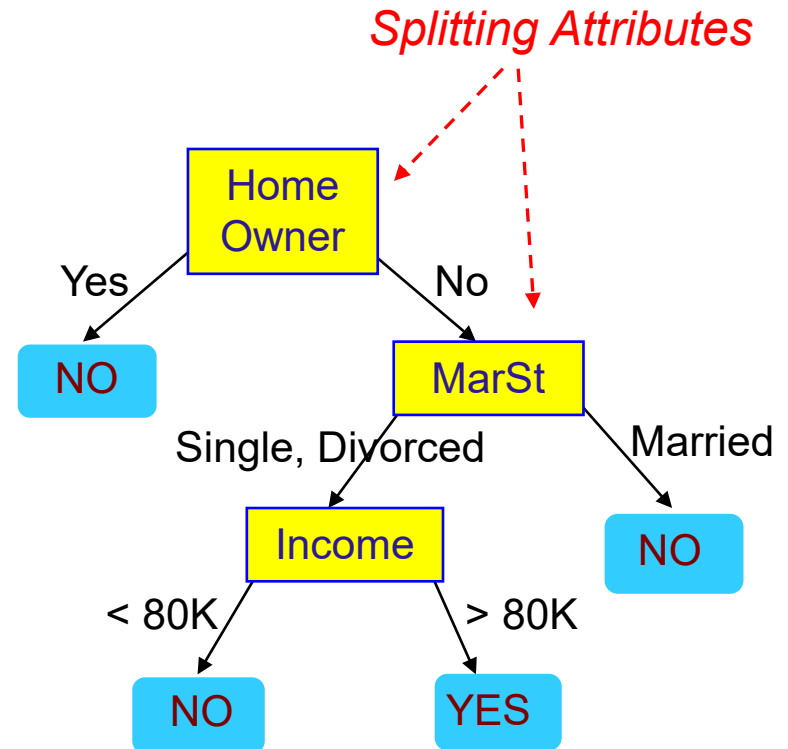


# Example of a Decision Tree

categorical  
categorical  
continuous  
class

ID	Home Owner	Marital Status	Annual Income	Defaulted Borrower
1	Yes	Single	125K	No
2	No	Married	100K	No
3	No	Single	70K	No
4	Yes	Married	120K	No
5	No	Divorced	95K	Yes
6	No	Married	60K	No
7	Yes	Divorced	220K	No
8	No	Single	85K	Yes
9	No	Married	75K	No
10	No	Single	90K	Yes

Training Data



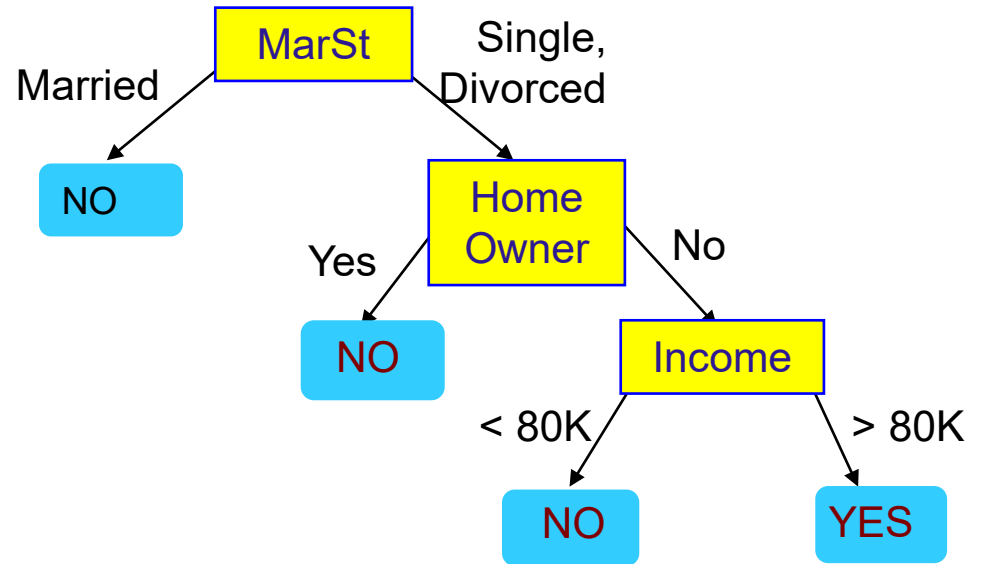
Model: Decision Tree

# Example of a Decision Tree

categorical  
categorical  
continuous  
class

ID	Home Owner	Marital Status	Annual Income	Defaulted Borrower
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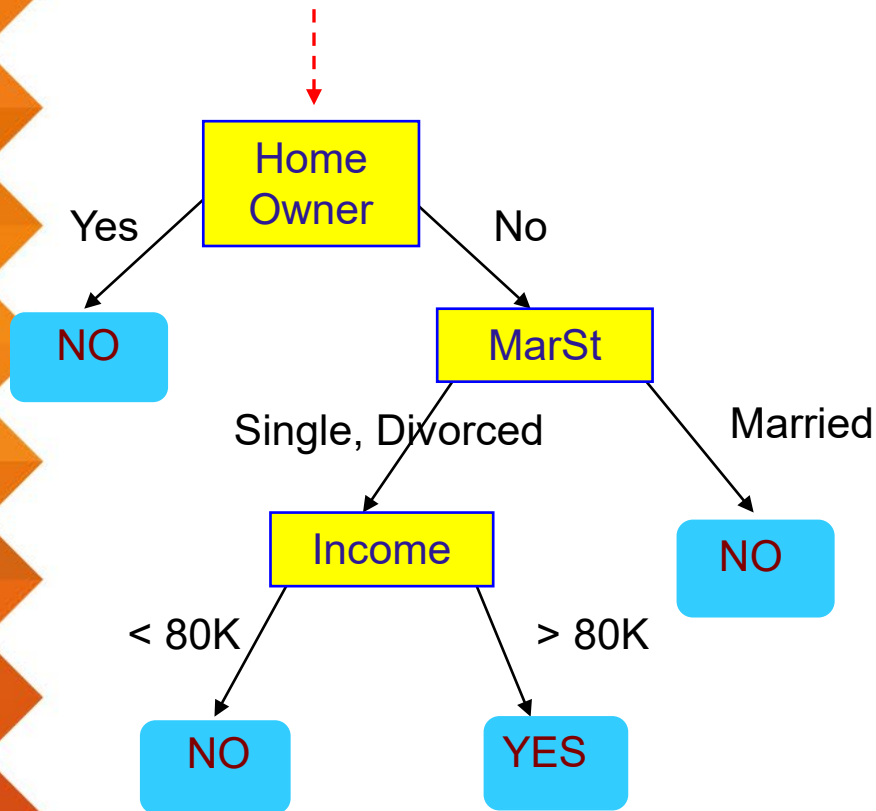
Training Data



There could be more than one tree that fits the same data!

# Apply Model to Test Data

Start from the root of tree.

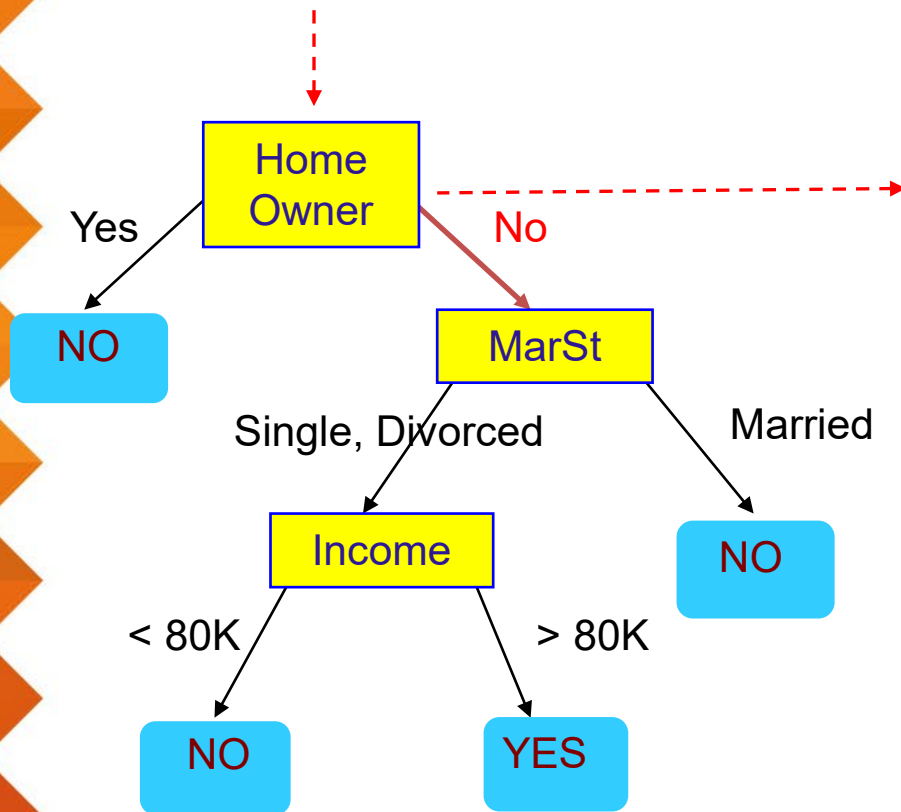


Test Data

Home Owner	Marital Status	Annual Income	Defaulted Borrower
No	Married	80K	?

# Apply Model to Test Data

Start from the root of tree.



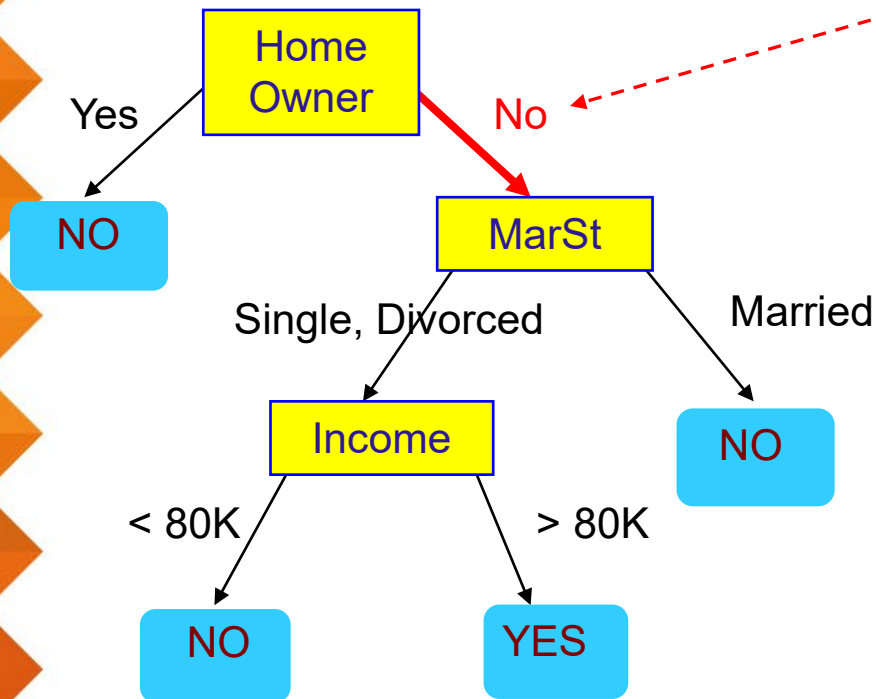
Test Data

Home Owner	Marital Status	Annual Income	Defaulted Borrower
No	Married	80K	?

# Apply Model to Test Data

Test Data

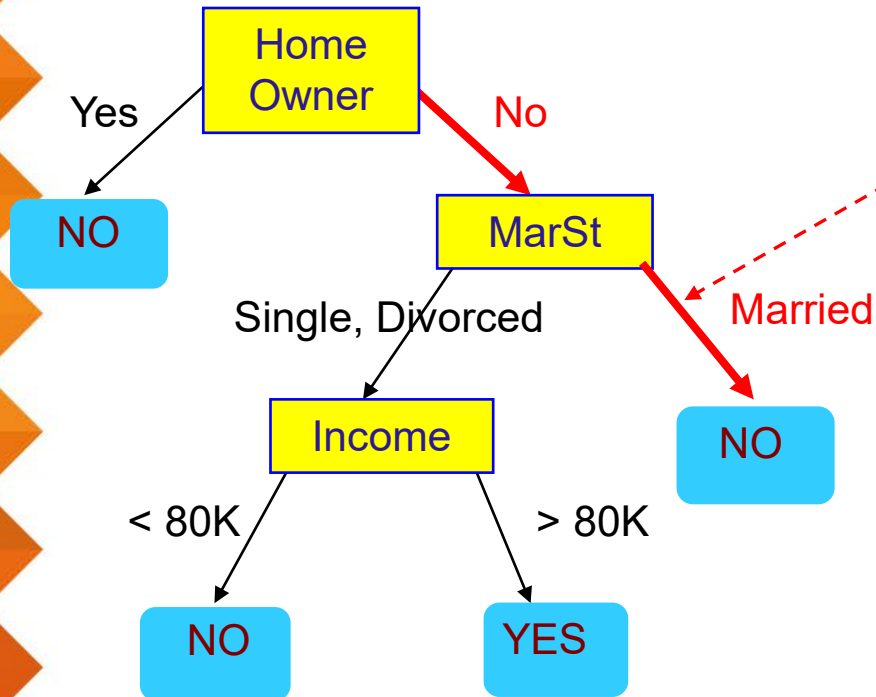
Home Owner	Marital Status	Annual Income	Defaulted Borrower
No	Married	80K	?



# Apply Model to Test Data

Test Data

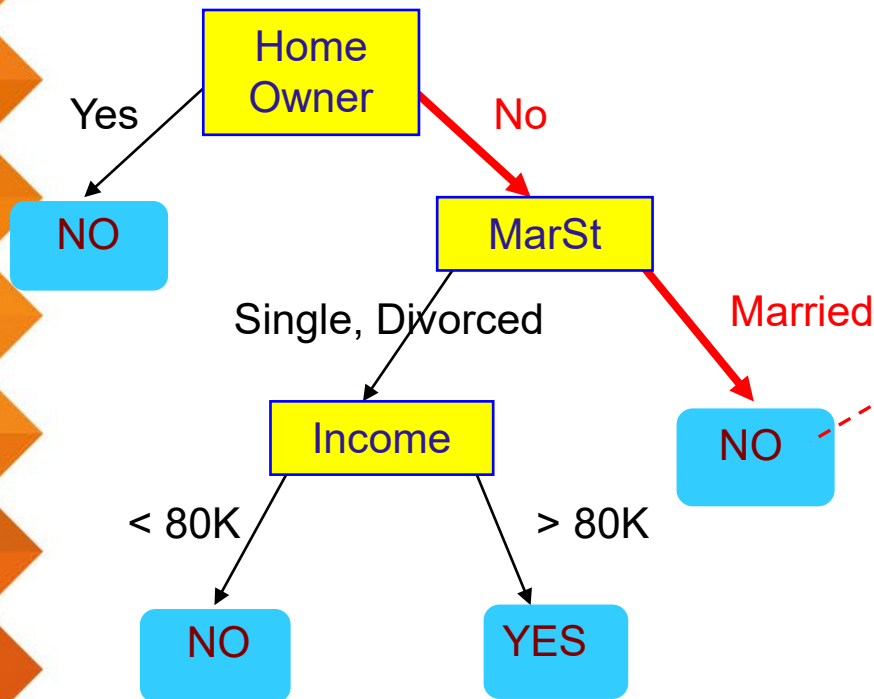
Home Owner	Marital Status	Annual Income	Defaulted Borrower
No	Married	80K	?



# Apply Model to Test Data

Test Data

Home Owner	Marital Status	Annual Income	Defaulted Borrower
No	Married	80K	?



Assign Defaulted to  
"No"



# Decision Tree Induction

- Many Algorithms:
  - *Hunt's Algorithm (one of the earliest)*
  - *CART*
  - *ID3, C4.5*
  - *SLIQ, SPRINT*

# Predict if John will play tennis

Training examples: 9 yes/ 5 no

- Hard to guess
- Try to understand when John plays
- Divide & conquer:
  - *split into subsets*
  - *are they pure? (all yes or all no)*
  - *if yes: stop*
  - *if not: repeat*
- See which subset
  - *New data falls into*

Day	Outlook	Humidity	Wind	Play
D1	Sunny	High	Weak	No
D2	Sunny	High	Strong	No
D3	Overcast	High	Weak	Yes
D4	Rain	High	Weak	Yes
D5	Rain	Normal	Weak	Yes
D6	Rain	Normal	Strong	No
D7	Overcast	Normal	Weak	Yes
D8	Sunny	High	Weak	No
D9	Sunny	Normal	Weak	Yes
D10	Rain	Normal	Strong	Yes
D11	Sunny	Normal	Strong	Yes
D12	Overcast	High	Strong	Yes
D13	Overcast	Normal	Weak	Yes
D14	Rain	High	Strong	No

New data:

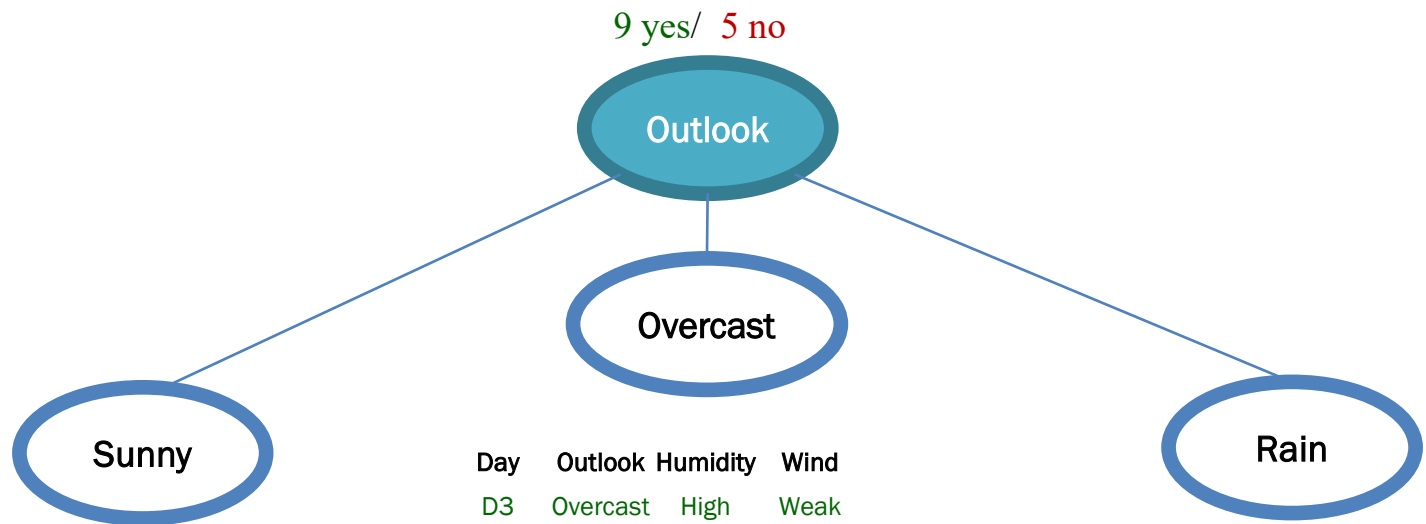
D15	Rain	High	Weak	?
-----	------	------	------	---

# Predict if John will play tennis

- Hard to guess
- Try to understand when John plays
- Divide & conquer:
  - *split into subsets*
  - *are they pure? (all yes or all no)*
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Training examples: 9 yes/ 5 no

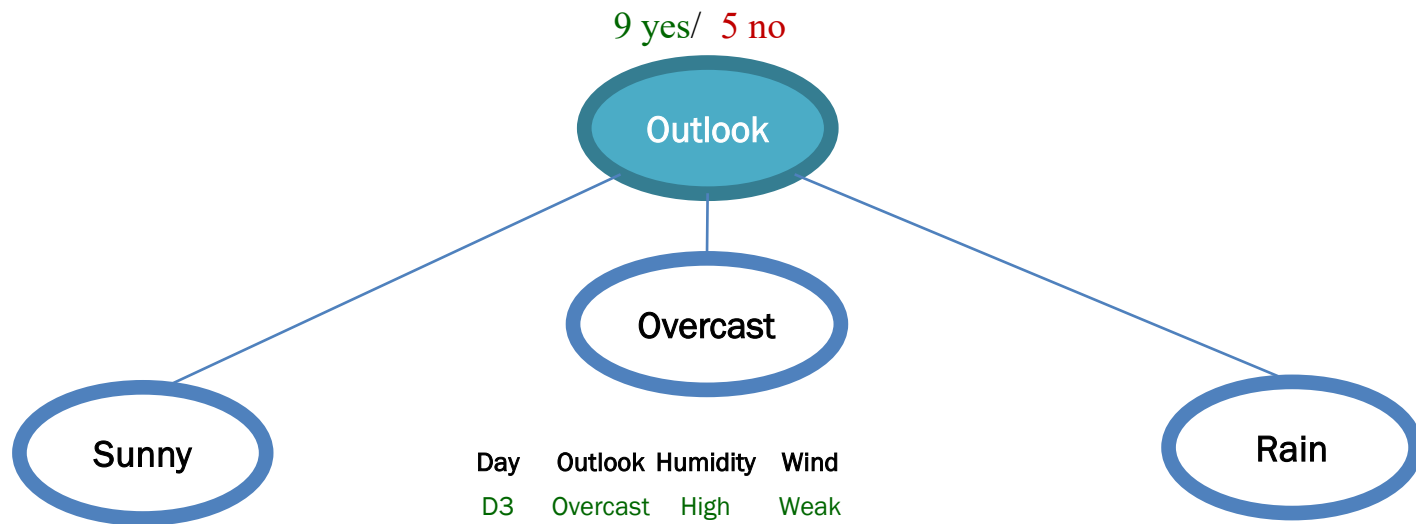
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D1	Sunny	High	Weak	No
D2	Sunny	High	Strong	No
D3	Overcast	High	Weak	Yes
D4	Rain	High	Weak	Yes
D5	Rain	Normal	Weak	Yes
D6	Rain	Normal	Strong	No
D7	Overcast	Normal	Weak	Yes
D8	Sunny	High	Weak	No
D9	Sunny	Normal	Weak	Yes
D10	Rain	Normal	Weak	Yes
D11	Sunny	Normal	Strong	Yes
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D13	Overcast	Normal	Weak	Yes
D14	Rain	High	Strong	No
New data:				
D15	Rain	High	Weak	?



Day	Outlook	Humidity	Wind
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D9	Sunny	Normal	Weak
D11	Sunny	Normal	Strong

Day	Outlook	Humidity	Wind
D3	Overcast	High	Weak
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D12	Overcast	High	Strong
D13	Overcast	Normal	Weak

Day	Outlook	Humidity	Wind
D4	Rain	High	Weak
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D14	Rain	High	Strong



Day	Outlook	Humidity	Wind
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D9	Sunny	Normal	Weak
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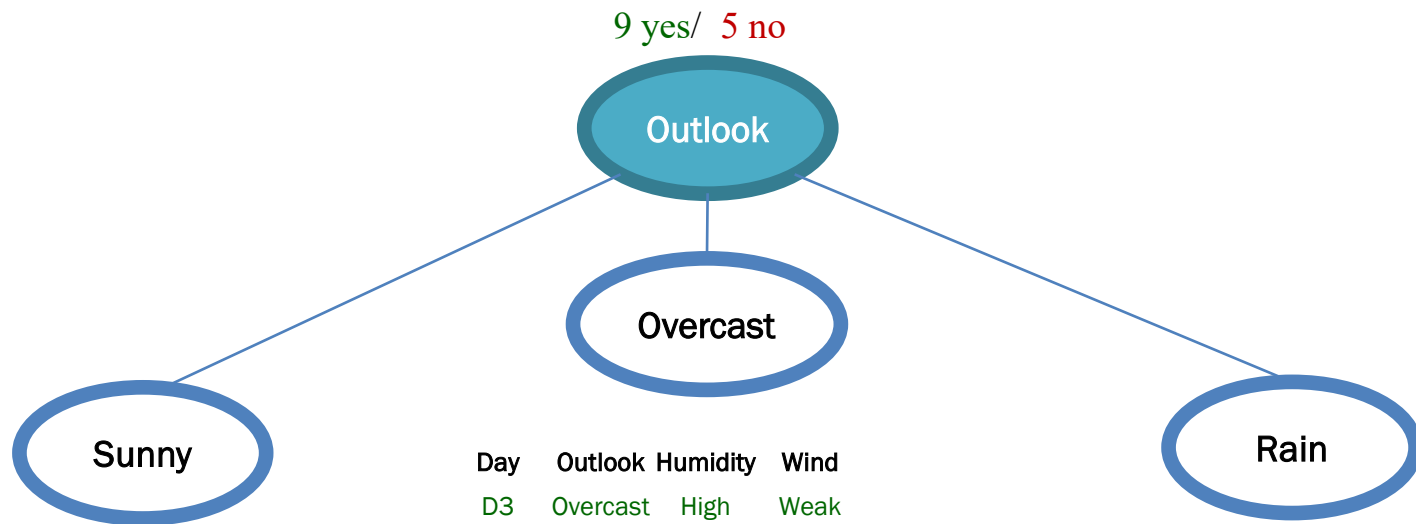
2 yes/ 3 no  
Split further

Day	Outlook	Humidity	Wind
D3	Overcast	High	Weak
D7	Overcast	Normal	Weak
D12	Overcast	High	Strong
D13	Overcast	Normal	Weak

4 yes/ 0 no  
pure subset

Day	Outlook	Humidity	Wind
D4	Rain	High	Weak
D5	Rain	Normal	Weak
D10	Rain	Normal	Weak
D6	Rain	Normal	Strong
D14	Rain	High	Strong

3 yes/ 2 no  
Split further



Day	Outlook	Humidity	Wind
D1	Sunny	High	Weak
D2	Sunny	High	Strong
D8	Sunny	High	Weak
D9	Sunny	Normal	Weak
D11	Sunny	Normal	Strong

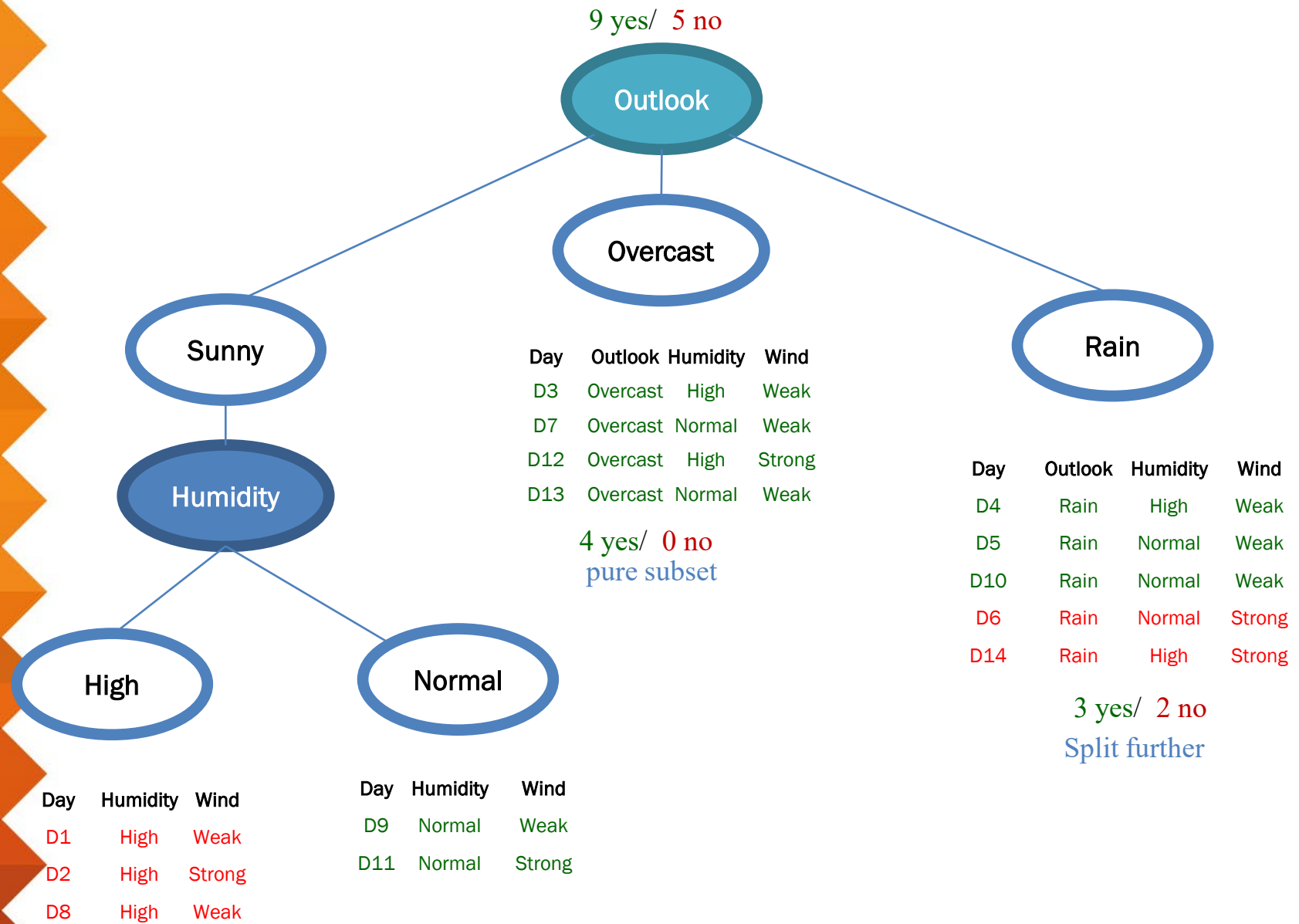
2 yes/ 3 no  
Split further

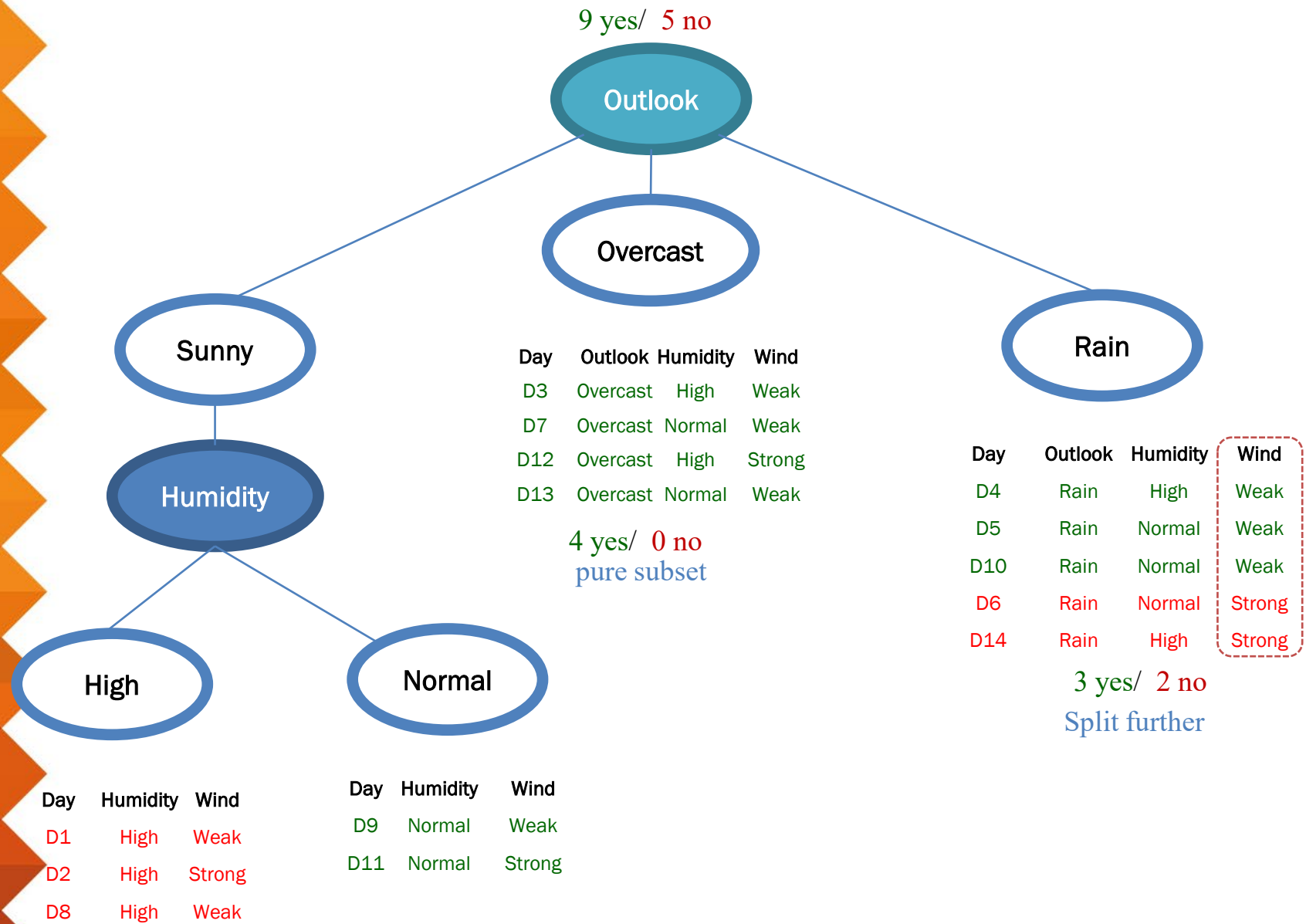
Day	Outlook	Humidity	Wind
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4 yes/ 0 no  
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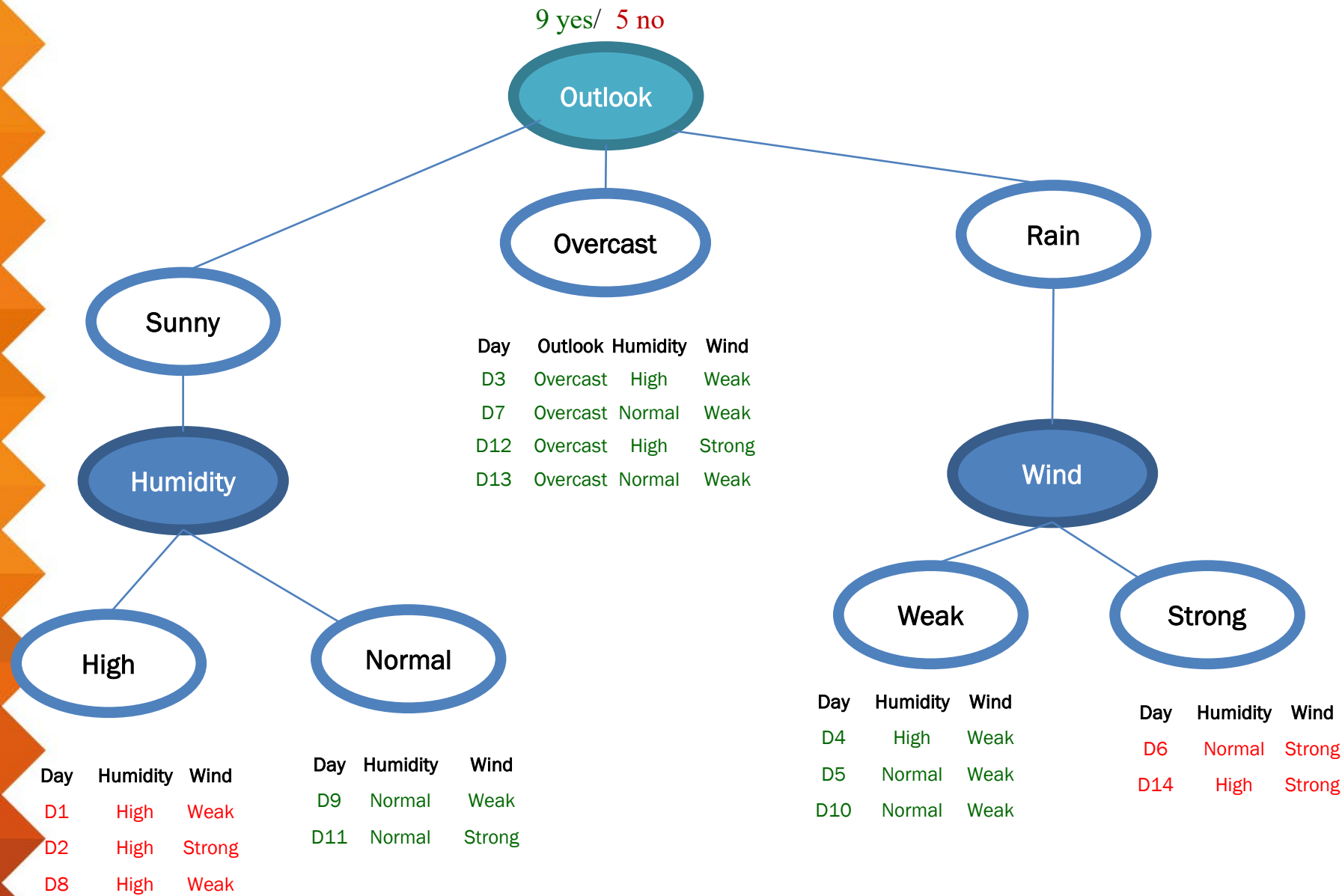
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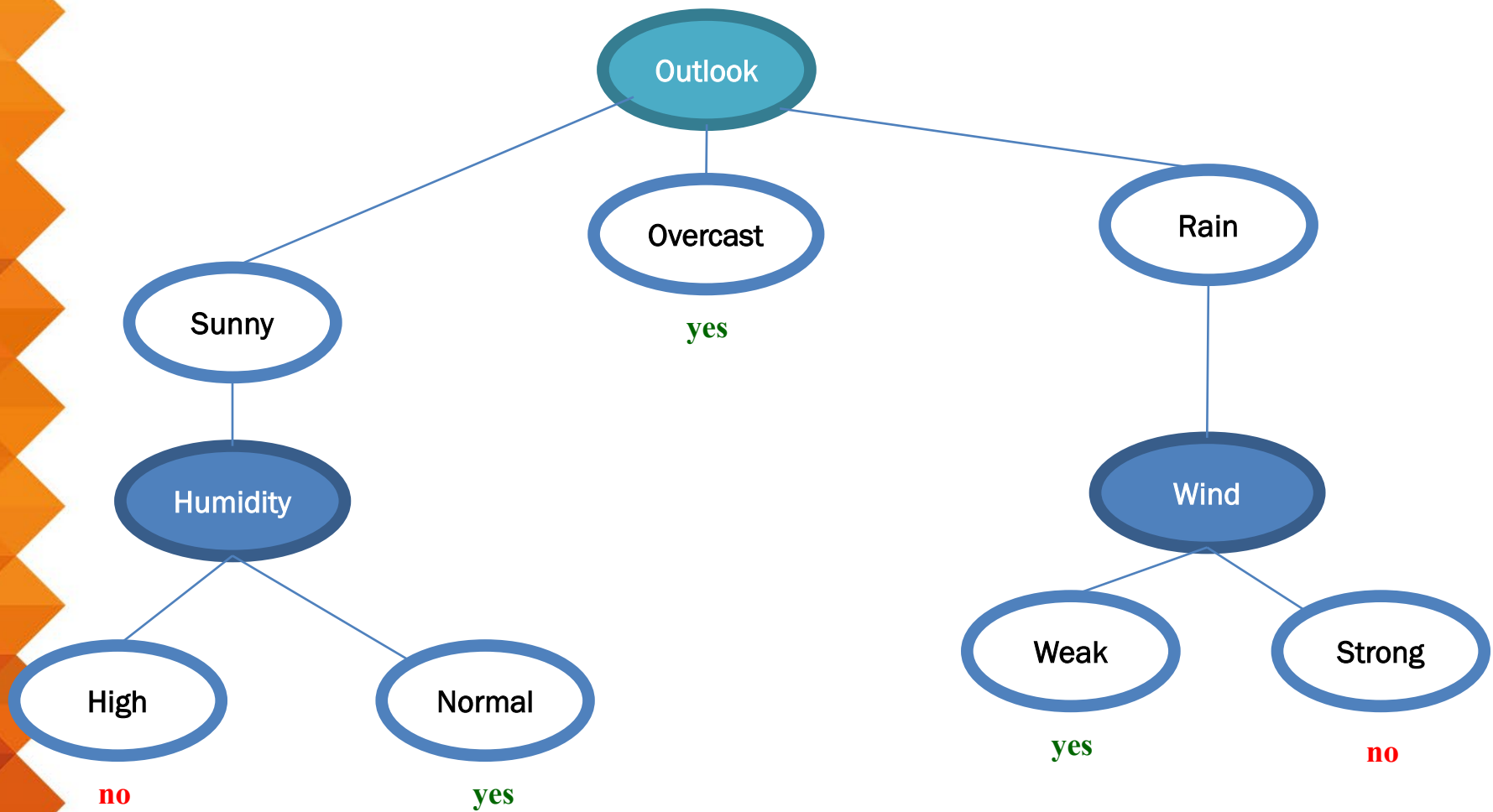
3 yes/ 2 no  
Split further

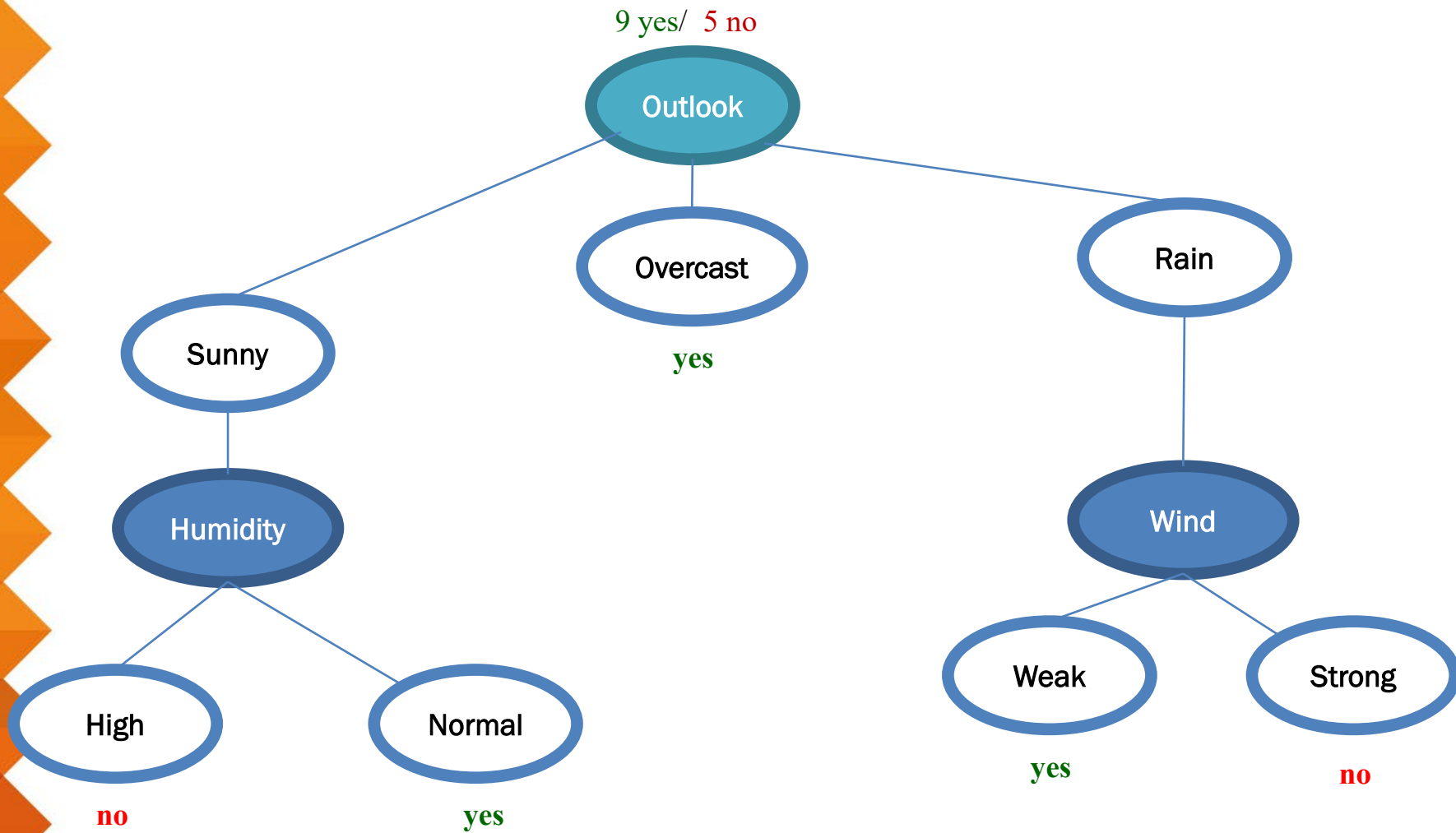






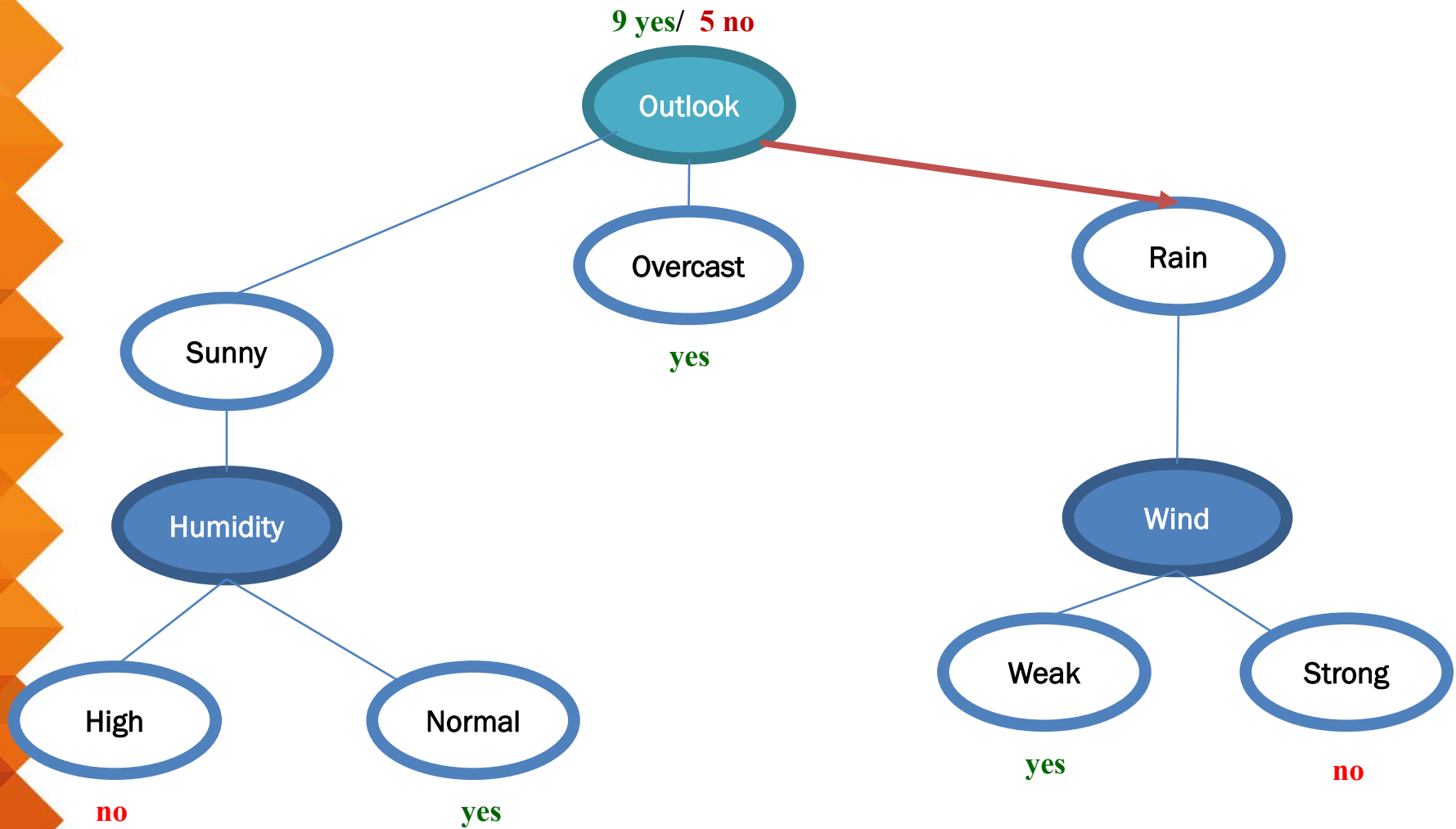






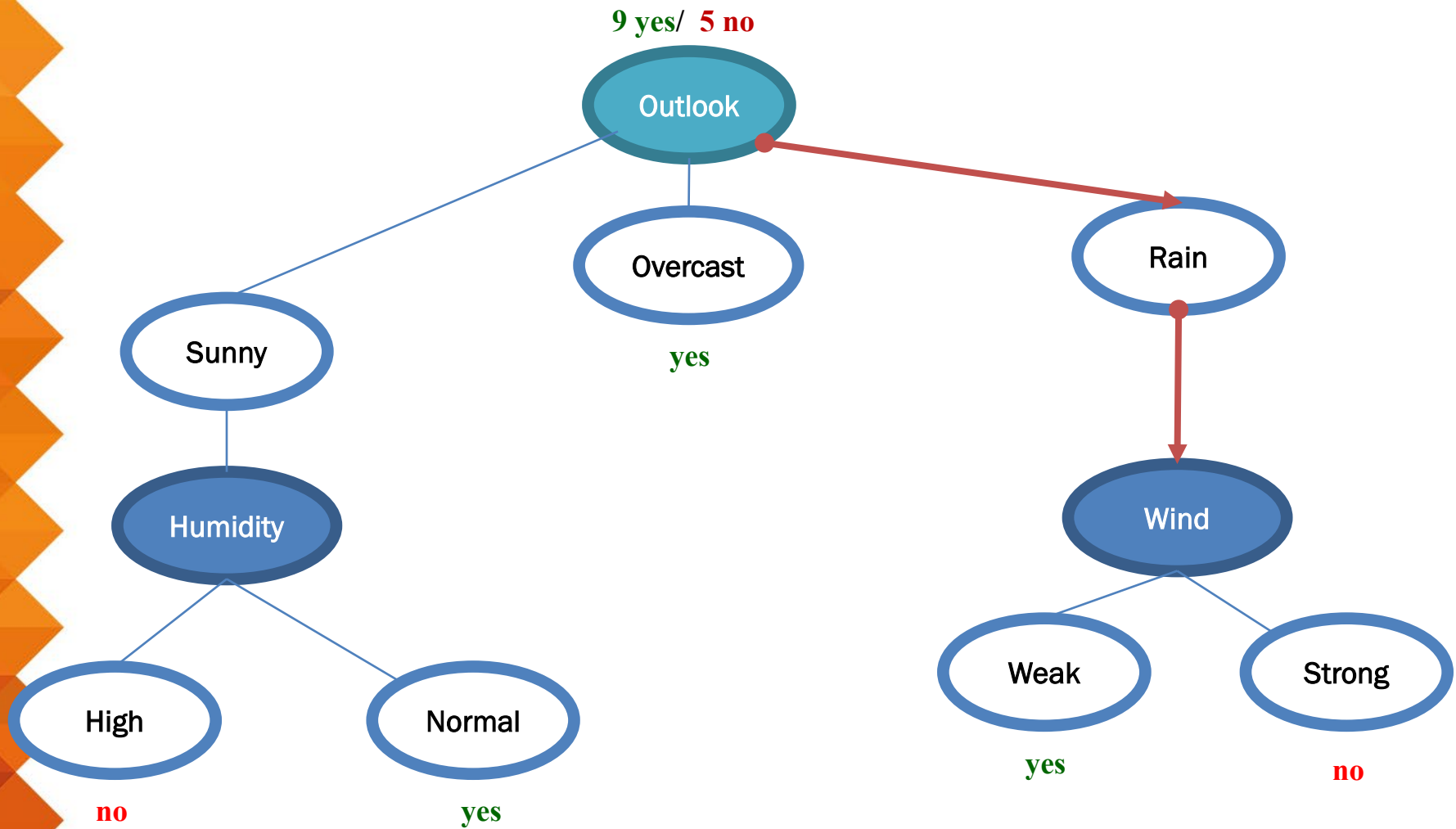
New data:

Day	Outlook	Humidity	Wind	Play
D15	Rain	High	Weak	?



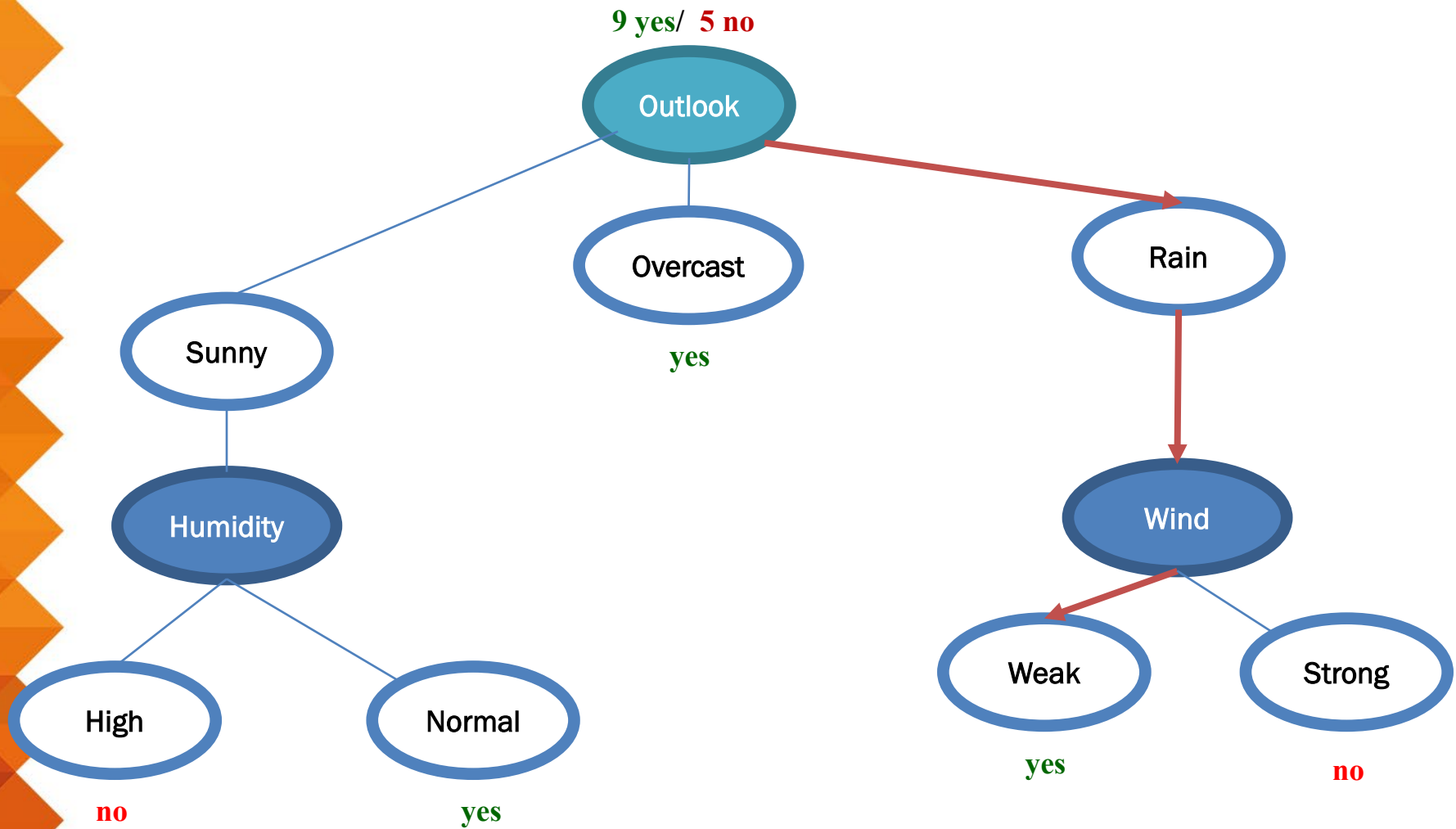
New data:

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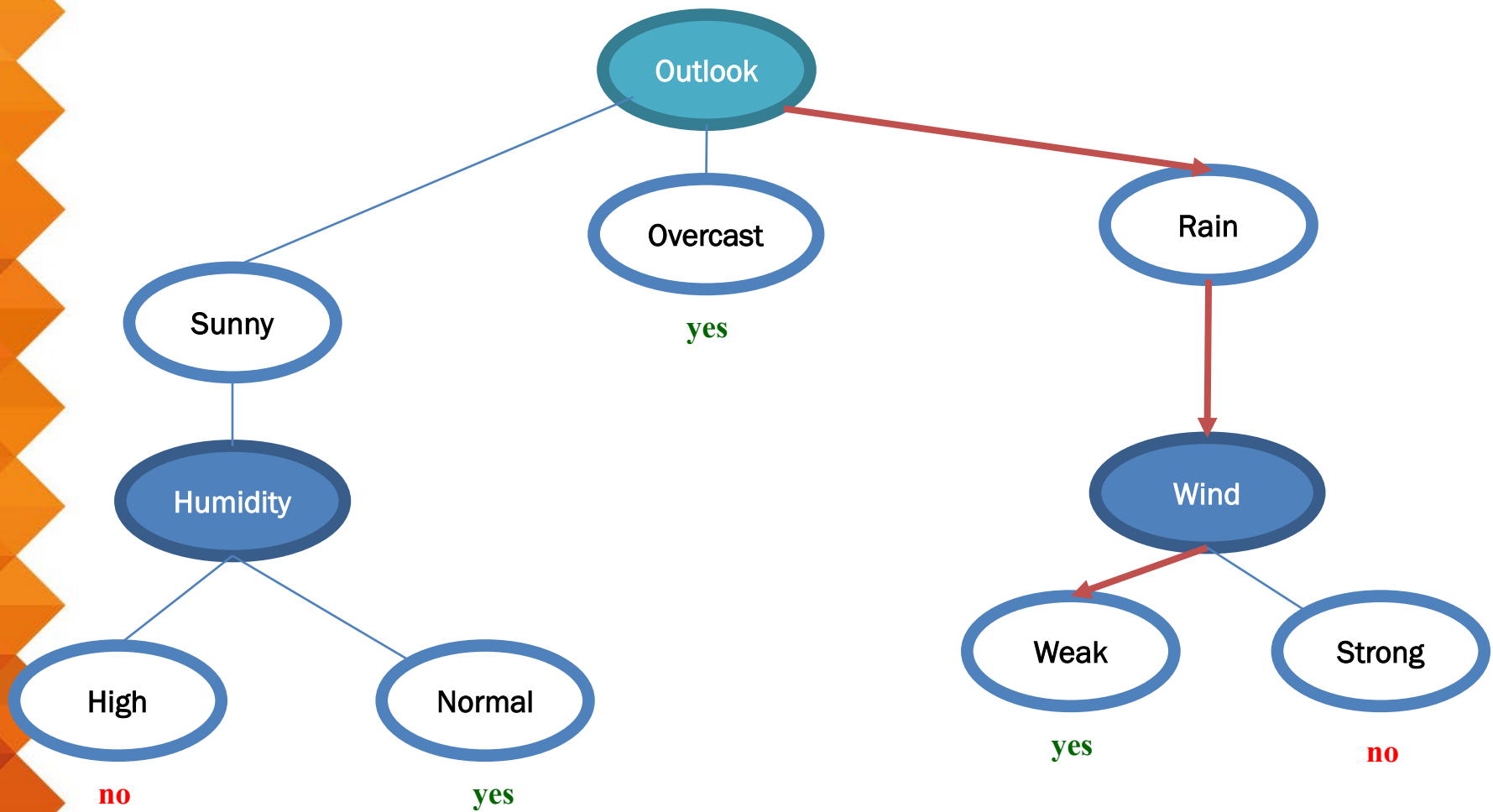
New data:

Day	Outlook	Humidity	Wind	Play
D15	Rain	High	Weak	?



New data:

Day	Outlook	Humidity	Wind	Play
D15	Rain	High	Weak	yes



New data:

Day	Outlook	Humidity	Wind	Play
D15	Rain	High	Weak	yes

# Tree Induction

- Greedy strategy.
  - *Split the records based on an attribute test that optimizes certain criterion.*
- Issues
  - *Determine how to split the records*
    - How to specify the attribute test condition?
    - How to determine the best split?
  - *Determine when to stop splitting*



# Tree Induction

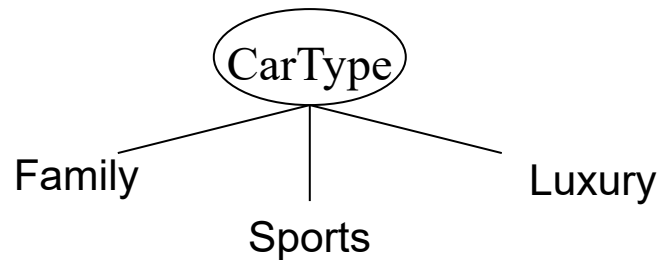
- Greedy strategy.
  - *Split the records based on an attribute test that optimizes certain criterion.*
- Issues
  - *Determine how to split the records*
    - *How to specify the attribute test condition?*
    - *How to determine the best split?*
  - *Determine when to stop splitting*

# How to Specify Test Condition?

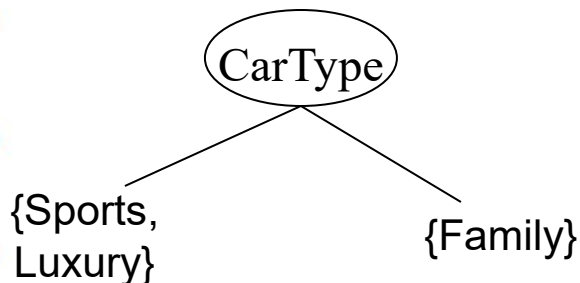
- Depends on attribute types
  - *Nominal*
  - *Ordinal*
  - *Continuous*
- Depends on number of ways to split
  - *2-way split*
  - *Multi-way split*

# Splitting Based on Nominal Attributes

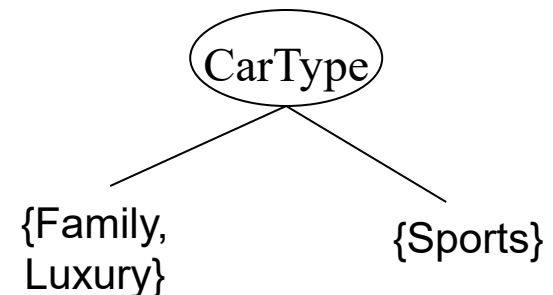
- **Multi-way split:** Use as many partitions as distinct values.



- **Binary split:** Divides values into two subsets.  
Need to find optimal partitioning.

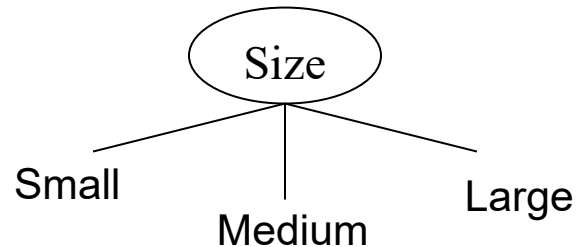


OR

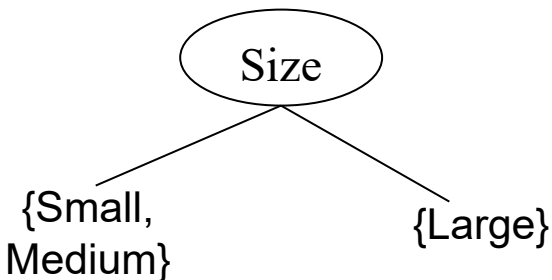


# Splitting Based on Ordinal Attributes

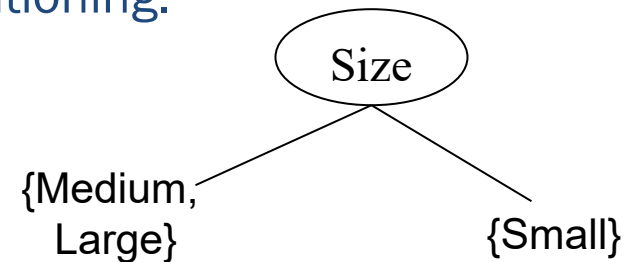
- **Multi-way split:** Use as many partitions as distinct values.



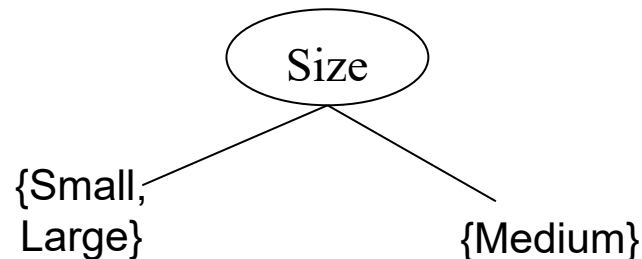
- **Binary split:** Divides values into two subsets.  
Need to find optimal partitioning.



OR



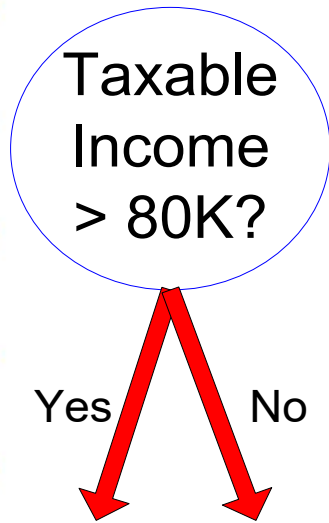
- What about this split?



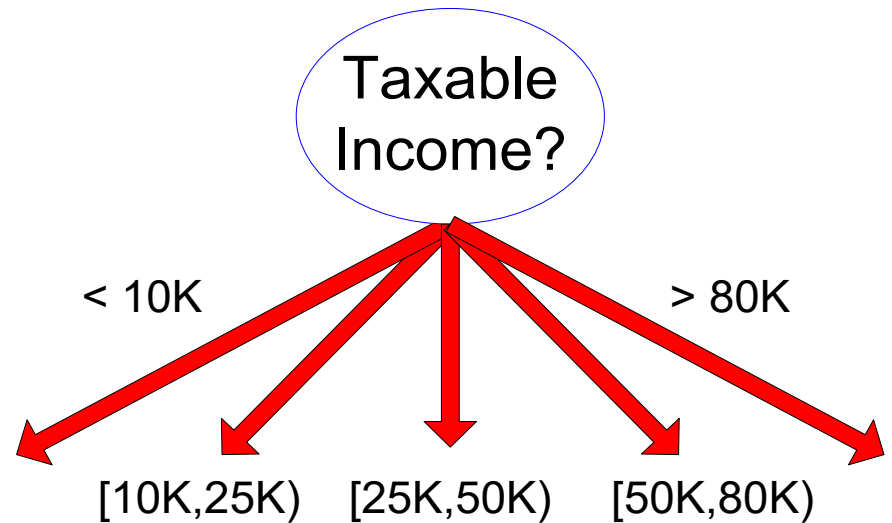
# Splitting Based on Continuous Attributes

- Different ways of handling
  - *Discretization* to form an ordinal categorical attribute
    - Static – discretize once at the beginning
    - Dynamic – ranges can be found by equal interval bucketing, equal frequency bucketing (percentiles), or clustering.
  - *Binary Decision*:  $(A < v)$  or  $(A \geq v)$ 
    - consider all possible splits and finds the best cut
    - can be more computationally intensive

# Splitting Based on Continuous Attributes



(i) Binary split

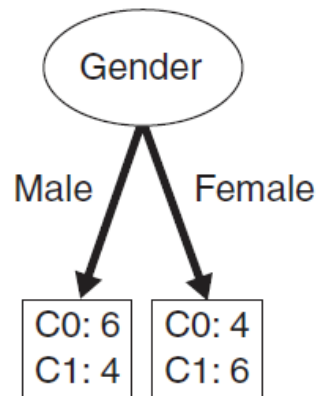


(ii) Multi-way split

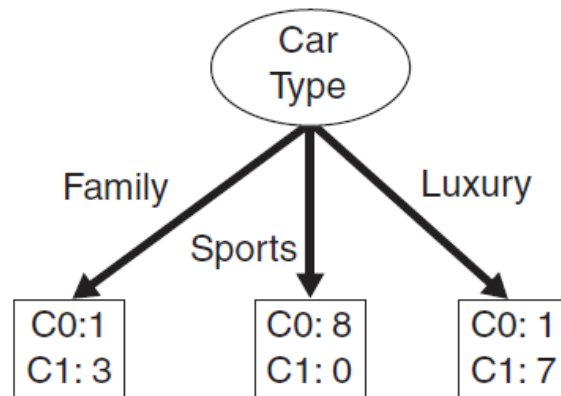
# How to determine the Best Split?

**Before Splitting: 10 records of class 0,  
10 records of class 1**

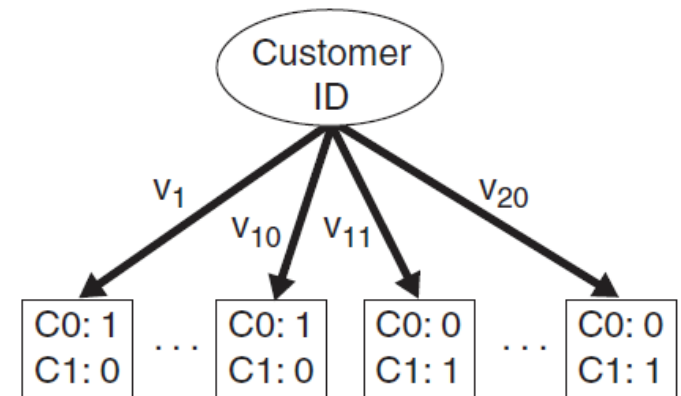
C0: 10
C1: 10



(a)



(b)



(c)

**Which test condition is the best?**

# How to determine the Best Split?

- Greedy approach:
  - Nodes with *homogeneous class distribution* are preferred
- Need a measure of node **impurity**:

C0: 5
C1: 5

**Non-homogeneous,  
High degree of impurity**

C0: 9
C1: 1

**Homogeneous,  
Low degree of impurity**



# Measures of Node Impurity

$p(i|t)$ : fraction of records associated with node  $t$  belonging to class  $i$

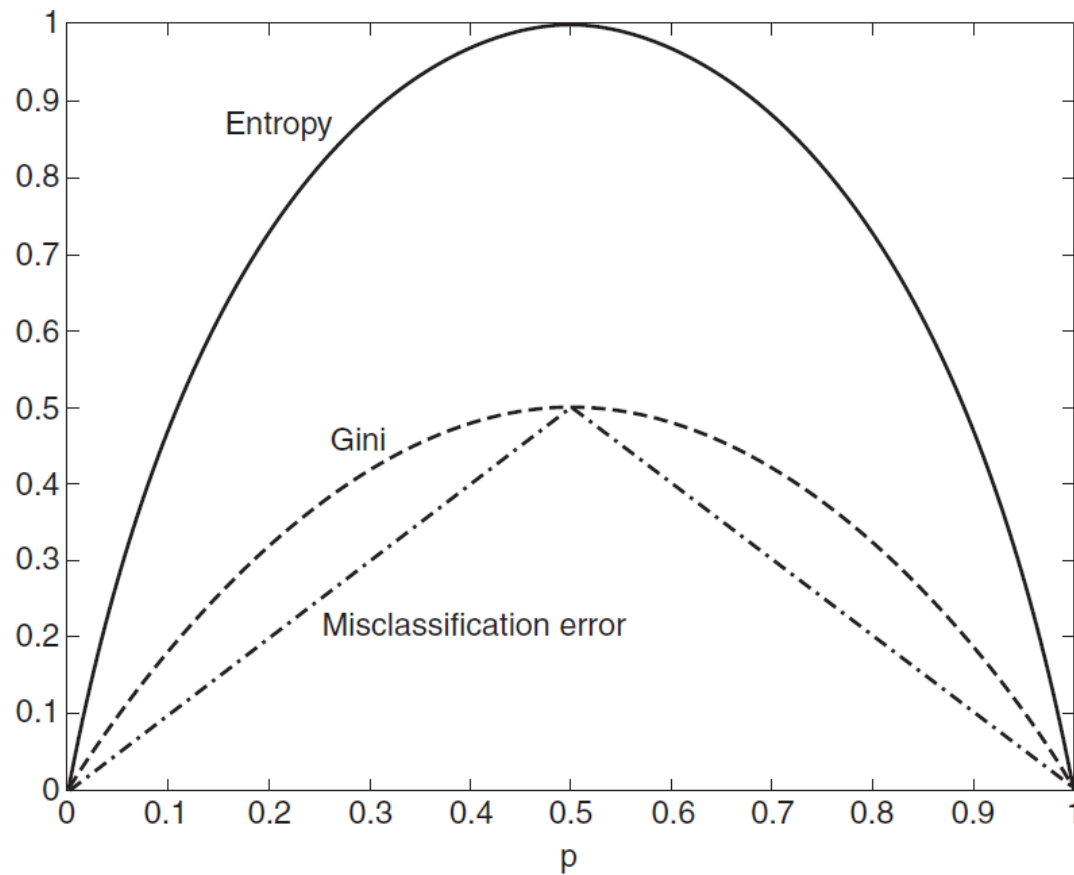
$$\text{Entropy}(t) = - \sum_{i=0}^{c-1} p(i|t) \log_2 p(i|t),$$

$$\text{Gini}(t) = 1 - \sum_{i=0}^{c-1} [p(i|t)]^2,$$

$$\text{Classification error}(t) = 1 - \max_i [p(i|t)],$$

# Comparison

- For a 2-class problem:
  - *Probability of the majority class  $p$  is always  $> .5$*



## Splitting Criteria based on Classification Error

- Classification error at a node  $t$  :

$$Error(t) = 1 - \max_i P(i | t)$$

- Measures misclassification error made by a node.
  - Maximum ( $1 - 1/n_c$ ) when records are equally distributed among all classes, implying least interesting information
  - Minimum (0.0) when all records belong to one class, implying most interesting information

# Computing Error of a Single Node

$$Error(t) = 1 - \max_i P(i | t)$$

C1	<b>0</b>
C2	<b>6</b>

$$P(C1) = 0/6 = 0 \quad P(C2) = 6/6 = 1$$

$$Error = 1 - \max(0, 1) = 1 - 1 = 0$$

C1	<b>1</b>
C2	<b>5</b>

$$P(C1) = 1/6 \quad P(C2) = 5/6$$

$$Error = 1 - \max(1/6, 5/6) = 1 - 5/6 = 1/6$$

C1	<b>2</b>
C2	<b>4</b>

$$P(C1) = 2/6 \quad P(C2) = 4/6$$

$$Error = 1 - \max(2/6, 4/6) = 1 - 4/6 = 1/3$$

# Example

C1	<b>0</b>
C2	<b>6</b>

$$P(C1) = 0/6 = 0 \quad P(C2) = 6/6 = 1$$

$$\text{Entropy} = -0 \log 0 - 1 \log 1 = -0 - 0 = 0$$

$$\text{Error} = 1 - \max(0, 1) = 1 - 1 = 0$$

C1	<b>1</b>
C2	<b>5</b>

$$P(C1) = 1/6 \quad P(C2) = 5/6$$

$$\text{Entropy} = - (1/6) \log_2 (1/6) - (5/6) \log_2 (5/6) = 0.65$$

$$\text{Error} = 1 - \max(1/6, 5/6) = 1 - 5/6 = 1/6 = 0.16$$

C1	<b>2</b>
C2	<b>4</b>

$$P(C1) = 2/6 \quad P(C2) = 4/6$$

$$\text{Entropy} = - (2/6) \log_2 (2/6) - (4/6) \log_2 (4/6) = 0.92$$

$$\text{Error} = 1 - \max(2/6, 4/6) = 1 - 4/6 = 1/3 = 0.33$$

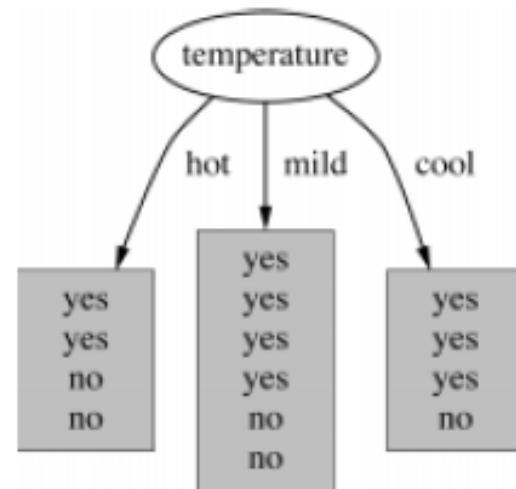
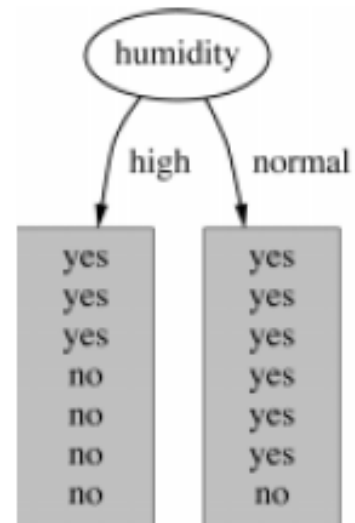
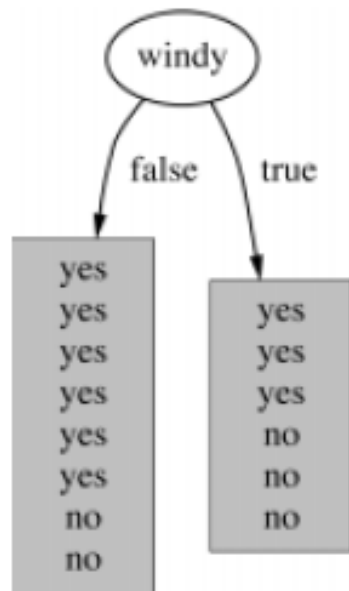
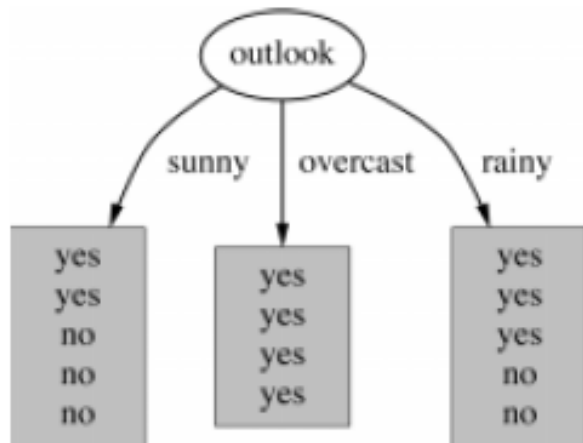
# Impurity measures

- All of the impurity measures take value zero (**minimum**) for the case of a pure node where a single value has probability 1
- All of the impurity measures take **maximum** value when the class distribution in a node is **uniform**.

# Decision Trees Classifiers

- ▶ Normal procedure: top down in recursive divide and conquer fashion
- ▶ First: attribute is selected for root node and branch is created for each possible attribute value
- ▶ Then: the instances are split into subsets (one for each branch extending from the node)
- ▶ Finally: procedure is repeated recursively for each branch, using only instances that reach the branch
- ▶ Process stops if all instances have the same class

# Decision Trees: which attribute to select?





# Criterion for attribute selection

- ▶ Which is the best attribute?
  - *The one which will result in the smallest tree*
  - *Heuristic: choose the attribute that produces the “purest” nodes*
- ▶ Popular purity criterion: information gain
  - *Information gain increases with the average purity of the subsets that an attribute produces*
- ▶ Strategy: choose attribute that results in greatest information gain

# Computing Information Gain

- Information is measured in bits
  - *Given a probability distribution, the info required to predict an event is the distribution's entropy*
  - *Entropy gives the information required in bits (this can involve fractions of bits!)*
- Formula for computing the entropy:

$$\text{entropy}(p_1, p_2, \dots, p_n) = -p_1 \log p_1 - p_2 \log p_2 \dots - p_n \log p_n$$

Where  $p_1, p_2, \dots, p_n$  denote the probability of occurrence of outcomes for class 1, 2, ..., n respectively

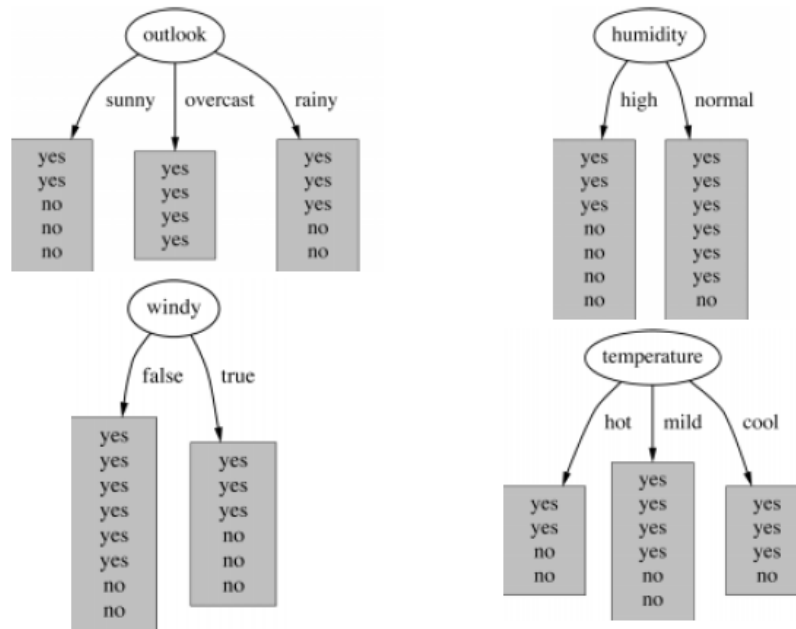
# Example: Attribute Outlook

- “Outlook” = “Sunny”:

$\text{info}([2,3]) = \text{entropy}(2/5, 3/5) = -2/5 \log(2/5) - 3/5 \log(3/5) = 0.971 \text{ bits}$

- “Outlook” = “Overcast”:

$\text{info}([4,0]) = \text{entropy}(1, 0) = -1 \log(1) - 0 \log(0) = 0 \text{ bits}$



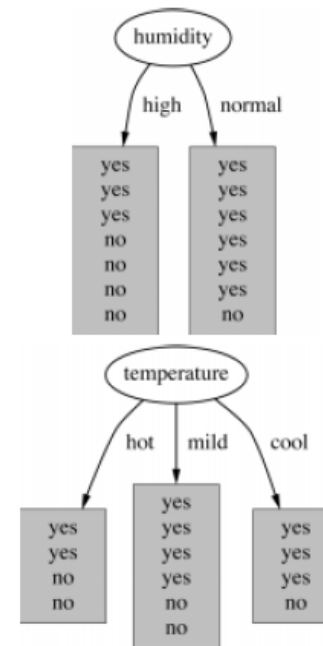
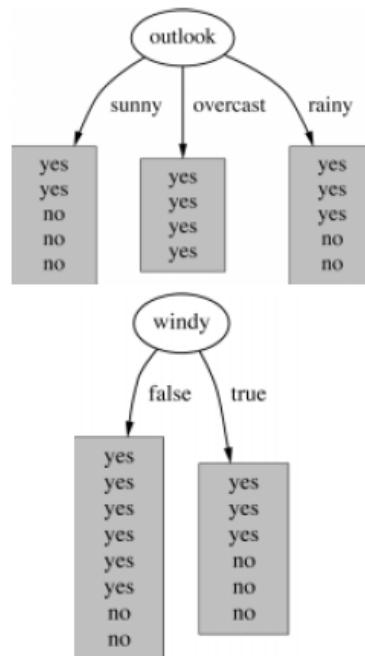
# Example: Attribute Outlook

- “Outlook” = “Rainy”:

$$\text{info}([3,2]) = \text{entropy}(3/5, 2/5) = -3/5 \log(3/5) - 2/5 \log(2/5) = 0.971 \text{ bits}$$

- Expected information for attribute:

$$\begin{aligned} \text{info}([3,2], [4,0], [3,2]) &= (5/14) \times 0.971 + (4/14) \times 0 + (5/14) \times 0.971 \\ &= 0.693 \text{ bits} \end{aligned}$$



# Computing Gain

- Information gain: information before splitting – information after splitting

$$\begin{aligned}\text{gain}(\text{"Outlook"}) &= \text{info}([9,5]) - \text{info}([3,2],[4,0],[3,2]) \\ &= 0.940 - 0.693 \\ &= 0.247 \text{ bits}\end{aligned}$$

- Information gain for attributes from weather data:

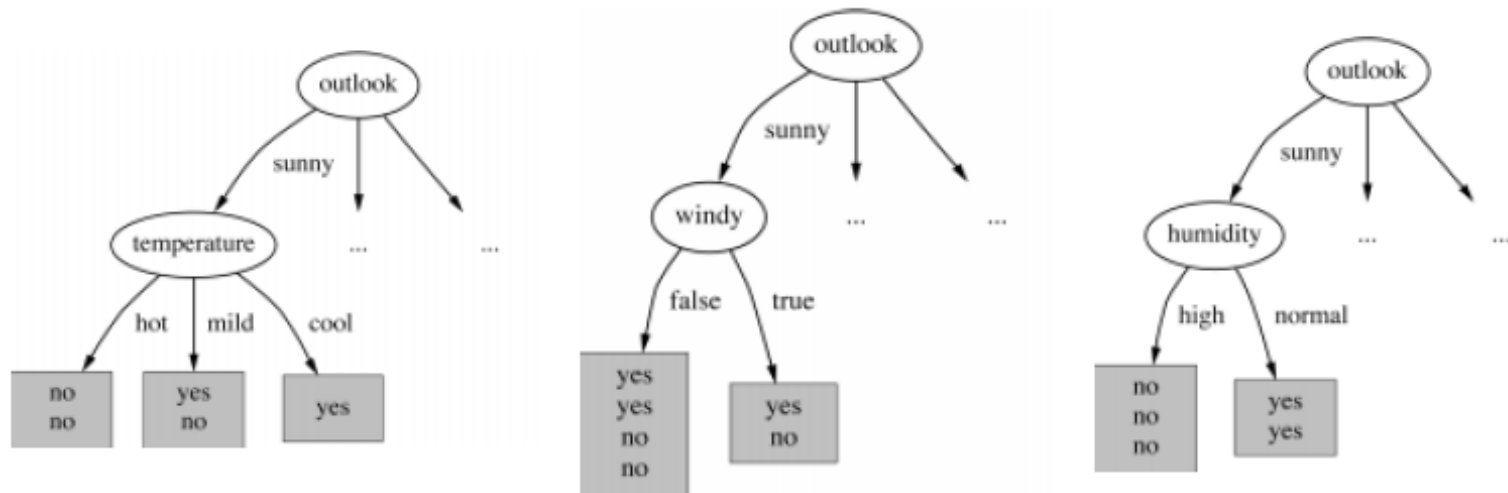
$$\text{gain}(\text{"Outlook"}) = 0.247 \text{ bits}$$

$$\text{gain}(\text{"Temperature"}) = 0.029 \text{ bits}$$

$$\text{gain}(\text{"Humidity"}) = 0.152 \text{ bits}$$

$$\text{gain}(\text{"Windy"}) = 0.048 \text{ bits}$$

# Continuing the split



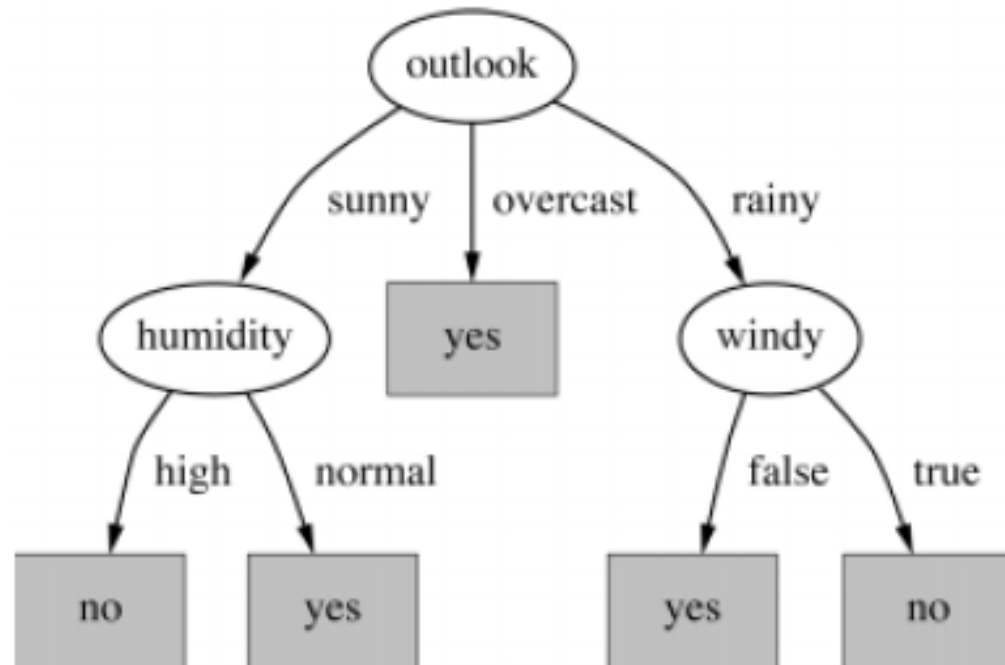
$\text{gain}(\text{"Temperature"}) = 0.571 \text{ bits}$

$\text{gain}(\text{"Humidity"}) = 0.971 \text{ bits}$

$\text{gain}(\text{"Windy"}) = 0.020 \text{ bits}$

# The final tree

- Note: not all leaves need to be pure
- Some leaf nodes will have instances belonging to more than one class – in this case the leaf node is labelled with the *majority* class
- Splitting stops when data can't be split any further – when we run out of attributes to split on or the number of instances at the node to be split is less than a specified minimum number.

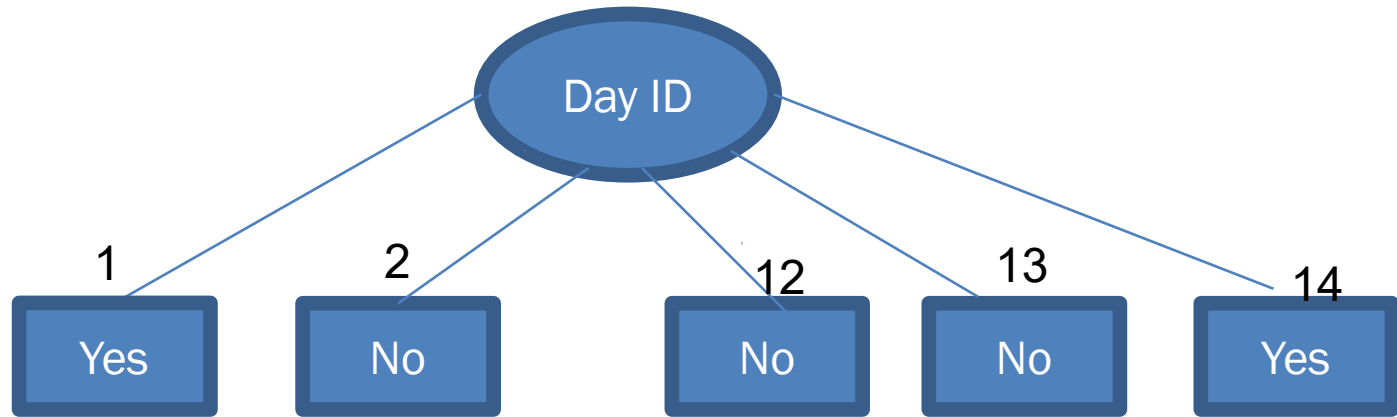


# Getting rid of bias introduced by Information Gain (IG)

- IG is a good method of identifying splits in the tree but it can cause problems for features with high cardinality
- In the weather example the Day ID perfectly correlates to the right play decision in *historical data*
- This means that Day ID has the highest IG value
- However, it has no power whatsoever to predict decisions, hence no generalization capability!
- The solution in this case is to use Gain Ratio



# Problem with highly branching features



- Remember there were 9 playing days and 5 non playing days and so:

$$\text{info}([0,1]) + \text{info}([0,1]) + \text{info}([1,0]) + \dots + \text{info}([1,0]) + \text{info}([0,1])$$

- Thus it follows that Day ID has the maximum information gain (=0.94)

(See Weka Book 2011: pp105-108)

# Intrinsic Information

- ▶ The problem occurs due to the high intrinsic information held by highly branching features
- ▶ Intrinsic Information contained by a split  $S$  created by a feature  $F$  is given by:

$I(F, S) = -\sum_i \frac{|B_i|}{|B|} \log \left( \frac{|B_i|}{|B|} \right)$  where  $|B_i|$  is the number of data instances on branch  $i$  and  $|B|$  is the total number across all branches (size of training dataset)

- ▶ For day ID,  $I = 14 \left( -\frac{1}{14} \log \left( \frac{1}{14} \right) \right) = 3.807$  which is the highest out of all the features

# Gain Ratio

- We are now in a better position to get rid of troublesome features
- ▶ We define Gain Ratio (GR) as the ratio of Information Gain (IG) to Intrinsic Information (IS)

$$GR(F, S) = \frac{IG(F, S)}{I(F, S)} \text{ where } F \text{ is the feature and } S \text{ is the split}$$

Feature\Measure	Information Gain	Gain Ratio
Outlook	0.247	0.157
Humidity	0.152	0.152
Windy	0.029	0.049
Temperature	0.048	0.019
Day ID	0.940	0.246

# Gain Ratio vs Information Gain

- Thus it can be seen that GR has narrowed the gap between Day ID and Outlook
- However, Day ID *still wins* as it has the highest Gain Ratio
- The lesson here is that the data miner *has to use his/her judgement and not rely totally on automated tools*
- It is easy for the human data miner to identify that Day ID is not suitable as it has no predictive power (generalizability).

# References

1. *Data Mining: Practical Machine Learning Tools and Techniques* (3<sup>rd</sup> edition) / Ian Witten, Eibe Frank; Elsevier, 2011, Chapter 4