# 1 Math: prove that $L(\mathrm{h})$ is a strictly convex function

Because If a function $L(\mathbf{h})$ is differentiable up to at least second order, $L$ is strictly convex if its Hessian matrix is positive definite.

Therefore, we can prove Hessian matrix of $L(\mathbf{h})$ is positive definite.

First, compute derivative of $L(\mathbf{h})$ to the first order.(Jacobian matrix )

$$\frac{\partial L(\mathbf{h})}{\partial \mathbf{h}} = (\mathbf{J}(\mathbf{x}))^T f(\mathbf{x}) + (\mathbf{J}(\mathbf{x}))^T \mathbf{J}(\mathbf{x})\mathbf{h} + \mu\mathbf{h}$$

Second, compute derivative of $L(\mathbf{h})$ to the second order.(Hessian matrix )

$$\frac{\partial L^2(\mathbf{h})}{\partial \mathbf{h}\mathbf{h}^T} = (\mathbf{J}(\mathbf{x}))^T \mathbf{J}(\mathbf{x}) + \mu\mathbf{I}$$

Let $\mathbf{H}(\mathbf{h}) = \frac{\partial L^2(\mathbf{h})}{\partial \mathbf{h}\mathbf{h}^T}$. To prove it is positive definite.

For all $\mathbf{x} \in \mathbb{R}^{n\times 1}$, and $\mathbf{x} \neq \mathbf{0}$.

$$\mathbf{x}^T\mathbf{H}(\mathbf{h})\mathbf{x} = \overbrace{\mathbf{x}^T(\mathbf{J}(\mathbf{x}))^T\mathbf{J}(\mathbf{x})\mathbf{x}}^{\geq 0} + \overbrace{\mu\mathbf{x}^T\mathbf{x}}^{>0} > 0$$

And we can know the Hessian Matrix of $L(\mathbf{h})$, $\mathbf{H}$ is positive definite. So $L$ is strictly convex.