1.Calculate the gradient of the following multivariate function

(1) $u = xy + y^2 + 5$

$$\frac{\partial u}{\partial x} = y$$
$$\frac{\partial u}{\partial y} = x + 2y$$
$$\nabla u = (y, x + 2y)$$

(2)$u = ln\sqrt{x^2 + y^2 + z^2}$

$$\frac{\partial u}{\partial x} = \frac{1}{\sqrt{x^2 + y^2 + z^2}} \times \frac{1}{2} \times \frac{1}{\sqrt{x^2 + y^2 + z^2}} \times 2x = \frac{x}{x^2 + y^2 + z^2}$$
$$\frac{\partial u}{\partial y} = \frac{1}{\sqrt{x^2 + y^2 + z^2}} \times \frac{1}{2} \times \frac{1}{\sqrt{x^2 + y^2 + z^2}} \times 2y = \frac{y}{x^2 + y^2 + z^2}$$
$$\frac{\partial u}{\partial z} = \frac{1}{\sqrt{x^2 + y^2 + z^2}} \times \frac{1}{2} \times \frac{1}{\sqrt{x^2 + y^2 + z^2}} \times 2z = \frac{z}{x^2 + y^2 + z^2}$$
$$\nabla u = (\frac{x}{x^2 + y^2 + z^2}, \frac{y}{x^2 + y^2 + z^2}, \frac{z}{x^2 + y^2 + z^2})$$
$$\nabla u_{(1,2,-2)} = (\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial u}{\partial z})_{(1,2,-2)} = (\frac{1}{9}, \frac{2}{9}, \frac{-2}{9})$$

2.

According to:

$$\operatorname{Ent}(D) = -\sum_{k=1}^{|\mathcal{Y}|} p_k \log_2 p_k$$

$$\operatorname{Gain}(D, a) = \operatorname{Ent}(D) - \sum_{v=1}^{V} \frac{|D^v|}{|D|} \operatorname{Ent}\left(D^v\right)$$

Node 1:

$$Ent(D) = -\frac{3}{12} log_2(\frac{3}{12}) - \frac{9}{12} log_2(\frac{9}{12}) = 0.8113$$

a=Season,

$$\sum_{v=1}^{V}\frac{|D^v|}{|D|}\mathrm{Ent}\left(D^v\right)=\frac{2}{12}(-\frac{1}{2}log_2(\frac{1}{2})-\frac{1}{2}log_2(\frac{1}{2}))+\frac{3}{12}(-\frac{2}{3}log_2(\frac{2}{3})-\frac{1}{3}log_2(\frac{1}{3}))$$
$$+\frac{2}{12}(-\frac{2}{2}log_2(\frac{2}{2}))+\frac{5}{12}(-\frac{5}{5}log_2(\frac{5}{5}))$$
$$=\frac{2}{12}\times 1+\frac{3}{12}\times 0.9183+\frac{2}{12}\times 0+\frac{5}{12}\times 0$$
$$=0.3962$$

a=After 8:00,

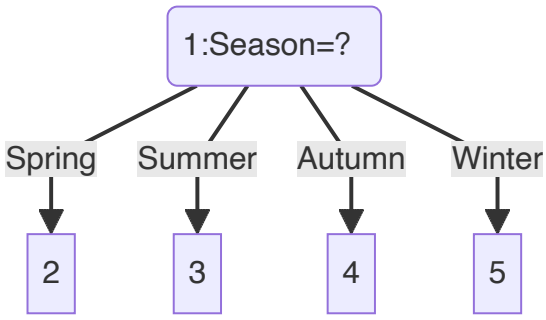
$$\sum_{v=1}^{V}\frac{|D^v|}{|D|}\mathrm{Ent}\left(D^v\right)=\frac{7}{12}(-\frac{1}{7}log_2(\frac{1}{7})-\frac{6}{7}log_2(\frac{6}{7}))+\frac{5}{12}(-\frac{2}{5}log_2(\frac{2}{5})-\frac{3}{5}log_2(\frac{3}{5}))$$
$$=\frac{7}{12}\times 0.5917+\frac{5}{12}\times 0.9710$$
$$=0.7497$$

a=Wind,

$$\sum_{v=1}^{V}\frac{|D^v|}{|D|}\mathrm{Ent}\left(D^v\right)=\frac{4}{12}(-\frac{1}{4}log_2(\frac{1}{4})-\frac{3}{4}log_2(\frac{3}{4}))+\frac{5}{12}(-\frac{5}{5}log_2(\frac{5}{5}))+\frac{3}{12}(-\frac{2}{3}log_2(\frac{2}{3})-\frac{1}{3}log_2(\frac{1}{3}))$$
$$=\frac{4}{12}\times 0.8113+\frac{5}{12}\times 0+\frac{3}{12}\times 0.9183$$
$$=0.5$$

So, Node split on feature **Season** with gain $\mathrm{Ent}(D)-\sum_{v=1}^{V}\frac{|D^v|}{|D|}\mathrm{Ent}\left(D^v\right)=0.4150$.

To generate **Node 2,3,4,5**.



Node 2:

$$Ent(D)=-\frac{1}{2}log_2(\frac{1}{2})-\frac{1}{2}log_2(\frac{1}{2})=1$$

a=After 8:00,

$$\sum_{v=1}^{V}\frac{|D^v|}{|D|}\mathrm{Ent}\left(D^v\right)=\frac{1}{2}(-\frac{1}{1}log_2(\frac{1}{1}))+\frac{1}{2}(-\frac{1}{1}log_2(\frac{1}{1}))=0$$

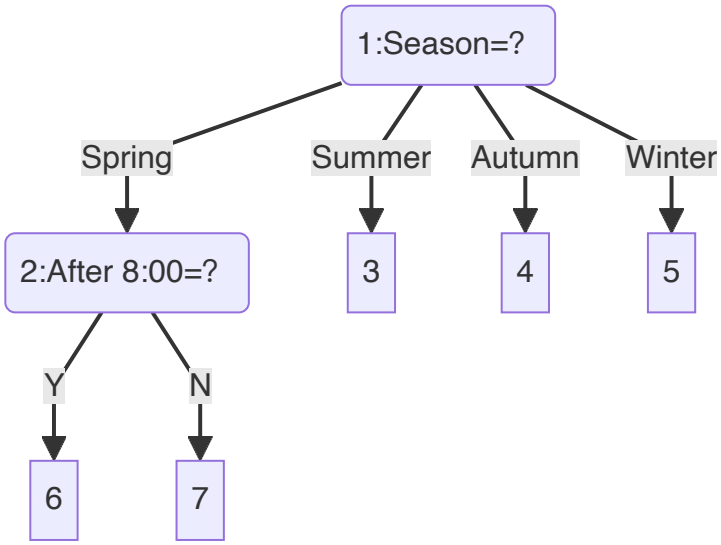
a=Wind

$$\sum_{v=1}^{V}\frac{|D^v|}{|D|}\mathrm{Ent}\left(D^v\right)=\frac{1}{2}(-\frac{1}{1}log_2(\frac{1}{1}))+\frac{1}{2}(-\frac{1}{1}log_2(\frac{1}{1}))=0$$

We can choose both of them.

So, Node split on feature **After 8:00** with gain 1.0.

To generate **Node 6,7**.



Node 6:

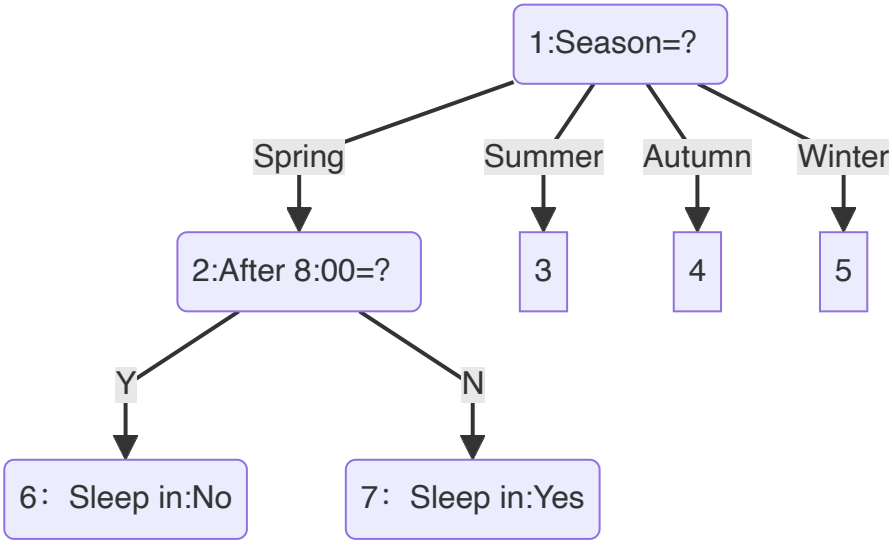
All target_values have the same value.

So this Node is `Sleep in:No`

Node 7:

All target_values have the same value.

So this Node is `Sleep in:Yes`



Node 3:

$$Ent(D) = -\frac{2}{3}log_2(\frac{2}{3}) - \frac{1}{3}log_2(\frac{1}{3}) = 0.9183$$

a=After 8:00,

$$\sum_{v=1}^{V}\frac{|D^v|}{|D|}\text{Ent}\left(D^v\right) = \frac{2}{3}(-\frac{1}{2}log_2(\frac{1}{2}) - \frac{1}{2}log_2(\frac{1}{2})) + \frac{1}{3}(-\frac{1}{1}log_2(\frac{1}{1})) = \frac{2}{3}\times 1 + \frac{1}{3}\times 0 = 0.6667$$

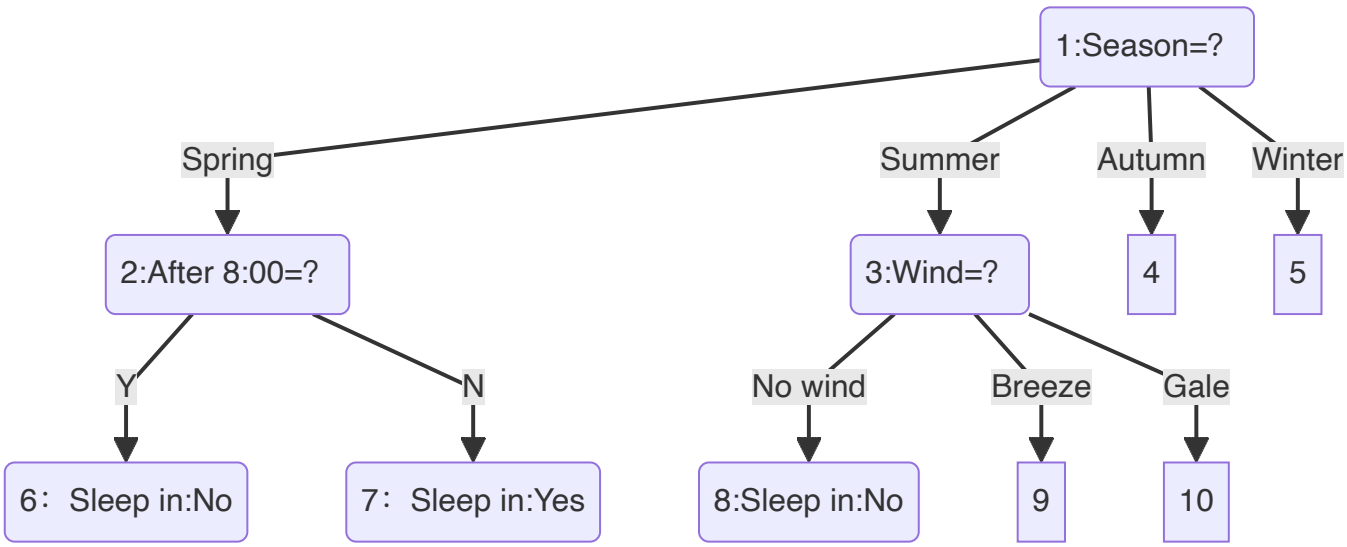
a=Wind,

$$\sum_{v=1}^{V}\frac{|D^v|}{|D|}\text{Ent}\left(D^v\right) = \frac{1}{3}(-\frac{1}{1}log_2(\frac{1}{1})) + \frac{2}{3}(-\frac{2}{2}log_2(\frac{2}{2})) = 0$$

So,Node split on feature Wind with gain 0.9183.

Because $D_{No\ wind}$ is empty, **Node 8 is marked as the class in D with largest proportion: yes**.

To generate **Node 9,10**.



Node 9:

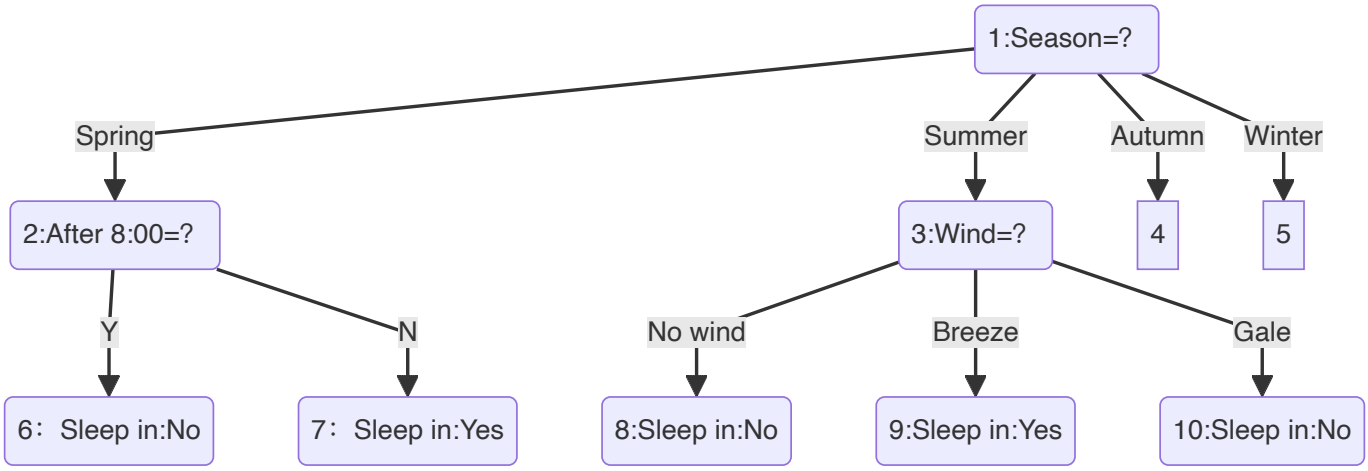
All target_values have the same value.

So this Node is `Sleep in:Yes`

Node 10:

All target_values have the same value.

So this Node is `Sleep in:No`



Node 4:

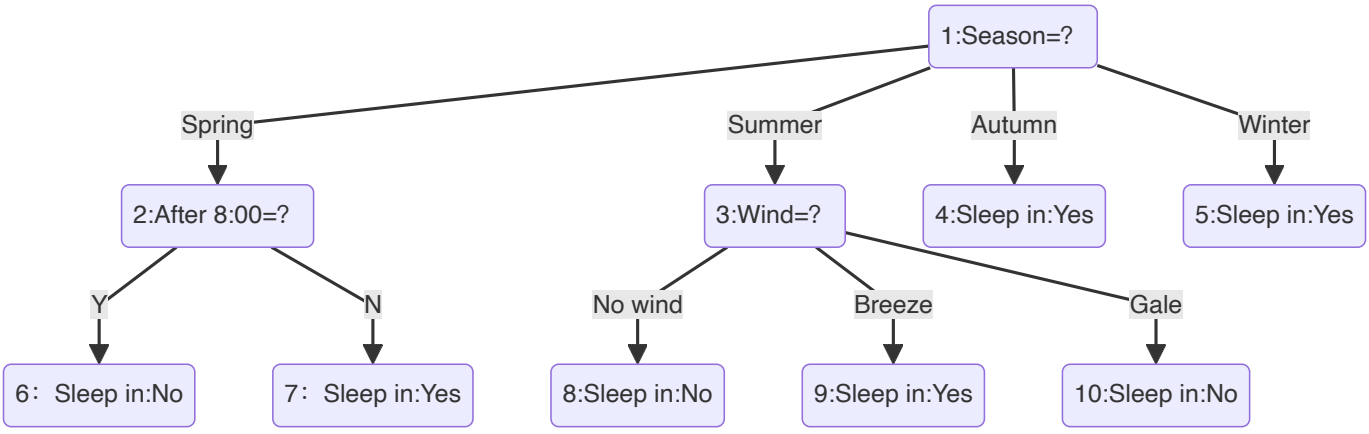
All target_values have the same value.

So this Node is `Sleep in:Yes`

Node 5:

All target_values have the same value.

So this Node is `Sleep in:Yes`



3.

The Prior probability：

$$P(y=-1)=\frac{5}{15}$$

$$P(y=1)=\frac{10}{15}$$

The condition probability:

$$P(x_1 = 2|y = -1) = \frac{\frac{1}{15}}{\frac{5}{15}} = \frac{1}{5}$$

$$P(x_1 = 2|y = 1) = \frac{\frac{4}{15}}{\frac{10}{15}} = \frac{4}{10}$$

$$P(x_2 = S|y = -1) = \frac{\frac{3}{15}}{\frac{5}{15}} = \frac{3}{5}$$

$$P(x_2 = S|y = 1) = \frac{\frac{1}{15}}{\frac{10}{15}} = \frac{1}{10}$$

And we have:

$$P(y = -1|x_1 = 2, x_2 = S) = P(y = -1) \times P(x_1 = 2|y = -1) \times P(x_2 = S|y = -1) = \frac{5}{15}\frac{1}{5}\frac{3}{5} = \frac{1}{25}$$

$$P(y = 1|x_1 = 2, x_2 = S) = P(y = 1) \times P(x_1 = 2|y = 1) \times P(x_2 = S|y = 1) = \frac{10}{15}\frac{4}{10}\frac{1}{10} = \frac{2}{75}$$

Because $P(y = -1|x_1 = 2, x_2 = S) > P(y = 1|x_1 = 2, x_2 = S)$,So predict $y = -1$