Assignment 3 (Due: Dec. 31, 2023)

1. (**Math**) Nonlinear least-squares. Suppose that $\mathbf{f}(\mathbf{x})=(f_1(\mathbf{x}),f_2(\mathbf{x}),\ldots,f_m(\mathbf{x})):\mathbb{R}^n\rightarrow\mathbb{R}^m$, $\mathbf{x}\in\mathbb{R}^n,\mathbf{f}\in\mathbb{R}^m$ and some $f_i(\mathbf{x}):\mathbb{R}^n\rightarrow\mathbb{R}$ is a (are) non-linear function(s). Then, the problem,

   $$\mathbf{x}^*=\arg\min_{\mathbf{x}}\frac{1}{2}\|\mathbf{f}(\mathbf{x})\|_2^2=\arg\min_{\mathbf{x}}\frac{1}{2}(\mathbf{f}(\mathbf{x}))^T\mathbf{f}(\mathbf{x})$$

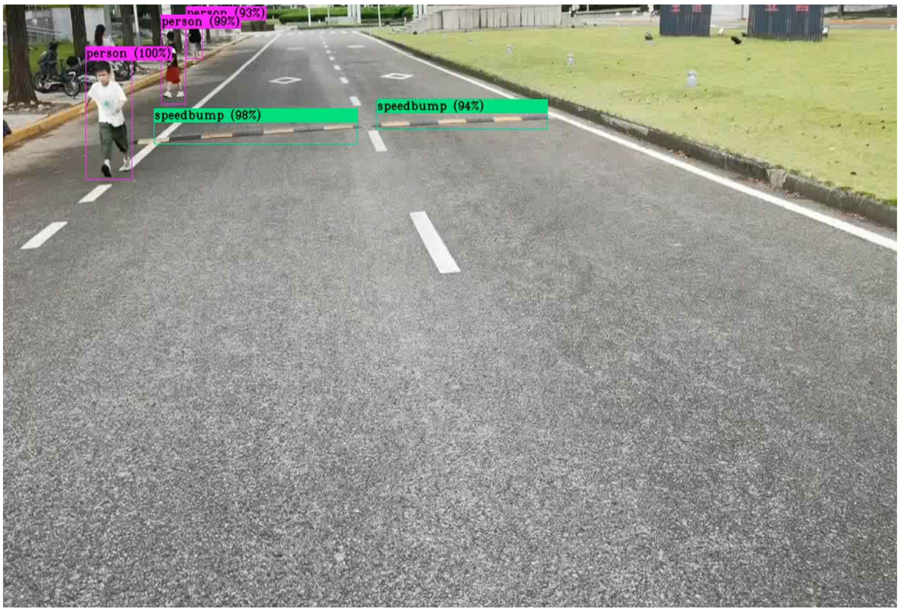
   is a nonlinear least-squares problem. In our lecture, we mentioned that Levenberg-Marquardt algorithm is a typical method to solve this problem. In L-M algorithm, for each updating step, at the current **x**, a local approximation model is constructed as,

   $$\begin{aligned}L(\mathbf{h})&=\frac{1}{2}(\mathbf{f}(\mathbf{x}+\mathbf{h}))^T\mathbf{f}(\mathbf{x}+\mathbf{h})+\frac{1}{2}\mu\mathbf{h}^T\mathbf{h}\\&=\frac{1}{2}(\mathbf{f}(\mathbf{x}))^T\mathbf{f}(\mathbf{x})+\mathbf{h}^T(\mathbf{J}(\mathbf{x}))^T\mathbf{f}(\mathbf{x})+\frac{1}{2}\mathbf{h}^T(\mathbf{J}(\mathbf{x}))^T\mathbf{J}(\mathbf{x})\mathbf{h}+\frac{1}{2}\mu\mathbf{h}^T\mathbf{h}\end{aligned}$$

   where **J**(**x**) is **f**(**x**)'s Jacobian matrix, and $\mu>0$ is the damped coefficient. Please prove that $L(\mathbf{h})$ is a strictly convex function. (Hint: If a function $L(\mathbf{h})$ is differentiable up to at least second order, L is strictly convex if its Hessian matrix is positive definite.)

2. (**Programming**) I have established a dataset for training models for detecting speed-bumps and persons. This dataset can be downloaded from https://github.com/csLinZhang/CVBook/tree/main/chapter-15-YOLO/For-yolov4. Using this dataset, please train a speed-bump detection model and test your model on the provided test video (on the course website). For this question, you only need to hand in your video with detected bounding-boxes to the TA. A sample

frame of our result video may like the following image.



3. (**Experiment**) 3D face scan and editing. Please refer to the files on the course website.