1.Calculate the gradient of the following multivariate function

(1) $u = xy + y^2 + 5$

$$\begin{gathered}\frac{\partial u}{\partial x} = y \\ \frac{\partial u}{\partial y} = x + 2y\end{gathered} \tag{1}$$

(2)$u = ln\sqrt{x^2 + y^2 + z^2}$

$$\begin{gathered}\frac{\partial u}{\partial x} = \frac{1}{\sqrt{x^2+y^2+z^2}} \times \frac{1}{2} \times \frac{1}{\sqrt{x^2+y^2+z^2}} \times 2x = \frac{x}{x^2+y^2+z^2} \\ \frac{\partial u}{\partial y} = \frac{y}{x^2+y^2+z^2}\end{gathered} \tag{2}$$

2.

According to:

$$\begin{gathered}\operatorname{Ent}(D) = -\sum_{k=1}^{|\mathcal{Y}|} p_k \log_2 p_k \\ \operatorname{Gain}(D,a) = \operatorname{Ent}(D) - \sum_{v=1}^{V} \frac{|D^v|}{|D|} \operatorname{Ent}\left(D^v\right)\end{gathered} \tag{3}$$

Node 1:

$$Ent(D) = -\frac{3}{12}log_2(\frac{3}{12}) - \frac{9}{12}log_2(\frac{9}{12}) = 0.8113 \tag{4}$$

a=Season,

$$\begin{aligned}\sum_{v=1}^{V} \frac{|D^v|}{|D|} \operatorname{Ent}\left(D^v\right) &= \frac{2}{12}(-\frac{1}{2}log_2(\frac{1}{2}) - \frac{1}{2}log_2(\frac{1}{2})) + \frac{3}{12}(-\frac{2}{3}log_2(\frac{2}{3}) - \frac{1}{3}log_2(\frac{1}{3})) \\ &+ \frac{2}{12}(-\frac{2}{2}log_2(\frac{2}{2})) + \frac{5}{12}(-\frac{5}{5}log_2(\frac{5}{5})) \\ &= \frac{2}{12} \times 1 + \frac{3}{12} \times 0.9183 + \frac{2}{12} \times 0 + \frac{5}{12} \times 0 \\ &= 0.3962\end{aligned} \tag{5}$$

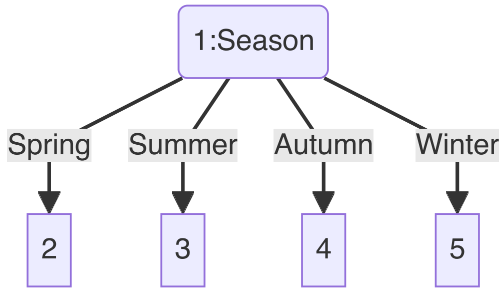
a=After 8:00,

$$\begin{aligned}\sum_{v=1}^{V} \frac{|D^v|}{|D|} \operatorname{Ent}\left(D^v\right) &= \frac{7}{12}(-\frac{1}{7}log_2(\frac{1}{7}) - \frac{6}{7}log_2(\frac{6}{7})) + \frac{5}{12}(-\frac{2}{5}log_2(\frac{2}{5}) - \frac{3}{5}log_2(\frac{3}{5})) \\ &= \frac{7}{12} \times 0.5917 + \frac{5}{12} \times 0.9710 \\ &= 0.7497\end{aligned} \tag{6}$$

a=Wind,

$$
\begin{aligned}
\sum_{v=1}^{V} \frac{|D^v|}{|D|} \operatorname{Ent}\left(D^v\right) &= \frac{4}{12}(-\frac{1}{4}log_2(\frac{1}{4}) - \frac{3}{4}log_2(\frac{3}{4})) + \frac{5}{12}(-\frac{5}{5}log_2(\frac{5}{5})) + \frac{3}{12}(-\frac{2}{3}log_2(\frac{2}{3}) - \frac{1}{3}log_2(\frac{1}{3})) \\
&= \frac{4}{12} \times 0.8113 + \frac{5}{12} \times 0 + \frac{3}{12} \times 0.9183 \\
&= 0.5
\end{aligned} \tag{7}
$$

So, Node split on feature **Season** with gain 0.4150.



Node 2:

$$
Ent(D) = -\frac{1}{2}log_2(\frac{1}{2}) - \frac{1}{2}log_2(\frac{1}{2}) = 1 \tag{8}
$$

a=After 8:00,

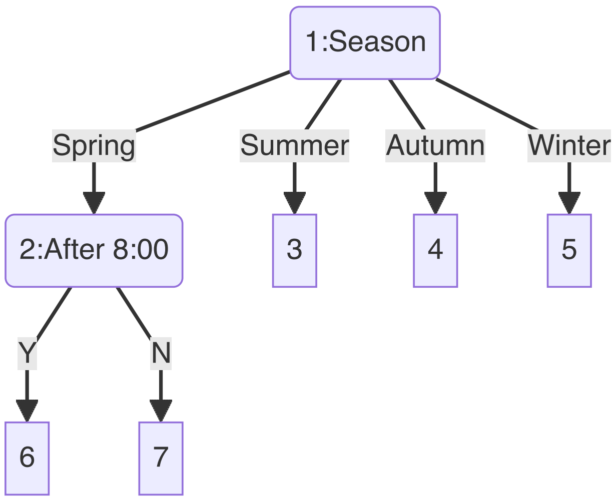
$$
\sum_{v=1}^{V} \frac{|D^v|}{|D|} \operatorname{Ent}\left(D^v\right) = \frac{1}{2}(-\frac{1}{1}log_2(\frac{1}{1})) + \frac{1}{2}(-\frac{1}{1}log_2(\frac{1}{1})) = 0 \tag{9}
$$

a=Wind

$$
\sum_{v=1}^{V} \frac{|D^v|}{|D|} \operatorname{Ent}\left(D^v\right) = \frac{1}{2}(-\frac{1}{1}log_2(\frac{1}{1})) + \frac{1}{2}(-\frac{1}{1}log_2(\frac{1}{1})) = 0 \tag{10}
$$

We can choose both of them.

So, Node split on feature **After 8:00** with gain 1.0.



Node 3:

$$
Ent(D) = -\frac{2}{3}log_2(\frac{2}{3}) - \frac{1}{3}log_2(\frac{1}{3}) = 0.9183 \tag{11}
$$

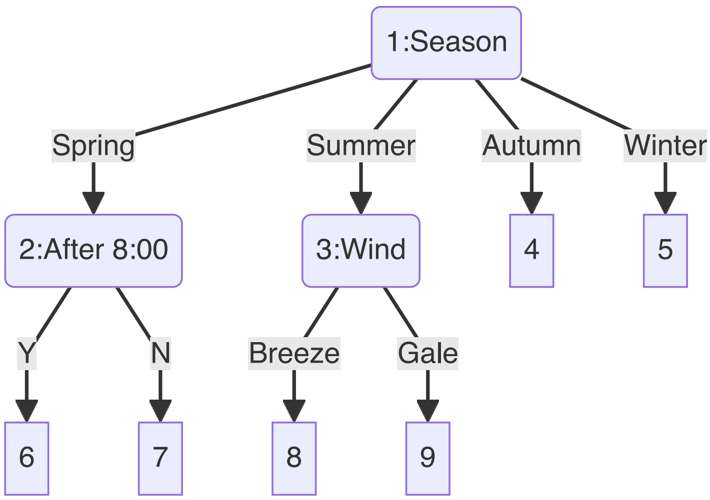
a=After 8:00,

$$\sum_{v=1}^{V}\frac{|D^v|}{|D|}\text{Ent}\left(D^v\right)=\frac{2}{3}(-\frac{1}{2}log_2(\frac{1}{2})-\frac{1}{2}log_2(\frac{1}{2}))+\frac{1}{3}(-\frac{1}{1}log_2(\frac{1}{1}))=\frac{2}{3}\times 1+\frac{1}{3}\times 0=0.6667 \tag{12}$$

a=Wind,

$$\sum_{v=1}^{V}\frac{|D^v|}{|D|}\text{Ent}\left(D^v\right)=\frac{1}{3}(-\frac{1}{1}log_2(\frac{1}{1}))+\frac{2}{3}(-\frac{2}{2}log_2(\frac{2}{2}))=0 \tag{13}$$

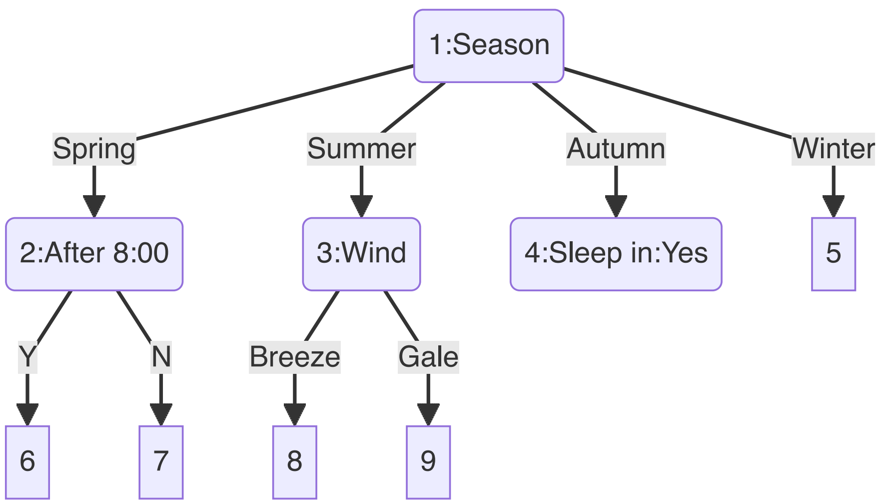
So,Node split on feature Wind with gain 0.9183.



Node 4:

All target_values have the same value.

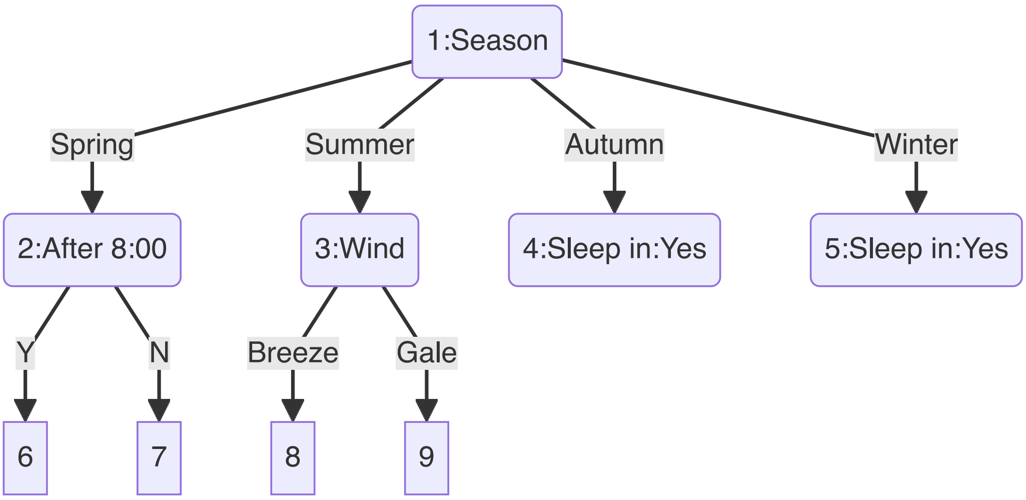
So this Node is `Sleep in:Yes`



Node 5:

All target_values have the same value.

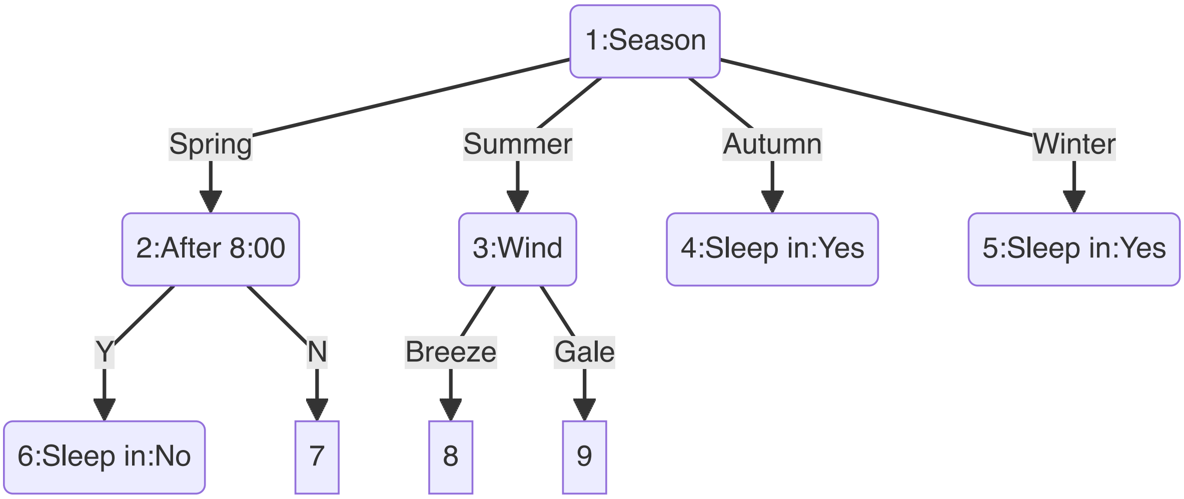
So this Node is `Sleep in:Yes`



Node 6:

All target_values have the same value.

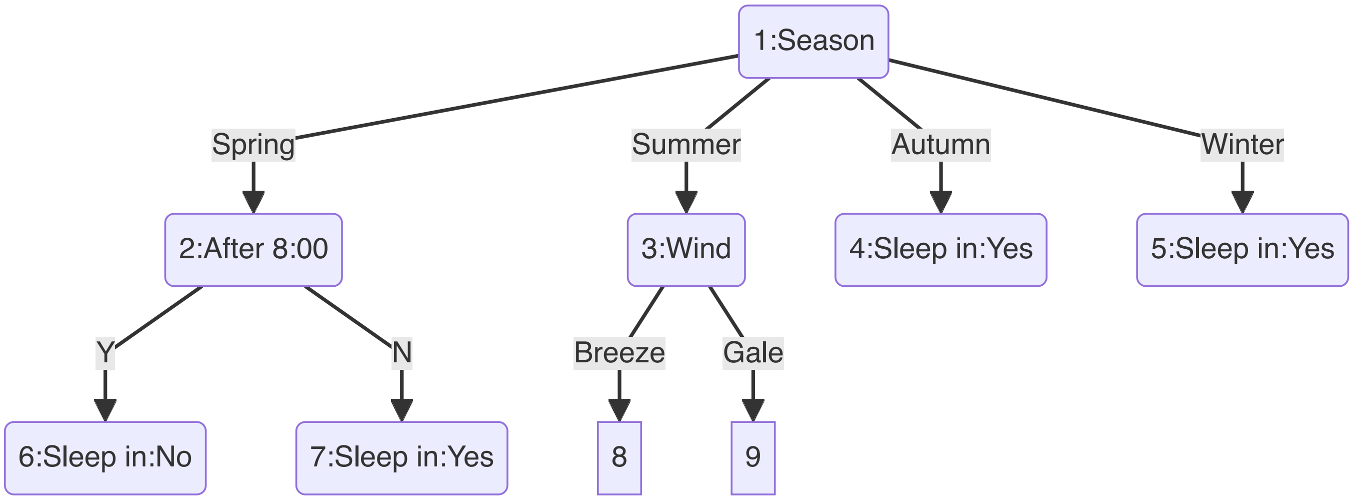
So this Node is `Sleep in:No`



Node 7:

All target_values have the same value.

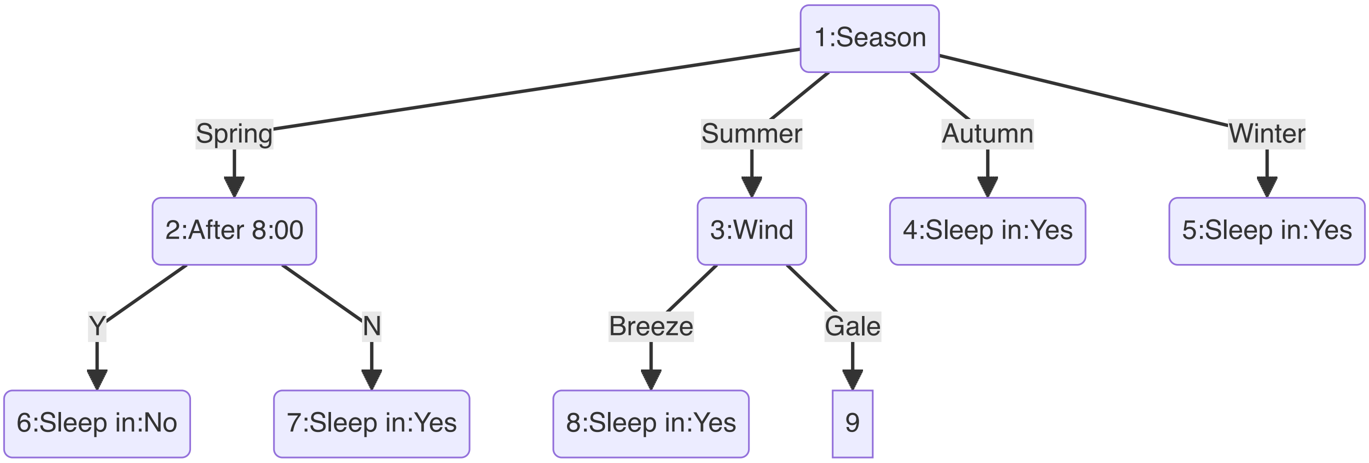
So this Node is `Sleep in:Yes`



Node 8:

All target_values have the same value.

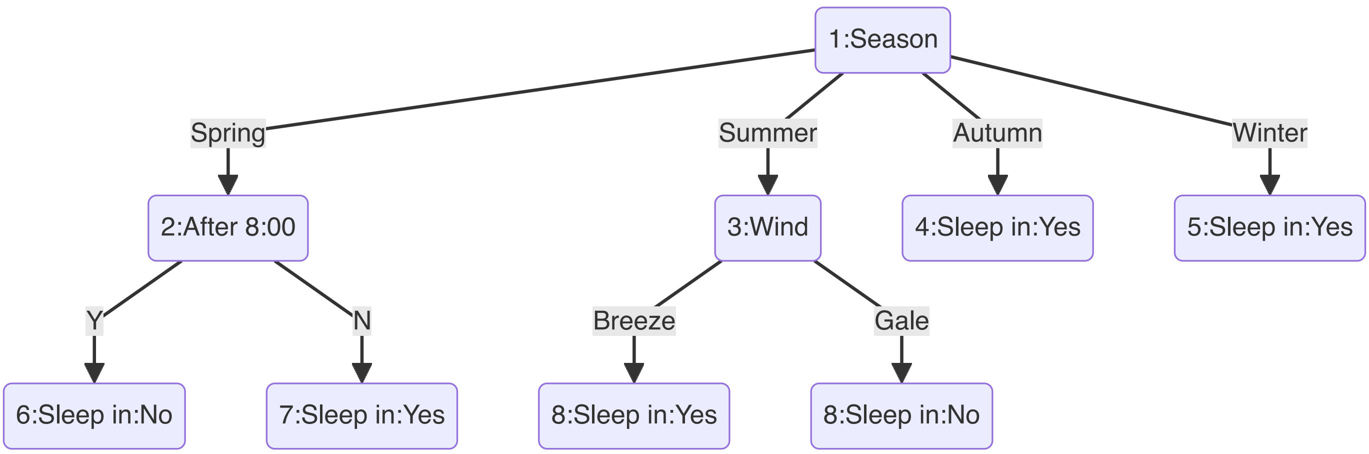
So this Node is `Sleep in:Yes`



Node 9:

All target_values have the same value.

So this Node is `Sleep in:No`



3.

The Prior probability：

$$
\begin{aligned}
P(y=-1) &= \frac{5}{15} \\
P(y=1) &= \frac{10}{15}
\end{aligned} \tag{14}
$$

The condition probability:

$$
\begin{aligned}
P(x_1=2|y=-1) &= \frac{\frac{1}{15}}{\frac{5}{15}} = \frac{1}{5} \\
P(x_1=2|y=1) &= \frac{\frac{1}{15}}{\frac{10}{15}} = \frac{1}{10} \\
P(x_2=S|y=-1) &= \frac{\frac{3}{15}}{\frac{5}{15}} = \frac{3}{5} \\
P(x_2=S|y=1) &= \frac{\frac{1}{15}}{\frac{10}{15}} = \frac{1}{10}
\end{aligned} \tag{17}
$$

And we have:

$$
\begin{aligned}
P(y=-1) \times P(x_1=2|y=-1) \times P(x_2=S|y=-1) &= \frac{5}{15}\frac{1}{5}\frac{3}{5} = \frac{1}{25} \\
P(y=1) \times P(x_1=2|y=1) \times P(x_2=S|y=1) &= \frac{10}{15}\frac{1}{10}\frac{1}{10} = \frac{1}{150}
\end{aligned} \tag{18}
$$

So predict $y=-1$