6.15 . 6.16

6.39 3.5 . 6.7 . 8

6.15

(1) 令 $\hat{x}(t)=x(t+2)$，$\hat{x}(t)$ 为实偶信号，则 $\hat{\varphi}(w)=\begin{cases}0, & \hat{X}(w)\geq 0\\ \pm\pi, & \hat{X}(w)<0\end{cases}$

$\hat{X}(w)=X(w)e^{j2w}$，$X(w)=\hat{X}(w)e^{-2jw}$

$\therefore \varphi(w)=\begin{cases}-2w, & \hat{X}(w)\geq 0\\ -2w\pm\pi, & \hat{X}(w)<0\end{cases}$

(2) $X(0)=\int_{-\infty}^{+\infty}x(t)dt=13$

(3) $\int_{-\infty}^{+\infty}X(w)dw=2\pi x(0)=4\pi$

(4) $\because \frac{dx(t)}{dt}\xrightarrow{F} jwX(w)$ $\therefore \int_{-\infty}^{+\infty}wX(w)dw=2\pi\frac{1}{j}\frac{dx(t)}{dt}\Big|_{t=0}=0$

(5) $\int_{-\infty}^{+\infty}|X(w)|^2dw=2\pi\int_{-\infty}^{+\infty}|x(t)|^2dt=\frac{140}{3}\pi$

(6) $\int_{-\infty}^{+\infty}X(w)e^{j2w}dw=2\pi x(2)=2\pi$

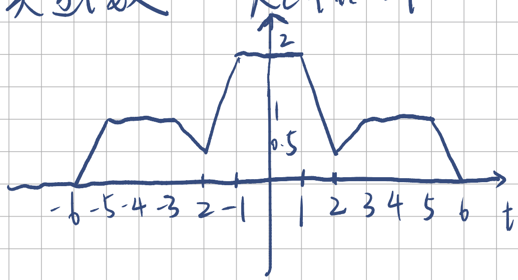
(7) $r(t)=\begin{cases}1, & |t|<1\\ 0, & t>1\end{cases}\xrightarrow{F} 2\frac{\sin w}{w}$

$\therefore \int_{-\infty}^{+\infty}2X(w)\frac{\sin w}{w}e^{j2w}dw=2\pi[x(t)*r(t)]\Big|_{t=2}=6\pi$

(8) $-jtx(t+2)\xrightarrow{F}\frac{dX(w)e^{j2w}}{dw}$

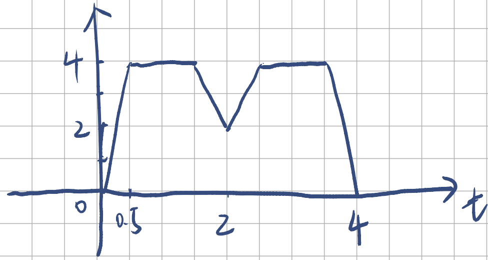
$\therefore \int_{-\infty}^{+\infty}\frac{dX(w)e^{j2w}}{dw}dw=2\pi[-jtx(t+2)]\Big|_{t=0}=0$

(9) $\because x(t)$ 是实函数 $\therefore Re\{X(t)\}\xrightarrow{F^{-1}}x_e(t)=\frac{x(t)+x(-t)}{2}$



(10) $x(t-2) \xrightarrow{F} X(\omega)e^{-j2\omega}$

$2x(2t-2) \xrightarrow{F} X(\frac{\omega}{2})e^{-j\omega}$



6.16

(1) $\tilde{X}(2\pi)=\tilde{X}(0)=\sum_{n=-\infty}^{+\infty}x[n]=0$

(2) $\tilde{X}(\pi)=\sum_{n=-\infty}^{+\infty}x[n]e^{j\pi n}=0$

(3) $\int_{-\pi}^{\pi}\tilde{X}(\Omega)d\Omega=2\pi x[0]=4\pi$

(4) $\int_{-\pi}^{\pi}|\tilde{X}(\Omega)|^2d\Omega=2\pi\sum_n|x[n]|^2=28\pi$

(5) 令 $\hat{x}[n]=x[n+2]$，则 $\tilde{\hat{X}}(\Omega)=\tilde{X}(\Omega)e^{j2\Omega}$, $\tilde{X}(\Omega)=\tilde{\hat{X}}e^{-j2\Omega}$

而 $\hat{\varphi}(\Omega)=\begin{cases}\frac{\pi}{2}, & Im\{\tilde{\hat{X}}(\Omega)\geq 0\}\\ -\frac{\pi}{2}, & Im\{\tilde{\hat{X}}(\Omega)<0\}\end{cases}$ $\therefore \varphi(\Omega)=\begin{cases}-2\Omega+\frac{\pi}{2}, & Im\{\tilde{\hat{X}}(\Omega)\geq 0\}\\ -2\Omega-\frac{\pi}{2}, & Im\{\tilde{\hat{X}}(\Omega)<0\}\end{cases}$

(6) $\int_{-\pi}^{\pi}\tilde{X}(\Omega)e^{-j\Omega}d\Omega=2\pi x[-1]=2\pi$

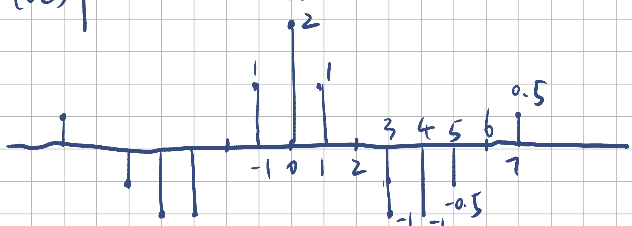
(7) $-jnx[n]\xrightarrow{F}\frac{d\tilde{X}(\Omega)}{d\Omega}$ $\therefore \int_{-\pi}^{\pi}\left|\frac{d\tilde{X}(\Omega)}{d\Omega}\right|^2d\Omega=2\pi\sum_n|nx[n]|^2=316\pi$

(8) $r[n]=\begin{cases}1, & |n|\leq 2\\ 0 & |n|>2\end{cases}\xrightarrow{F}\frac{\sin\frac{5}{2}\Omega}{\sin\frac{1}{2}\Omega}$

$\therefore \int_{-\pi}^{\pi}\tilde{X}(\Omega)\frac{\sin\frac{5}{2}\Omega}{\sin\frac{1}{2}\Omega}d\Omega=2\pi[x[n]*r[n]]\Big|_{n=0}=8\pi$

(9) $\int_{-\pi}^{\pi}\tilde{X}(\Omega)(1-e^{-j\Omega})d\Omega=2\pi[x[0]-x[1]]=2\pi$

(10) $Re\{\tilde{X}(\Omega)\}\xrightarrow{F^{-1}}x_e[n]=\frac{x[n]+x[-n]}{2}$



6.39

(3) $\lim\limits_{t\to 0^+} x(t) = 10$, $\lim\limits_{t\to \infty} x(t) = 4$

(5) $sX(s) = \dfrac{e^{-s}}{5s(s-2)^3}$, $t\to 0^+ : 0$ $\quad t\to\infty : \infty$

(6) $x[0] = 1$, $\lim\limits_{n\to\infty} x[n] = \lim\limits_{z\to 1}(z-1)\cdot \dfrac{z^4-1}{z^3(z-1)} = 0$

(7) $x[0] = 1$, $\lim\limits_{n\to\infty} x[n] = \lim\limits_{z\to 1}(z-1)\cdot \dfrac{z^2}{(z-0.5)(z+0.5)} = 0$

(8) $x[0] = 0$, $\lim\limits_{n\to\infty} x[n] = \lim\limits_{z\to 1}(z-1)\cdot \dfrac{1}{z^3-1.5z^2+0.5z} = \lim\limits_{z\to 1} \dfrac{z-1}{z\cdot(z-\frac{1}{2})(z-1)} = 2$