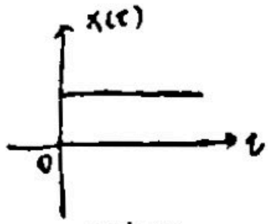
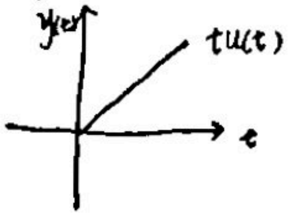
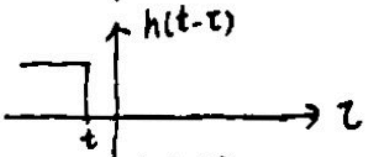
3.3

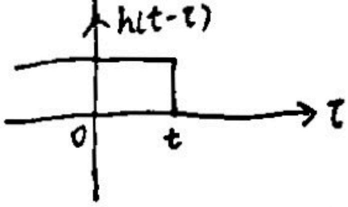
(1) $y(t)=u(t)*u(t)=\int_{-\infty}^{+\infty}u(t-\tau)u(\tau)d\tau=\int_0^t d\tau\, u(t)=t\,u(t)$







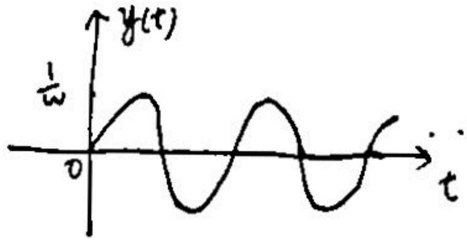
$t<0,\ y(t)=0$

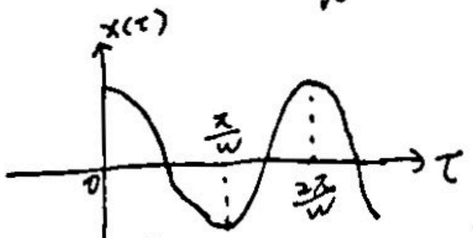


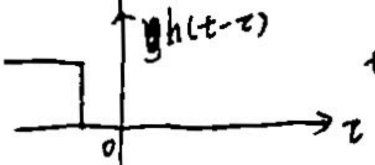
$t\geqslant 0,\ y(t)=\int_0^t d\tau=t$

(2) $y(t)=x(t)*h(t)=\int_{-\infty}^{+\infty}x(\tau)h(t-\tau)d\tau=\int_{-\infty}^{+\infty}\cos\omega\tau\, u(\tau)\, u(t-\tau)d\tau$

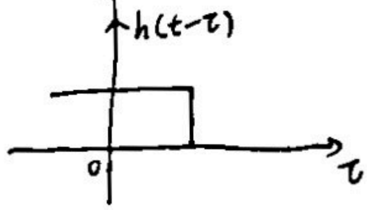
$=\int_0^t\cos\omega\tau d\tau\cdot u(t)=\frac{\sin\omega t}{\omega}u(t)$







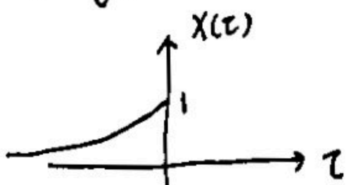
$t<0,\ y(t)=0$

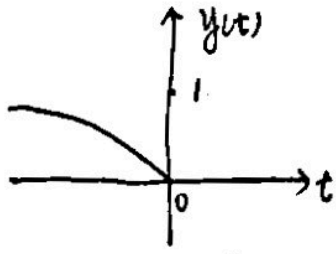


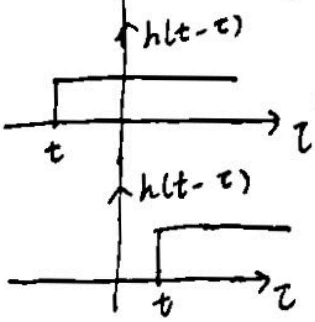
$t\geqslant 0,\ y(t)=\int_0^t x(\tau)h(t-\tau)=\frac{\sin\omega t}{\omega}$

(3) $y(t)=x(t)*h(t)=\int_{-\infty}^{+\infty}x(\tau)h(t-\tau)d\tau=\int_{-\infty}^{+\infty}e^{\tau}u(-\tau)u(\tau-t)d\tau=\int_t^0 e^{\tau}d\tau\cdot u(-t)$

$=(1-e^{t})u(-t)$







$t\leqslant 0\quad y(t)=1-e^{t}$

$t>0\quad y(t)=0$

(4) $y[n] = x[n]*h[n] = \sum_{k=-\infty}^{+\infty} u[n-k]u[k] = \sum_{k=0}^{n} 1 \cdot u[n] = (n+1)u[n]$

(5) $y(t) = \int_{-\infty}^{+\infty} \sin w\tau\, u(\tau)\, u(t-\tau)\, d\tau = \int_0^t \sin w\tau\, d\tau\, u(t) = \frac{1}{w}(1-\cos wt)u(t)$

(6) $y[n] = \sum_{k=-\infty}^{+\infty} u[k-n]\, 2^k u[-k] = \sum_{k=n}^{0} 2^k u[-n] = (2-2^n)u[-n]$

3.4 (1) $a \neq b$时, $a^n u[n] * b^n u[n] = \sum_{m=-\infty}^{+\infty} a^{n-m} u[n-m]\, b^m u[m] = \sum_{m=0}^{n} a^{n-m} b^m u[n]$

$$= a^n u[n] \cdot \sum_{m=0}^{n} (a^{-1}b)^m = a^n \cdot \frac{1-(a^{-1}b)^{n+1}}{1-a^{-1}b} u[n] = \frac{a^{n+1}-b^{n+1}}{a-b} u[n]$$

$a=b$时, $a^n u[n] * b^n u[n] = a^n u[n] \sum_{m=0}^{n} 1 = (n+1)a^n u[n]$

(2) $a \neq b$时, $e^{-at}u(t) * e^{-bt}u(t) = \int_{-\infty}^{+\infty} e^{-a(t-\tau)} u(t-\tau) \cdot e^{-b\tau} u(\tau)\, d\tau = e^{-at}u(t) \cdot \int_0^t e^{a\tau-b\tau} d\tau$

$$= \frac{e^{-bt}-e^{-at}}{a-b} u(t)$$

$a=b$时, $e^{-at}u(t) * e^{-bt}u(t) = e^{-at}u(t)\int_0^t d\tau = te^{-at}u(t)$

3.5 (1)

		$x[0]$ 1	$x[1]$ 0	$x[2]$ -2	$x[3]$ 1
$h[-1]$	1	1	0	-2	1
$h[0]$	-1	-1	0	2	-1
$h[1]$	1	1	0	-2	1

n	<-2	-1	0	1	2	3	4	>5
$y[n]$	0	1	-1	-1	3	-3	1	0

(2)

		$x[0]$ 1	$x[1]$ -1	$x[2]$ 0	$x[3]$ 0	$x[4]$ 1	$x[5]$ 1
$h[-1]$	1	1	-1	0	0	1	1
$h[0]$	2	2	-2	0	0	2	2
$h[1]$	1	1	-1	0	0	1	1
$h[2]$	1	1	-1	0	0	1	1

n	<-2	-1	0	1	2	3	4	5	6	7	>8
$y[n]$	0	1	1	-1	0	0	3	3	2	1	0

3-9 $y[n] = x[n] * h_1[n] * h_2[n] = (\delta[n] - a\delta[n-1]) * a^n u[n] * \sin 8n$

$= a^n u[n] * \delta[n] * \sin 8n - a^n u[n] * a\delta[n-1] * \sin 8n$

$= a^n u[n] * \sin 8n - a^n u[n-1] * \sin 8n$

$= \sin 8n$

3.11 (1)

$$h(t) = \left\{ \left[(h_1 * h_2 - h_3 * h_4) * h_3 + h_3 * h_2 \right] * h_5 - h_3 \right\} * h_6$$

(2) $(h_1 * h_2 - h_3 * h_4) * h_3 = [u(t) * \delta(t-1) - e^{-t}u(t) * u(t-1)] * e^{-t}u(t)$

$= [u(t) - e^{-t}u(t) * u(t)] * \delta(t-1) * e^{-t}u(t)$

$h_3(t) * h_2(t) = e^{-t}u(t) * \delta(t-1)$

$\therefore (h_1 * h_2 - h_3 * h_4) * h_3 - h_3 * h_2 = [u(t) - e^{-t}u(t) * u(t) + \delta(t)] * \delta(t-1) * e^{-t}u(t)$

$\therefore [(h_1 * h_2 - h_3 * h_4) * h_3 + h_3 * h_2] * h_5 = [u(t) - e^{-t}u(t) * u(t) + \delta(t)] * e^{-t}u(t)$
$= g(t)$

$[g(t) - h_3] * h_6 = \{[u(t) - e^{-t}u(t) * u(t) + \delta(t)] * e^{-t}u(t) - e^{-t}u(t)\} * \delta'(t)$

$= [u(t) - e^{-t}u(t) * u(t)] * e^{-t}u(t) * \delta'(t)$

$= [\delta(t) - e^{-t}u(t)] * e^{-t}u(t) = e^{-t}u(t) - te^{-t}u(t)$

$= (1-t)e^{-t}u(t) = h(t)$

(3) $y(t) = u(t) * (1-t)e^{-t}u(t)$

$= \int_0^t (1-\tau)e^{\tau} d\tau \; u(t)$

$= te^{-t}u(t)$