HW4.

2.7 $[W_r \mid N(t)=n] \overset{d}{=} U_r$

$$f_{W_r \mid N(t)=n}(w_r \mid n) = \frac{1}{t^n} \frac{n!}{(r-1)!(n-r)!} w_r^{r-1}(t-w_r)^{n-r}$$

2.11 $$P(T>t) = P(N(t)=0) + \sum_{n=1}^{\infty} P(X(t)<\alpha \mid N(t)=n) P(N(t)=n)$$

$$= e^{-\lambda t} + \sum_{n=1}^{\infty} \int_0^{\alpha} \frac{\mu^n}{(n-1)!} x^{n-1} e^{-\mu x} \cdot \frac{(\lambda t)^n}{n!} e^{-\lambda t} dx$$

$$E[T] = \int_0^{\infty} P(T>t) dt = \int_0^{\infty} e^{-\lambda t} dt + \int_0^{\infty} \sum_{n=1}^{\infty} \int_0^{\alpha} \frac{\mu^n}{(n-1)!} x^{n-1} e^{-\mu x} \frac{(\lambda t)^n}{n!} e^{-\lambda t} dx dt$$

$$= \frac{1}{\lambda} + \sum_{n=1}^{\infty} \int_0^{\alpha} \frac{\mu^n}{(n-1)!} x^{n-1} e^{-\mu x} dx \int_0^{\infty} \frac{(\lambda t)^n}{n!} e^{-\lambda t} dt$$

$$= \frac{1}{\lambda} + \frac{1}{\lambda} \sum_{n=1}^{\infty} \int_0^{\alpha} \frac{\mu^n}{(n-1)!} x^{n-1} e^{-\mu x} dx$$

$$= \frac{1}{\lambda} + \frac{1}{\lambda} \int_0^{\alpha} \mu \sum_{n=1}^{\infty} \frac{(\mu x)^{n-1}}{(n-1)!} e^{-\mu x} dx$$

$$= \frac{1}{\lambda} + \frac{1}{\lambda} \int_0^{\alpha} \mu e^{\mu x} \cdot e^{-\mu x} dx$$

$$= \frac{1}{\lambda} + \frac{1}{\lambda} \mu \alpha$$

$$= \frac{\mu \alpha + 1}{\lambda}$$