第四次作业

3.22

3.27 (1)(5)

3.29

3.34

4.4 (1)

4.7 (1)(2)

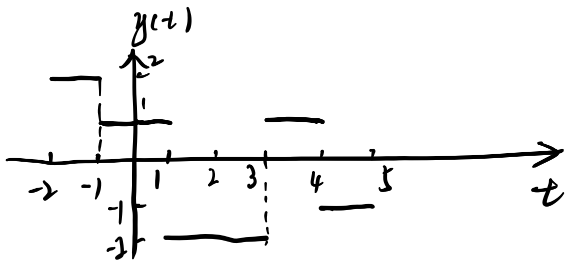
T3.22

(1) $x(t)=2x_0(t+1)+x_0(t)+3x_0(t-1)-x_0(t-2)+x_0(t-3)$

则

n	-1	0	1	2	3	others
a_n	2	1	3	-1	1	0

(2) $y(t)=2y_0(t+1)+y_0(t)+3y_0(t-1)-y_0(t-2)+y_0(t-3)$

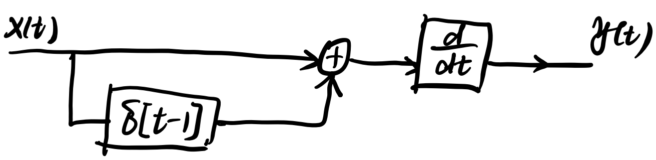


(3) $u_{-2}(t)=\sum_{k=1}^{\infty} k x_0(t-k) \Rightarrow h_{-2}(t)=\sum_{k=1}^{\infty} k y_0(t-T)=u(t)+u(t-1)$

(4) $\delta(t)=\frac{d}{dt}h_{-2}(t)=\delta(t)+\delta(t-1)$

$h(t)=\frac{d}{dt}\delta(t)=\delta'(t)+\delta'(t-1)$

(5)



T3.27 (1) $y(t)=\frac{1}{2T}\int_{-T}^{T}x(t-\tau)d\tau=\frac{1}{2T}\int_{-\infty}^{+\infty}x(t-\tau)[u(\tau+T)-u(\tau-T)]d\tau$

$=x(t)*\frac{u(t+T)-u(t-T)}{2T}$

线性、时不变、非因果、稳定 $h(t)=\frac{1}{2T}[u(t+T)-u(t-T)]$

(5) $y[n] = a^n \sum_{k=n-3}^{n} x[k] a^{-k} = x[n-3]a^3 + x[n-2]a^2 + x[n-1]a + x[n]$

线性、时不变、因果、稳定、$h(t) = \delta[n-3]a^3 + \delta[n-2]a^2 + \delta[n-1]a$
$+ \delta[n]$

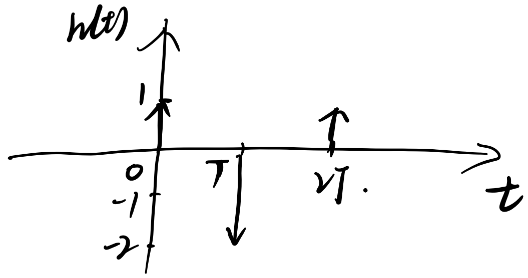
T3.28

(1) $y(t) = \sum_{n=0}^{\infty} y_n(t) = \sum_{n=0}^{\infty} h_n x(t-nT)$

当 $x(t) = \delta(t)$ 时：$h(t) = \sum_{n=0}^{\infty} h_n \delta(t-nT)$

$\because h_0 = 1, h_1 = -2, h_2 = 1, h_n = 0, n > 2$

$\therefore h(t) = \delta(t) - 2\delta(t-T) + \delta(t-2T)$



(2) 由(1)得 $h(t) = \sum_{n=0}^{\infty} h_n \delta(t-nT)$

$x(t) = \sum_{n=-\infty}^{+\infty} x_n \delta(t-nT) \Rightarrow y(t) = x(t) * h(t) = \sum_{k=0}^{\infty} h_k \sum_{m=-\infty}^{\infty} x_m \delta[t-(m+k)T]$

令 $m+k=n$, $\Rightarrow y(t) = \sum_{n=-\infty}^{+\infty} \left(\sum_{k=-\infty}^{\infty} x_{n-k} h_k \right) \delta[t-nT]$

$\therefore y(t) = \sum_{n=-\infty}^{+\infty} y_n \delta(t-nT)$

3.34 (1) $s(t)=(e^{-3t}-2e^{-2t}+1)u(t)$

$\Downarrow$ $h(t)=s'(t)=(-3e^{-3t}+4e^{-2t})u(t)$

$y(t)=h(t)*x(t)=(\frac{5}{2}e^{-t}+\frac{1}{2}e^{-3t}-4e^{-2t})u(t)$.

(2) $s[n]=(\frac{1}{2})^n u[n+1]$

$h[n]=s[n]-s[n-1]=(\frac{1}{2})^n u[n+1]-(\frac{1}{2})^{n-1}u[n]=-(\frac{1}{2})^n u[n]+2\delta[n+1]$

$x[n]*h[n]=(-\frac{1}{2})^n u[n]*[-(\frac{1}{2})^n u[n]]+2\delta[n+1]*(-\frac{1}{2})^n u[n]$

$=\sum_{k=-\infty}^{\infty}(-\frac{1}{2})^k u[k]\cdot[-(\frac{1}{2})^{n-k}u[n-k]]-(-\frac{1}{2})^n u[n+1]$

$=[-(\frac{1}{2})^n\sum_{k=0}^{n}(-1)^k]u[n]-(-\frac{1}{2})^n u[n+1]$

$=-(\frac{1}{2})^n[\frac{1}{2}+\frac{1}{2}(-1)^n]u[n]-(-\frac{1}{2})^n u[n]+2\delta[n+1]$

$=[-(\frac{1}{2})^{n+1}+(-\frac{1}{2})^{n+1}-(-\frac{1}{2})^n]u[n]+2\delta[n+1]$

$=[-(\frac{1}{2})^{n+1}+3(-\frac{1}{2})^{n+1}]u[n]+2\delta[n+1]$

T4.4

4.4 $y[n]=x[n]+\frac{3}{4}y[n-1]-\frac{1}{8}y[n-2]$, $y[n-2]=8x[n]-8y[n]+6y[n-1]$

(1) $y[-1]=0$, $y[-2]=0$

① 当输入为 $x_1[n]=(\frac{1}{3})^n$ 时

$y[0]=x[0]+\frac{3}{4}y[-1]-\frac{1}{8}y[-2]=1$

$y[1]=x[1]+\frac{3}{4}y[0]-\frac{1}{8}y[-1]=\frac{13}{12}$

$y[2]=x[2]+\frac{3}{4}y[1]-\frac{1}{8}y[0]=\frac{115}{144}$

$y[3]=x[3]+\frac{3}{4}y[2]-\frac{1}{8}y[1]=\frac{865}{1728}$

$\vdots$

$y[-3]=8x[-1]-8y[-1]+6y[-2]=24$

$y[-4]=8x[-2]-8y[-2]+6y[-3]=216$

$y[-5]=8x[-3]-8y[-3]+6y[-4]=1320$

$\vdots$

② 当输入为 $x_2[n]=(\frac{1}{3})^n u[n]$ 时

$y[0]=x[0]+\frac{3}{4}y[-1]-\frac{1}{8}y[-2]=1$

$y[1]=x[1]+\frac{3}{4}y[0]-\frac{1}{8}y[-1]=\frac{13}{12}$

$y[2]=x[2]+\frac{3}{4}y[1]-\frac{1}{8}y[0]=\frac{115}{144}$

$y[3]=x[3]+\frac{3}{4}y[2]-\frac{1}{8}y[1]=\frac{865}{1728}$

$\vdots$

$y[-3]=8x[-1]-8y[-1]+6y[-2]=0$

$y[-4]=8x[-2]-8y[-2]+6y[-3]=0$

$y[-5]=8x[-3]-8y[-3]+6y[-4]=0$

$\vdots$

T4.7 (1) $y''(t)+2y'(t)+y(t)=x(t)$; $y(0)=2$, $y'(0)=-1$

特征方程 $\lambda^2+2\lambda+1=0 \Rightarrow \lambda_1=\lambda_2=-1$

$\hat{y}(t)=(C_1+C_2t)e^{-t}$

特解: $y''(t)+2y'(t)+y(t)=u(t)$

当 $t\geq 0$ 时, $u(t)=1 \Rightarrow y''(t)+2y'(t)+y(t)=1$

$\therefore \hat{y}(t)=1$

$\therefore y(t)=(C_1+C_2t)e^{-t}+1 \xrightarrow[y'(0)=-1]{y(0)=2} y(t)=e^{-t}+1$

当 $t<0$ 时, $u(t)=0 \Rightarrow y''(t)+2y'(t)+y(t)=0$

$\therefore y(t)=(C_1+C_2t)e^{-t} \xrightarrow[y'(0)=-1]{y(0)=2} y(t)=(t+2)e^{-t}$

综上：$y(t)=\begin{cases}(t+2)e^{-t}, & t<0\\ e^{-t}+1, & t\geq 0\end{cases}$.

(2) 特征方程 $\lambda^2-\lambda+\frac{1}{4}=0 \Rightarrow \lambda_1=\lambda_2=\frac{1}{2}$

特解：当 $n\geq 0$ 时 $x[n]=1$. $\therefore p-p+\frac{1}{4}p=1 \Rightarrow p=4$

$\therefore y_p[n]=4$

$\therefore y[n]=\begin{cases}(C_1+C_2n)(\frac{1}{2})^n+4, & n\geq 0\\ (C_1+C_2n)(\frac{1}{2})^n, & n<0\end{cases}$

当 $n\geq 0$ 时

$y[0]-y[-1]+(\frac{1}{4})y[-2]=1$

$\therefore y[0]=3=C_1+4$

$y[1]-y[0]+(\frac{1}{4})y[-1]=1$

$\therefore y[1]=3=(C_1+C_2)\times\frac{1}{2}+4$

$\therefore\begin{cases}C_1=-1\\ C_2=-1\end{cases}$

即 $y[n]=(-1-n)(\frac{1}{2})^n+4,\ n\geq 0$

当 $n<0$ 时

$y[-1]=(C_1-C_2)\times 2=4$

$y[-2]=(C_1-2C_2)\times 4=8$

$\therefore\begin{cases}C_1=2\\ C_2=0\end{cases}$

即 $y[n]=2\cdot(\frac{1}{2})^n,\ n<0$

综上：$y[n]=\begin{cases}-(n+1)(\frac{1}{2})^n+4, & n\geq 0\\ 2\cdot(\frac{1}{2})^n, & n<0\end{cases}$