

## 第二章：随机信号与系统：

1. 随机信号（序列）通过线性时不变系统：时域频域的分析方法

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} h(\tau) x(t-\tau) d\tau, \quad Y(\omega) = H(j\omega) X(\omega)$$

$$S_y(\omega) = \int_{-\infty}^{\infty} R_y(\tau) e^{-j\omega\tau} d\tau = |H(j\omega)|^2 S_x(\omega)$$

2. 平稳随机序列的参数模型：ARMA 模型，AR 模型，MA 模型：自相关函数，功率谱密度函数，三种模型之间的联系，模型的建立

$$y(n) = \sum_{k=0}^q b_k w(n-k) - \sum_{k=1}^p a_k y(n-k) \quad \text{滑动平均分量+自回归分量}$$

3. 随机信号的复表示：正弦型信号的复表示，任意信号的复表示

**第三章：信号检测：**判断是否存在信号或者存在哪个信号问题：假设检验问题处理

1. 几种准则下的判决规则都具有如下似然比检验形式：  $\lambda(x) = \frac{f(x|H_1)}{f(x|H_0)} \underset{H_0}{\geq} th$

最大后验概率准则：  $P(H_1|x) \underset{H_0}{\geq} P(H_0|x), \quad th = \frac{P(H_0)}{P(H_1)}$

最小错误概率准则：  $\bar{P}_e = P(H_0)P(D_1|H_0) + P(H_1)P(D_0|H_1) \xrightarrow{R0,R1} \min, \quad th = \frac{P(H_0)}{P(H_1)}$

$$P(D_1|H_0) = \int_{R_1} f(x|H_0) dx$$

贝叶斯准则：

$$\bar{C} = P(H_0)[C_{00}P(D_0|H_0) + C_{10}P(D_1|H_0)] + P(H_1)[C_{01}P(D_0|H_1) + C_{11}P(D_1|H_1)] \rightarrow \min$$

$$th = \frac{P(H_0)(C_{10} - C_{00})}{P(H_1)(C_{01} - C_{11})}$$

极小极大准则：

$$\bar{C}(p, p_1) = C_{00}p + C_{11}(1-p) + (C_{10} - C_{00})\alpha(p_1)p + (C_{01} - C_{11})\beta(p_1)(1-p) \geq \bar{C}(p, p) = \bar{C}_{\min}(p)$$

$$p_1 = \arg \max_p \bar{C}_{\min}(p), \quad \frac{\partial \bar{C}(p, p_1)}{\partial p} = 0, \quad th = \frac{p_1(C_{10} - C_{00})}{(1-p_1)(C_{01} - C_{11})}$$

纽曼-皮尔逊准则：  $\min P(D_0|H_1) \quad s.t. \quad P(D_1|H_0) = \alpha, \quad th$  由给定的虚警概率确定。

2. 对于  $M$  种假设的假设检验问题：

$$\bar{C} = \sum_{i=0}^{M-1} \sum_{j=0}^{M-1} C_{ij} P(D_i | H_j) P(H_j) \xrightarrow{R_0, R_1, \dots, R_{M-1}} \min$$

当  $C_{ii} = 0, C_{ij} = 1, i \neq j$ : 似然比检验形式  $\lambda(x) = \frac{f(x|H_i)}{f(x|H_j)} \geq \frac{P(H_j)}{P(H_i)}$ ,  $j = 0, \dots, M-1, \neq i$

3. 多样本的假设检验问题:

$$\lambda(\mathbf{x}) = \frac{f(\mathbf{x}|H_1)}{f(\mathbf{x}|H_0)} = \frac{f(x_1, x_2, \dots, x_N | H_1)}{f(x_1, x_2, \dots, x_N | H_0)} \underset{H_0}{\overset{H_1}{\geq}} \frac{P(H_0)(C_{10} - C_{00})}{P(H_1)(C_{01} - C_{11})} = th$$

4. 判决性能的计算

根据判决规则, 确定检验统计量, 计算检验统计量在两种假设下的概率密度函数。

$$P(D_1 | H_1) = \int_{th}^{+\infty} f(\lambda | H_1) d\lambda$$

5. 复合的假设检验问题

$$\lambda(\mathbf{x}) = \frac{f(\mathbf{x}|H_1)}{f(\mathbf{x}|H_0)} = \frac{\int_{(\Theta)} f(\mathbf{x}|\Theta, H_1) f_1(\Theta) d\Theta}{\int_{(\Phi)} f(\mathbf{x}|\Phi, H_0) f_0(\Phi) d\Phi} \underset{H_0}{\overset{H_1}{\geq}} \frac{(C_{10} - C_{00}) P(H_0)}{(C_{01} - C_{11}) P(H_1)} = th$$

6. 高斯白噪声中已知信号的检测: 根据似然比判决设计最佳接收机, 计算系统性能。

$$f(x(t) | H_i) = \lim_{N \rightarrow \infty} f(x_1, x_2, \dots, x_N | H_i) = F \exp \left\{ -\frac{1}{N_0} \int_0^T [x(t) - s_i(t)]^2 dt \right\}$$

$$\lambda(x(t)) \underset{H_0}{\overset{H_1}{\geq}} th \Rightarrow \int_0^T [s_1(t) - s_0(t)] x(t) dt \underset{H_0}{\overset{H_1}{\geq}} \frac{N_0}{2} \ln th + \frac{1}{2} \int_0^T [s_1^2(t) - s_0^2(t)] dt$$

通信接收机 (最小错误概率准则):

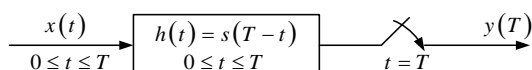
$$P_e = 1 - \Phi \left( \sqrt{(1-\rho)E/N_0} \right), \quad \rho = \frac{1}{E} \int_0^T s_0(t) s_1(t) dt, \quad E = \frac{1}{2} (E_0 + E_1)$$

雷达接收机 (NP 准则):  $P_D = 1 - \Phi \left( \Phi^{-1}(1-\alpha) - \sqrt{2E_1/N_0} \right)$

7. 匹配滤波器

$$t = t_0 \text{ 时刻的瞬时输出信噪比 } SNR_o = \frac{|s_o(t_0)|^2}{E\{n_o^2(t)\}} \leq \frac{E}{N_0/2}。 \quad H(j\omega) = S^*(\omega) e^{-j\omega t_0}。$$

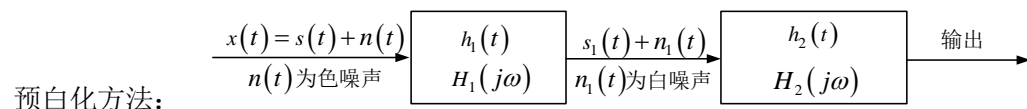
$$y(T) = \int_0^T x(t) s(t) dt$$



8. 信号的分集接收：设计最佳接收机，计算系统性能

$$\lambda(\mathbf{x}(t)) = \frac{f(x_1(t), \dots, x_M(t) | H_1)}{f(x_1(t), \dots, x_M(t) | H_0)} = \prod_{i=1}^M \frac{f(x_i(t) | H_1)}{f(x_i(t) | H_0)} = \prod_{i=1}^M \lambda(x_i(t)) \underset{H_0}{\overset{H_1}{\geq}} th$$

9. 高斯色噪声中已知信号的检测



$$H(j\omega) = H_1(j\omega)H_2(j\omega) = \frac{1}{S_n^+(\omega)} \frac{S^*(\omega)}{S_n^-(\omega)} e^{-j\omega T} = \frac{S^*(\omega)}{S_n(\omega)} e^{-j\omega T}$$

卡亨南-洛维展开：  $\int_0^T R_n(t_1 - t_2) f_j(t_2) dt_2 = \lambda_j f_j(t_1), \quad 0 \leq t_1 \leq T$

实信号下，  $x(t) = \sum_k x_k f_k(t)$ ,  $x_k = \int_0^T x(t) f_k(t) dt$ ,  $x_k$  间互不相关。

$$\lambda(x(t)) = \lim_{N \rightarrow \infty} \lambda(x_1, x_2, \dots, x_N) \underset{H_0}{\overset{H_1}{\geq}} th$$

$$\int_0^T x(t) \eta_1(t) dt - \int_0^T x(t) \eta_0(t) dt \underset{H_0}{\overset{H_1}{\geq}} th', \quad th' = \ln th + \frac{1}{2} \int_0^T s_1(t) \eta_1(t) dt - \frac{1}{2} \int_0^T s_0(t) \eta_0(t) dt$$

10. 高斯白噪声中随机相位信号的检测：设计最佳接收机，计算系统性能

$$\lambda(x(t)) = \frac{f(x(t) | H_1)}{f(x(t) | H_0)} = \frac{\int_0^{2\pi} f(x(t) | \theta, H_1) f(\theta) d\theta}{f(x(t) | H_0)} = \exp\left(-\frac{A^2 T}{2N_0}\right) I_0\left(\frac{2Aq}{N_0}\right) \underset{H_0}{\overset{H_1}{\geq}} th$$

判决规则：  $q = \sqrt{a^2 + b^2} \underset{H_0}{\overset{H_1}{\geq}} th'$  -----门限  $th'$  由  $\alpha$  确定（NP 准则）

$$a = q \sin \theta_0 = \int_0^T x(t) \cos \omega_c t dt$$

$$b = q \cos \theta_0 = \int_0^T x(t) \sin \omega_c t dt$$

计算性能：  $f(a, b | \theta, H_i) \rightarrow f(q, \theta_0 | \theta, H_i) \rightarrow f(q | \theta, H_i) \rightarrow f(q | H_i)$

非相干频移键控信号的检测：设计最佳接收机，计算系统性能

$$\lambda(x(t)) = \frac{f(x(t)|H_1)}{f(x(t)|H_0)} = \frac{\int_0^{2\pi} f(x(t)|\theta, H_1) f(\theta) d\theta}{\int_0^{2\pi} f(x(t)|\phi, H_0) f(\phi) d\phi} = \frac{I_0\left(\frac{2Aq_1}{N_0}\right)}{I_0\left(\frac{2Aq_0}{N_0}\right)} \stackrel{H_1}{\geq} \stackrel{H_0}{\leq} 1$$

$$q_i = \left[ \left( \int_0^T x(t) \sin \omega_i t dt \right)^2 + \left( \int_0^T x(t) \cos \omega_i t dt \right)^2 \right]^{1/2}, \quad i=1,2, \quad q_i > 0$$

## 第四章：参量估计

### 1. 估计准则

最大后验概率估计：  $\hat{\theta}_{MAP} = \arg \max_{\theta} f(\theta | \mathbf{x}) = \arg \max_{\theta} \ln f(\theta | \mathbf{x})$

最大似然估计：  $\hat{\theta}_{ML} = \arg \max_{\theta} f(\mathbf{x} | \theta) = \arg \max_{\theta} \ln f(\mathbf{x} | \theta)$

最小均方误差估计：  $E\{e^2(\hat{\theta})\} = \int \int_{(\theta)(\mathbf{x})} (\theta - \hat{\theta})^2 f(\theta, \mathbf{x}) d\theta d\mathbf{x} \rightarrow \min, \quad \hat{\theta}_{MS} = \int_{(\theta)} \theta f(\theta | \mathbf{x}) d\theta$

线性最小均方误差估计：  $E\left\{\left[\theta - \left(a + \sum_{k=1}^N b_k x_k\right)\right]^2\right\} \rightarrow \min$

$$\hat{\theta}_{LMS} = a + \mathbf{b}^T \mathbf{x} = E\{\theta\} + Cov\{\theta, \mathbf{x}\} Cov^{-1}\{\mathbf{x}, \mathbf{x}\} [\mathbf{x} - E\{\mathbf{x}\}]$$

正交条件（充要）：  $E\{(\theta - \hat{\theta}_{LMS}) \mathbf{x}^T\} = 0$

最小平均绝对误差估计：  $E\{|e(\hat{\theta})|\} \rightarrow \min, \quad \int_{-\infty}^{\hat{\theta}_{ABS}} f(\theta | \mathbf{x}) d\theta = \int_{\hat{\theta}_{ABS}}^{\infty} f(\theta | \mathbf{x}) d\theta$

贝叶斯估计：  $E\{c(\hat{\theta})\} \rightarrow \min$

最小二乘估计：线性观测模型下  $\hat{\theta}_{LS} = \arg \min_{\hat{\theta}} [\mathbf{x} - \mathbf{H}\hat{\theta}]^T [\mathbf{x} - \mathbf{H}\hat{\theta}]$ ,

$$\hat{\theta}_{LS} = (\mathbf{H}^T \mathbf{H})^{-1} \mathbf{H}^T \mathbf{x} = \mathbf{H}^{\#} \mathbf{x}$$

### 2. 多参量估计

$$\left[ \frac{\partial}{\partial \theta_i} \ln f(\boldsymbol{\theta} | \mathbf{x}) \right] \Big|_{\boldsymbol{\theta} = \hat{\boldsymbol{\theta}}_{MAP}} = 0, \quad \left[ \frac{\partial}{\partial \theta_i} \ln f(\mathbf{x} | \boldsymbol{\theta}) \right] \Big|_{\boldsymbol{\theta} = \hat{\boldsymbol{\theta}}_{ML}} = 0, \quad i = 1, 2, \dots, M$$

$$E\{[\boldsymbol{\theta} - \mathbf{a} - \mathbf{B}\mathbf{x}]^T [\boldsymbol{\theta} - \mathbf{a} - \mathbf{B}\mathbf{x}]\} \rightarrow \min$$

$$\hat{\boldsymbol{\theta}}_{LMS} = E\{\boldsymbol{\theta}\} + Cov\{\boldsymbol{\theta}, \mathbf{x}\} Cov^{-1}\{\mathbf{x}, \mathbf{x}\} [\mathbf{x} - E\{\mathbf{x}\}], \quad E\{(\boldsymbol{\theta} - \hat{\boldsymbol{\theta}}_{LMS}) \mathbf{x}^T\} = 0$$

$$\hat{\boldsymbol{\theta}}_{LS} = (\mathbf{H}^T \mathbf{H})^{-1} \mathbf{H}^T \mathbf{x} = \mathbf{H}^{\#} \mathbf{x}$$

$$3. \text{ 计算估计性能: } E\{\hat{\theta}\}, E\{(\hat{\theta} - E\{\hat{\theta}\})^2\}, E\{(\theta - \hat{\theta})^2\}$$

$$\text{无偏性: } E\{\hat{\boldsymbol{\theta}}\} = \boldsymbol{\theta} \text{ 或 } E\{\hat{\boldsymbol{\theta}}\} = E\{\boldsymbol{\theta}\}$$

有效性: 无偏估计量的均方误差达到最小。对于确定单参量无偏估计量  $\hat{\theta}$ , 有

$$E\{(\hat{\theta} - \theta)^2\} = \left\{ E \left\{ \left[ \frac{\partial}{\partial \theta} \ln f(\mathbf{x} | \theta) \right]^2 \right\} \right\}^{-1} = - \left\{ E \left\{ \frac{\partial^2}{\partial \theta^2} \ln f(\mathbf{x} | \theta) \right\} \right\}^{-1}$$

## 第五章：波形估计

### 1. 连续维纳滤波器

$$E\{e^2(t)\} = E\left\{[g(t) - y(t)]^2\right\} = E\left\{[g(t) - \int_{-\infty}^{\infty} h(\tau)x(t-\tau)d\tau]^2\right\} \xrightarrow{h(t)} \min$$

$$\text{线性最小均方误差估计的正交条件: } E\{e(t)x(\tau')\} = 0, \begin{cases} -\infty < \tau' < \infty & \text{非因果} \\ -\infty < \tau' \leq t & \text{因果} \end{cases}$$

$$\text{维纳-霍夫方程: } R_{gx}(\eta) = \int_{-\infty}^{\infty} h(\lambda)R_x(\eta - \lambda)d\lambda, \begin{cases} -\infty < \eta < \infty & \text{非因果} \\ 0 \leq \eta < \infty & \text{因果} \end{cases}$$

$$\text{求解: } H(j\omega) = \frac{S_{gx}(\omega)}{S_x(\omega)} \text{-----物理不可实现}$$

$$H(s) = \frac{1}{S_x^+(s)} \left[ \frac{S_{gx}(s)}{S_x^-(s)} \right]^+ \text{----- 物理可实现}$$

$$E\{e^2(t)\}_{\min} = E\{e(t)g(t)\} = R_g(0) - \int_{-\infty}^{\infty} h(\lambda)R_{gx}(\lambda)d\lambda$$

### 2. 离散维纳滤波器

$$E\{e^2(k)\} = E\left\{[g(k) - y(k)]^2\right\} = E\left\{[g(k) - \sum_{i=-\infty}^{\infty} h(i)x(k-i)]^2\right\} \rightarrow \min$$

$$\text{线性最小均方误差估计的正交条件: } E\{e(k)x(j)\} = 0, \begin{cases} -\infty < j < \infty & \text{非因果} \\ -\infty < j \leq k & \text{因果} \end{cases}$$

维纳-霍夫方程:  $\sum_{i=-\infty}^{+\infty} h(i) R_x(l-i) = R_{gx}(l), \begin{cases} -\infty < l < \infty & \text{非因果} \\ 0 \leq l < \infty & \text{因果} \end{cases}$

求解:  $H(z) = \frac{S_{gx}(z)}{S_x(z)}$  -----物理不可实现

$$H(z) = \frac{1}{S_x^+(z)} \left[ \frac{S_{gx}(z)}{S_x^-(z)} \right]^+ \text{----- 物理可实现}$$

对应有限样本的维纳滤波器:  $\sum_{i=0}^{N-1} h(i) R_x(l-i) = R_{gx}(l), l = 0, 1, \dots, N-1$

$$\mathbf{R}_x \mathbf{h} = \mathbf{r}_{gx}, \quad \mathbf{h} = \mathbf{R}_x^{-1} \mathbf{r}_{gx}$$