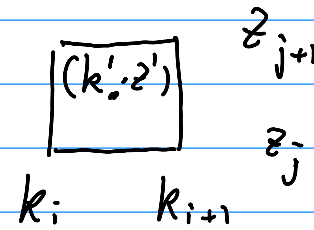
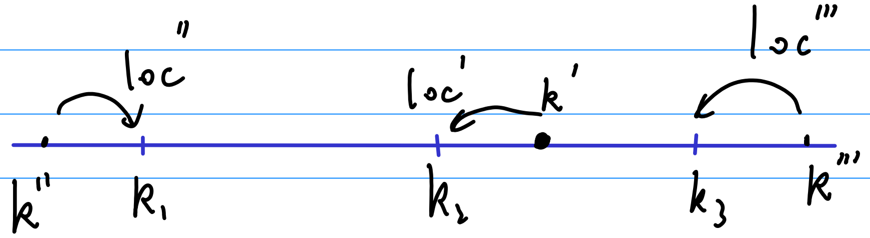
Linear Interpolation (2-D case)

Step 1 Find the policy value $c(k', z')$

1.1 Find out the nearest nodes of (k', z'), s.t



e.g



1.2 Use the value $[c(k_i, z_j)\ c(k_i, z_{j+1}),\ c(k_{i+1}, z_j),\ c(k_{i+1}, z_{j+1})]$ to compute $c(k', z')$

- Interpolate in the k direction $(\forall z)$

$$c(k', z) = \frac{k_{i+1} - k'}{k_{i+1} - k_i} c(k_i, z) + \frac{k' - k_{i+1}}{k_{i+1} - k_i} c(k_{i+1}, z)$$

(note: when $k' = k_i$, make sure $c(k', z) = c(k_i, z)$)

- Intepolate in the z direction $(\forall k)$

$$c(k, z') = \frac{z_{i+1} - z'}{z_{i+1} - z_i} c(k, z_i) + \frac{z' - z_{i+1}}{z_{i+1} - z_i} c(k, z_{i+1})$$

- Combine the above two results to get $c(k', z')$

$$c(k', z') = \frac{\overbrace{k_{i+1} - k'}^{\omega_{k_i}}}{k_{i+1} - k_i} c(k_i, z') + \frac{\overbrace{k' - k_i}^{\omega_{k_{i+1}}}}{k_{i+1} - k_i} c(k_{i+1}, z')$$

$$= \omega_{k_i}\left(\overbrace{\frac{z_{i+1} - z'}{z_{i+1} - z_i}}^{\omega_{z_i}} c(k_i, z_i) + \overbrace{\frac{z' - z_i}{z_{i+1} - z_i}}^{\omega_{z_{i+1}}} c(k_i, z_{i+1}) \right)$$

$$+ \omega_{k_{i+1}}\left(\frac{z_{i+1} - z'}{z_{i+1} - z_i} c(k_{i+1}, z_i) + \frac{z' - z_i}{z_{i+1} - z_i} c(k_{i+1}, z_{i+1}) \right)$$

$$= \omega_{k_i}\omega_{z_i} c(k_i, z_i) + \omega_{k_i}\omega_{z_{i+1}} c(k_i, z_{i+1})$$

$$+ \omega_{k_{i+1}}\omega_{z_i} c(k_{i+1}, z_i) + \omega_{k_{i+1}}\omega_{z_{i+1}} c(k_{i+1}, z_{i+1})$$

$$= \sum_{a=0}^{1}\sum_{b=0}^{1} \omega_{k_{i+a}}\omega_{z_{j+b}} c(k_{i+a}, z_{j+b})$$